MHD Stability Analysis Consistent to Transport Properties

presented by T.Ozeki
Thanks to N.Hayashi, S.Tokuda, N.Aiba
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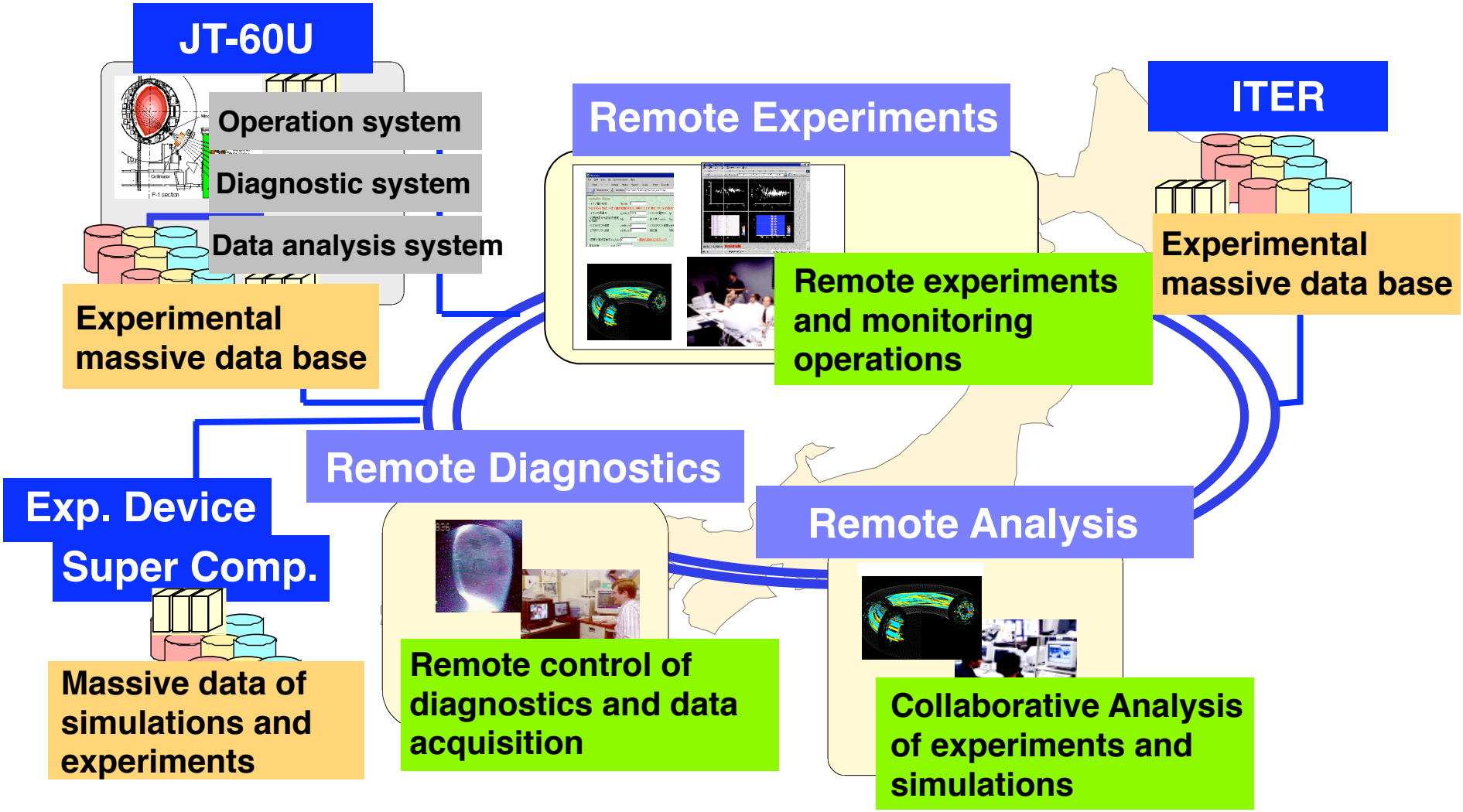
Introduction

- For estimations and analyses of burning plasmas in ITER, analysis and simulation codes for JT-60U experiments are being reconstructed and developed based on the transport code of TOPICS in JAERI.
- Plan of the reconstruction and development of the codes
  - Fusion research grid
  - Burning Plasma Simulation Code Cluster
  -
- Results of stability analysis based on the transport code are discussed.
  - NTM, Beta-collapse, ELM

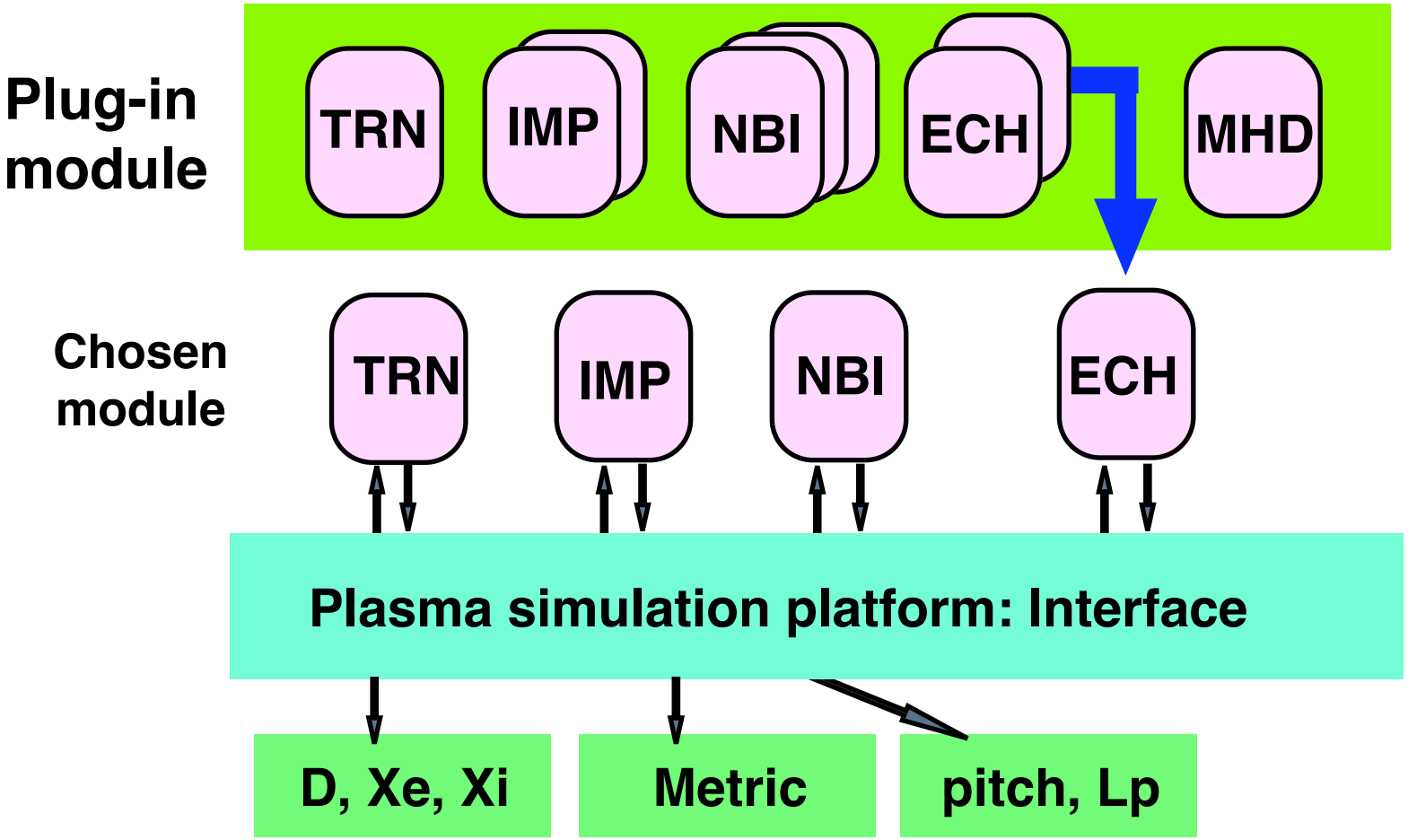
Fusion Research Grid

Establish the remote research environment using the IT technology

- Remote Experiments, Remote diagnostics, and Remote collaborative analysis
- Communication, information shearing and much presence with high security



- For the collaboration to BPSI, plug-in modules of the element of the code is proposed and investigated.



Burning Plasma Simulation Code Cluster in JAERI

Transport code TOPICS

Tokamak Preduction and Interpretation Code
Time dependent/Steary state analyses
1D transport and 2D equilibrium
Matrix Inversion Method for NeoClassical Trans.

Current Drive

ECCD/ECH (Ray tracing, Relativistic F-P), NBCD(1 or 2D F-P)

Impurity Transport

1D transport for each impurities, Radiation: IMPACT

Edge Pedestal

Perp. and para. transport in SOL and Divertor, Neutral particles, Impurity transport on SOL/Div. : SOLDOR, NEUT2D, IMPMC

Divertor

MHD

Tearing/NTM, High-n ballooning, Low-n: ERATO-J, Low and Mid.-n MARG2D

High Energy Behaviour

OFMC

MHD Stability and Modeling

MHD Behavior	Stability	Modeling
Sawtooth	Ideal/Resistive m/n=1/1 mode	Kadomtsev Model
Island Evolution	Tearing/NTM	Modified Rutherford Eq.
Beta Limits/ Disruption	low n kink high n ballooning	ERATO-J Ballooing Eq.
ELM	Medium n modes high n ballooning	MARG2D Ballooning Eq.
High energy induced instability & particles	TAE/EAE/EPM... Particles loss	Under consideration

NTM StabilityAnalysis

1. Purpose of NTM Analysis

The Neoclassical tearing mode (NTM) emerges by the luck of the bootstrap current inside the island and it induces the transport degradation. Therefore, NTM is important issue to improve beta limit. Also, NTM stabilization is crucial in ITER.
Local current driven by electron cyclotron wave (ECCD) is one of effective methods to stabilize NTM. Important issues for the NTM stabilization by ECCD

(1) Sensitivity of the stabilization to the EC current location
(2) Necessary ECCD power for stabilization

For the design of ECCD in ITER, EC power required for the stabilization should be examined.

To analyzed the NTM, the modified Rutherford equation should be analyzed with the transport property, the bootstrap current and the current drive of ECCD.

The following three topics have been studied.

(1) Estimation of parameters in modified Rutherford eq. by comparing with JT-60U experiments
(2) Sensitivity of the stabilization to the EC current location
(3) EC current necessary for stabilization (Power evaluation on ITER)

2. Numerical model

Modified Rutherford equation ← EC current ← **ECCD code**

Physical values at rational surface ↑

Current profile (Bootstrap current, etc) ↓

Transport degradation ↓

Geometry & Plasma profiles ↑

Additional current source ↓

1.5D tokamak simulation code (TOPICS)

Normalized minor radius defined by toroidal flux :

$$\rho = (\Phi(\rho)/\Phi(1))^{0.5} \quad (0 < \rho < 1)$$

1D transport equations for density and temperatures

1D current diffusion equation

$$\frac{\partial}{\partial t}\left(\rho\frac{\partial\Psi}{\partial\Phi}\right) = \frac{\partial}{\partial\rho}\left\{D_c\frac{\partial}{\partial\rho}\left(E_c\frac{\partial\Psi}{\partial\Phi}\right) - S_c(j_{BS}, j_{EC})\right\}$$

Bootstrap current : Matrix Inversion method for Hirshman & Sigmar formula

Neoclassical resistivity : Hirshman & Hawryluk model

2D MHD equilibrium : Grad-Shafranov equation (Fixed boundary)

NTM model: the modified Rutherford equation

$$\frac{\mu_0 a^2}{\eta}\frac{dW}{dt} = \Gamma_{\Delta'} + \Gamma_{BS} + \Gamma_{GGJ} + \Gamma_{pol} + \Gamma_{ECCD}$$

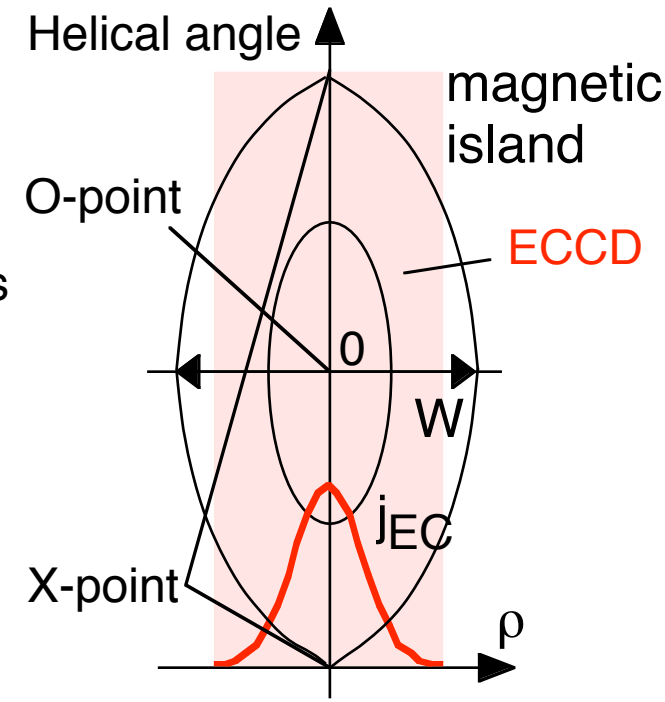
$\Gamma_{\Delta'}$: Classical tearing stability index term

Γ_{BS} : Disappearance of bootstrap current due to plasma profiles flattened in the magnetic island destabilizes the mode.

Γ_{GGJ} : Stabilization by magnetic well (Glasser-Green-Johnson effect)

Γ_{pol} : Stabilization or destabilization by ion polarization current

Γ_{ECCD} : EC current compensating bootstrap current lost in the magnetic island stabilizes the mode.



Degradation model in TOPICS

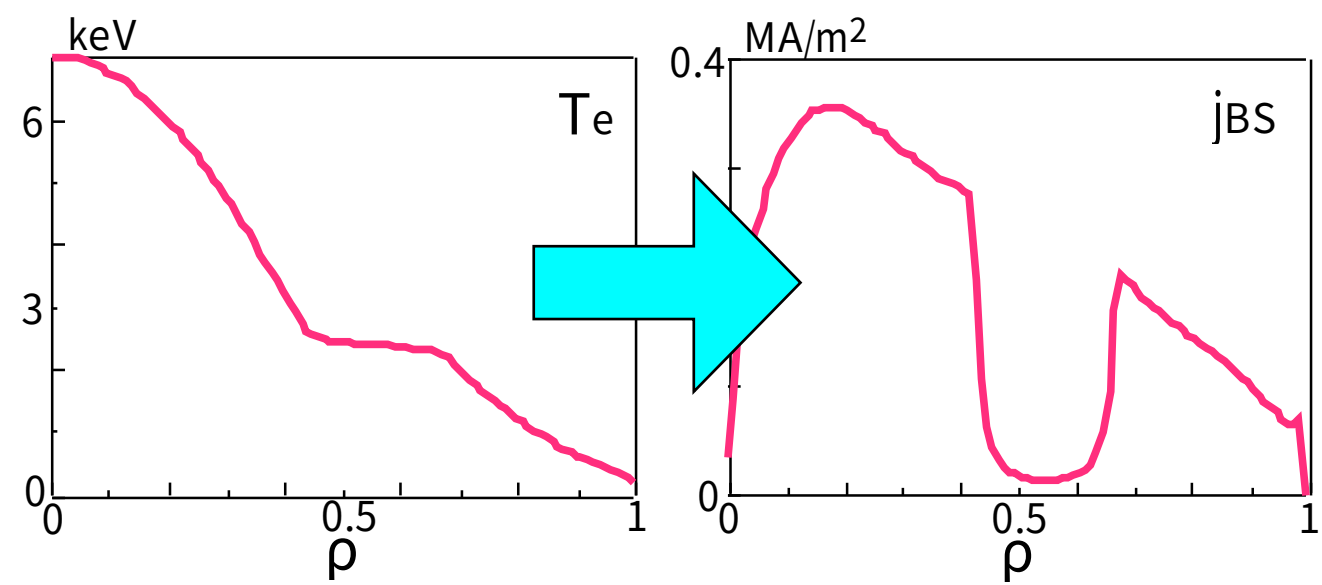
χ : Diffusivities Di, χ_e, χ_i

Flattening effect model: Amplified diffusivities is enlarged inside island

χ in island (ρ) = C(ρ) • χ(ρ)

$$C(\rho) = 1 + C_0\left\{1 - \left(\frac{\rho - \rho_s}{W/2}\right)^2\right\}^2$$

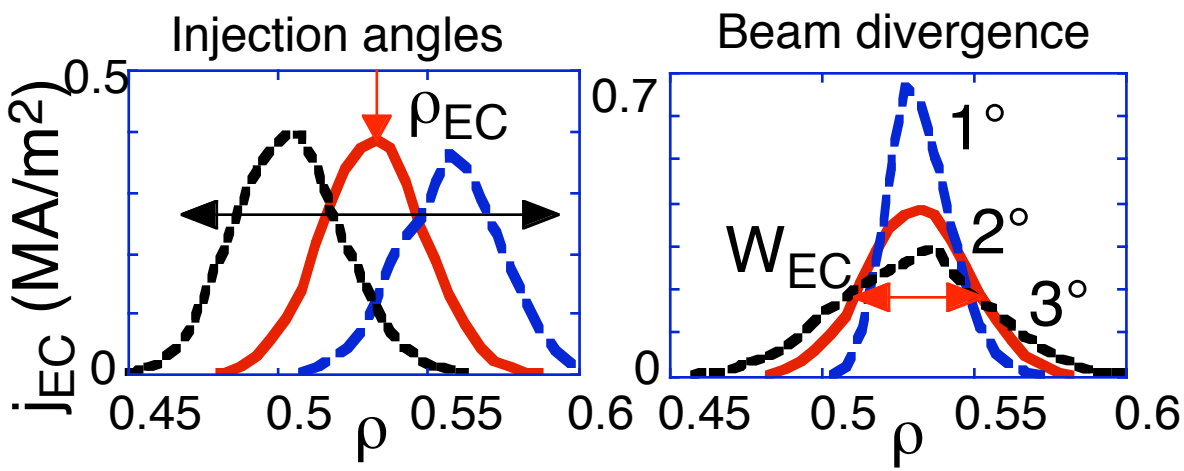
$$C_0 = Min\left[20,\ 10^4\left(W - W_{init}\right)^3\right]$$

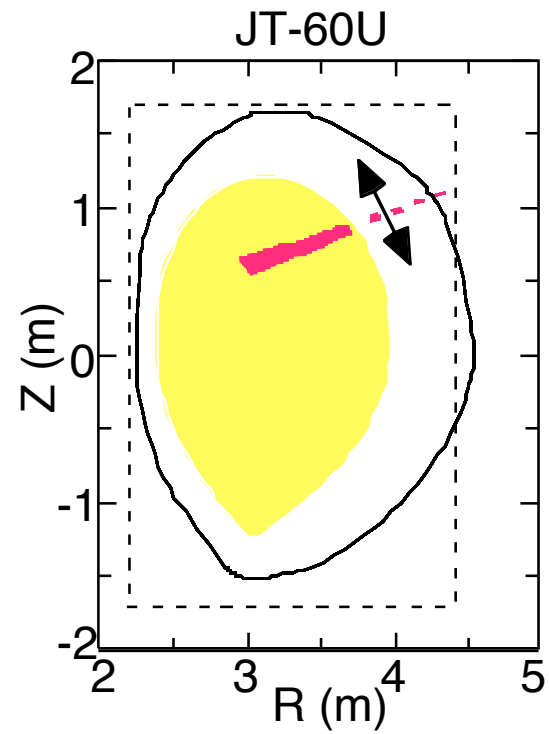


ECCD model: EC current profile based on results of EC code

EC code of ray tracing method and Fokker-Planck eq.

(K.Hamamatsu, et al., Plasma Phys.Control. Fusion 42(2000)1309.)





Based on the result of the EC code,
Total EC current is evaluated by the EC power.
EC current profile can be modeled by a Gaussian distribution.

$$j_{EC} \propto \exp\left(-\left(\frac{\rho-\rho_{EC}}{W_{EC}}\right)^2\right)$$

ρ_{EC} : Peak location of EC current

W_{EC} : Full width at half maximum

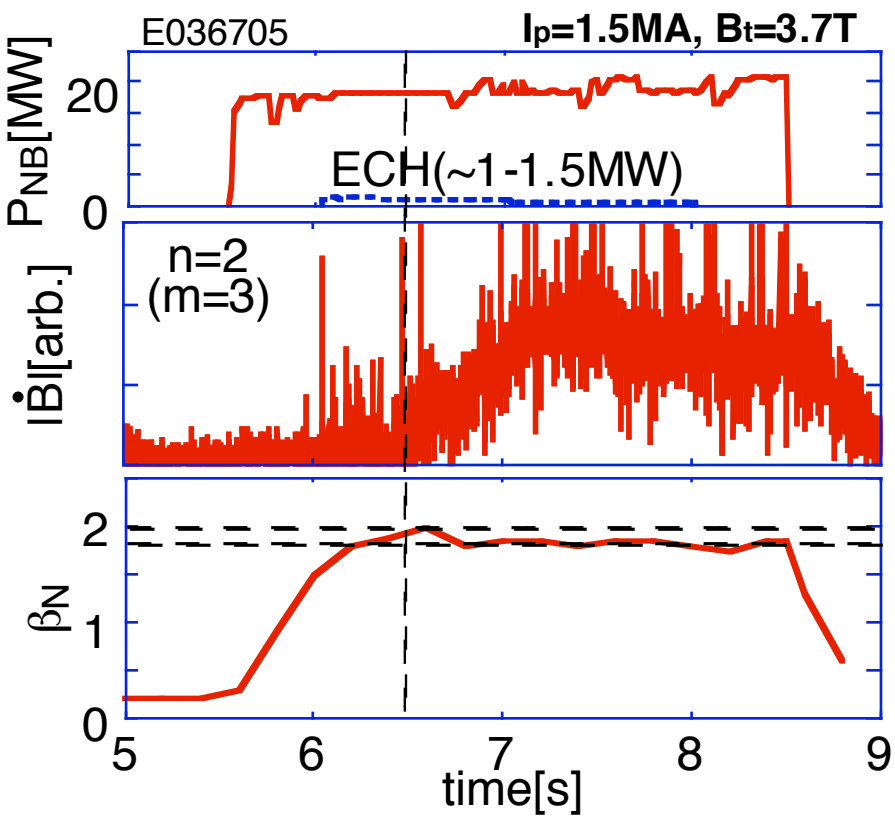
3. Results of Analysis

NTM growth and its stabilization by ECCD in JT-60U experiments

NTM island growth (E36705)

m/n=3/2 mode NTM grew from t=6.4 s and was saturated at t=7.2 s.

Normalized beta was decreased by about 10%.

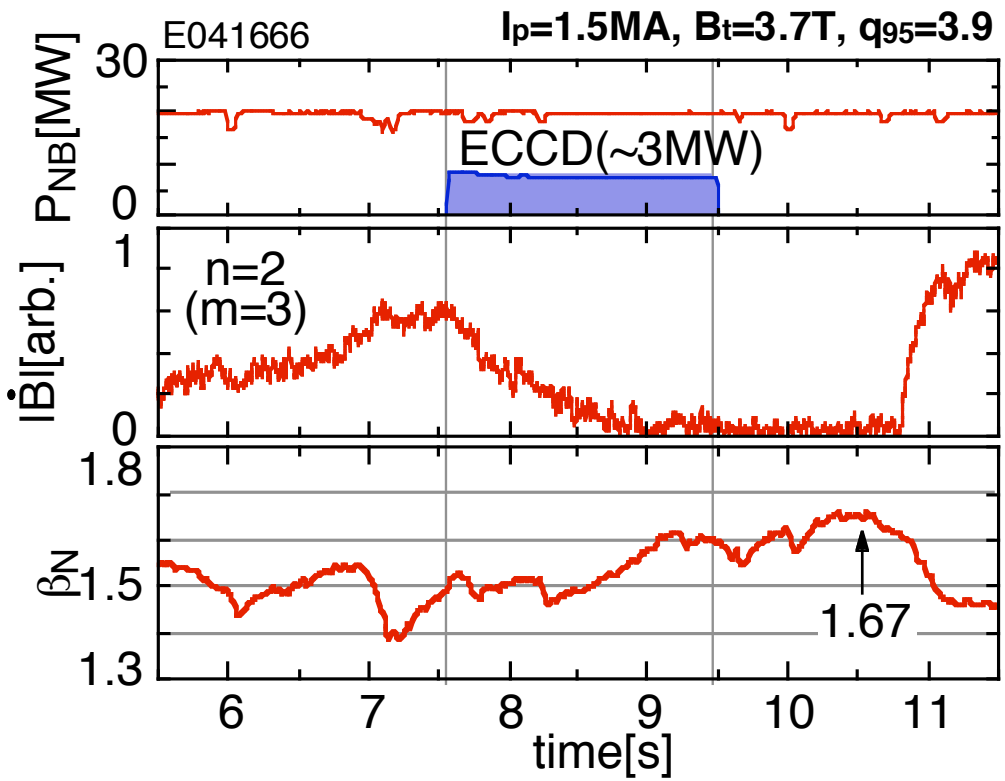


Stabilization by ECCD (E41666)

Fundamental O-mode EC wave of 110GHz was injected to 3/2 mode NTM.

Real-time control of the EC current location (island is detected from T_e perturbation) could completely stabilize the NTM.

A.Isayama, et al. 19th IAEA conf.



Determination of coefficients of Modified Rutherford eq.

$$\frac{\mu_0}{\eta}\frac{dW}{dt} = k_c \Delta'(W)\left\langle |\nabla\rho|^2 \right\rangle + k_{BS}\mu_0 L_q j_{BS}\left\langle \frac{|\nabla\rho|}{B_p}\right\rangle \frac{W}{W^2+W_d^2} - k_{GGJ}\varepsilon_s^2\beta_p \frac{L_q^2}{\rho_s L_p}\left(1-\frac{1}{q^2}\right)\left\langle |\nabla\rho|^2\right\rangle \frac{1}{W}$$

Classical Bootstrap GGJ

$$-k_{pol}\varepsilon_s^{1.5}\beta_p\left(\frac{\rho_{pi}L_q}{L_p}\right)^2\left\langle |\nabla\rho|^2\right\rangle\frac{1}{W^3} - k_{EC}\mu_0\frac{L_q}{\rho_s}\left\langle\frac{|\nabla\rho|}{B_p}\right\rangle \eta_{EC}\frac{I_{EC}}{a^2}\frac{1}{W^2}$$

Polarization ECCD

W : Magnetic island width in ρ coordinate

k_c=1.2, $\Delta'(W)$: Cylindrical model

$L_q = q / (dq/d\rho)$, ρ_s : Rational surface position

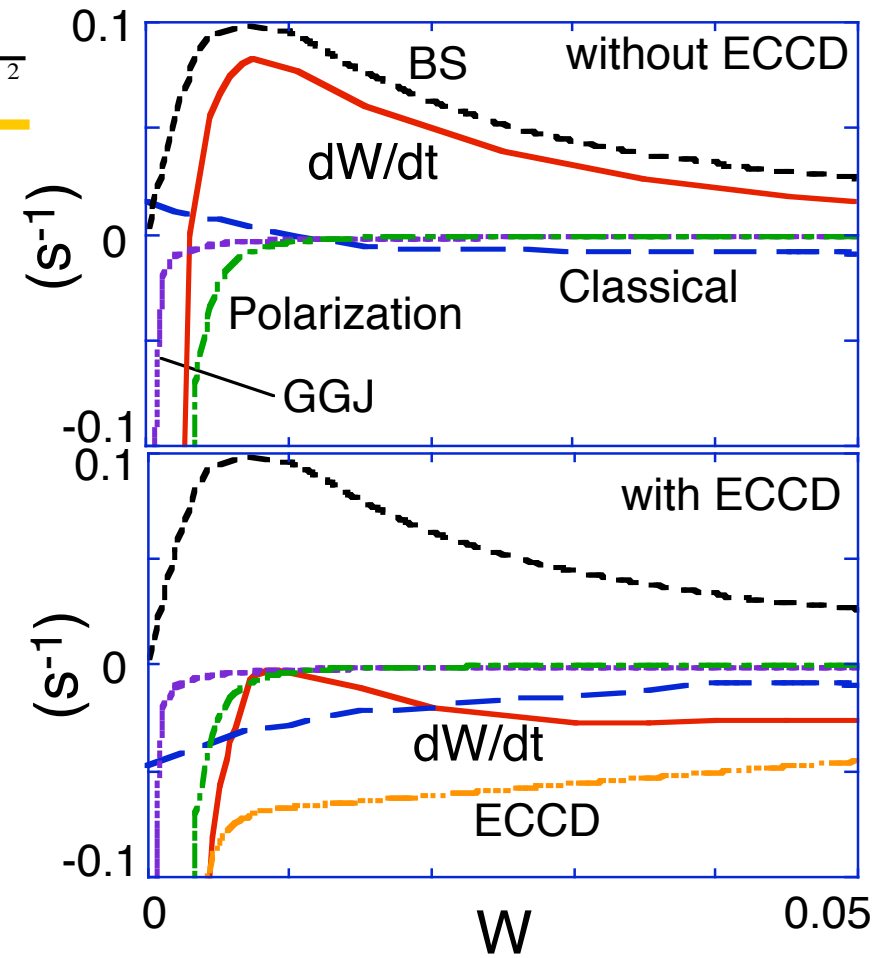
W_d : Finite $\chi_\perp / \chi_{//}$ effect (R.Fitzpatrick, 1995)

η_{EC} : Localized efficiency of EC current

I_{EC} : Total EC current

Parameters of k_{BS}, k_{GGJ}, k_{pol}, k_{EC} are constant of order unity.
Value of W_d depends on theoretical models (to limit the parallel heat transport).

These coefficenets should be estimated by fitting to experiments.



k_{BS} can be estimated from large W.

k_{GGJ}, k_{pol}, W_d from small W

Unknown Parameters is estimated by fitting of JT-60 experiments

EC current (I_{EC}=52 kA, W_{EC}=0.12)

k_{BS} ~ 4-5 from the saturated island width at 7.5s

Real-time control of the EC current location was applied in the experiment.

Misalignment between the EC current location and the rational surface due to an interval of T_e measured points (2 cm).

In numerical calculations,

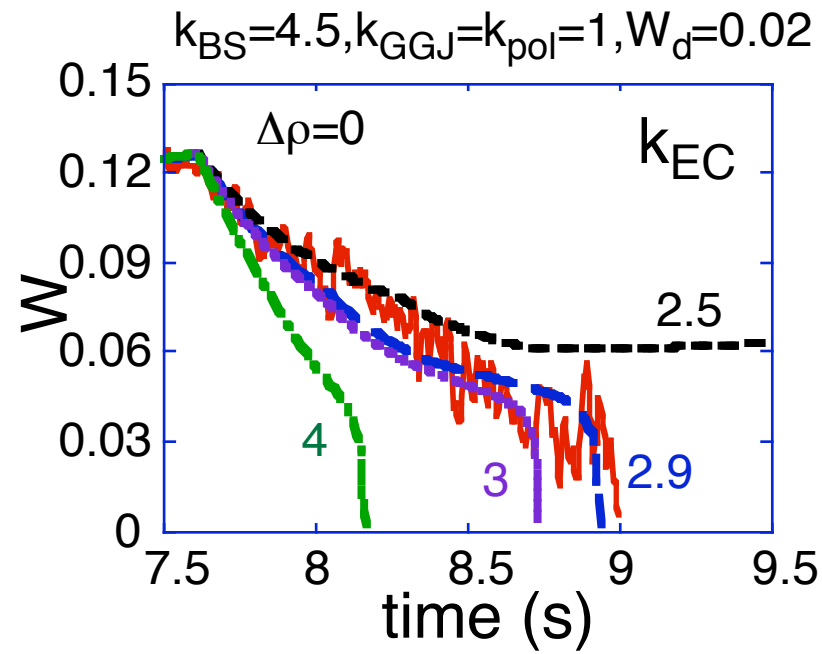
EC current location traces the rational surface with and without a constant misalignment of 0.025 (~2 cm).

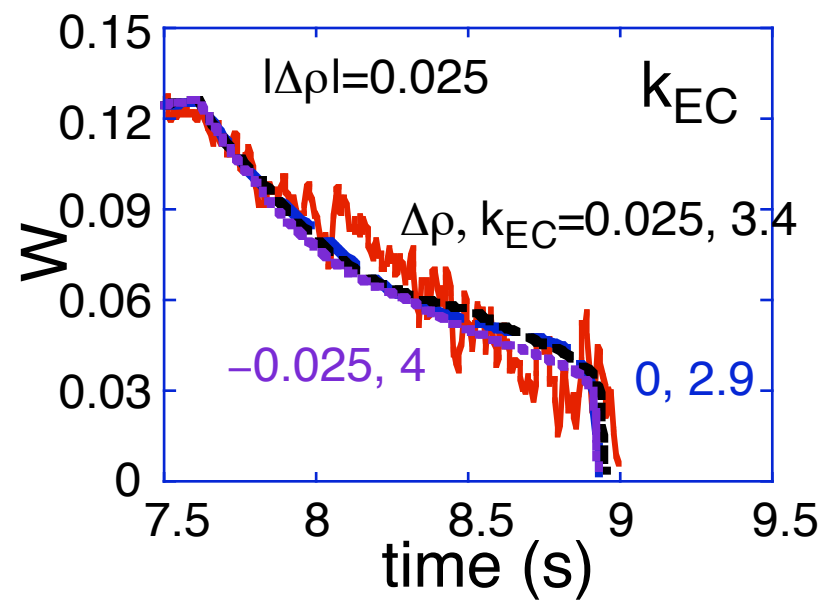
$$\Delta\rho = \rho_{EC} - \rho_s$$

ρ_{EC} : peak location of EC current profile

k_{EC} ~ 3-4 from fitting to experimental results

Other parameters (k_{pol}, W_d) also can be estimated.





Evaluation of EC current necessary for stabilization in ITER

Based on the JT-60 expeiments, ITER Plasma was simulated for the inductive scenario #2 :

$I_p = 15$ MA, $B_T = 5.3$ T, $R_0 = 6.2$ m, $a = 2$ m,

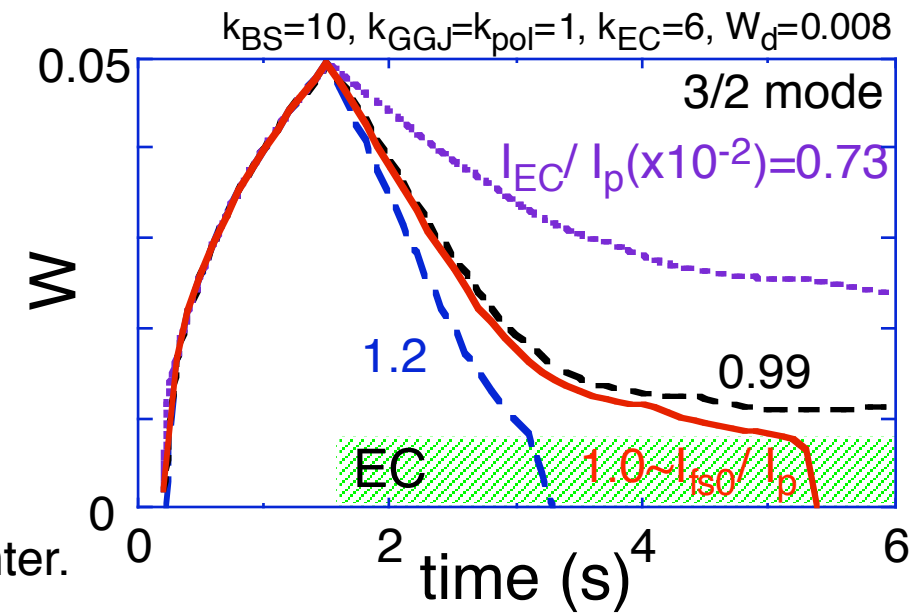
$\beta_N \approx 1.8$, $\bar{n}_e = 1.0 \times 10^{20}$ m^{-3}, $\bar{T}_e = 9.1$ keV, $\bar{T}_i = 8.3$ keV

n_e, T_e, T_i profiles : Fixed profiles

EC current (W_{EC}=0.04)

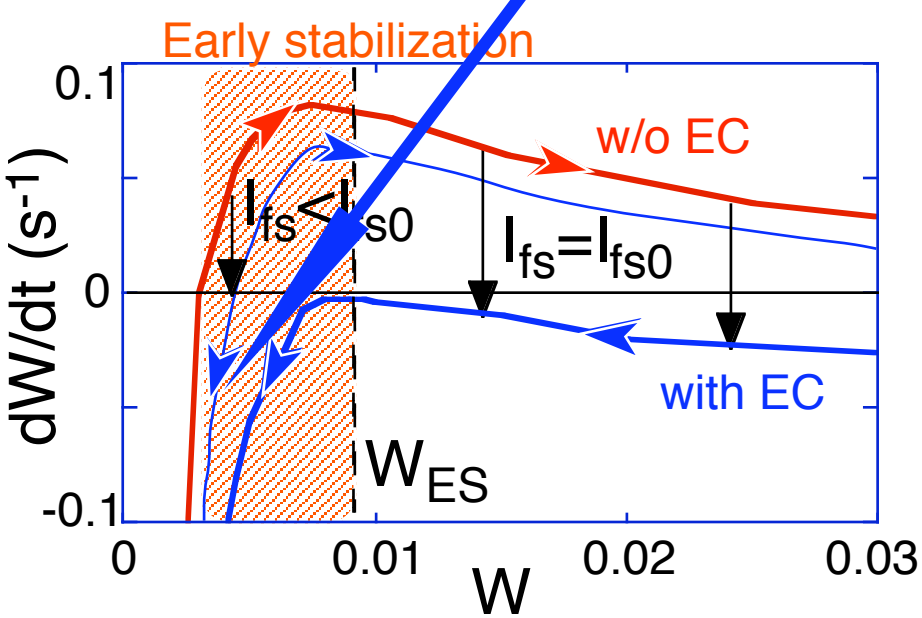
Fundamental O-mode EC wave of 170 GHz

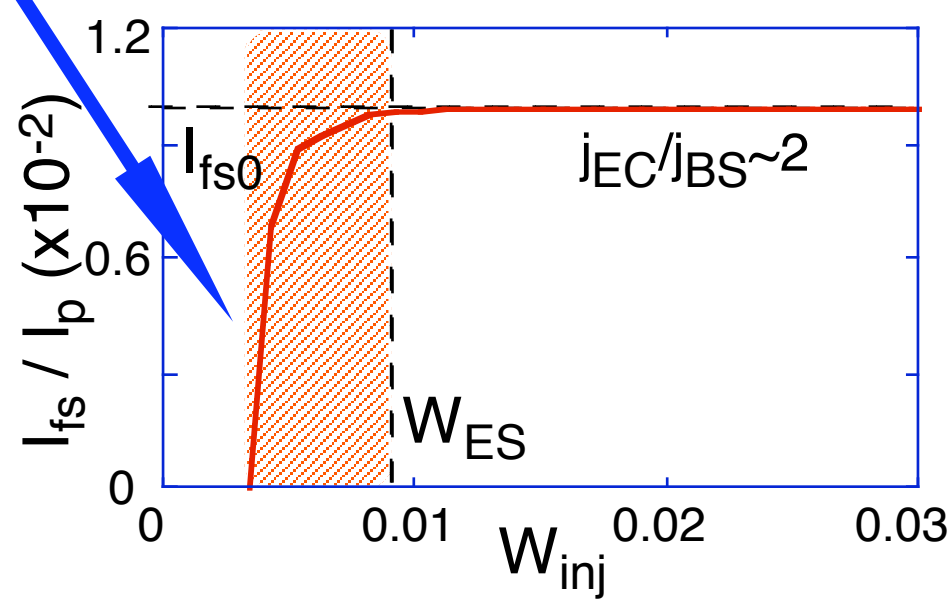
EC current location is assumed to be just island center.



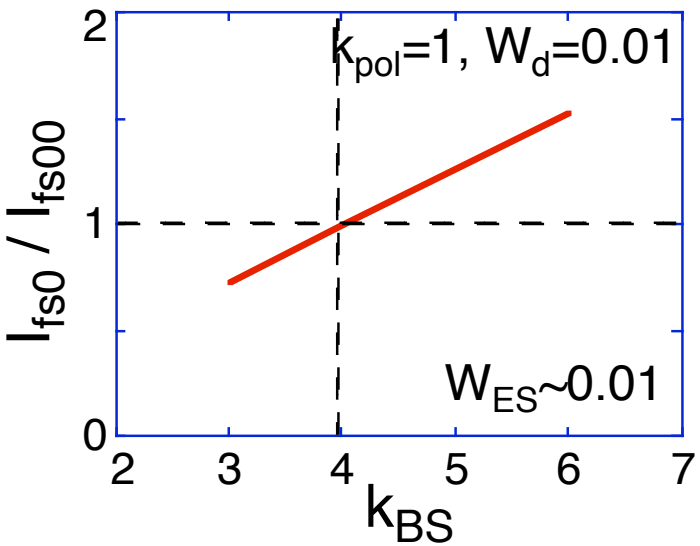
EC wave was injected during the island growing: **Early stabilization** ($W_{inj} < W_{ES}$)

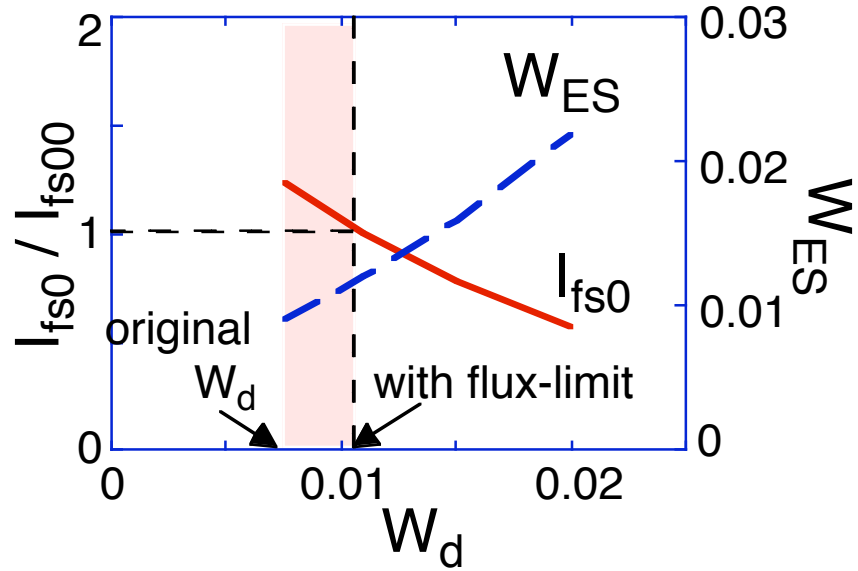
The EC current necessary for the full stabilization, I_{fs} , can be reduced.





Results : Necessary power for stabilization was obtained
Dependence on the parameters of the modified Rutherford eq.





For k_{BS}~4, k_{pol}~1, k_{EC}~4, W_d~0.01 estimated from JT-60U experiment,
I_{fs0} ~ 74 kA for 3/2 mode and ~ 54 kA for 2/1 mode on ITER (error~20%) .

ECCD power necessary for both 3/2 and 2/1 modes
NTM stabilization on ITER is 30 MW.

Necessary ECCD power can be reduced to 12 MW when the EC current width decreased by optimizing both toroidal and poloidal injection angles.

4. Summary of NTM analysis

- **This method is useful to evaluate the suppression by ECCD.**
- **It is need to develop the model including the basic model, for example, to clarify the mechanism of the mergence of the island.**

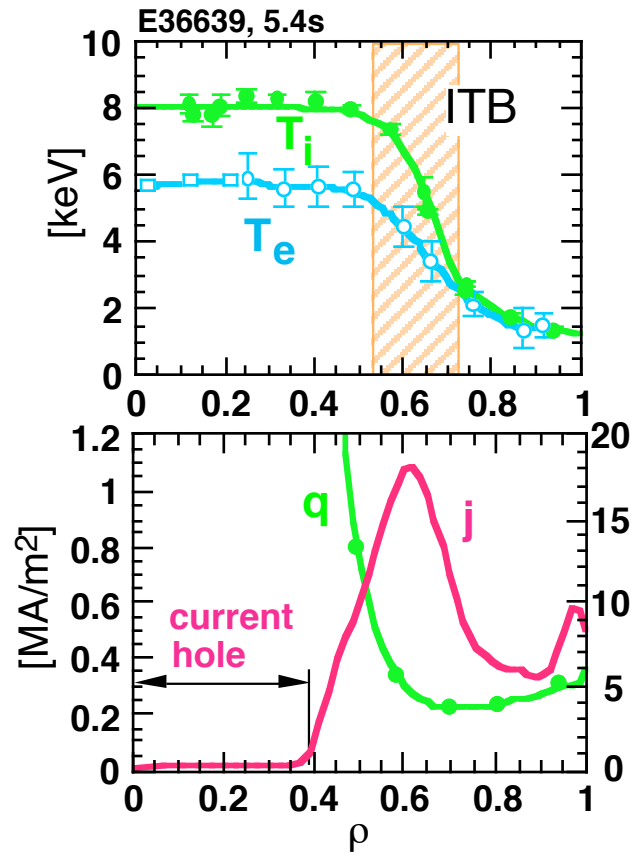
Beta limits Analysis

1. Purpose of CH Analysis

Current hole (CH) with nearly zero toroidal current in the central region has been observed form MSE measurements.

In the CH plasma, high confinement performance is achieved due to the formation of strong internal transport barrier (ITB) in the reversed-shear (RS) region.

The CH plasma has the autonomous property because the pressure profile and the current profile are strongly coupled each other. And discharges are mostly terminated by the disruption frequently.



The physical mechanism of profile formation and sustainment of the CH plasma which is consistent MHD stability should be clarified.

T.Fujita, et al., Phys. Rev. Lett. **87**(2001)245001.

2. Simulation model

Profile formation and sustainment of CH plasma by 1.5D time-dependent transport simulations for JT-60U parameters are investigated.

1.5D transport code

1D transport equations : Normalized minor radius defined by toroidal flux :

$\rho = (\Phi(\rho)/\Phi(1))^{0.5}$ $(0 < \rho < 1)$

$$\frac{\partial n_i}{\partial t} = \frac{\partial}{V'\partial\rho}\left(V'\left\langle|\nabla\rho|^2\right\rangle D_i \frac{\partial n_i}{\partial\rho}\right) + S$$

$$\frac{\partial}{\partial t}\left(\frac{3}{2}n_j T_j\right) = \frac{\partial}{V'\partial\rho}\left(V'\left\langle|\nabla\rho|^2\right\rangle n_j \chi_j \frac{\partial T_j}{\partial\rho}\right) + P_j \qquad (j = e,i) \qquad V' = \frac{dV}{d\rho}$$

1D current diffusion equation : Parallel current density

$$\frac{\partial}{\partial t}\left(\rho\frac{\partial\Psi}{\partial\Phi}\right) = \frac{\partial}{\partial\rho}\left\{D\frac{\partial}{\partial\rho}\left(E\frac{\partial\Psi}{\partial\Phi}\right) - S(j_{BS})\right\}$$

$$j = \frac{2\Phi_1}{\mu_0} D\left\langle R^{-2}\right\rangle^{1/2} \rho\frac{\partial}{\partial\rho}\left(E\frac{\partial\Psi}{\partial\Phi}\right)$$

Impurity : C^{6+}, $T_{imp}=T_i$, assumed profile Z_{eff}

NBI source : Fixed profile, Given deposition ratio of ion to electron, R_{NB}

Neutral : Monte-Carlo method, Given recycling coefficient, R

2D MHD equilibrium : Grad-Shafranov equation (Fixed boundary)

Model of current limit inside CH is applied. (q_{limit}= constant)

↓ 2D Equilibrium data

low n MHD stability: ERATO/MARG2D, High n ballooning and Interchange

Model of CH was proposed: Axisymmetric Tri-Magnetic-Islands (ATMI) equilibrium

T.Takizuka, et al., J. Plasma Fusion Res. **78**(2002)1282.

ATMI has three islands along the R direction (a central-negative-current island and two side-positive-current islands) and two x-points along the Z direction.

ATMI equilibrium is stable with the elongation coils when the current in the ATMI region is limited to be small.

Stable condition :

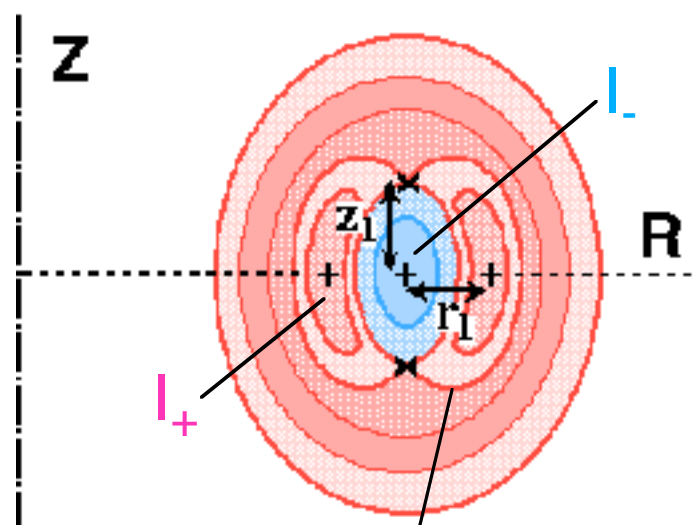
$$\frac{q_{ATMI}}{q_a} > \kappa_1^{\,2} \frac{Z_c^{\,2}}{a\left(Z_c - Z_x\right)} \qquad \kappa_1 = \frac{z_1}{r_1}$$

q_{ATMI} : Effective safety factor at the surface of ATMI

q_a : Engineering safety factor at the surface of a whole plasma

Z_x, Z_c : Position of x-points and elongation coils

$$\frac{2I_+}{I_-} \approx \frac{1+\kappa_1^{\,2}}{\kappa_1^{\,2}}$$



$I_{ATMI} = 2I_+ - I_- \sim I_- > 0$

($\kappa_1 \sim 1$)

JT-60U parameters : q_{ATMI}>30 for Z_c=2, Z_c-Z_x=0.6, a=0.8, q_a=4, $\kappa_1 = 1$

Upper limit of q inside the current hole is modeled based on the ATMI model

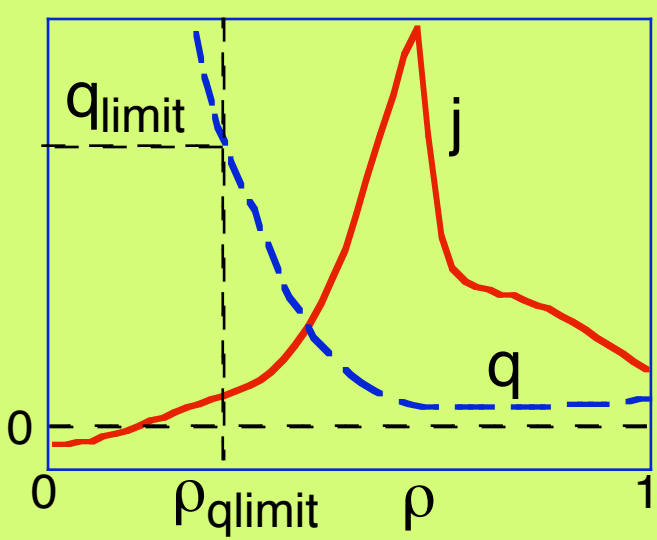
Safety factor at the surface of stable ATMI equilibrium : q_{ATMI}

Safety factor inside CH is limited by $q_{limit} = q_{ATMI}$ in the calculation of the MHD equilibrium and the neoclassical transport.

CH radius is at $q=q_{limit}$ and moves in the simulation.

1D Transport

Negative current inside CH



$q_{eq}(\rho), n(\rho), T(\rho)$

Geometry

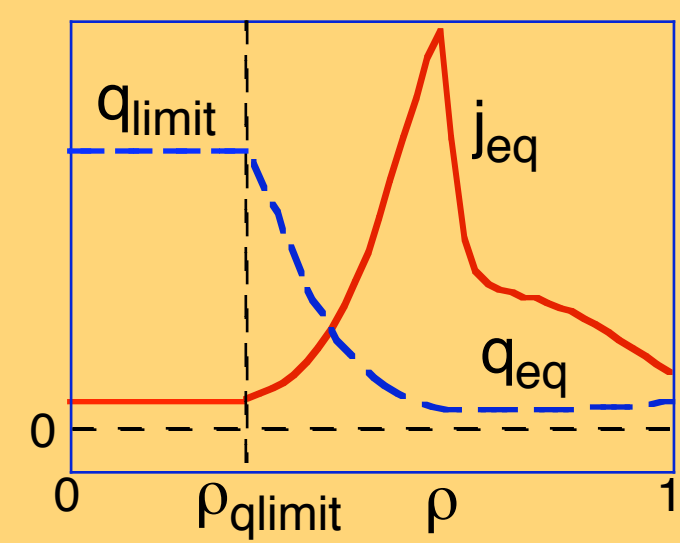
Bootstrap current

Neoclassical diffusivity

Neoclassical resistivity

2D MHD equilibrium

Neoclassical transport (Bootstrap current, diffusivity, resistivity)



Model of transport: Neoclassical inside of s~0 and anomalous outside of s~0

Diffusivities in the transport eqs. :

$$\chi_i = \chi_e = \chi_{neo,i} + \chi_{ano}$$
$$D_i = C_D D_{neo,i} + D_{ano}$$

Neoclassical transport :

Diffusivity and bootstrap current : Matrix inversion method for Hirshman & Sigmar formula (M.Kikuchi, et al., Nucl. Fusion **30**(1990)343.)

Neoclassical resistivity : Hirshman & Hawryluk model (Nucl. Fusion **17**(1977)611.)

Inside the CH region, model of current limit is applied. (q_{limit}= constant)

Anomalous transport : Negative magnetic shear is effective to stabilize the ballooning mode and micro-instabilities. **CDBM-type model**

$$\chi_{ano} = D_{ano} = \chi_0 F(s - k\alpha)$$

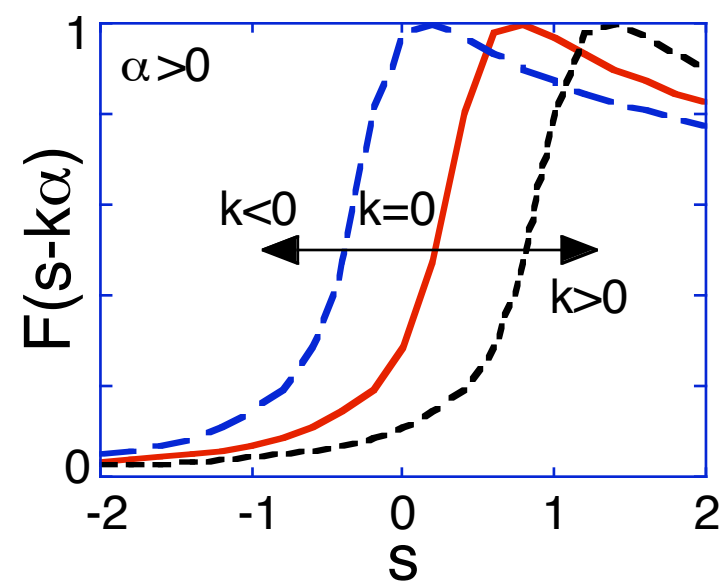
χ_0 : constant anomalous diffusivity

k : arbitrary constant

Function F depends on

s : magnetic shear

α : normalized pressure gradient



CDBM model : F with k=1 was originally developed for the ballooning mode turbulence. (A.Fukuyama, et al., Plasma Phys. Control. Fusion **37**(1995)611.)

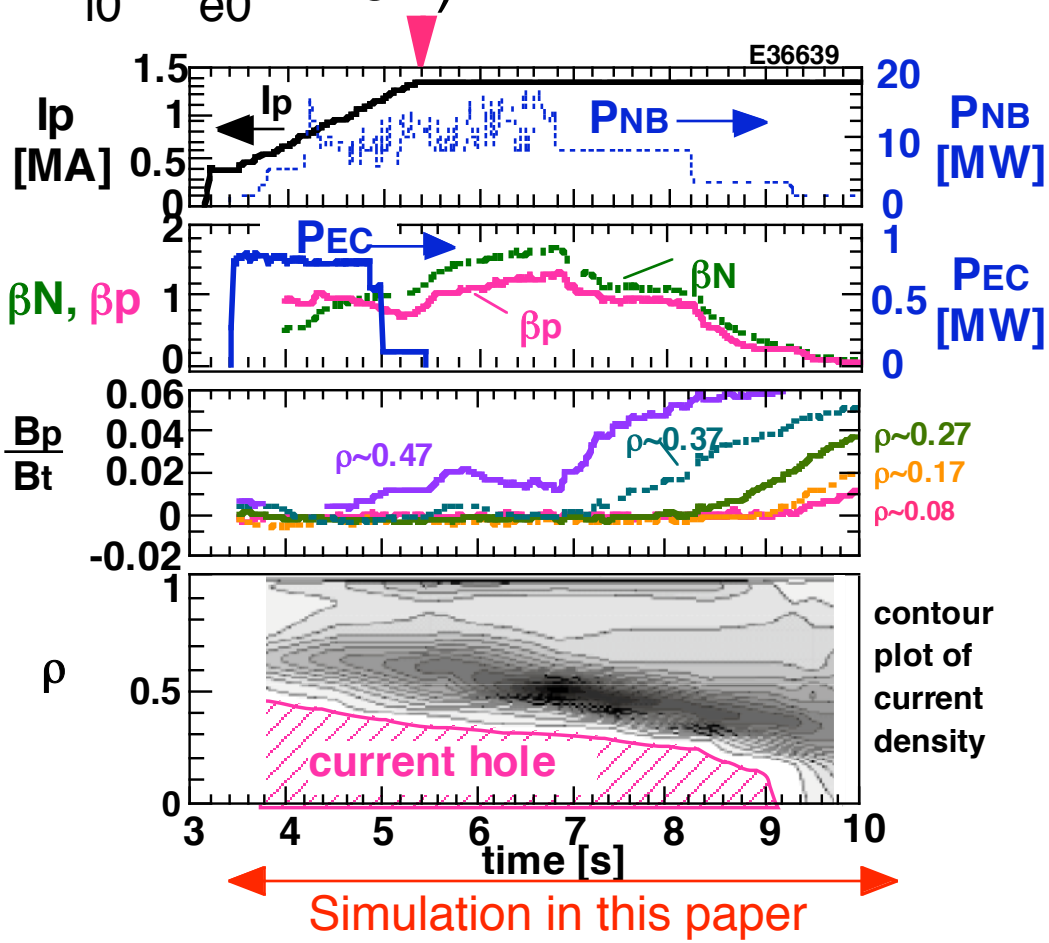
3. Results

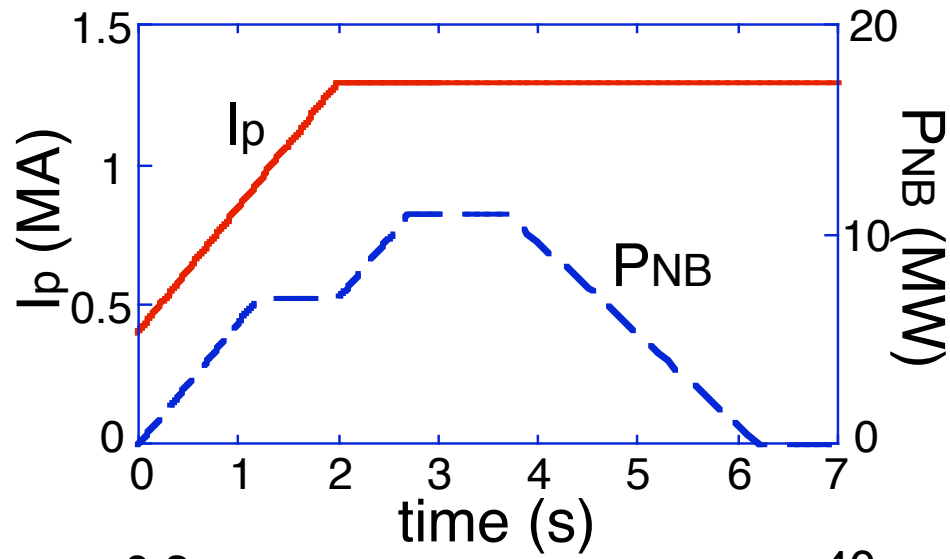
Simulation was done for the shot of E36639.

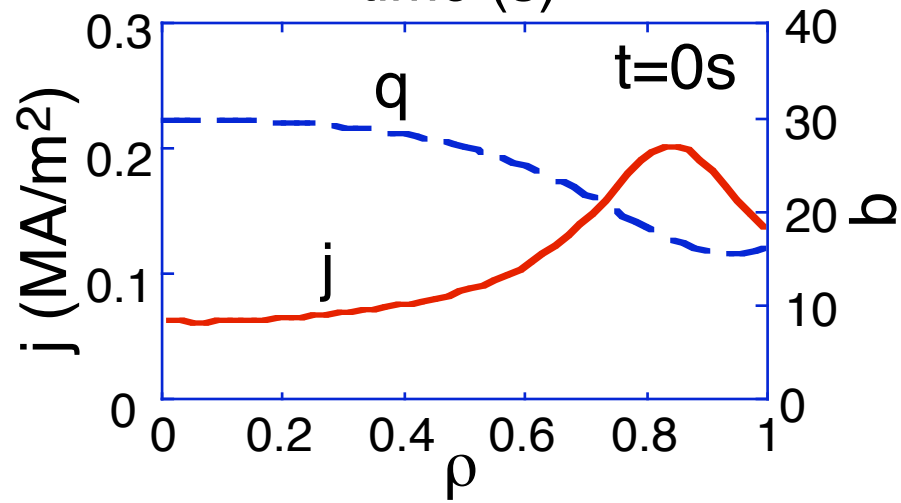
Neutral-beam (NB) is injected during the current ramp-up.

Initial current profile is assumed to be off-centered like that measured in a similar shot. $B_{t0} = 3.7\ \mathrm{T}$, $R_0 = 3.3\ \mathrm{m}$, $a = 0.8\ \mathrm{m}$

Initial plasma profiles are assumed to be parabolic. ($n_{e0}=10^{19}\mathrm{m}^{-3}$, $T_{i0}=T_{e0}=1\mathrm{keV}$)







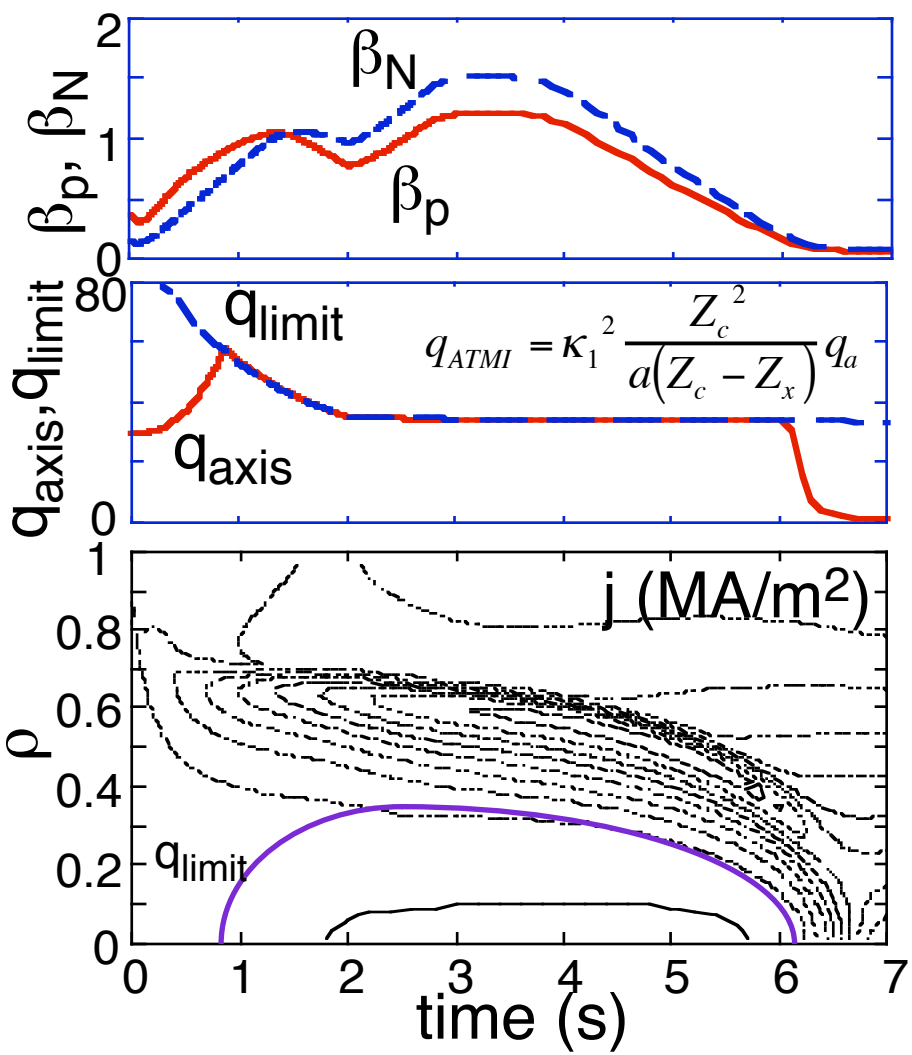
Transport simulation reproduces features of a JT-60U experiment.

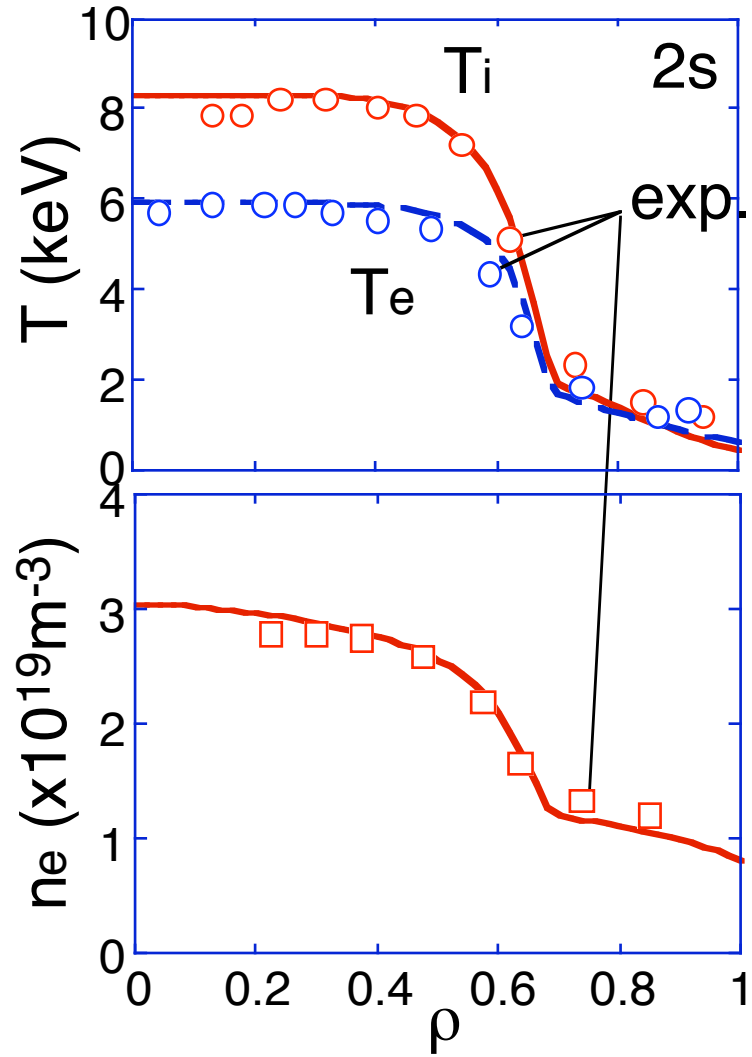
To validate the model, the simulation for a shot E36639 has been done.

Parameters set to simulate the experiment : $k = 0,\ \chi_0 = 2.6\ \mathrm{m^2/s},\ C_D = 1,\ R_{NB} = 2,\ R = 0.978$

for ITB — for T_i / T_e — for n_e

Normalized beta, poloidal beta, contour plot of current density and profiles almost agree with those in E36639.





Physical mechanism of current hole formation

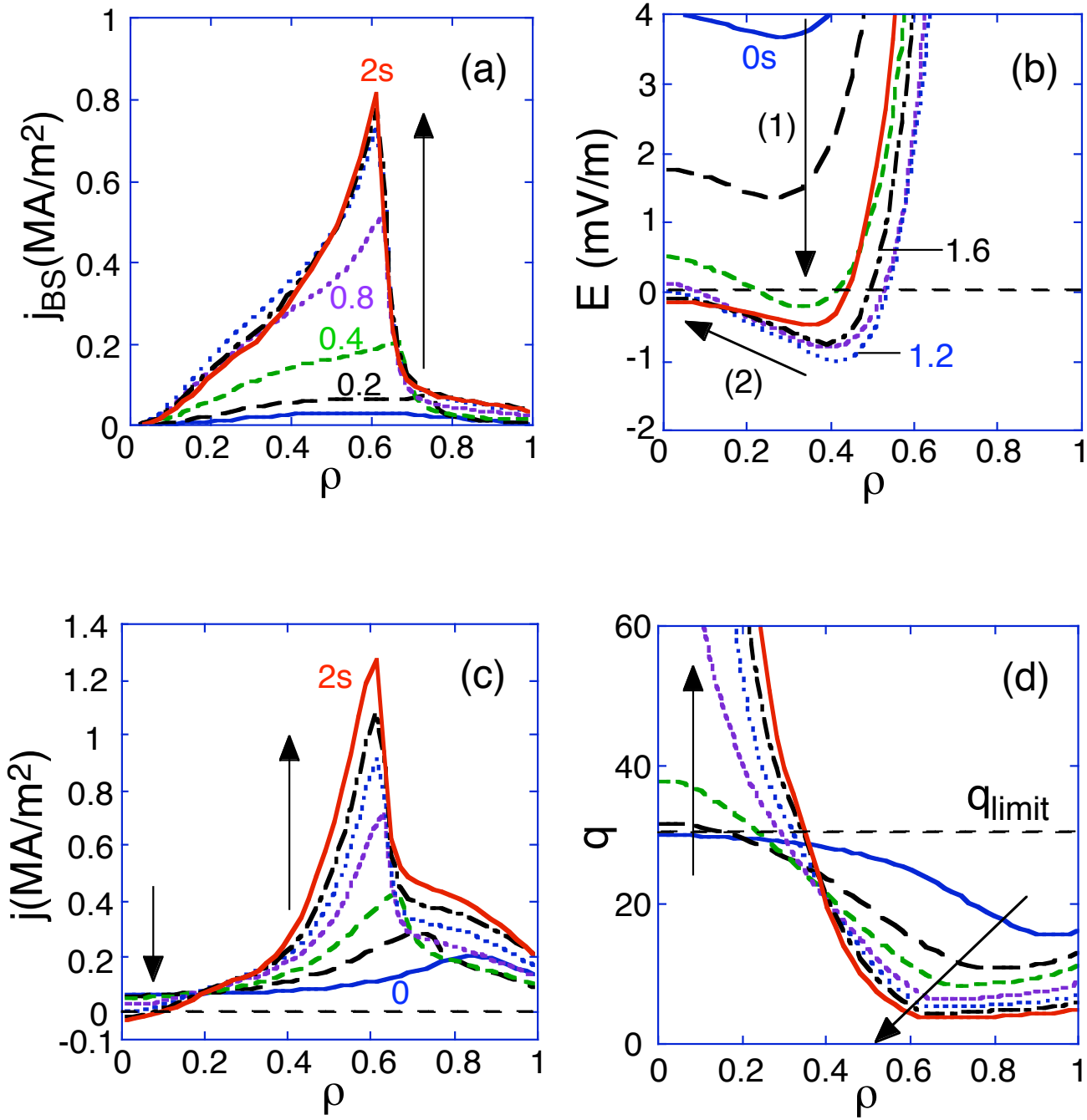
(a) Off-central bootstrap current increases due to the ITB growth in the reversed-shear region.

(b) (1) Large bootstrap current decreases the parallel electric field to negative. (2) The negative electric field diffuses into the central region.

$$\frac{\partial \mathbf{E}}{\partial t} = \underbrace{\frac{\eta}{\mu_0}\nabla^2\mathbf{E}}_{(2)} - \underbrace{\eta\frac{\partial \mathbf{j}_{BS}}{\partial t}}_{(1)}$$

(c) The current in the central region drops due to the negative electric field and becomes negative at t=1.2 s. Outer current increases due to the bootstrap current.

(d) The safety factor increases in the central region. The minimum q decreases and the radius at the minimum q surface moves inward.



Stability Analysis for results of transport simulation

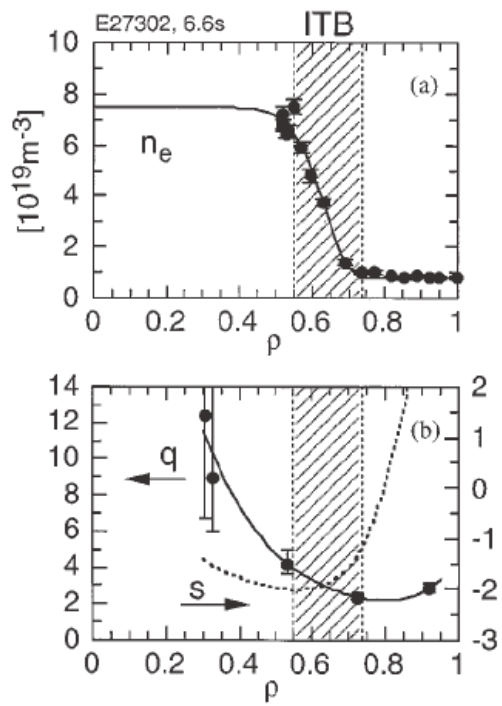
Low n modes stability was checked by ERATO-J:

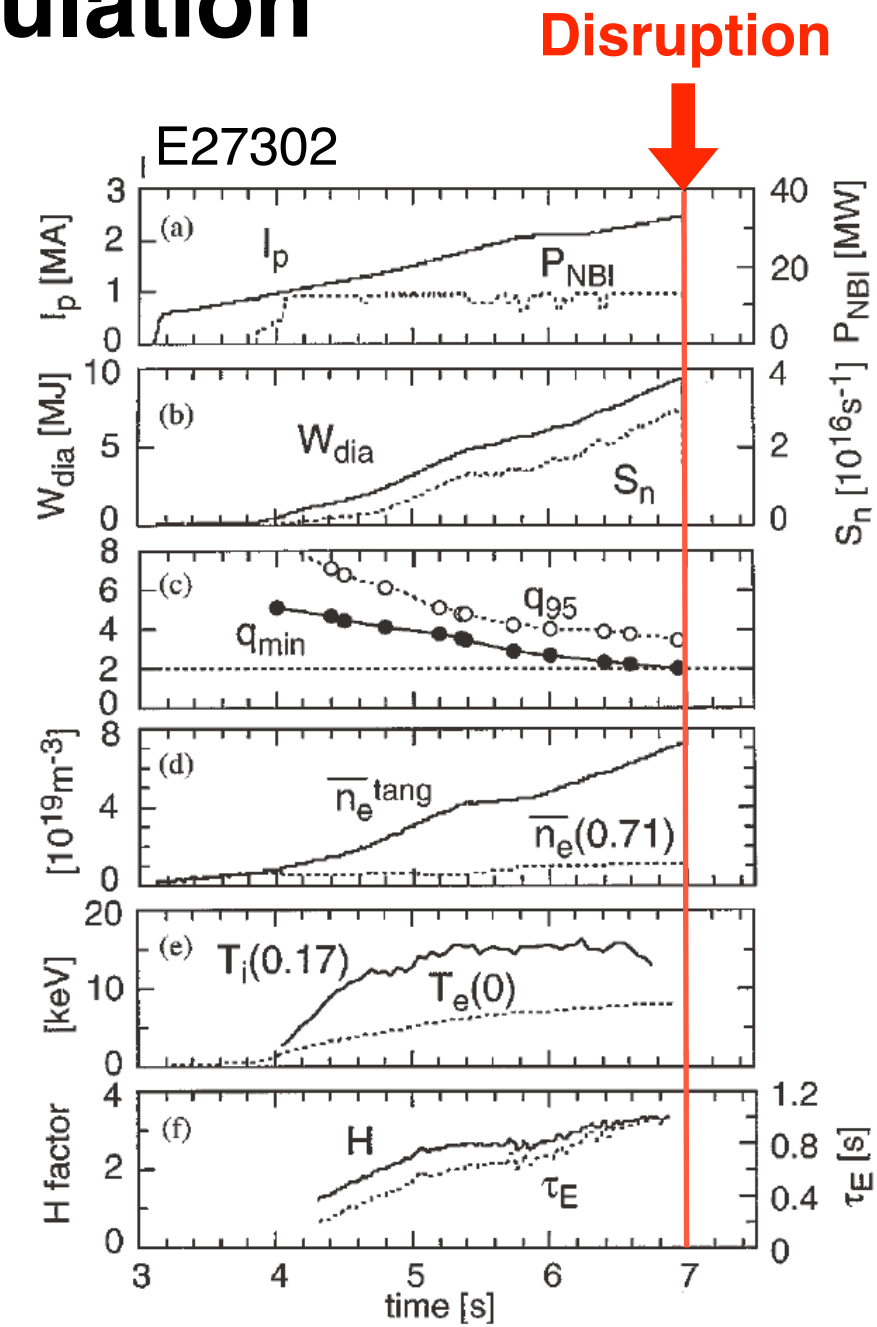
Equilibrium of each time steps in E36639 (previous discharge) are stable for low n mode.

These are consistent to the observation.

However, the discharge of E27302 terminated by the disruption.

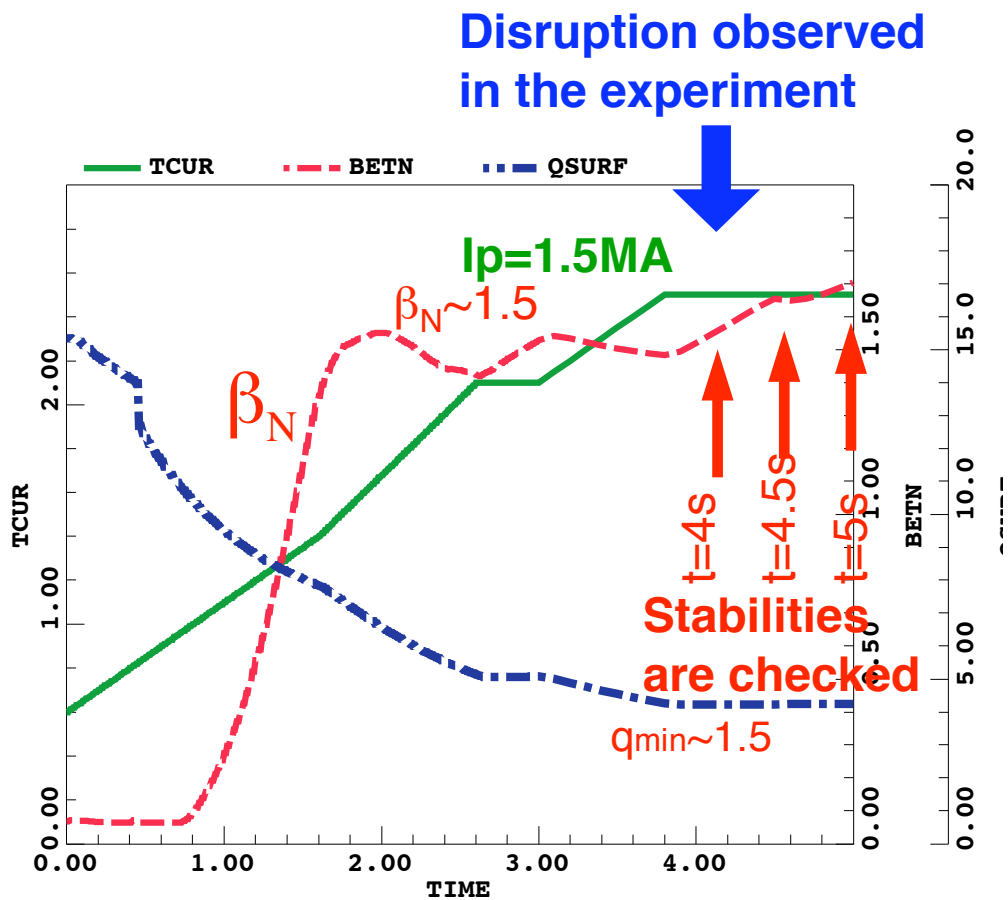
Disruption:
βN~1.6
Steep pressure grad.
t~4sec

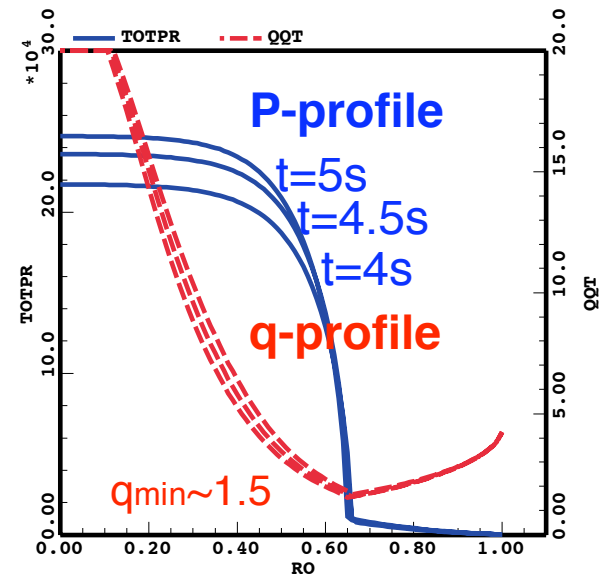


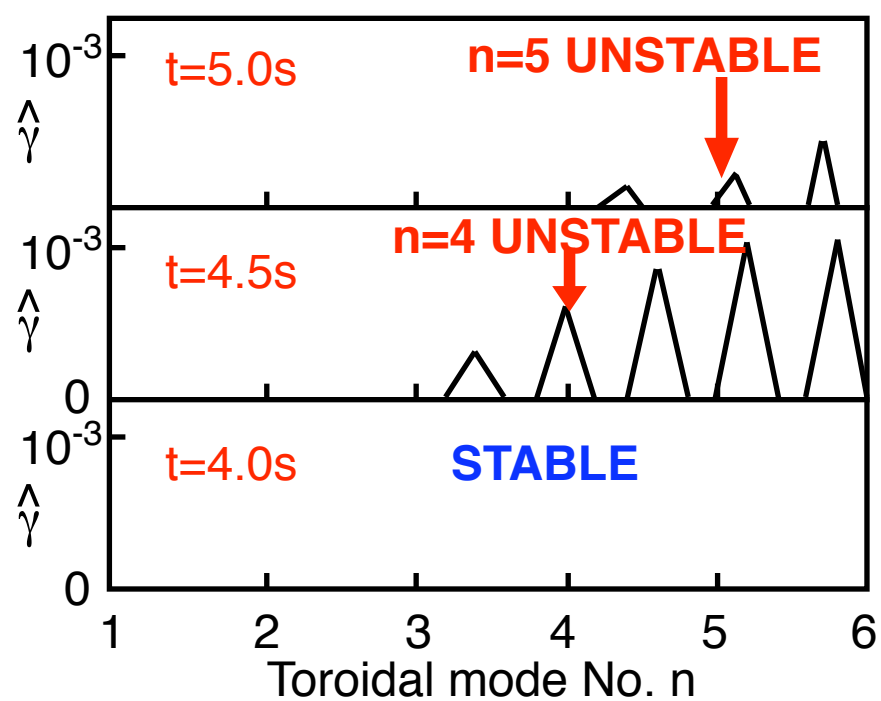


Stability analysis of low n modes

Result of the transport simulation of the case of disruption



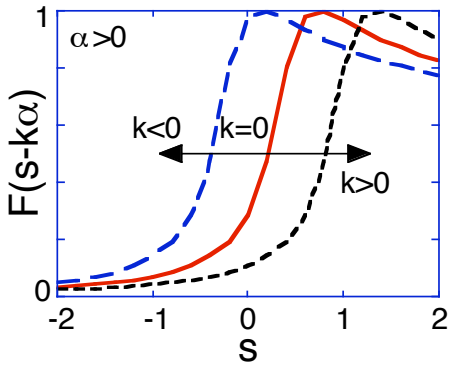




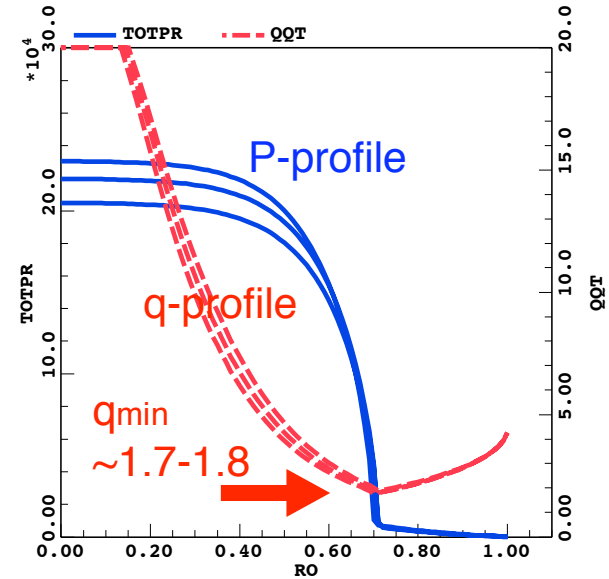
The unstable mode was obtained by the stability analysis. But results are very sensitive to profile.
Further investigations are required.

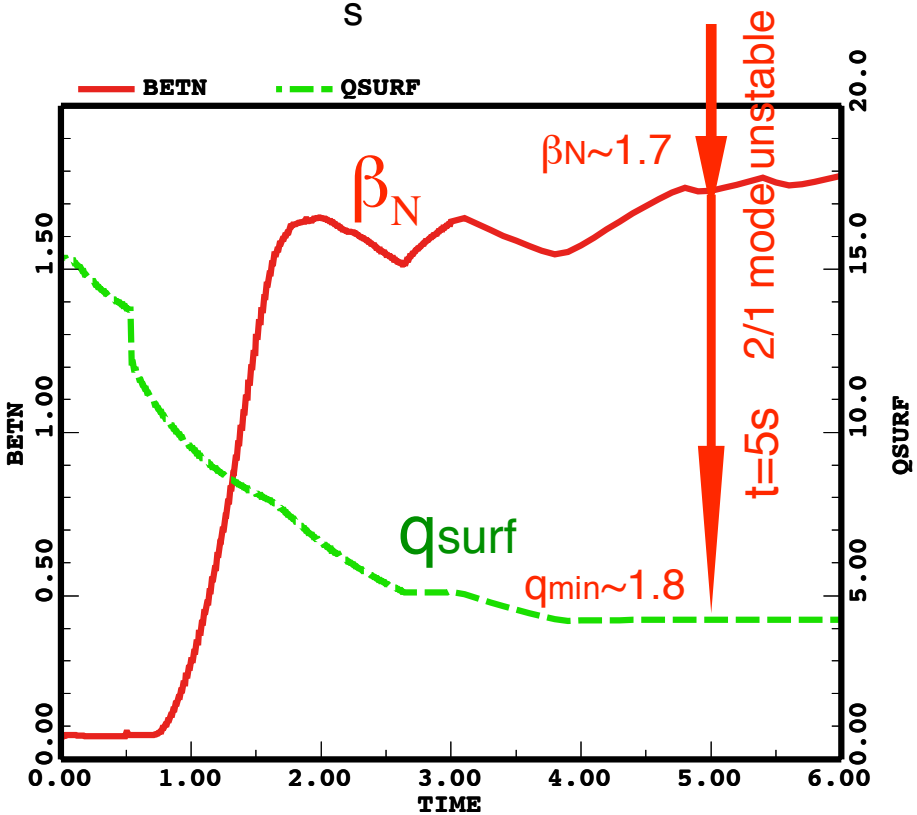
Influence of transport property on the MHD Stability: Case of k=0.3

$$\chi_{ano} = D_{ano} = \chi_0 F(s - k\alpha)$$

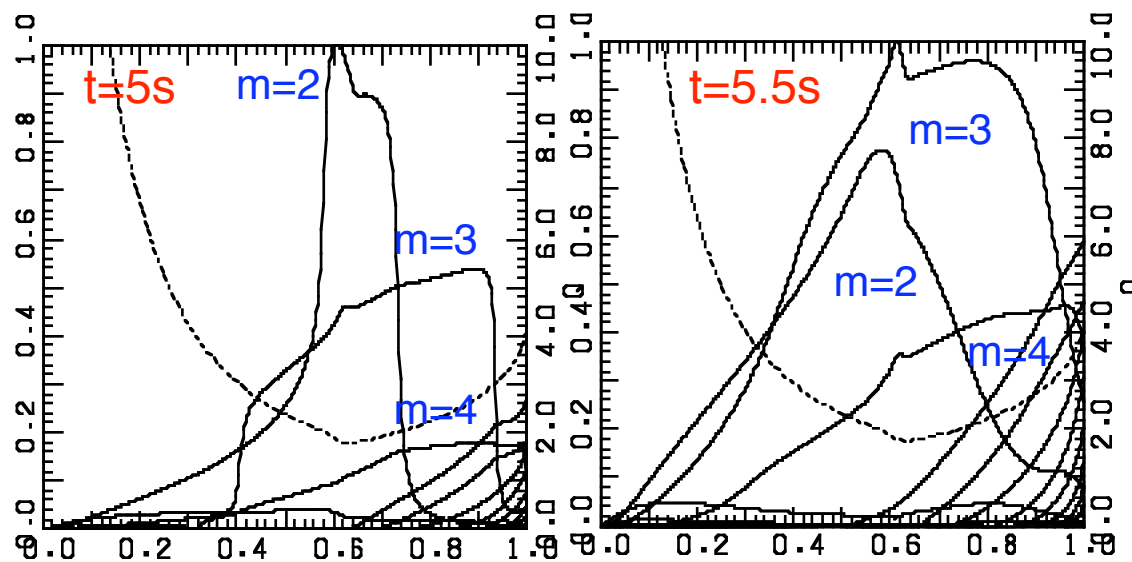


Pressure profiles become broader, β_N increases and qmin increases increase.





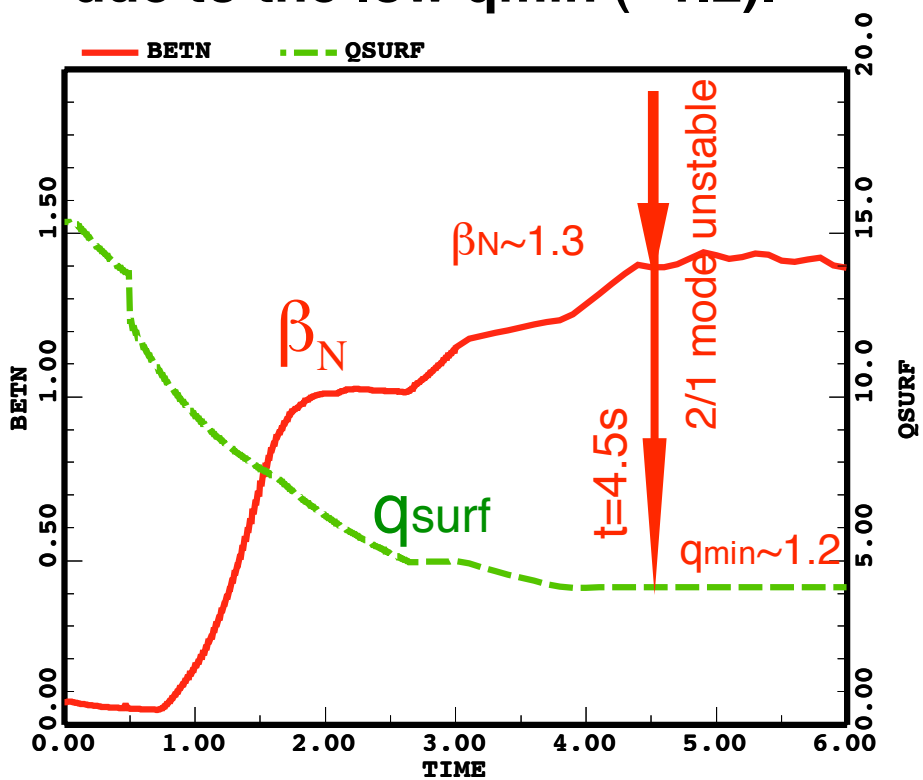
Internal and external modes modes become unstable

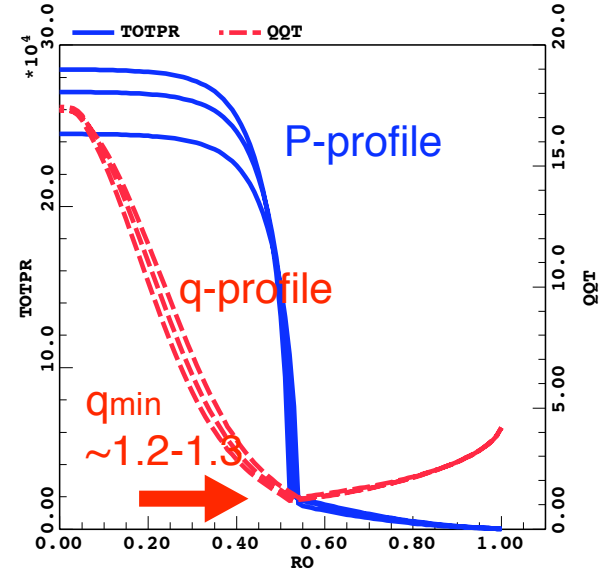


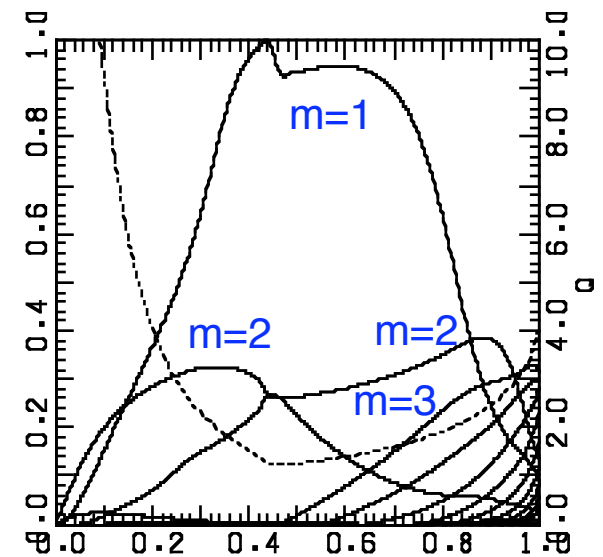
Influence of transport property on the MHD Stability: Case of k=-0.5

Pressure profiles become more peaked and the qmin decreased down to ~1.3

Internal modes of m/n=1/1,2/1,3/1 becomes unstable due to the low qmin (~1.2).







4. Summary of beta limits analysis

- There is a possibility to sustain the strongly reversed profile autonomously, but the beta limit due to the finite n mode may be low.
- It is need to explore the suitable current profile control to sustain the high beta.
- More careful analysis is required because the accuracy of the equilibrium and the stability analysis.

Edge Stability Analysis

(planing)

MARG2D code was developed for low-n and high-n mode stability

[S.Tokuda, Phys. Plasmas 6 (8) 1999]

MARG2D solves the 2D Newcomb equation

$$N\xi := -\frac{d}{dr}\left(L\frac{d\xi}{dr}\right) - \frac{d}{dr}(M^t\xi) + M\frac{d\xi}{dr} + K\xi = 0$$

associated with eigenvalue problem

$$N\xi = -\lambda R\xi$$

$$R : \text{diagonal matrix with } R_{m,m} \propto (n/m - 1/q)^2$$

Properties of the code

- **This method can avoid problems due to the continuum spectrum.**
- **Applicable for high n modes stabilities (more than n=50)**
- **Very fast calculation time (~85sec for n=40, NR=2800, NV=280,m=90 by Origin 3800,128cpu)**
- **Results of the stability of n=1 are agree to those of ERATO-J**

Possibility of utilization

- **Consistent stability analyses from low-n to high-n modes**
- **Apply to the burning plasma simulation as the ELM model**

Plan: Iterative calculation during 1.5D time-dependent transport simulations

1.5D transport code

1D transport equations :

1D current diffusion equation :

2D MHD equilibrium : Grad-Shafranov equation (Fixed boundary)
Model of current limit inside CH is applied. (q_{limit}= constant)

ELM/Pedestal : Modelling due to stability results ??

2D Equilibrium data ↓ ↑ Growth rate and eigenfunction

Finite n mode: MARG2D, High n ballooning

Summary

Issues of the integration of the transport and MHD analyses:

- Issues of modeling:
  - Modeling of the influence of the MHD instability on the transport? Mixing length approximation?
  - Difference of the dimension. How models 2D structure of the island to 1D transport model? 3D structure is more complicated.
- Issues of numeical calulation:
  - MHD code requires the high accuracy of the equilibrium, however the transport code does not. Small modulation of the profiles due to the transport effects are large influence on the stability calculation.
- Issues of integration:
  - Physically meaningful modeling is difficult.
  - Large integration is time consuming.

Further investigation is required.