coilenergy

Construct "energy" function, and constraints, and the derivatives with respect to coil geometry, including the first and second dirivatives

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contents

1.1 The second derivatives

1. The integrands in B^x, B^y and B^z can be expressed in a concise way:

$$B^i \equiv g^i r^{-3} \tag{1}$$

where,

$$g^i \equiv \varepsilon^{ijk}\, \Delta l^j \; \dot{l}^k \tag{2}$$

Here ε^{ijk} is Levi-Civita symbol and for simplification I will omit this symbol in later i, j, k cases.

2. Therefore, the derivatives of B^x, B^y and B^z integrands (both the first and second derivatives) can be expressed as:

$$\begin{aligned}
\frac{\partial B^i}{\partial x_l} &\equiv \frac{\partial g^i}{\partial x_l} r^{-3} - \frac{3}{r^4}\frac{\partial r}{\partial x_l} g^i \\
\frac{\partial^2 B^i}{\partial x_l \partial x_m} &\equiv \frac{\partial^2 g^i}{\partial x_l \partial x_m} r^{-3} - \frac{3}{r^4}\frac{\partial r}{\partial x_m}\frac{\partial g^i}{\partial x_l} + \frac{12}{r^5}\frac{\partial r}{\partial x_l}\frac{\partial r}{\partial x_m} g^i - \frac{3}{r^{-4}}\frac{\partial^2 r}{\partial x_l \partial x_m} g^i - \frac{3}{r^4}\frac{\partial r}{\partial x_l}\frac{\partial g^i}{\partial x_m}
\end{aligned} \tag{3}$$

3. In that case, $\frac{\partial^2 B^i}{\partial x_l \partial x_m}$ is just related to the derivatives of g^i and r.So we can also write out all the derivatives of g^i and r.

$$\begin{aligned}
\frac{\partial g^i}{\partial x_l} &\equiv \frac{\partial \Delta l^j}{\partial x_l}\dot{l}^k + \Delta l^j \frac{\partial \dot{l}^k}{\partial x_l} - \frac{\partial \Delta l^k}{\partial x_l}\dot{l}^j - \Delta l^k \frac{\partial \dot{l}^j}{\partial x_l} \\
\frac{\partial^2 g^i}{\partial x_l \partial x_m} &\equiv \frac{\partial^2 \Delta l^j}{\partial x_l \partial x_m}\dot{l}^k + \frac{\partial \Delta l^j}{\partial x_l}\frac{\partial \dot{l}^k}{\partial x_m} + \frac{\partial \Delta l^j}{\partial x_m}\frac{\partial \dot{l}^k}{\partial x_l} + \Delta l^j \frac{\partial^2 \dot{l}^k}{\partial x_l \partial x_m} \\
&\quad - \frac{\partial^2 \Delta l^k}{\partial x_l \partial x_m}\dot{l}^j - \frac{\partial \Delta l^k}{\partial x_l}\frac{\partial \dot{l}^j}{\partial x_m} - \frac{\partial \Delta l^k}{\partial x_m}\frac{\partial \dot{l}^j}{\partial x_l} - \Delta l^k \frac{\partial^2 \dot{l}^j}{\partial x_l \partial x_m} \\
\frac{\partial r}{\partial x^i_l} &\equiv \frac{\Delta l^i}{r}\frac{\partial \Delta l^i}{\partial x_l} \\
\frac{\partial^2 r}{\partial x^i_l \partial x^j_m} &\equiv \delta^i_j \frac{1}{r}\frac{\partial \Delta l^i}{\partial x_l}\frac{\partial \Delta l^j}{\partial x_m} - \frac{\Delta l^i}{r^2}\frac{\partial \Delta r}{\partial x_m}\frac{1}{r}\frac{\partial \Delta l^i}{\partial x_l}
\end{aligned} \tag{4}$$