Global Stability Modelling of Stellarators

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Stellarator anomalies

- Beta is higher in experiments than predicted by high mode number n ballooning codes
- Stellarators are resistive interchange unstable – but modes are not observed experimentally
- “Soft” beta limit is seen in experiments – confinement degrades above a critical beta

MHD and resistive MHD do not explain these results

Stellarator stability requires multiscale and multi level physics models

- Stellarator stability depends on intermediate and short wavelength modes (n >> 1) – unlike tokamaks, which depend on long wavelength (n = 1) (pressure driven vs. current driven)
- Ideal MHD:
- Resistive MHD:
- Two fluid model: explains the anomalies
- Free boundary: long wavelength modes – resistive MHD
- Kinetic model – TAE modes

Outline

- M3D code
- Ideal MHD and drift stabilization
- Resistive ballooning and drift stabilization
- Equilibrium Islands
- Free Boundary Stability
- TAE modes

M3D code

- Extended MHD multi level physics model
  - Ideal and resistive MHD
  - 2 fluid drift model (Sugiyama & Park, PoP 2000), similar to Braginskii equations
  - main effects produced by:
    - Ion Gyroviscous stress
    - Electron pressure in Ohm's Law
  - Gyrokinetic energetic ions

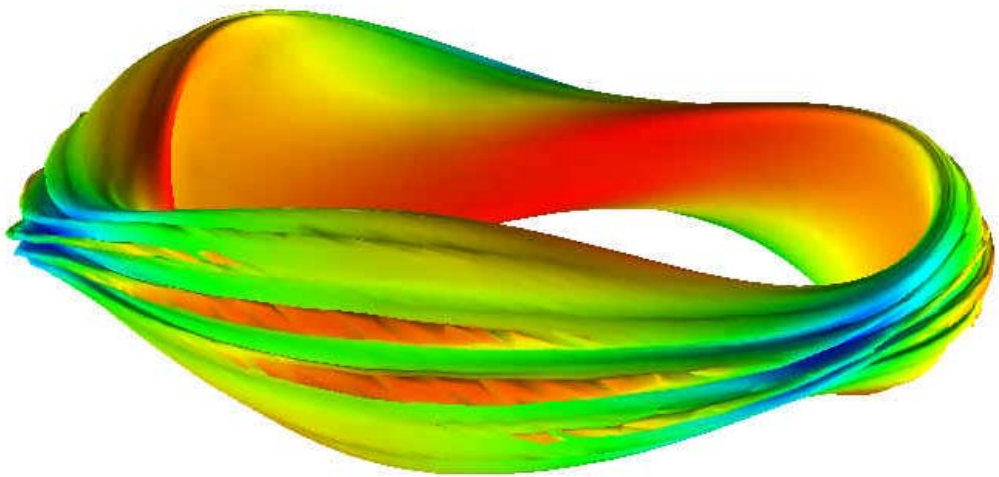
Ideal Stability: internal modes

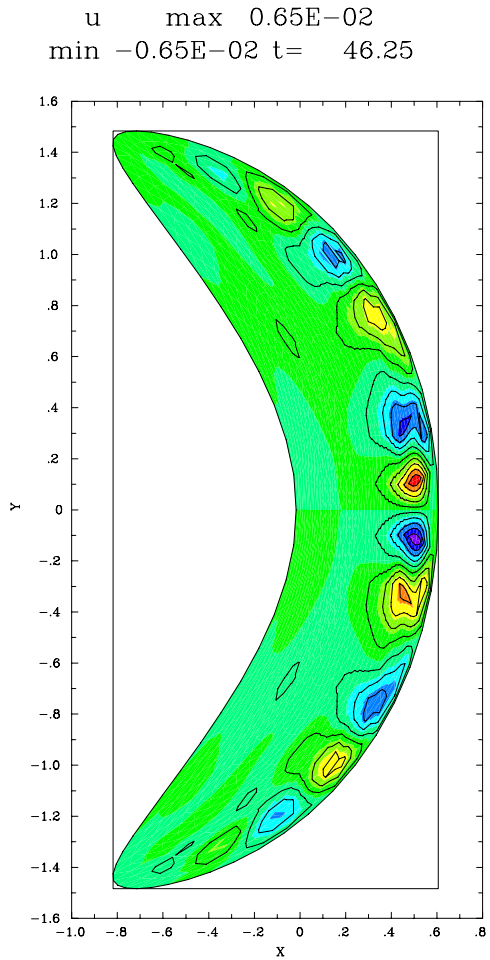
- Tokamaks: lowest toroidal mode n (=1) has lowest beta threshold
- Stellarators: highest toroidal mode is most unstable (Terpsichore: G.-Y. Fu)
- 2 fluid drift more important in stellarators than in tokamaks

Mode structure
M3D Nonlinear internal mode

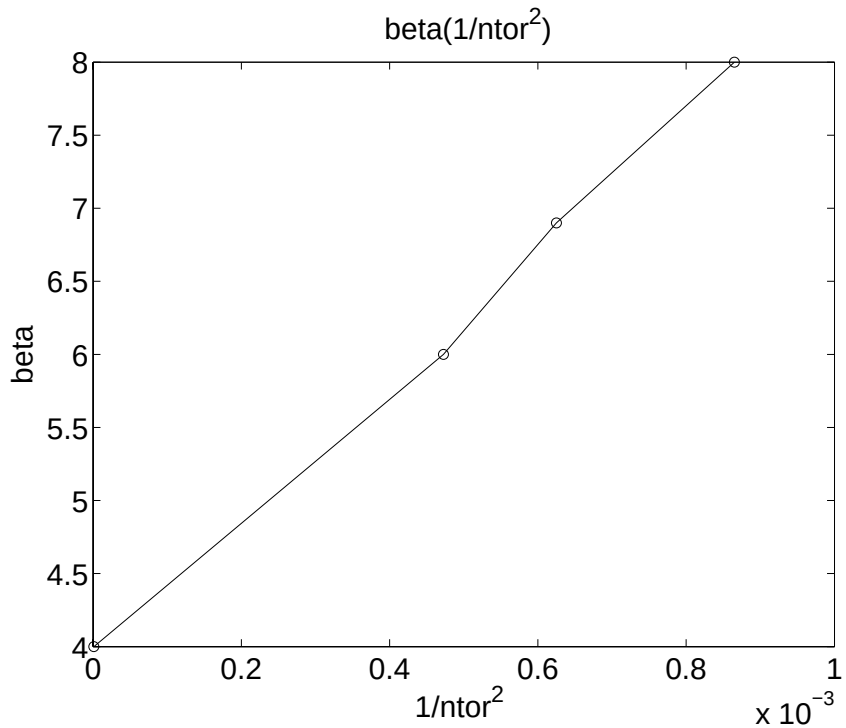
Velocity streamfunction

Pressure isosurface, NCSX
li383
Beta = 8%, n ~ 9





Ideal Ballooning stability depends on toroidal mode number n – NCSX li383



Scaling with n – Terpsichore – G-Y Fu

$$\beta_c = \beta_o \left(1 + \frac{n_0^2}{n^2} \right)$$

$$n_0 = 30, \beta_0 = 0.04$$

2 fluid: Gyroviscous term

- Hall parameter

$$H = \frac{c}{\omega_{pi} R} \approx .01$$

- Gyroviscous term in momentum equation

$$\rho \frac{\partial v}{\partial t} + \left(v_{di} + v\right) \bullet \nabla v = -\nabla p + \left(\nabla \times B\right) \times B$$

$$v_{di} = -H \frac{\nabla p_i \times B}{\rho B^2}$$

$$\omega_{*i} = k \bullet v_{di}$$

Gyroviscous stabilization

$$\frac{\omega_{*i}^2}{2} = -\gamma_{MHD}^2$$

$$\omega_* \tau_A = Hn \frac{\beta q R^2}{a^2}$$

$$\gamma_{MHD}^2 \tau_A^2 = \frac{R}{a}\left\{\beta - \beta_0\left(1 + \frac{n_0^2}{n^2}\right)\right\}$$

$$H = \frac{c}{\omega_{pi} R} \approx .01$$

$$\tau_A = \frac{R}{v_A}$$

Critical Beta depends on H

- Maximum unstable n ~ $H^{-1/2}$

$$n = \frac{2n_0^{1/2}(a/R)^{3/4}}{H^{1/2} q \beta_0^{1/2}\left(1+\frac{q}{2}\beta_0^{1/2} n_0 (R/a)^{3/2}\right)^{1/2}}$$

- Minimum stable beta as a function of H

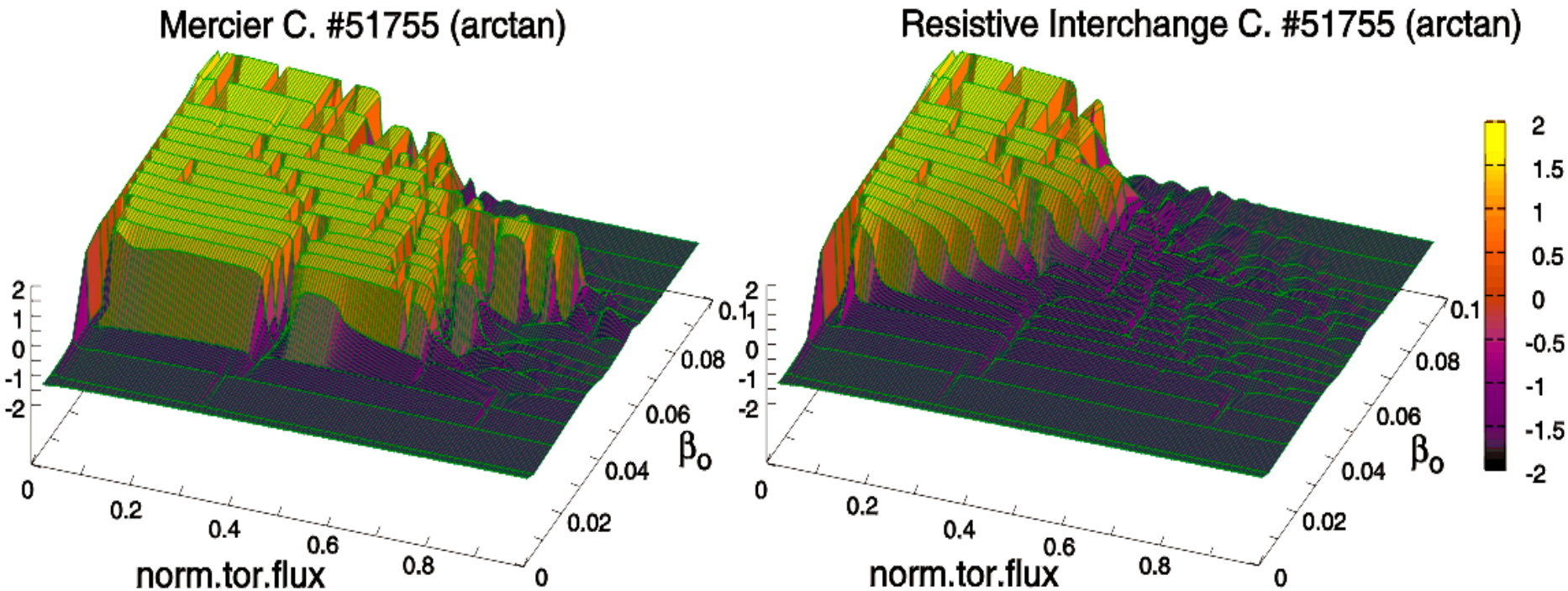
$$\beta = \beta_0\left(1+\frac{q}{2}\beta_0^{1/2} n_0 H \frac{R^{3/2}}{a^{3/2}}\right)$$

- H = 0.018 required to increase beta from 4% to 8%

In NCSX. Ballooning criterion is not important

Computational Stability Studies

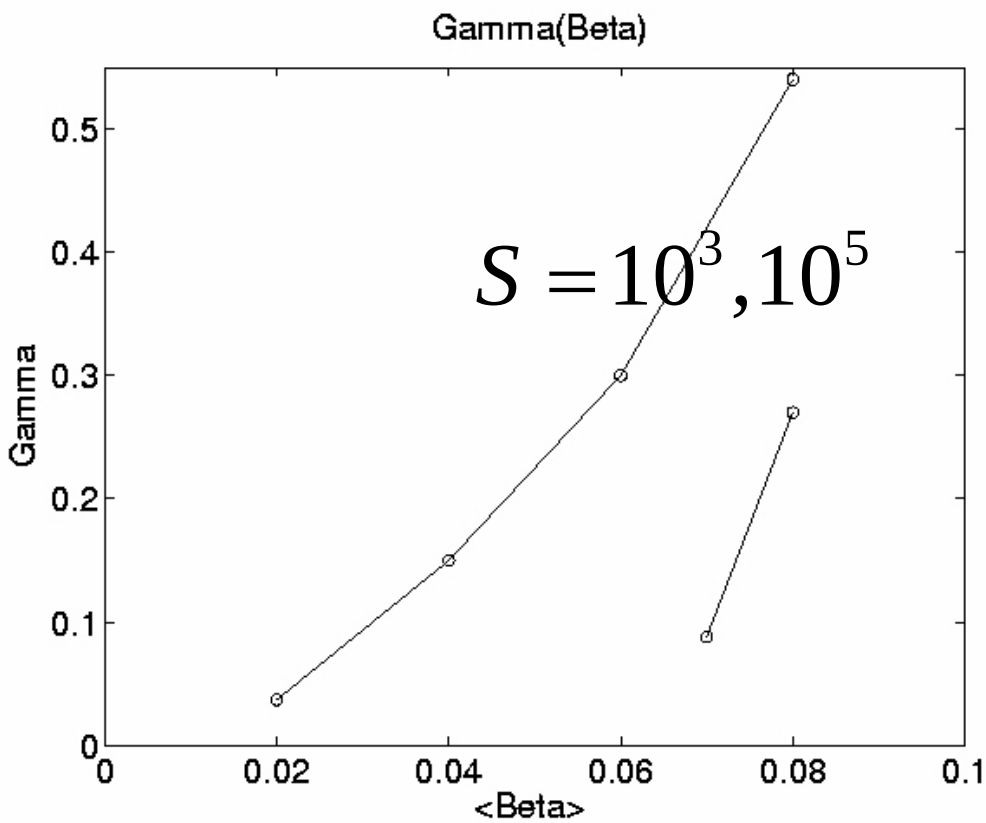
- A. Weller - local Mercier- and resistive interchange criteria, where is the stability limit -



- stable Region evolves in center and increases towards edge
- stable region shrinks again beyond $<\beta> \approx 3$ % (max. in experiment)
- experimentally observed pressure gradients can exceed theoretical stability limits !

Resistive Stability

- Stellarators tend to be resistive interchange unstable
- Resistive ballooning modes not stabilized by coupling to interchange, unlike tokamaks
- Stellarators unstable to resistive ballooning for all beta
- 2 Fluid stabilizes RBM



Resistive ballooning modes

- Long wavelength electromagnetic mode (Strauss, 1981, …)

$$\omega(\omega+\omega_{*i})(\omega-\omega_{*e})^3=\eta^3\Delta'^4 q^{-2}\tau_A^{-2}$$

- Short wavelength electrostatic mode (Carreras – Diamond, 1983)

$$\omega(\omega+\omega_{*i})(\omega-\omega_{*e})=\eta\beta^2 q^2(R/a)\tau_A^{-2}$$

- Growth rate linear in resistivity

$$\omega_* > \gamma$$

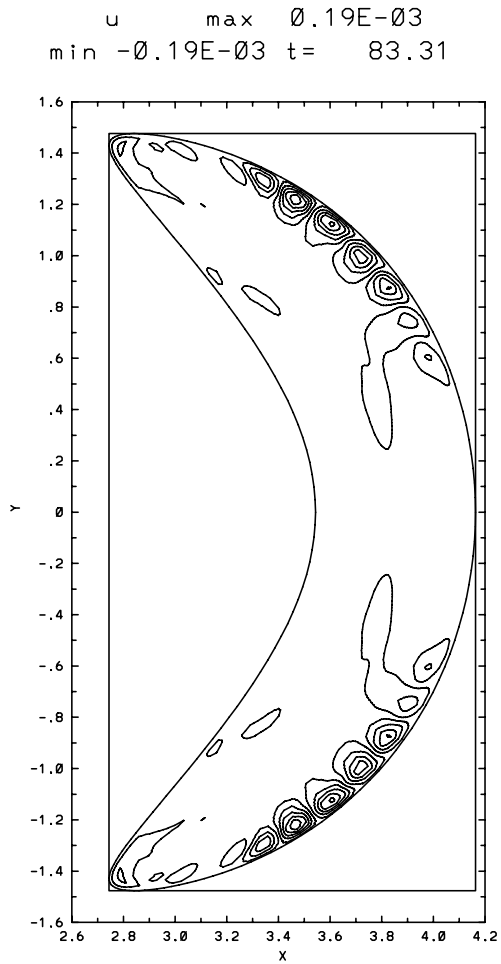
RBM stabilization

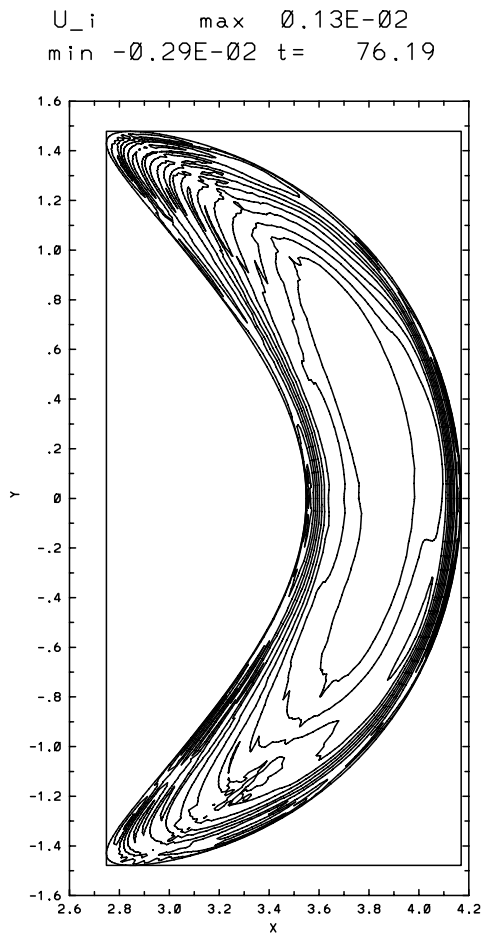
H can be lower because growth rate is usually small compared to ideal MHD
H = 0, 0.02

Drifts are large where pressure
gradient is large

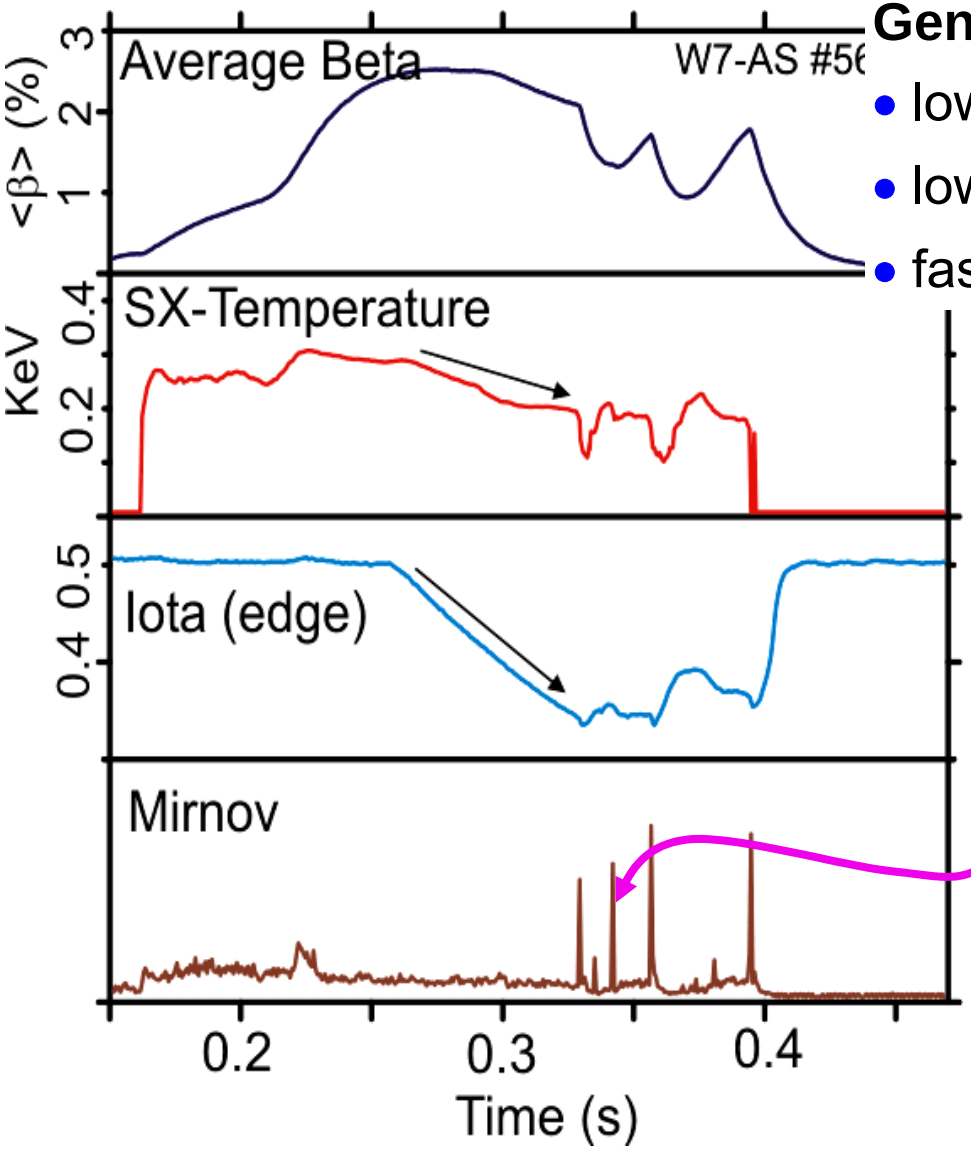
Ion flow may be important for stabilization

W7AS – may observe RBM in cold high density shots



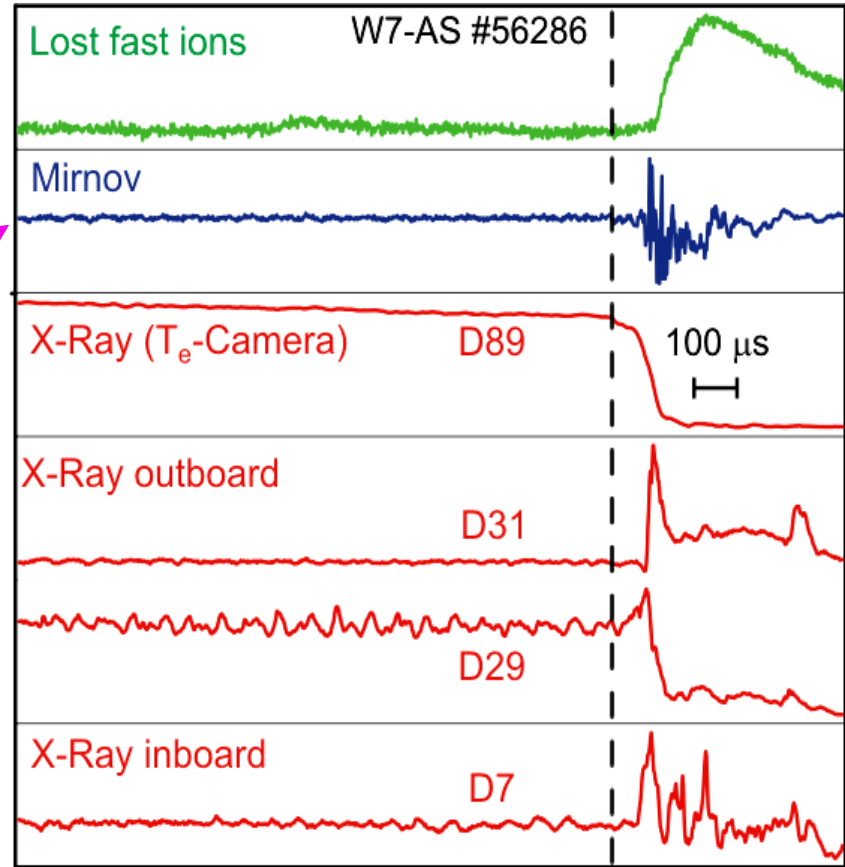


Fast MHD Crashes



General Characteristics:

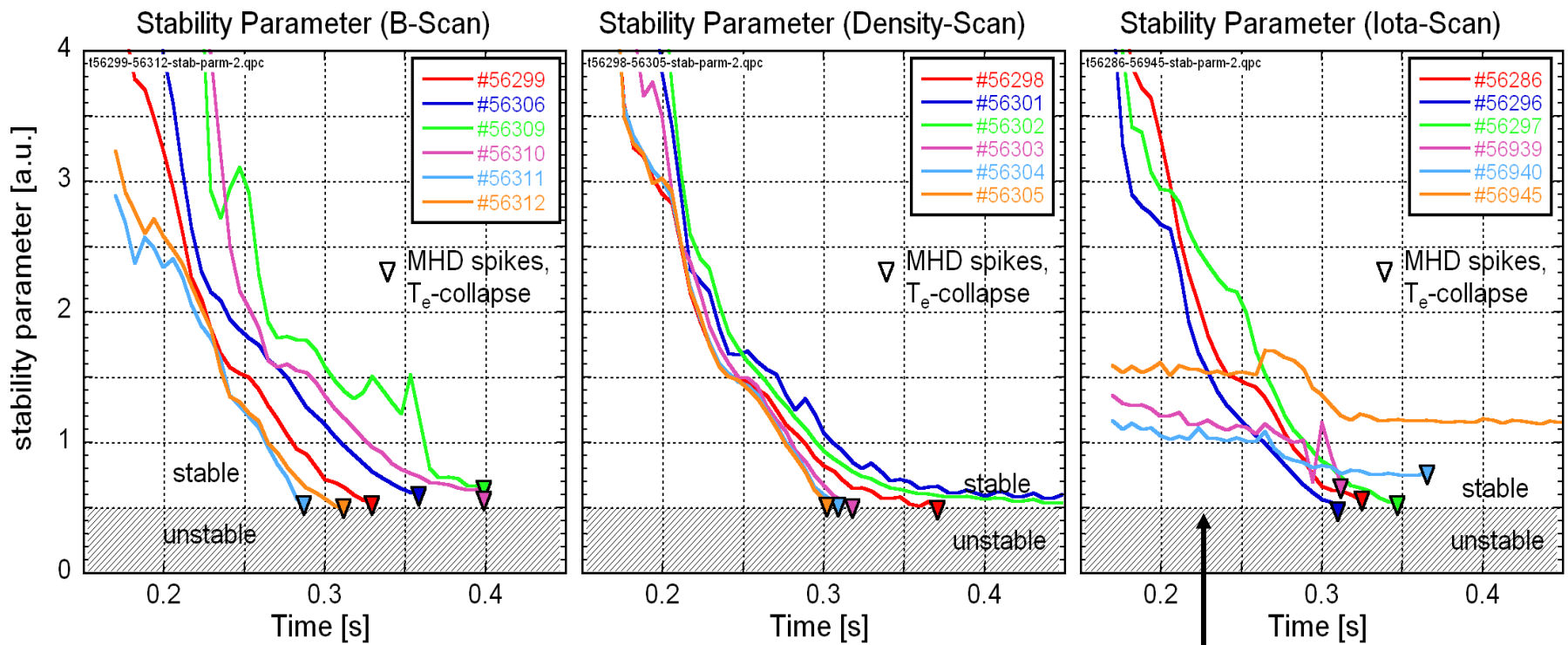
- low T_e preferred
- low iota preferred (with and w/o I_{OH})
- fast (~ 100 μs) MHD bursts, ELM-like



Fast MHD Crashes

Resistive ballooning Modes ?

Growth Rate for Resistive Ballooning Modes: $\gamma \propto (\langle\beta\rangle/\langle\beta\rangle_c)\eta k^2/\mu_0 \quad (\langle\beta\rangle/\iota^2)T_e^{-3/2}$

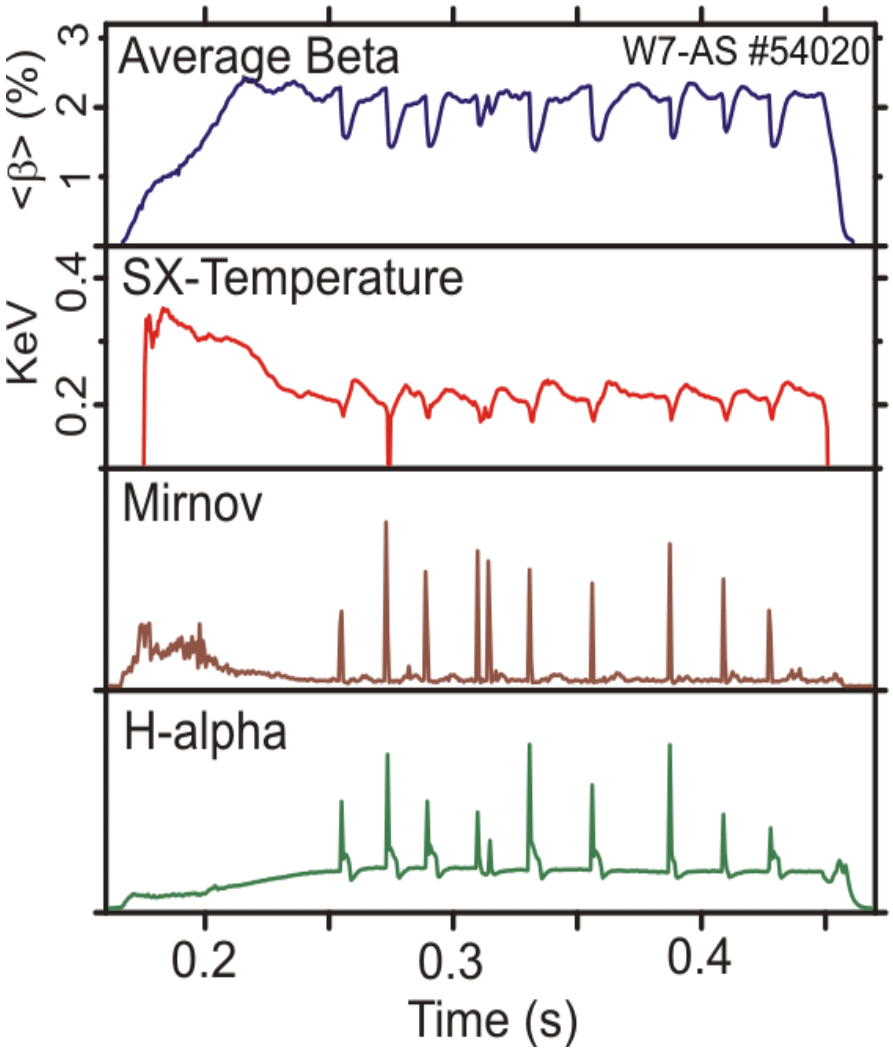


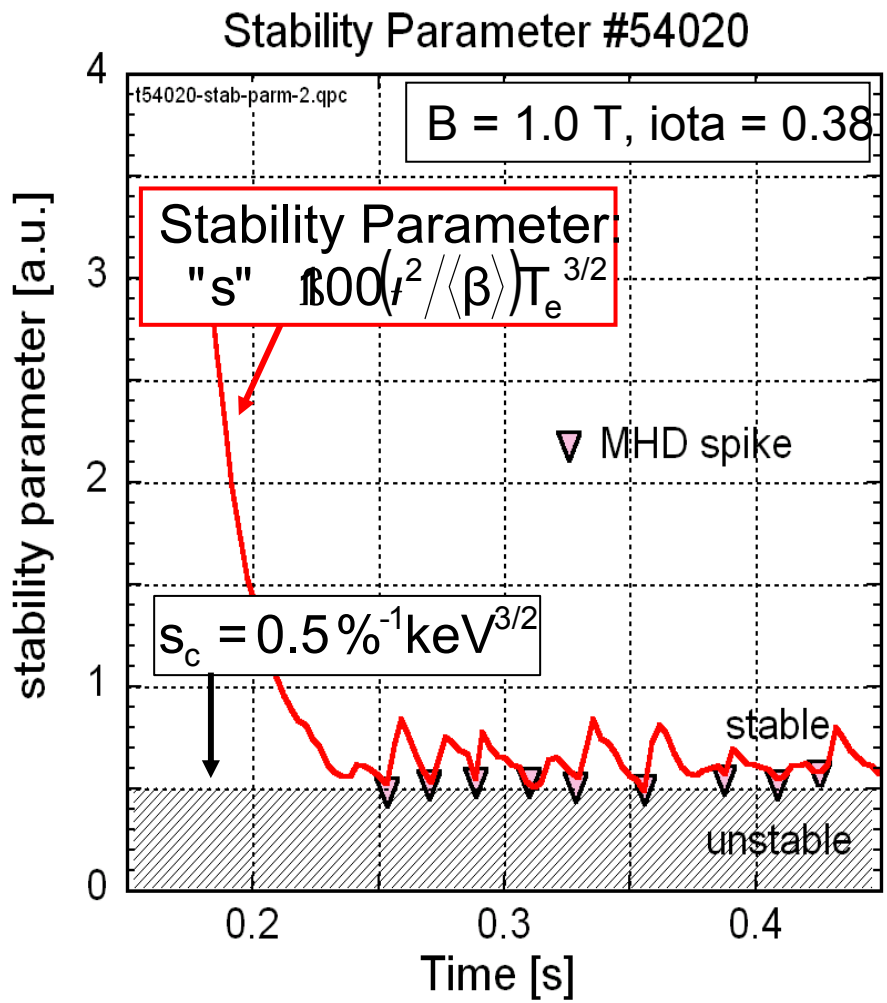
$s_c = 0.5\,\%^{-1}\,\mathrm{keV}^{3/2}$

Stability Parameter: "s" ß $100\,(\iota^2/\langle\beta\rangle)T_e^{3/2}$

Fast MHD Crashes

Resistive ballooning Modes ?





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Island growth – soft beta limit

- Islands can give soft beta limit
- Parallel electron pressure gradient in Ohm's Law enhances island growth

$$\frac{\partial A_{\parallel}}{\partial t} = \nabla_{\parallel}\Phi - H\frac{\nabla_{\parallel} p_e}{\rho} - \eta J_{\parallel}$$

$$p_e = \rho T_e$$

$$\frac{\partial T_e}{\partial t} = -v_e \bullet \nabla T_e - (\Gamma - 1) T_e \nabla \bullet v_e + \chi_{\parallel}\nabla_{\parallel}^2 T_e + \chi_{\perp}\nabla_{\perp}^2 T_e$$

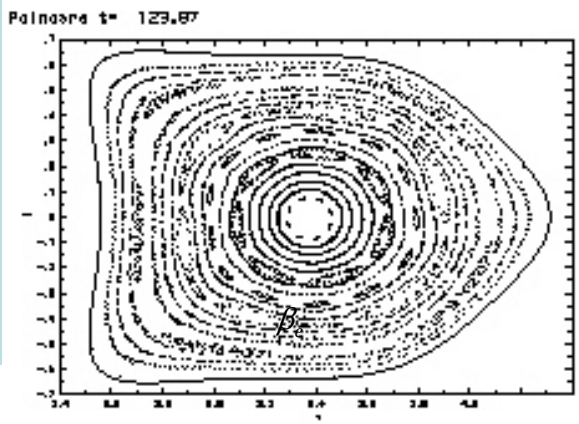
$$\frac{\partial \rho}{\partial t} = -\nabla \bullet (\rho v_i) \approx 0$$

Density evolution turned off

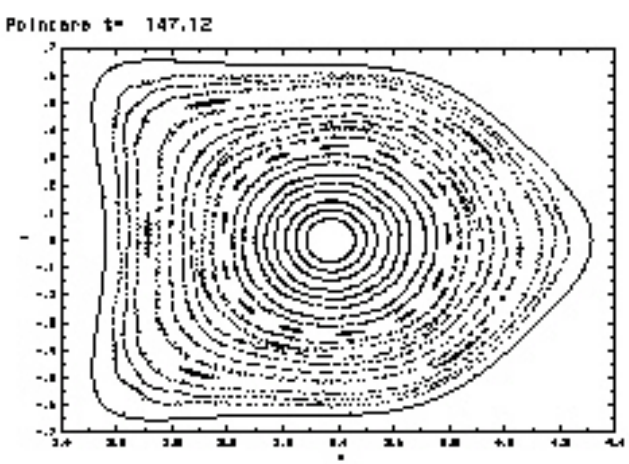
Island growth depends on H β_e

- Beta = 7%
- H = .02
- L. Sugiyama, APS03

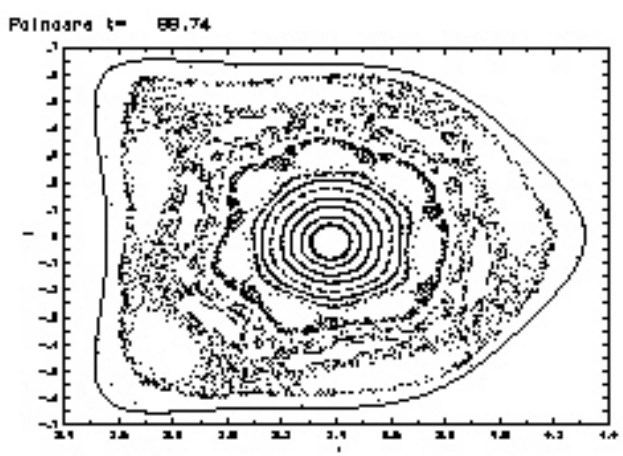
$p_e/p = 0.5$



$p_e/p = 0.05$



$p_e/p = 0.95$

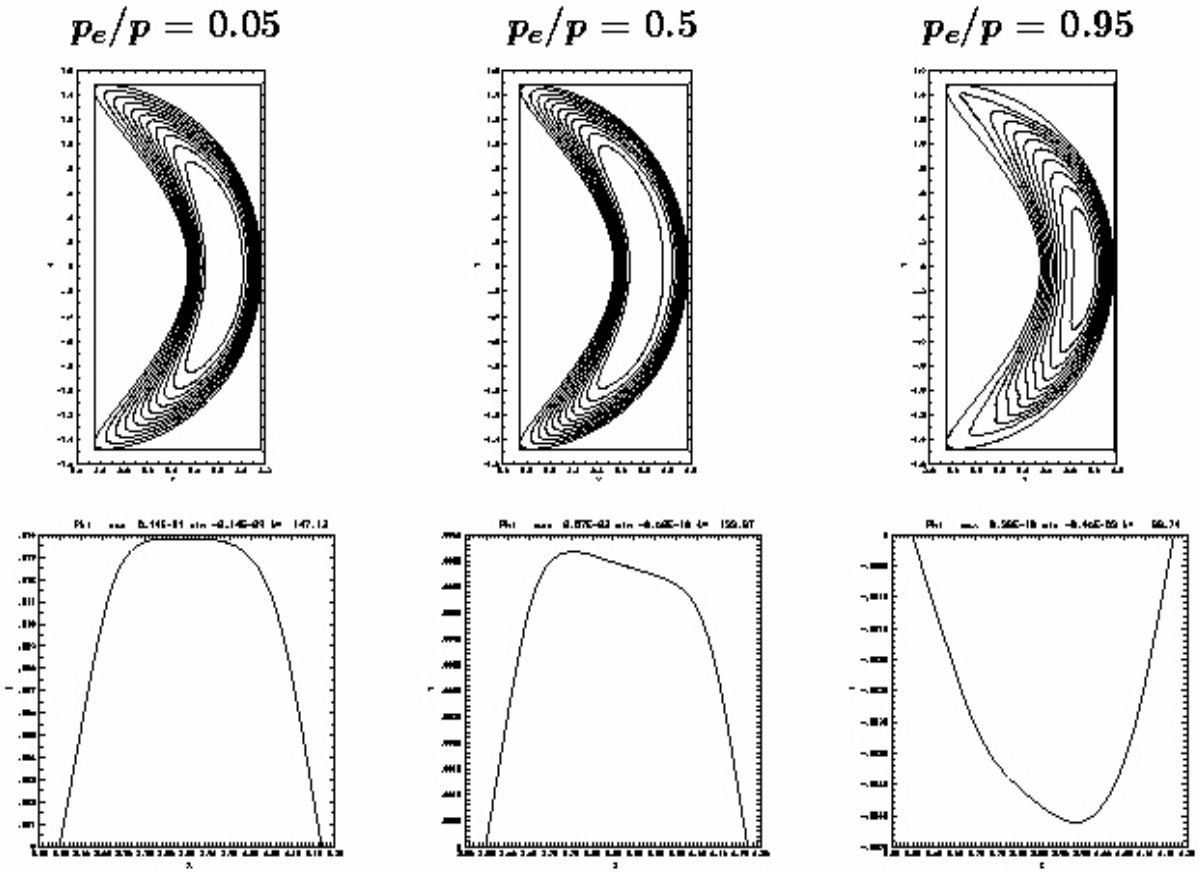


Island growth – reversal of ion flow

- Tokamak theory – Connor et al., 2001

$$\omega = kv_i$$

$$\frac{\partial W}{\partial t} = \eta\left(\Delta' + g\frac{\omega(\omega+\omega_{*i})L_s^2}{W^3k^2v_A^2}\right)$$



Free boundary modes

- Free boundary modes
  - Limit beta
  - Resistive halo model $10^5 > S > 10^2$

  (vacuum modeled as high resistivity region)
- Moderate n external kinks
  - Beta = 4% in NCSX
  - Medium beta n=1 mode in W7AS
  - Tokamak
    - VDE
    - RWM

MHD Activity in currentless High-β Discharges

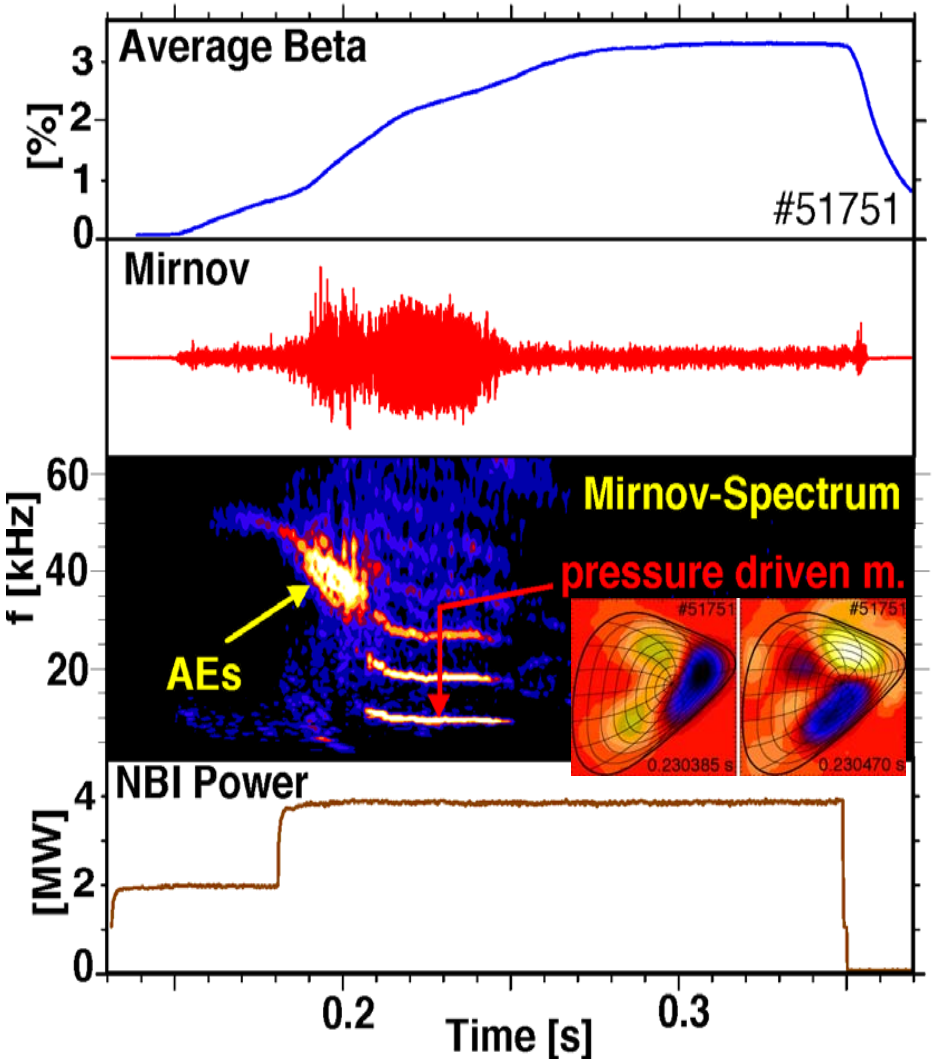
- generally very weak at highest β !
- transient activity at intermediate β

- **3 main MHD phenomena**:

  - pressure driven global low frequency m.

  - fast ion driven global Alfvén modes

  - fast crashes (ELM-like, low T_e, low iota)

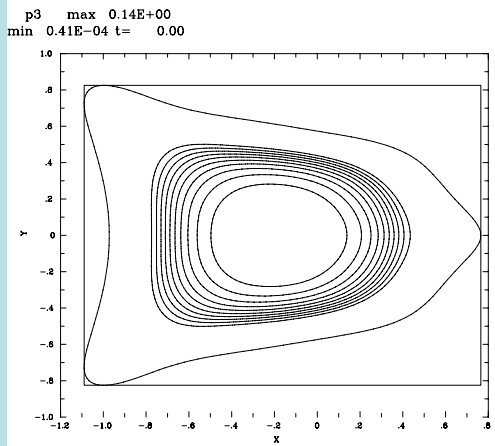


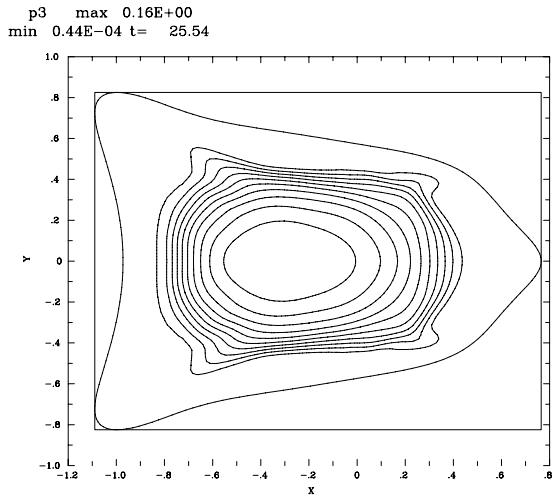
A. Weller, PPPL, 2003

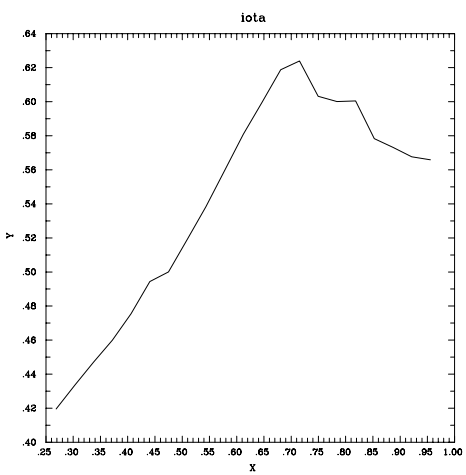
- Transient pressure driven m = 2 modes at $\langle\beta\rangle < 2.5$

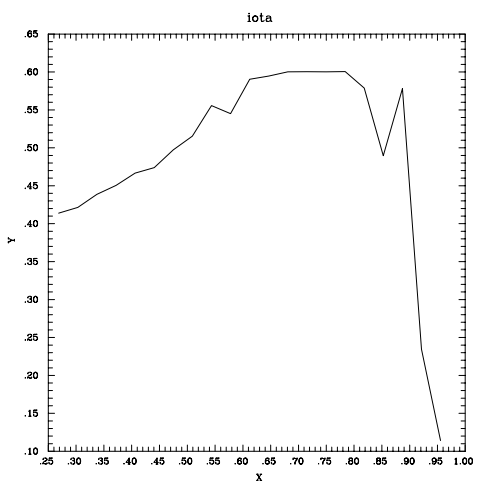
NCSX li383 5% beta

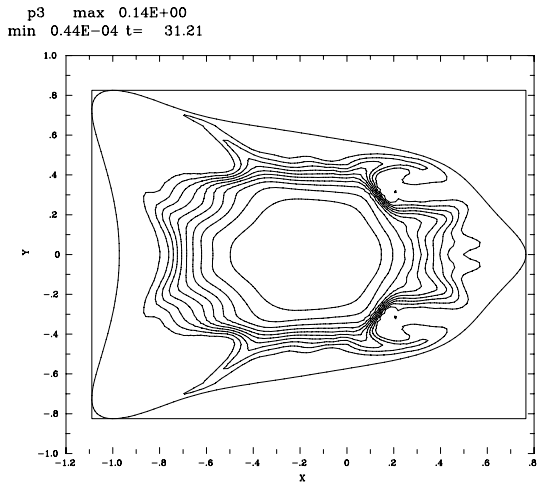
- Cold halo region
- 5/3 island in plasma and in halo
- Double tearing mode
- Might limit beta







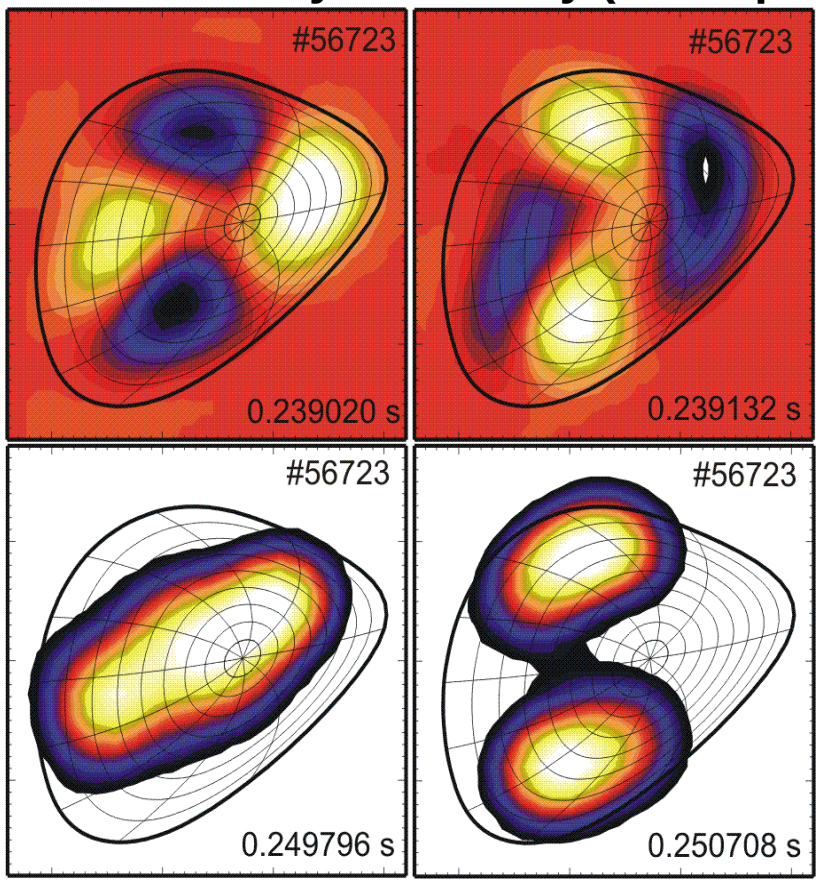




plasma resistivity caused 5/3 double tearing mode to grow faster than ideal free boundary edge kink

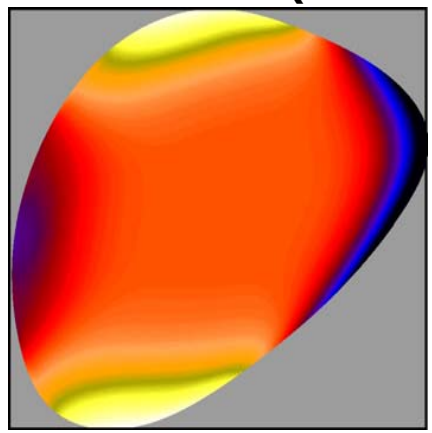
Transient (low-β) Collapse induced by large m = 2

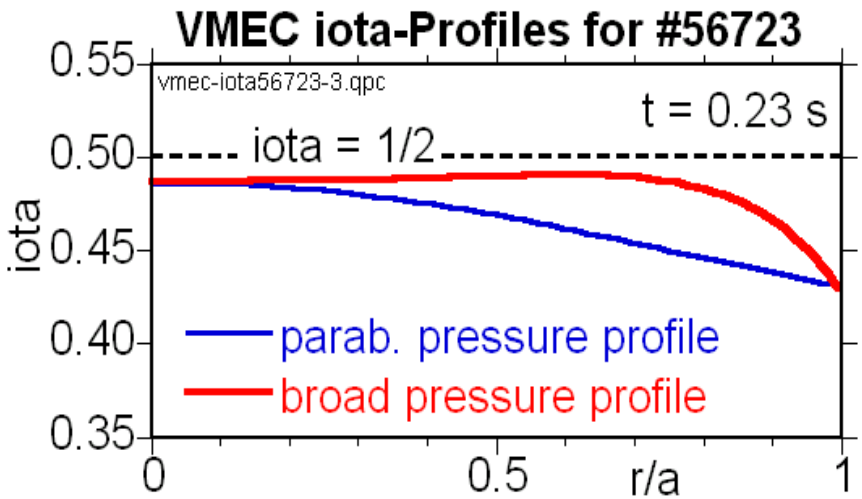
Perturbed X-Ray Emissivity (initial phase)



Total X-Ray Emissivity (late phase)

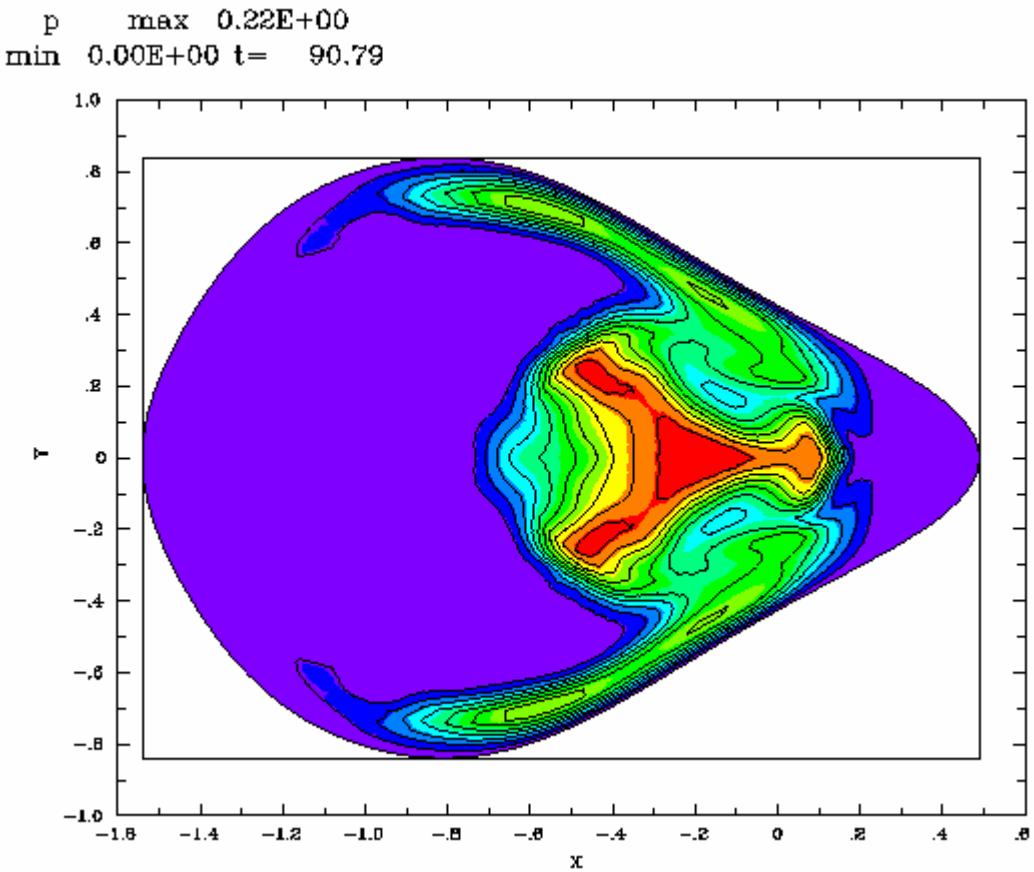
Perturbed Pressure (CAS3D Code)

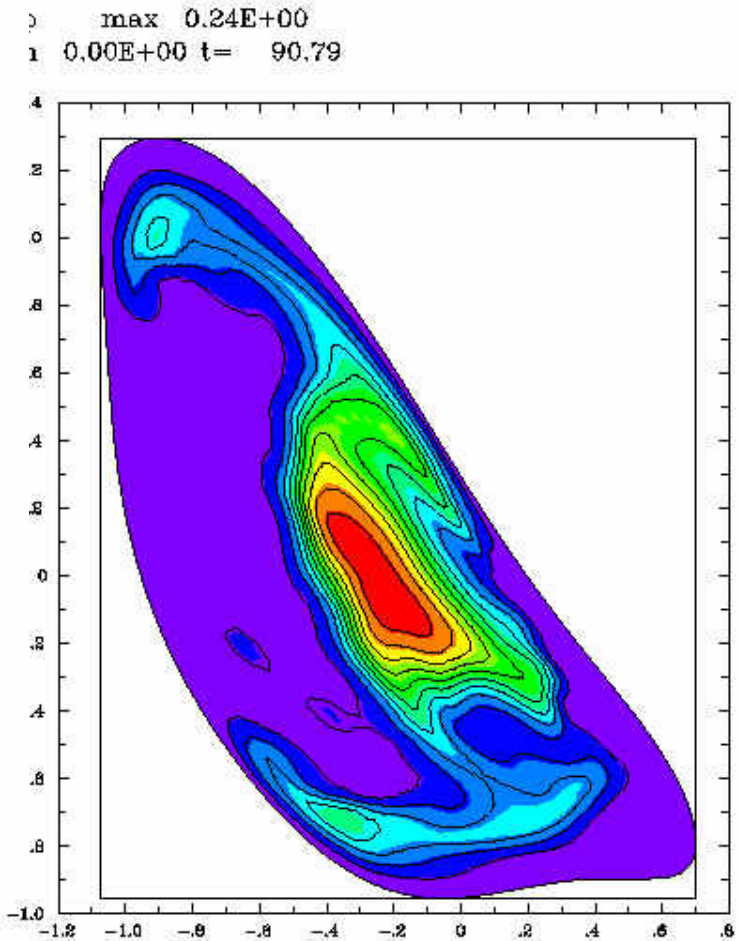




- Data indicate resistive features (m = 2 islands, locking to m = 2 error field)
- islands grow into central region (very low shear), hot core breaks apart

W7 free boundary modes





Kinetic effects- TAE in stellarator and tokamak

- Gyrokinetic hot ion species – G.-Y. Fu
- 2 period QAS stellarator
- Interpolate between stellarator and tokamak boundary shape with same profiles of p, q:
- Parameters:

$$R(\theta,\phi) = R_0(\theta) + \lambda R_1(\theta,\phi)$$

$$Z(\theta,\phi) = Z_0(\theta) + \lambda Z_1(\theta,\phi)$$

$$2.4 < q < 2.9, q_{gap} = 2.5$$

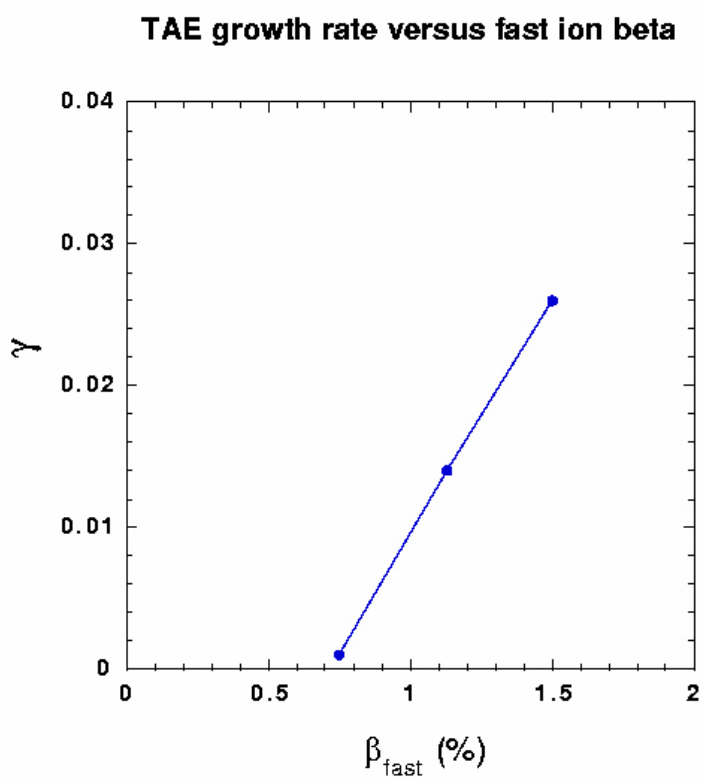
$$\beta = 0.014, R/a = 4$$

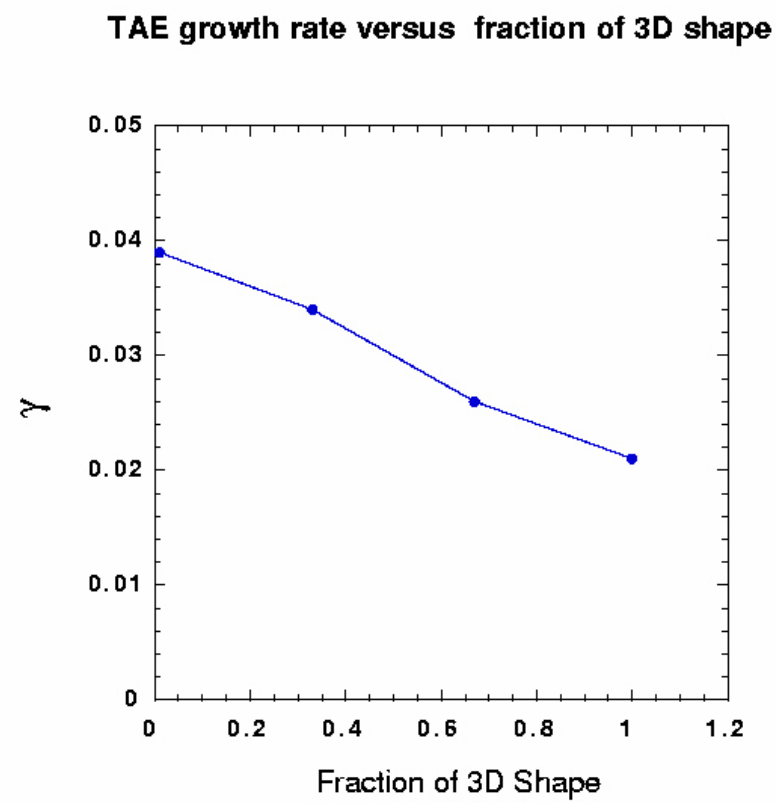
$$\rho_{fast} = 0.12a, v_{fast} = 1.6v_A$$

TAE modes

Growth rate proportional to hot ion beta, frequency is TAE

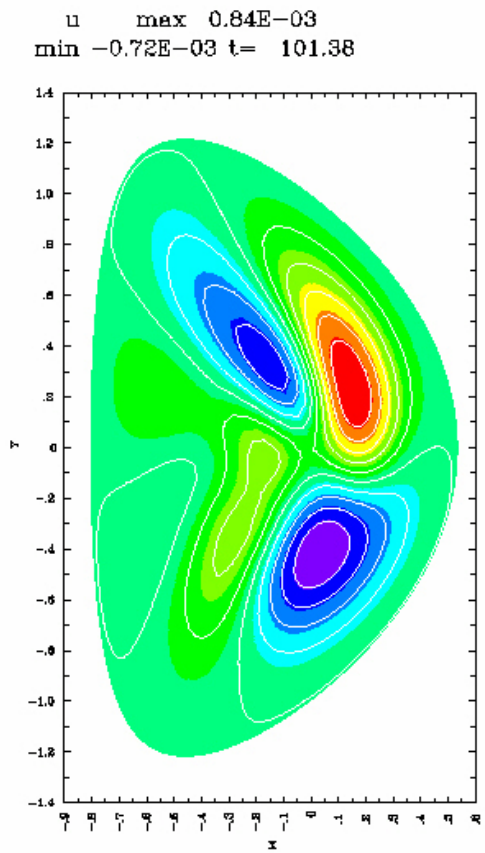
Growth rate decreases with increasing 3D shape

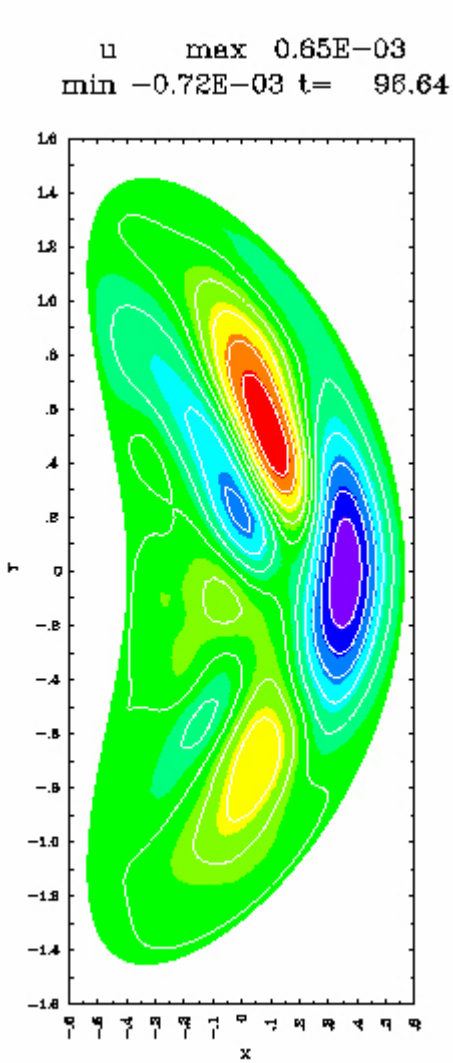




TAE modes – mode structure

Tokamak and stellarator have similar TAE eigenmode (electric potential)





Conclusions

- Ideal mode beta limit determined by intermediate n
  - 2 fluid (H = .02) raises beta by stabilizing n>20
  - Beta = 7% rather than 4% in NCSX
  - Resistive modes (H = 0) unstable for all beta
  - Resistive modes stabilized by ion flow and diamagnetic drifts
- Equilibrium islands depend on $H\beta_e$
  - Sensitive to details of model
  - Islands can give soft beta limit
- Free boundary
  - Model vacuum as resistive halo
- Kinetic hot particles
  - TAE modes may be more stable in stellarators than tokamaks