An Example "Reference Problem" for Code Verification

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2006 Transport Task Force Meeting
April 6, 2006

“Reference Problem”: Some Problem for Which We Know Solution with High Degree of Confidence

- **Recall that:**
  - Verification is ensuring code solves equations correctly.
  - Do this with series of tests.
  - Focus on testing against highly accurate solution – a “reference problem”.
- **How accurate:**
  - Ideally, exact analytic solutions,
    - But, these are rare & test only some code capabilities.
  - $\Rightarrow$ Also consider semi-analytic or simulated solutions.
  - Viable simulated solutions:
    - Codes based on more fundamental technique,
    - High accuracy (“hero”) runs,
    - Results confirmed by more than one independent code.

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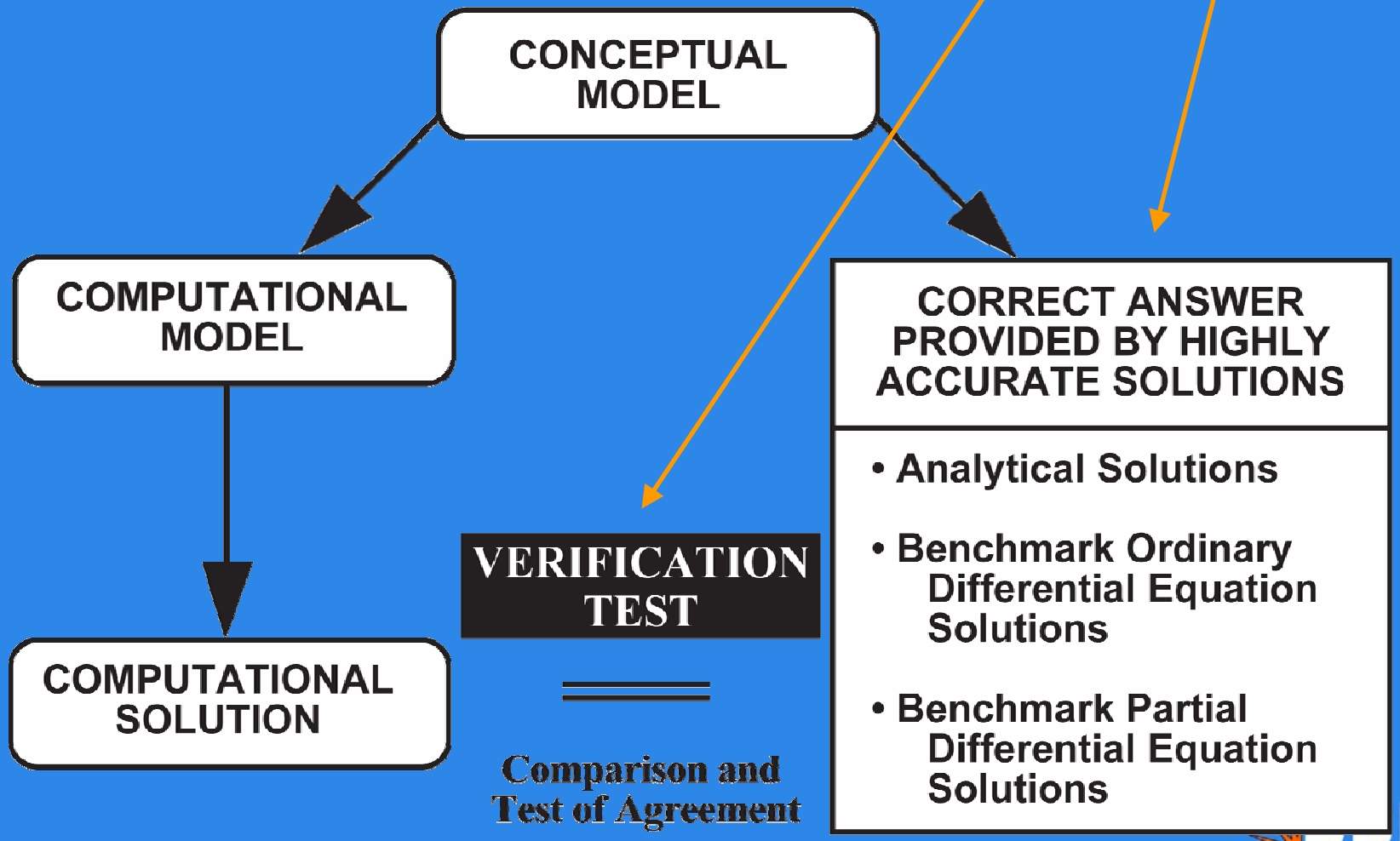
Note Distinction Between "Reference Problem" & "Verification Test"



W. L. Oberkampf & T. G. Trucano, Prog. Aero. Sci. 38 (2002) 209.

Edge Coordinating Committee Assembling Set of Edge Relevant Reference Problems

- **Many examples of verification exist in our community,**
  - But, rarely documented.
- **Gather underlying "reference problems" so can be shared by community.**
- **First goal: set standards for "reference problems".**

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Reference Problem Description Consists of Three Sections

1. **Summary**
   - Keyword-value pairs characterizing problem,
   - Could be stored in database.
2. **Detailed Description**
   - Bulk of the document.
3. **Model Results**
   - Suitable for both human & code consumption.

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1. Summary

- **Type of problem**
  - E.g., kinetic neutral transport, fluid plasma transport, MHD, …
- **Physics or algorithm stressed**
  - What aspect of a code's physics or numerics would be tested by doing a verification test with this problem?
- **Dimensionality & temporal variation**
  - 1-D or 3-D? Time varying or steady-state?
- **Type of solution (one of):**
  1. Analytic solutions
  2. Semi-analytic solutions (quadrature or ODE)
  3. Manufactured solution
  4. Simulation result confirmed by two or more independent codes
  5. Simulation result from one code
- **Relevance**

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2. Detailed Description

- **Equation(s) solved**
- **Specific Details**
  - Geometry,
  - Boundary conditions,
  - Sources / sinks,
  - Other pertinent details.
- **Description of Solution Method (one of):**
  - Derivation of analytic solution,
  - Derivation of semi-analytic solution and description of technique used to arrive at final result,
  - Derivation of manufactured solution,
  - Description of or reference to code used to compute simulated result & details pertaining to that run.

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3. Model Results

- **Include data needed as input for reference problem.**
  - As well as output data corresponding to its solution,
  - Non-exact solutions should also have error estimates for output data.
- **Establishing widely acceptable & effective format a priori is difficult,**
  - $\Rightarrow$ Will examine format(s) used in initial set of test problems for insight into possible standards.
- **Possibilities:**
  - Plots (1-D, 2-D, 3-D),
  - Tabular text (suitable for use with commercial packages),
  - Self-describing, device independent formats (netCDF, HDF),
  - MDS+,
  - Else?

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Example: *Fluid Neutral Momentum Transport Reference Problem*

1. Summary

Type of problem: kinetic or fluid neutral transport

Physics or algorithm stressed: thermal force term (spatial resolution) in momentum transport equations & treatment of collisions (charge exchange)

Dimensionality & temporal variation: 1-D in space; steady-state

Type of solution: semi-analytic

Relevance: Divertor plasmas cool & dense enough for atoms to be in or near fluid regime.

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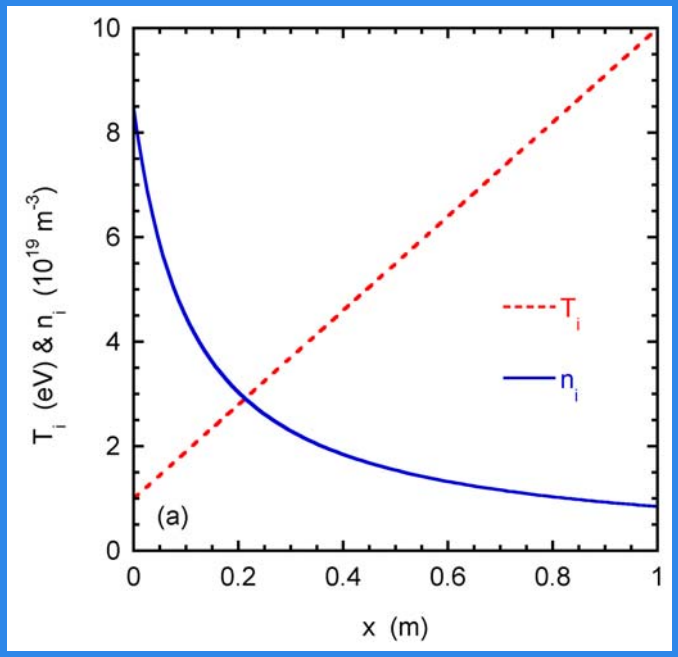
2. Detailed Description

- **Equations**
  - $T = T_n = T_i$
- **Geometry: L = 1 m box,**
- **Plasma parameters:**
  - T_i linear, $T_i(0) = 1$ eV, $T_i(L) = 10$ eV,
  - $n_i T_i$ = constant = 8.5×10^{19} eV m^{-3}.
- **Constant CX cross section**
  - $\sigma_{cx} = 4 \times 10^{-19}$ m^2.
- **Boundary conditions:**
  - x = 0 is neutral entrance,
  - x = L is exit (sink).

$$\frac{d}{dx}(Nv) = 0$$

$$\frac{d}{dx}\left(mNv^2 + NT\right) = \alpha_T N \frac{dT}{dx} - m\nu_{cx} N v$$



Non-Fluid Boundary Condition Requires Approximate Solution

- **Have Nv = constant,**
- **Method 1: Simple fluid**
  - $N(L) = 0$, $mv^2 << T$
  - Can reduce equation to quadrature for $N_1(x)/N_1(0)$.
- **Method 2: Extended fluid**
  - Write $v(L) = \gamma[T(L)/m]^{1/2}$,
    - $\gamma \sim 1$ & $\gamma < 1$.
  - Yields simple ODE for $N_2(x)/N_2(L)$,
  - Integrated with LSODE.

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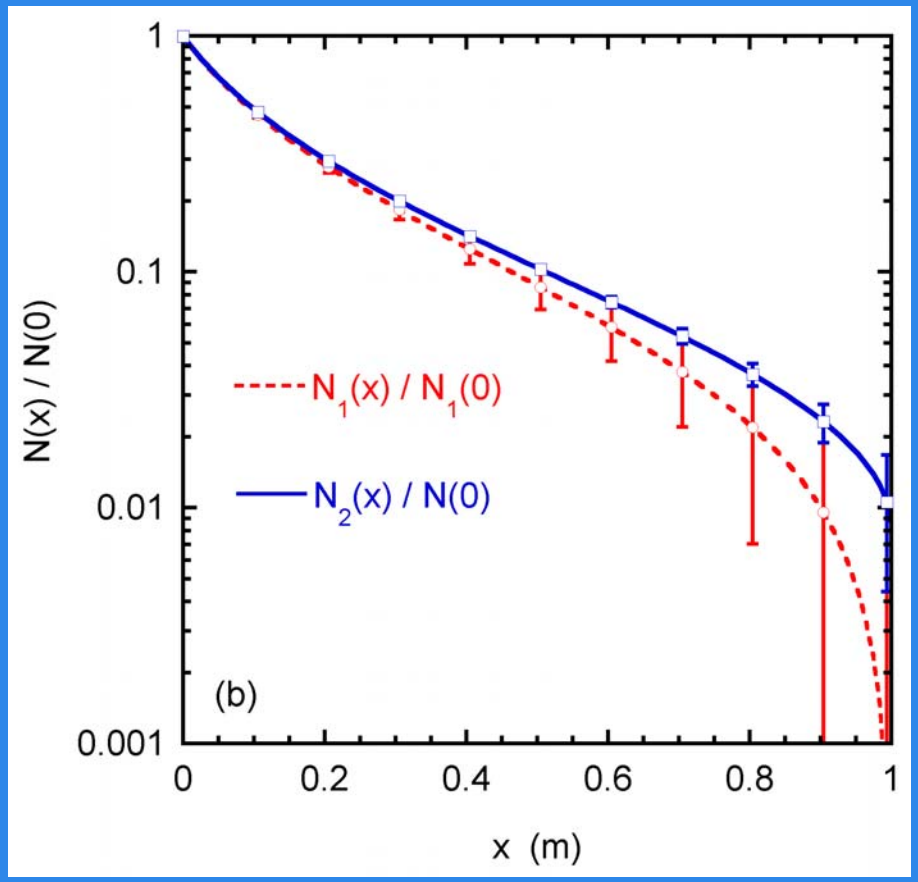
Model Results

- **Value of γ undetermined,**
  - Using $1/\gamma = 1.01$ here.
  - From solution behavior, $1 < 1/\gamma < 2$,
  - $\Rightarrow$ Use this range to set error bars.
- **Error bar for $N_1(x)$ is just difference with $N_2(x)$.**
- **"Reference problem" also comes with tabular text containing $n_i(x)$, $T_i(x)$, $N_1(x)/N_1(0)$, $N_2(x)/N(0)$, plus error bars.**