tlength

Calling tlength(nderiv) to calculate the length cost function w/o its derivatives on all degrees of freedom. When $nderiv = 0$, it only calculates the $0 - order$ length cost function, which is $/tlength = \sum_{i=1,Ncoils} \frac{1}{2}(L_i - L_o^i)^2/$. When $nderiv = 0$, it will calculate both the $0 - order$ and $1^{st} - order$ derivatives in array $t1L(1{:}Ncoils,0{:}Codf)$. When $nderiv = 0$, it will calculate all the $0 - order$, $1^{st} - order$ and $2^{nd} - order$ derivatives $t2L(1 : Ncoils, 0 : Codf, 1 : Ncoils, 0 : Codf)$.

[called by: denergy.]

contents

1.1 Total length cost function

1. The length of each coils can be calculated through,

$$L_i = \int_{icoil} \sqrt{\dot{x}(t)^2 + \dot{y}(t)^2 + \dot{z}(t)^2} dt \approx \sum_{kseg=0,Ndcoil-1} \sqrt{xt(kseg)^2 + yt(kseg)^2 + zt(kseg)^2} \frac{2\pi}{NDcoil} \tag{1}$$

   The results of user discretized method and NAG:D01EAF adaptive routine can be compared through subroutine descent.

2. And then the cost funtion on length is

$$ttlen = \sum_{i=1,Ncoils} \frac{1}{2} Lw_i (L_i - L_o^i)^2 \tag{2}$$

   where Lw_i is the weight of length of each coil, while there is another weight on the length cost function *weight_ttlen*

1.2 First dirivatives

Since the length of coils has no relationships with the current in the coil, the first derivatives of $ttlen$ on currents are all zero. An the derivatives on the geometry variables can be represented as,

$$\frac{\partial ttlen}{\partial x_n^i} \equiv (L_i - L_o^i) \frac{\partial L_i}{\partial x_n^i} \tag{3}$$

Here i is denoted to the i^{th} coil and x_n means the n^{th} DoF. And

$$\frac{\partial L_i}{\partial x_n^i} \equiv \int_{icoil} \frac{\dot{x}}{\sqrt{\dot{x}(t)^2 + \dot{y}(t)^2 + \dot{z}(t)^2}} \frac{\partial \dot{x}}{\partial x_n^i} dt \tag{4}$$

1.3 Second derivatives

The second derivatives of length cost function are only related to geometry variables of the same coil. That is,

$$\frac{\partial^2 ttlen}{\partial x_n^i \partial x_m^i} \equiv \frac{\partial L_i}{\partial x_m^i} \frac{\partial L_i}{\partial x_n^i} + (L_i - L_o^i) \frac{\partial^2 L_i}{\partial x_n^i \partial x_m^i} \equiv \int_{icoil} \frac{1}{\sqrt{\dot{x}(t)^2 + \dot{y}(t)^2 + \dot{z}(t)^2}} \frac{\partial \dot{x}}{\partial x_n^i} \frac{\partial \dot{x}}{\partial x_m^i} - \frac{\dot{x}\,\dot{x}}{(\sqrt{\dot{x}(t)^2 + \dot{y}(t)^2 + \dot{z}(t)^2})^3} dt \tag{5}$$

1.4 Normalization

While normalizing length cost function, we should divide it with the target length of each coils. That is,

$$ttlen = \sum_{i=1,Ncoils} \frac{1}{2} Lw_i \frac{(L_i - L_o^i)^2}{{L_o^i}^2} \tag{6}$$

Since the target lengths are user specified constant, the normalization of length cost function derivatives can be implemented by dividing them with each coil's target length.