Study of Transverse Dipole and Quadrupole Modes in a Pure Ion Plasma in a Linear Paul Trap to Study Beam Stability

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Greifswald

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The Paul Trap Simulator Experiment (PTSX) Simulates Nonlinear Beam Dynamics in Magnetic Alternating-Gradient Systems

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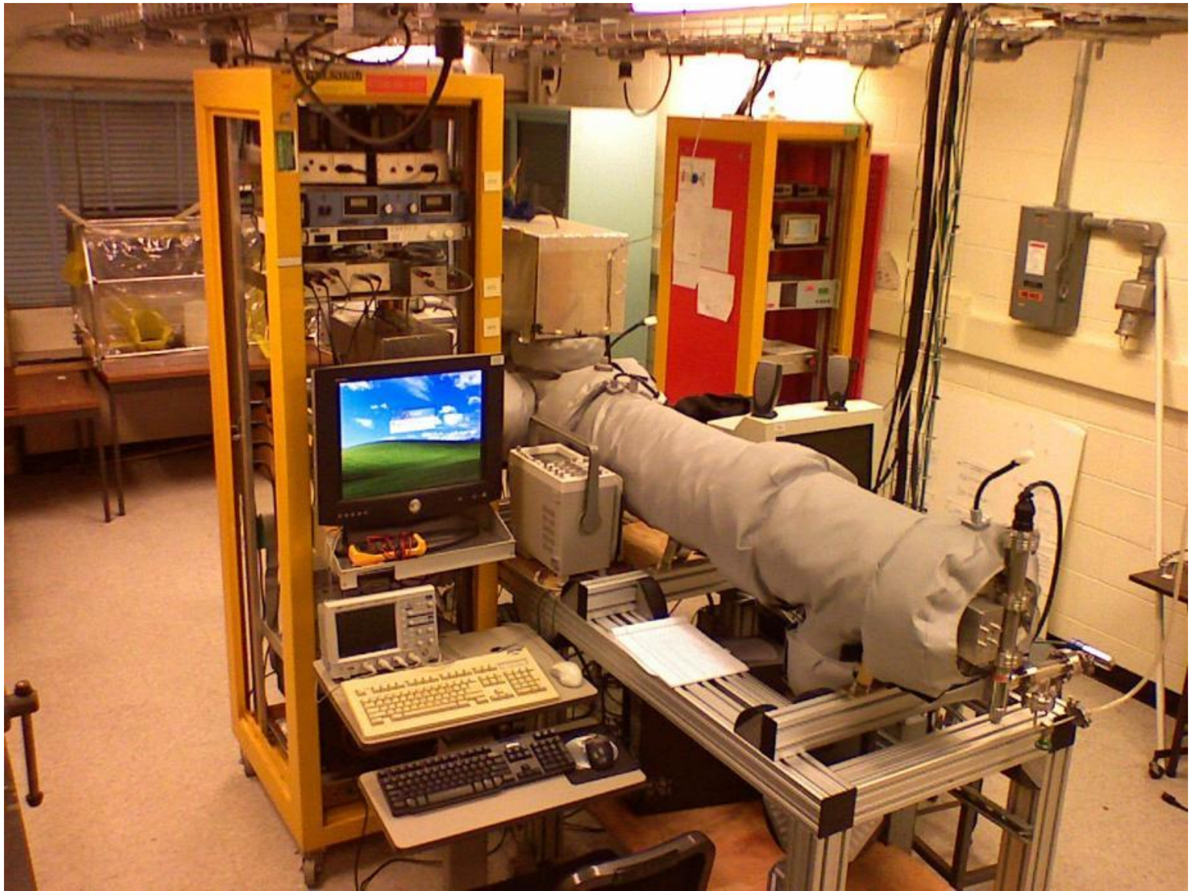
- Purpose: PTSX simulates, in a compact experiment, the transverse nonlinear dynamics of intense beam propagation over large distances through magnetic alternating-gradient transport systems.
- Applications: Accelerator systems for high energy and nuclear physics applications, heavy ion fusion, spallation neutron sources, and high energy density physics.

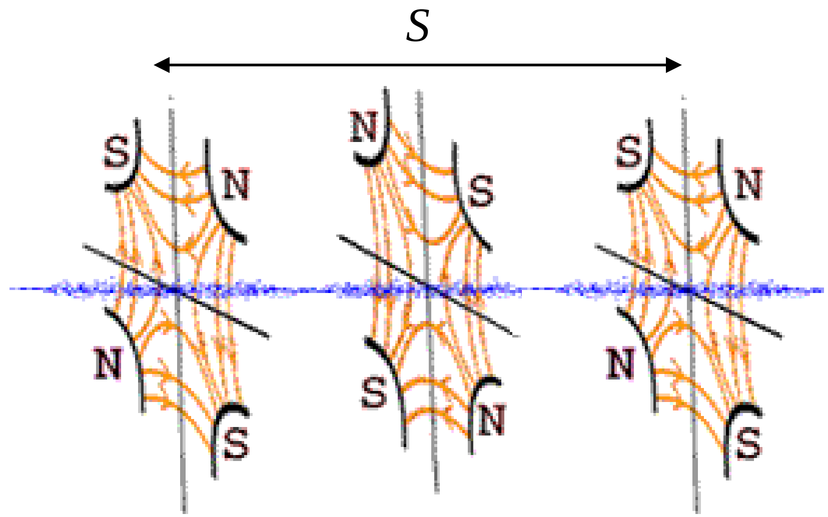


See also…

M. Drewsen
Aarhus

H. Okamoto
Hiroshima

Alternating-Gradient Transport Systems Use a Spatially Periodic Lattice of Quadrupole Magnets for Transverse Confinement

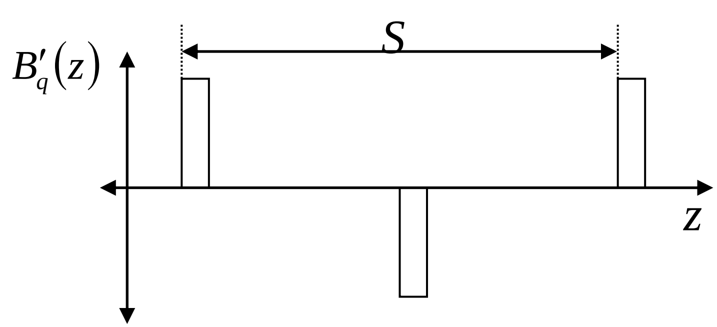


Focusing-Off-Defocusing-Off (FODO) Lattice

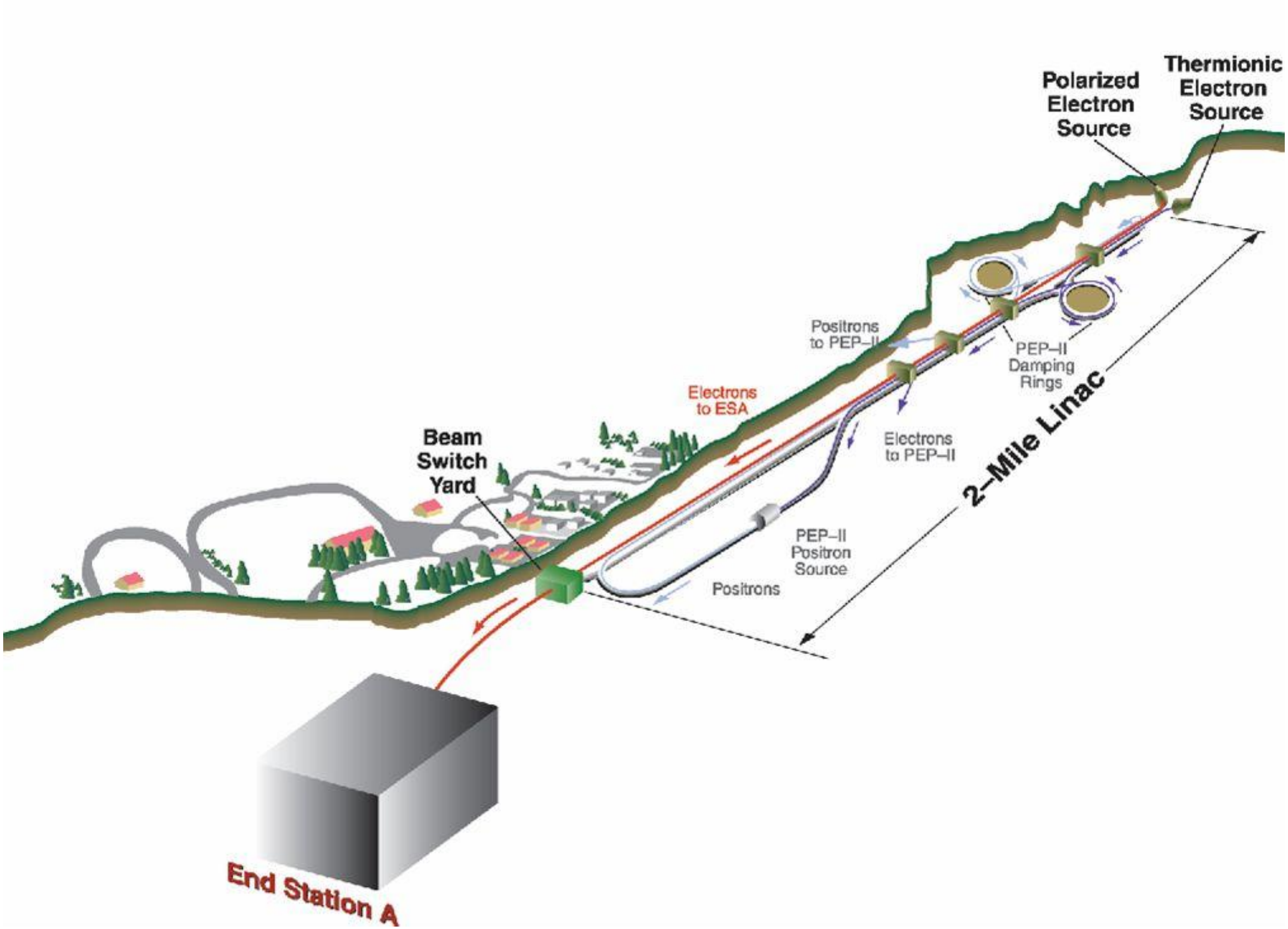
$$\boxed{\begin{aligned} \boldsymbol{B}_q^{foc}(\boldsymbol{x}) &= B_q'(z)\left(y\hat{\boldsymbol{e}}_x + x\hat{\boldsymbol{e}}_y\right) \\ \boldsymbol{F}_{foc}(\boldsymbol{x}) &= -\kappa_q(z)\left(x\hat{\boldsymbol{e}}_x - y\hat{\boldsymbol{e}}_y\right) \end{aligned}}$$

$$\boxed{\kappa_q(z) \equiv \frac{ZeB_q'(z)}{\gamma m \beta c^2}}$$

Example of FODO lattice

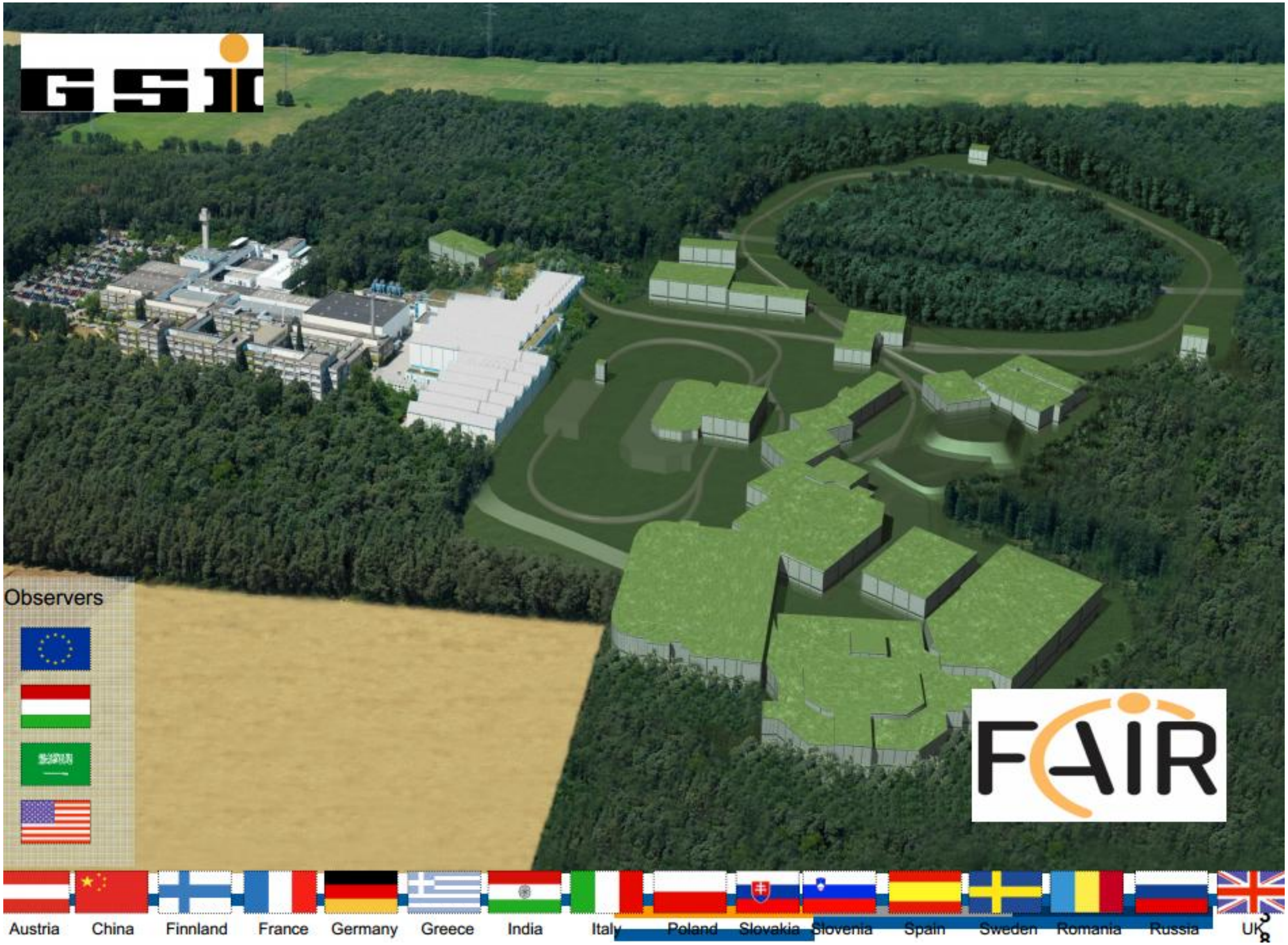


SLAC – ~ 3 km Length With About 3000 Lattice Periods

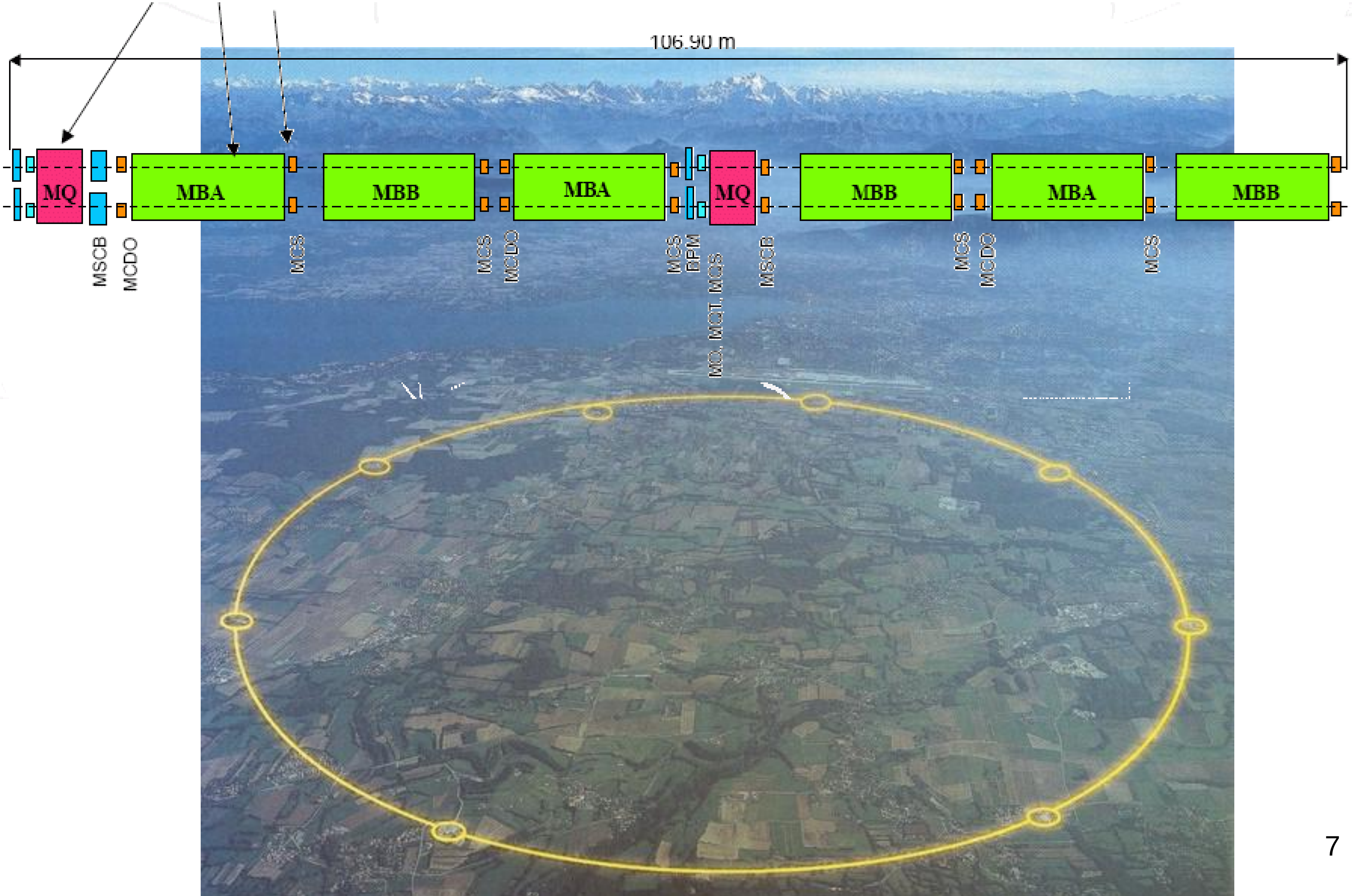


Spallation Neutron Source (SNS) Ring – 248 m Circumference With About 24 Lattice Periods

FAIR at GSI – 1 km Circumference With About 80 Lattice Periods

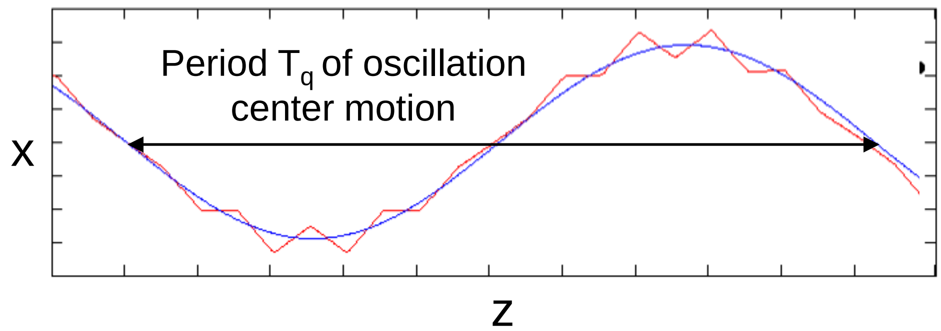


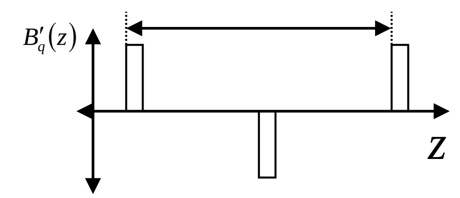
LHC – 27 km Circumference With About 200 Lattice Periods



Average Transverse Focusing Frequency and Phase Advance Characterize the Motion – Emittance is a Measure of Beam Quality

$\kappa_q(z)$: FODO lattice

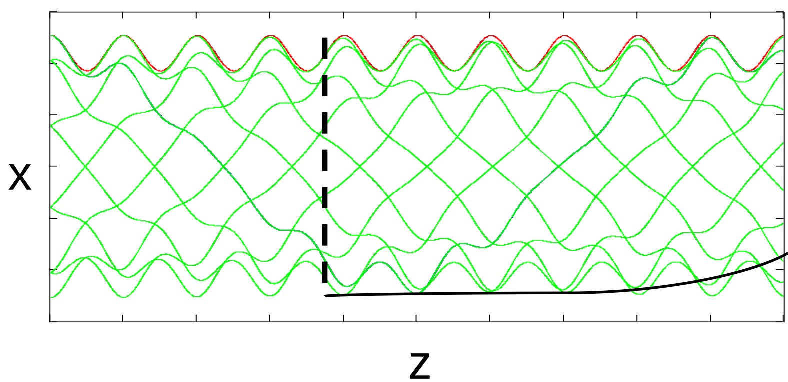




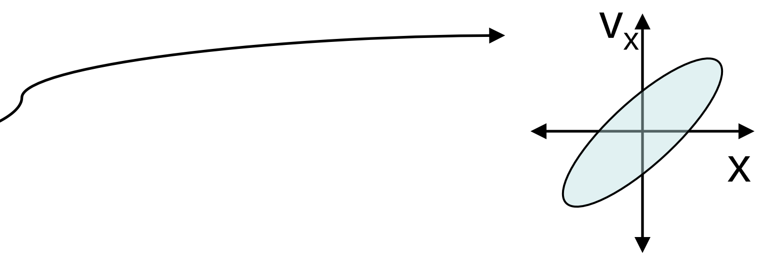
$\omega_q = 2\pi/T_q$ is the average transverse focusing frequency

$\sigma_v = \omega_q/f$ Here, the vacuum phase advance, σ_v, is 35°.

$\kappa_q(z)$: sinusoidal lattice



The (x,v_x) values of particles at the location of the dotted line gives an ellipse in phase space.



KV envelope equation $x'' + \kappa(z)x - \dfrac{2N}{x+y} - \dfrac{\varepsilon}{x^3} = 0$

Emittance ε is the phase space area of the beam and scales as $R_b\,(kT)^{1/2}$

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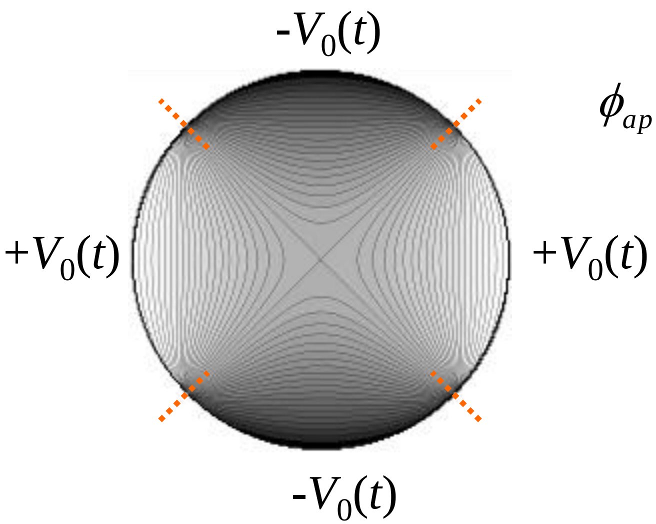
PTSX is a Cylindrical Paul Trap



$$\phi_{ap}(x,y,t) = \frac{4V_0(t)}{\pi} \sum_{\ell=1}^{\infty} \frac{\sin(\ell\pi/2)}{\ell} \left(\frac{r}{r_w}\right)^{2\ell} \cos(2\ell\theta)$$

$$e_b\phi_{ap}(x,y,t) = \frac{1}{2}\kappa_q(t)(x^2 - y^2)$$

$$\kappa_q(t) = \frac{8e_b V_0(t)}{m_b \pi\, r_w^2}$$

$$V_0(t) = V_{0\ \max} \sin(\omega t)$$

The ponderomotive force…

$$\overrightarrow{F_p} = -\frac{\omega_p^2}{\omega^2}\vec{\nabla}\frac{\langle \varepsilon_0 E^2 \rangle}{2}$$

…can be written as…

$$\vec{F}_p = -m_b \omega_q^2 \vec{r}$$

…where…

$$\omega_q = \frac{8e_b V_{0\max}}{m_b \pi r_w^2 f}\xi$$

…and

$$\xi = \frac{1}{2\sqrt{2}\pi}$$

$$\ddot{x} + 2q\cos(2t)x = 0$$

$$q = \frac{8eV_0}{m\pi^3 f^2 r_w^2} < 0.908$$

Analogy Between AG System and Paul Trap

$$\boldsymbol{B}_q^{foc}(\boldsymbol{x}) = B_q'(z)\left(y\hat{\boldsymbol{e}}_x + x\hat{\boldsymbol{e}}_y\right)$$
$$\boldsymbol{F}_{foc}(\boldsymbol{x}) = -\kappa_q(z)\left(x\hat{\boldsymbol{e}}_x - y\hat{\boldsymbol{e}}_y\right)$$

Quadrupolar Focusing

$$e\phi_{ap}(x,y,t) = \frac{1}{2} m\kappa_q'(t)(x^2 - y^2)$$

$$\kappa_q(z) = \frac{ZeB_q'(z)}{\gamma m\beta c^2}$$

$$\kappa_q'(t) = \frac{8eV_0(t)}{m\pi\, r_w^2}$$

$$\psi = \frac{Ze}{\gamma m\beta^2 c^2}\left[\phi(x,y,s) - \beta\, A_z(x,y,s)\right]$$

Self-Forces

usual $\phi_{\rm self}(x,y,t)$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\psi = -\frac{2\pi K}{N}\int dx'dy'\, f_b$$

Field Equations

Poisson's Equation

Vlasov Equation

$$\left\{\frac{\partial}{\partial s} + x'\frac{\partial}{\partial x} + y'\frac{\partial}{\partial y} - \left(\kappa_q(s)x + \frac{\partial\psi}{\partial x}\right)\frac{\partial}{\partial x'} - \left(-\kappa_q(s)y + \frac{\partial\psi}{\partial y}\right)\frac{\partial}{\partial y'}\right\} f_b = 0$$

The resulting ponderomotive force is a radial linear restoring force with characteristic frequency ω_q.

$$\omega_q = \frac{8eV_{0\,\max}}{m\pi r_w^2 f}\xi$$

$$\xi = \frac{1}{2\sqrt{2}\pi}$$ for a sinusoidal waveform $V(t)$.

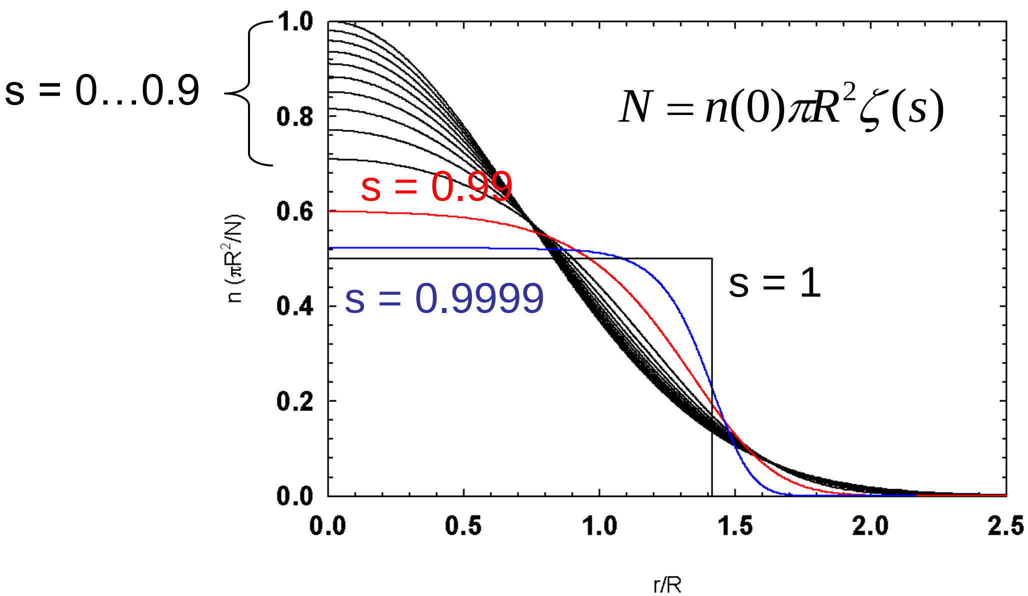
$$\xi = \frac{\eta\sqrt{3-2\eta}}{4\sqrt{3}}$$ for a periodic step function waveform $V(t)$ with fill factor η.

$$\sigma_v = \frac{\omega_q}{f} < \sigma_{v\max}$$

Smooth-Focusing Equilibria are Parameterized by *s*

In thermal equilibrium, $n(r) = n(0)\exp\left[-\frac{m\omega_q^2 r^2 + 2q\phi^s(r)}{2kT}\right]$

Poisson's equation $\frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial \phi^s}{\partial r} = \frac{qn(r)}{\varepsilon_0}$ becomes a nonlinear equation for ϕ^s that must be solved numerically.



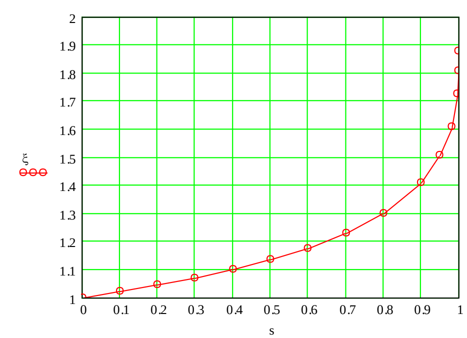
$$s \equiv \frac{\omega_p^2(0)}{2\omega_q^2} < 1$$

Normalized intensity parameter s.

s ~ 0.2 for SNS and Tevatron injector.

s ~ 0.99 for HIF

ζ(s) is determined numerically.
ζ(0) = 1
ζ(1) = 2



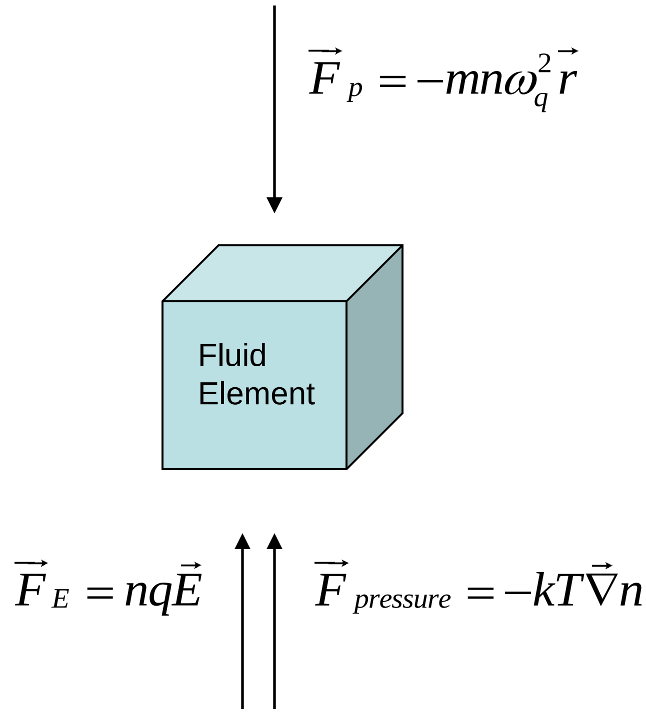
Global Energy Balance Provides a Method for Inferring Temperature From the Radial Profile

If $p = n\,kT$, then the statement of local force balance on a fluid element can be manipulated to give a global energy balance equation.

$$mn\omega_q^2 r = qnE - kT\frac{\partial n}{\partial r}$$

$$\int_0^\infty r^2 dr\left(mn\omega_q^2 r = qnE - kT\frac{\partial n}{\partial r}\right)$$

$$\boxed{m\omega_q^2 R^2 = \frac{Nq^2}{4\pi\varepsilon_o} + 2kT}$$



where R is the root-mean-squared radius and N is the line charge.

Note that N and R are measured by integrating the experimentally obtained density profiles $n(r)$.

kT is inferred from this global force balance.

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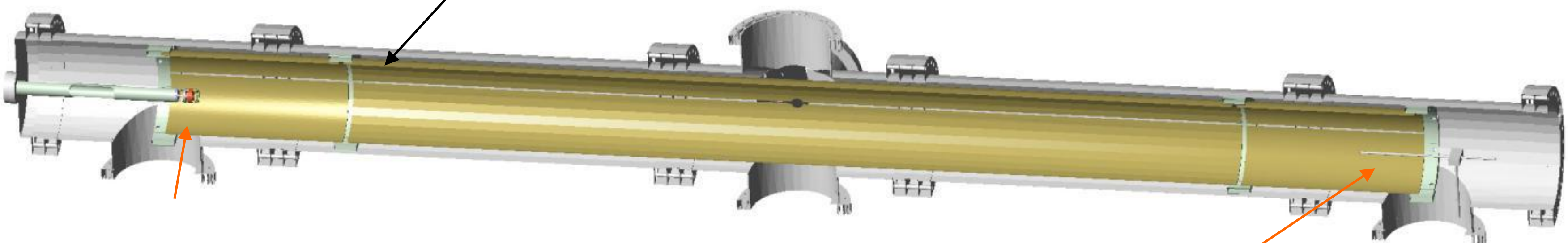
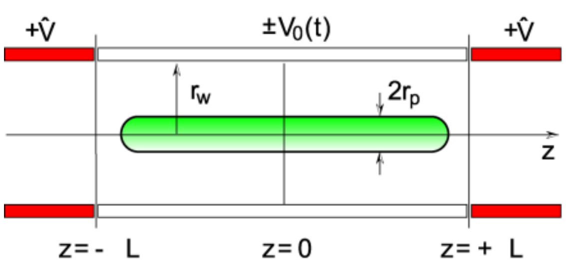
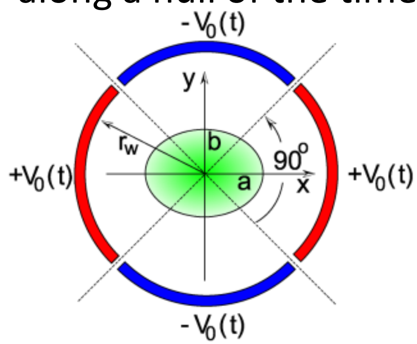
PTSX Apparatus



$$\boxed{e\phi_{ap}(x,y,t) = \frac{1}{2}\kappa_q'(t)(x^2 - y^2)}$$

The PTSX collector disk is a 5 mm diameter copper disk, held at ground, that is mounted to a linear motion feedthrough and moves along a null of the time-dependent oscillating potential $\pm V_0(t)$.





$$\boxed{\kappa_q'(t) = \frac{8eV_0(t)}{m\pi r_w^2}} \qquad \boxed{\omega_q = \frac{8eV_{0\,\max}}{m\pi r_w^2 f}\xi}$$

Plasma length	2 m	Wall voltage	140 V
Wall radius	10 cm	End electrode voltage	20 V
Plasma radius	~ 1 cm	Frequency	60 kHz
Cesium ion mass	133 amu	Pressure	$5\text{x}10^{-10}$ Torr
Ion source grid voltages	< 10 V	Trapping time	100 ms

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Studies of Beam Modes Begin with Expressions for Several Modes

Dipole mode: $$\omega_B = \omega_q$$

Breathing mode:
$$\omega_B = 2\omega_q \left(1 - \frac{1}{2}\widehat{s}\right)^{1/2}$$

Quadrupole mode:
$$\omega_Q = 2\omega_q \left(1 - \frac{3}{4}\widehat{s}\right)^{1/2}$$

$$\ddot{x} + \omega_q^2 x - \frac{N}{x+y} - \frac{\varepsilon}{x^3} = 0$$

$$\ddot{y} + \omega_q^2 y - \frac{N}{x+y} - \frac{\varepsilon}{y^3} = 0$$

Using KV distribution and smooth focusing approximation.

$$\hat{s} = \frac{\omega_p^2}{2\omega_q^2}$$

$$s \sim 0.23$$

$$f_0 = 60kHz$$

$$\sigma_v \sim 48.6\deg$$

$$2f_q = \frac{2\omega_q}{2\pi} \sim 16.08kHz$$

$$f_B = \frac{\omega_B}{2\pi} \sim 15.13kHz$$

$$f_Q = \frac{\omega_Q}{2\pi} \sim 14.63kHz$$

The Modes Can Be Excited Using the Beat Frequency Between f_0 and f_1

- Sum-of-sines is applied to arbitrary function generator

$$V(t) = \widehat{V}_0 \sin(2\pi f_0 t) + \delta V \sin(2\pi f_1 t)$$

where f_1 is near $f_0 \pm f_{mode}$

- Typical Operating Parameters

$\widehat{V}_0 \sim 140\text{ V}$

$\delta V \sim 0.7\text{ V } (0.5\% \widehat{V}_0)$

$f_0 \sim 60\text{ kHz}$

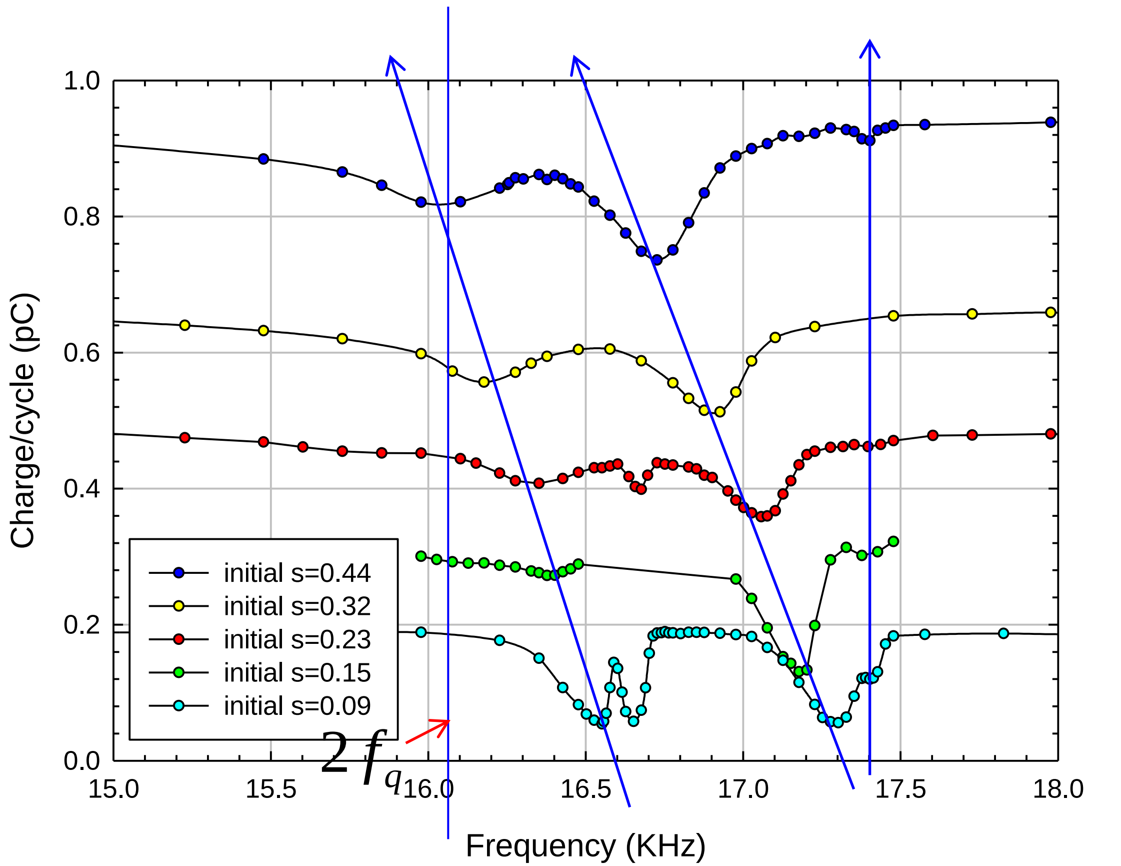
$f_1 \sim \text{varying}$

$2 f_q \sim 16.083\text{ kHz}$

$t_{perturbation} = 30\text{ ms}$

Beat-Method Frequency Scans With Different Initial Amounts of Space-Charge Attempt to Find the Space-Charge Dependence

Change of on-axis density under different perturbation amplitudes



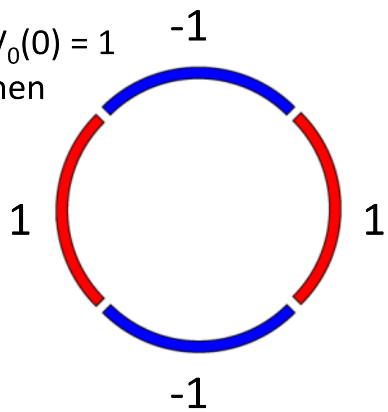
30ms, 1.0% perturbation

s increases

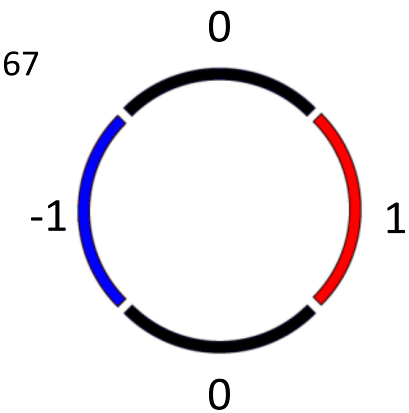
Using a Second Arbitrary Function Generator to Break the Quadrupole Symmetry

$$\phi(r,\theta) = \sum_n C_n \left(\frac{r}{r_w}\right)^n \cos(n\theta) \qquad A_n = \frac{1}{4}\int_0^{2\pi} V(\theta)\cos(n\theta)$$

At t = 0, with $V_0(0) = 1$
(1, -1, 1, -1), then
$A_2 = 1$
$A_6 = 0.333$

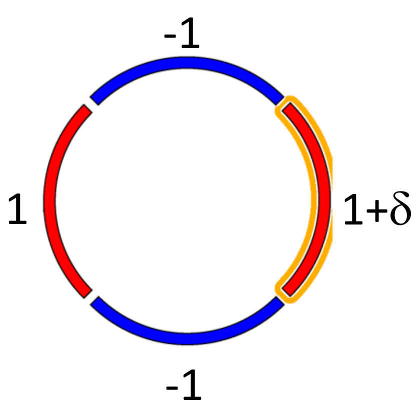


If a dipole is applied as (1, 0, -1, 0), then
$A_1 = 0.5$
$A_3 = 0.167$
$A_5 = 0.1$



A perturbation (1+δ, -1, 1, -1), can be decomposed as
(1+δ/4, -1-δ/4, 1+δ/4, -1-δ/4) + (δ/2, 0, -δ/2, 0) + (δ/4, δ/4, δ/4, δ/4) and then

$A_0 = \delta\pi/8$
$A_1 = \delta/4$
$A_2 = 1+\delta/4$
$A_3 = \delta/12$
$A_5 = \delta/20$
$A_6 = 1/3 + \delta/12$



The higher-order terms are less significant because the contribution of each term is proportional to $(r/r_w)^n A_n$.

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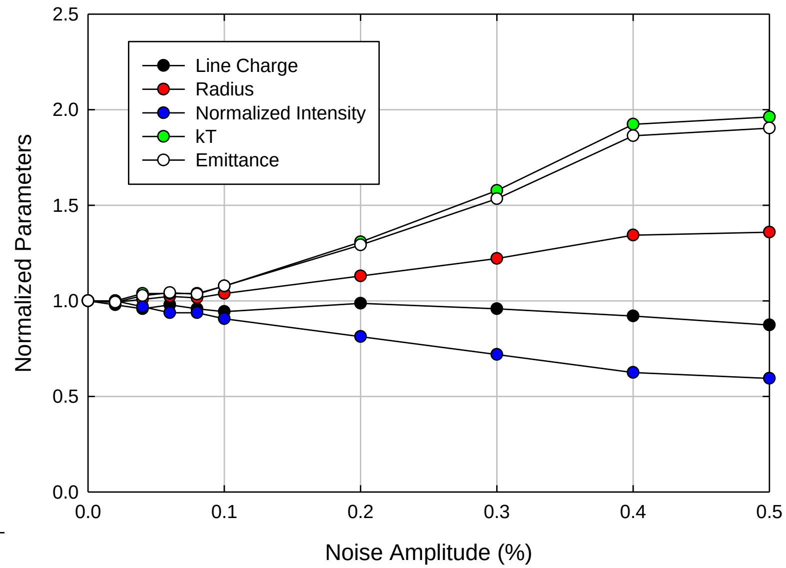
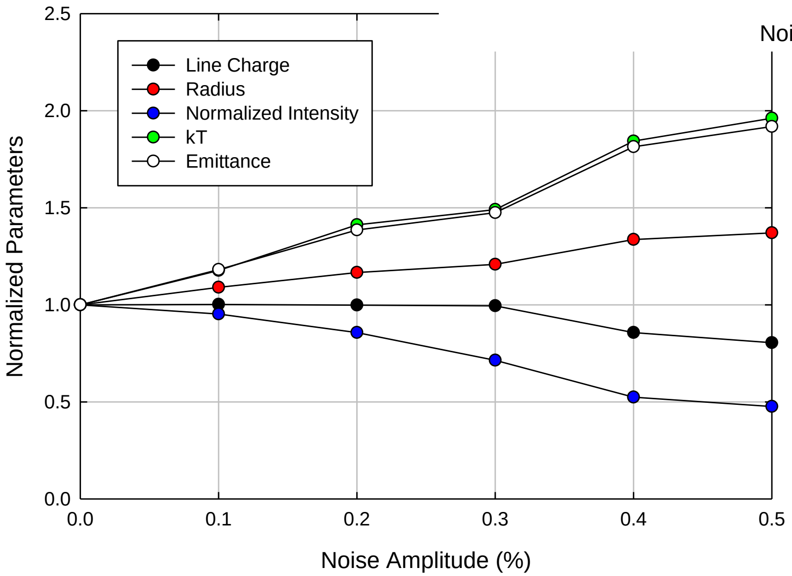
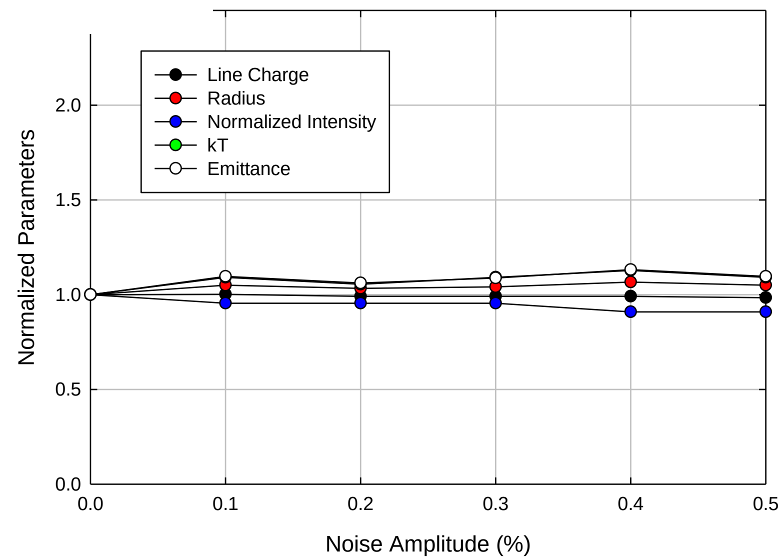
As Expected the Dipole Noise has a Larger Effect Than the Quadrupole Noise

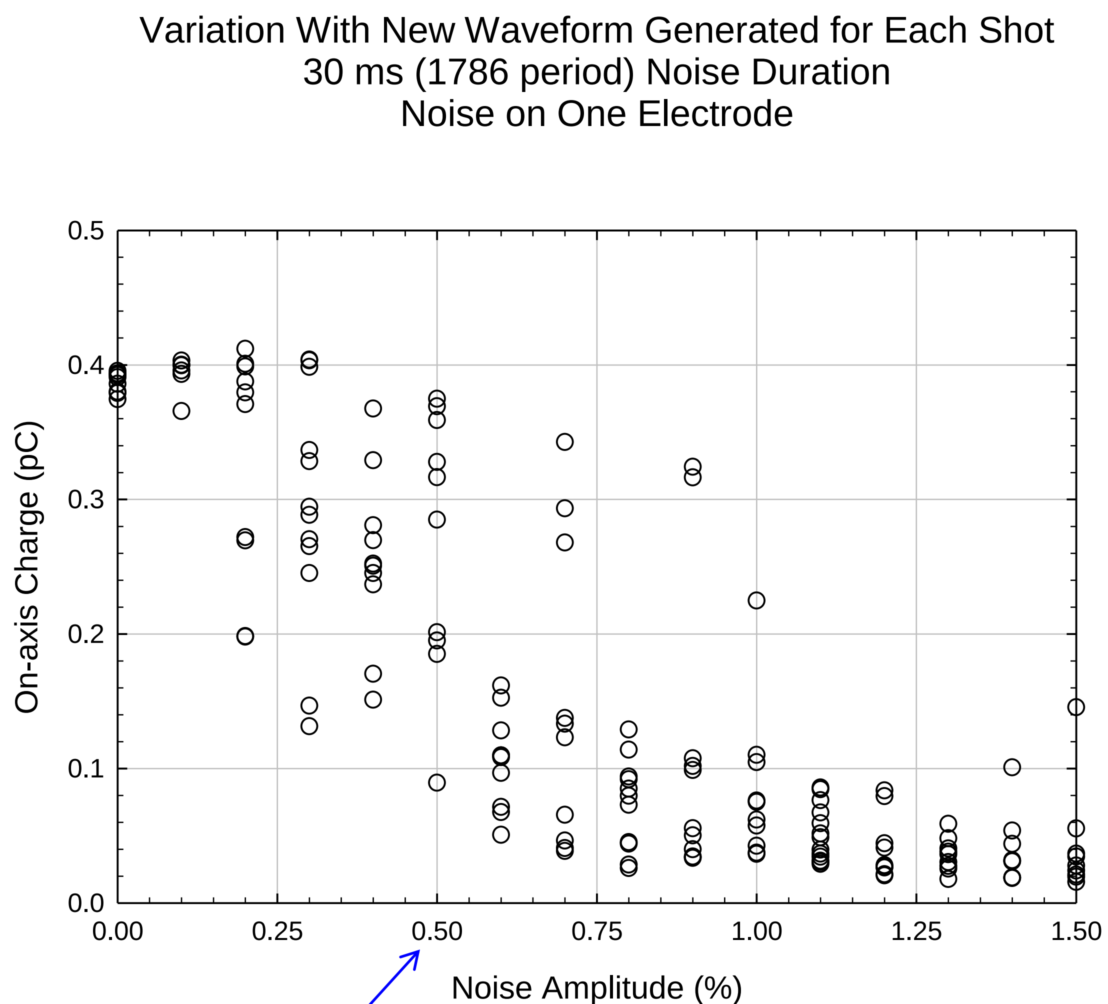






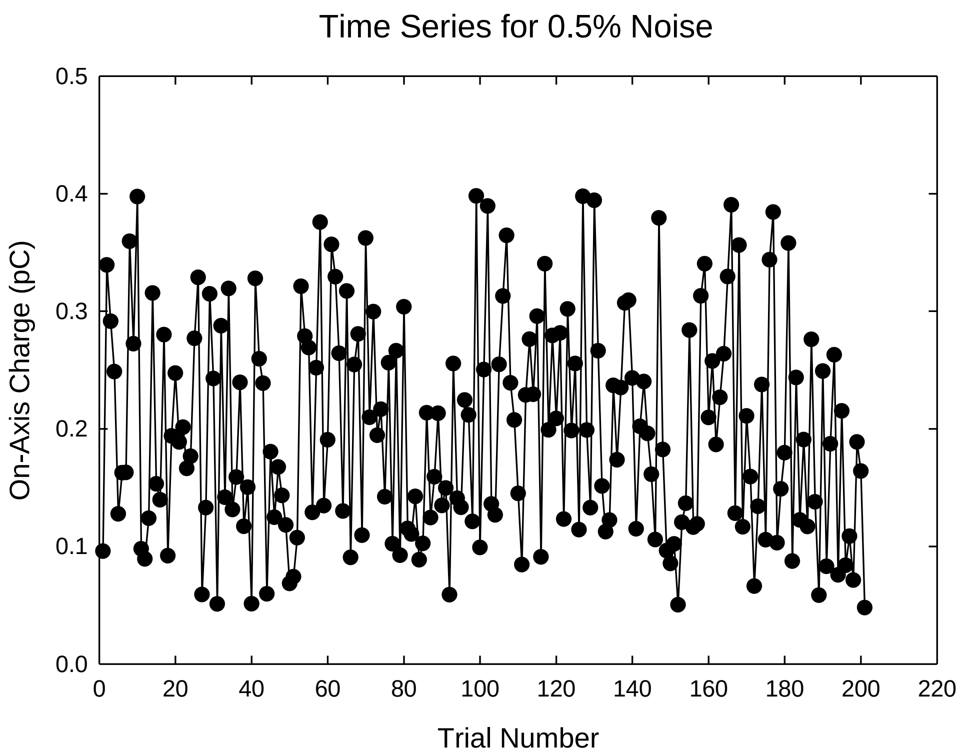
Using the arbitrary function generators, the dipole noise and the quadrupole noise can be considered separately.

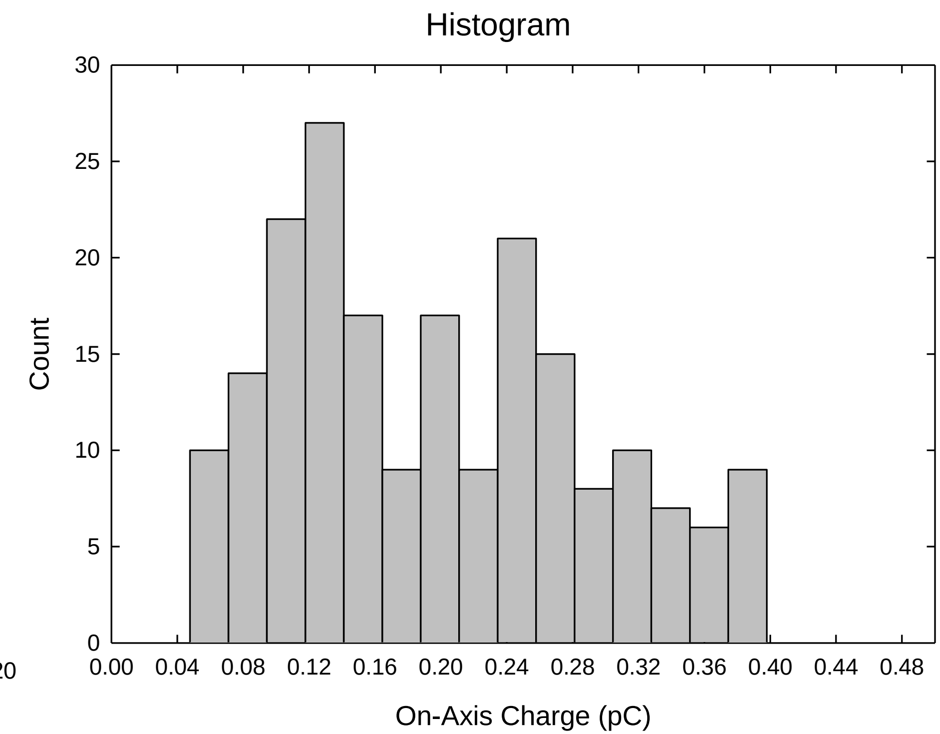
Investigating the Statistical Nature of Noise Applied to One Electrode



Focus on 0.50% noise amplitude on next slide…

Time Series and Histogram of On-axis Charge Measurements for 200 Sets of Random Numbers





As before, this is a predominantly the result of the dipole perturbation.

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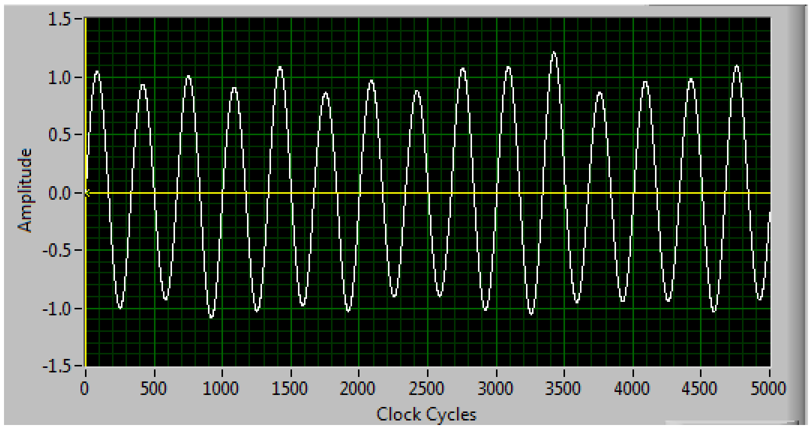
Fix the Waveform and Manipulate the Spectrum to See How the Noise Acts By Coupling to the Modes

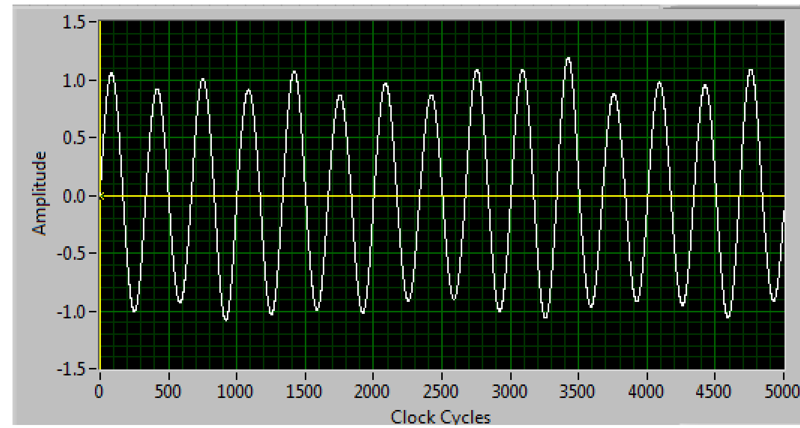
Example with 10% noise

Before



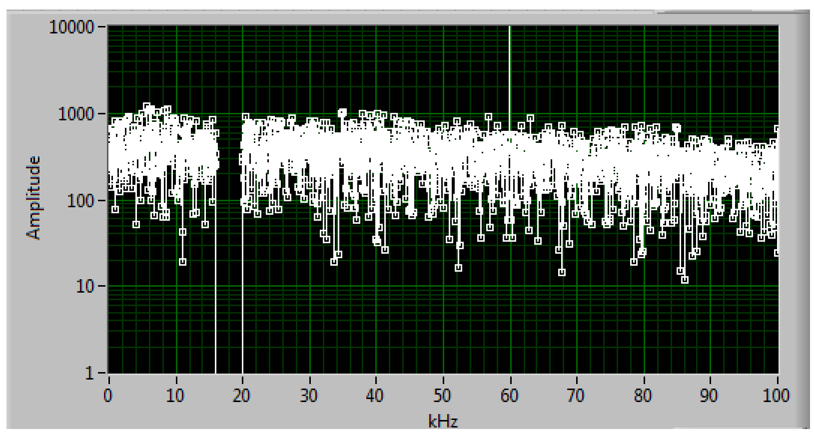
First 15 of 1000 periods

After



First 15 of 1000 periods

Manipulate

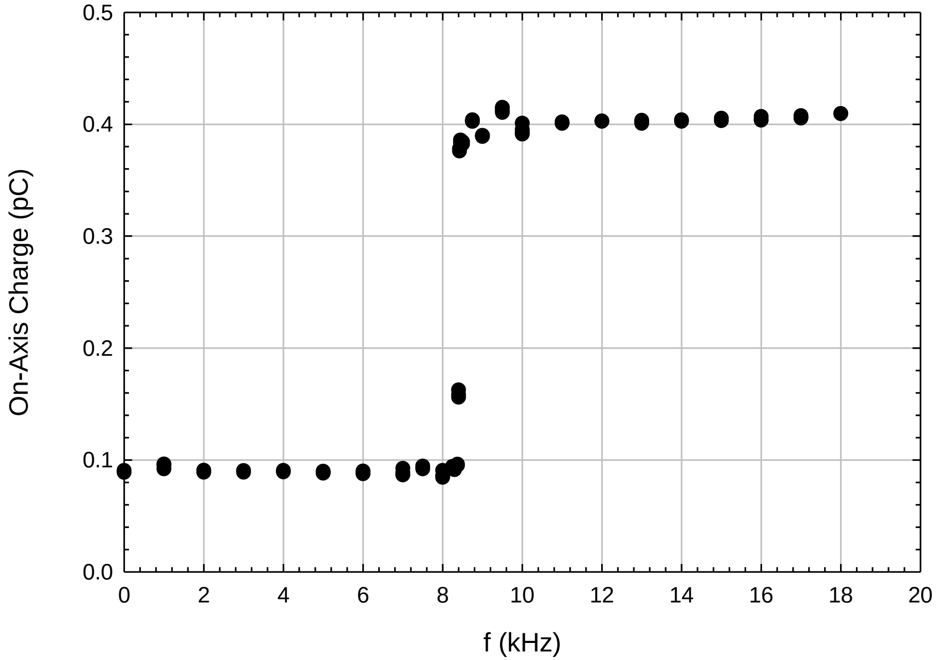


First 100 kHz of the FFT of the Waveform

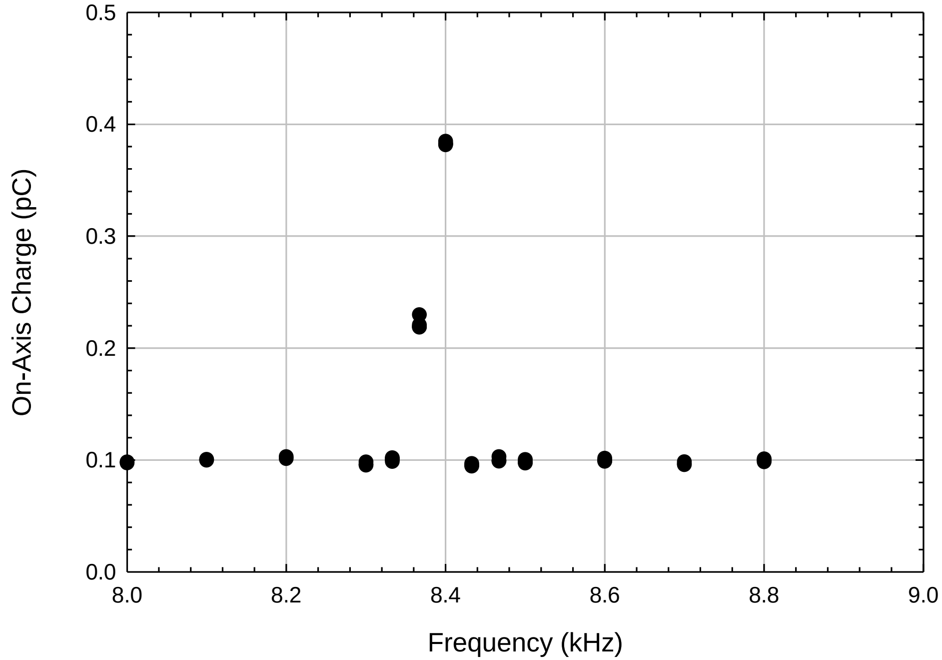
1786 lattice periods = 30 ms → 33.3 Hz resolution
20 MHz clock → 10 MHz maximum frequency

Noise Applied to One Electrode Damages the Beam Through Its Interaction with the ℓ = 1 Dipole Mode

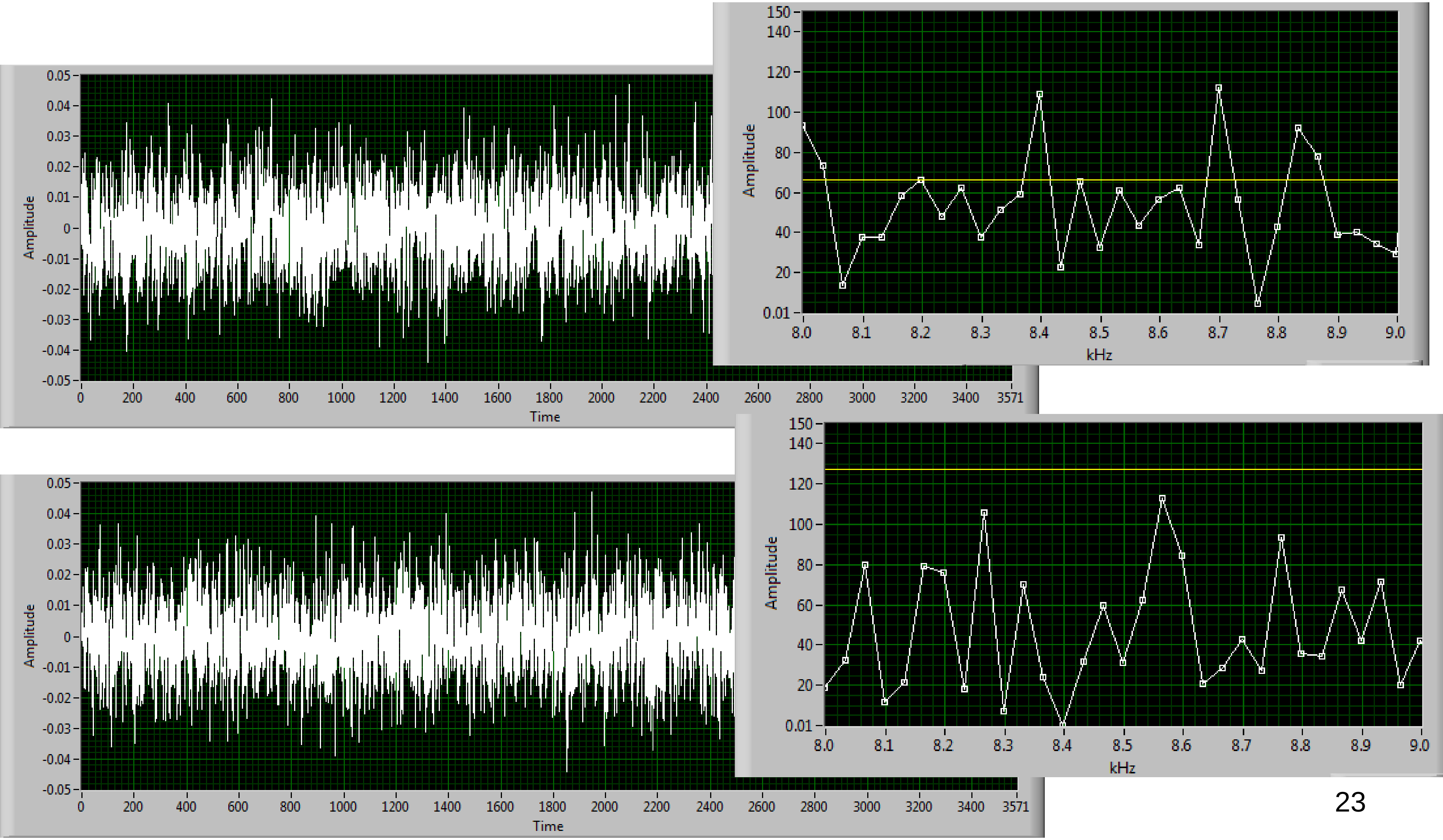
One Set of 0.5% Noise Applied to One Electrode
A Notch Filter Removes Frequencies from Zero to f kHz



One Set of 0.5% Noise Applied to One Electrode
A Notch Filter Removes One Frequency Component



The Lattice Can Be Reordered to Remove the Component at the Mode Frequency



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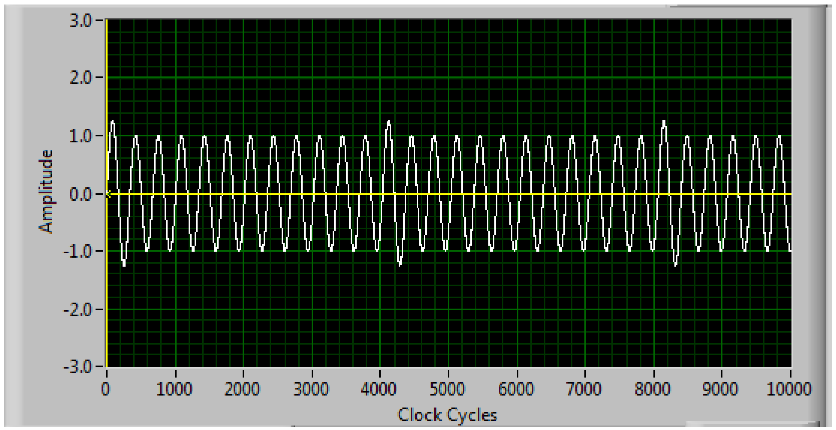
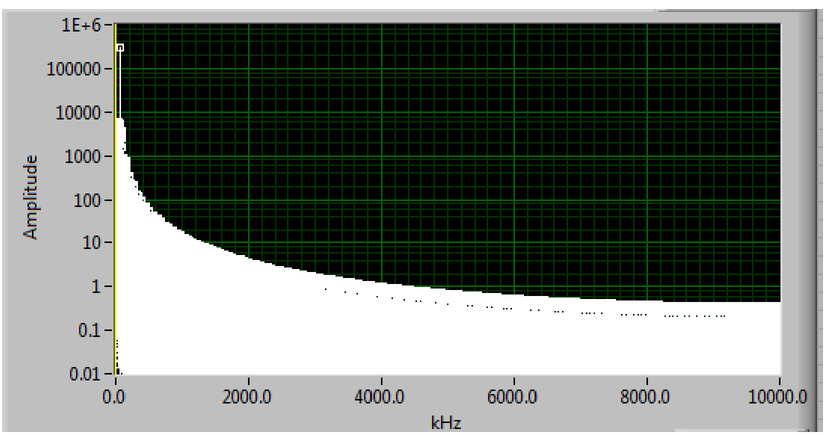
Resonance Between Beam Modes and Periodic Lattice Errors in a Ring

Example: ring period N = 12

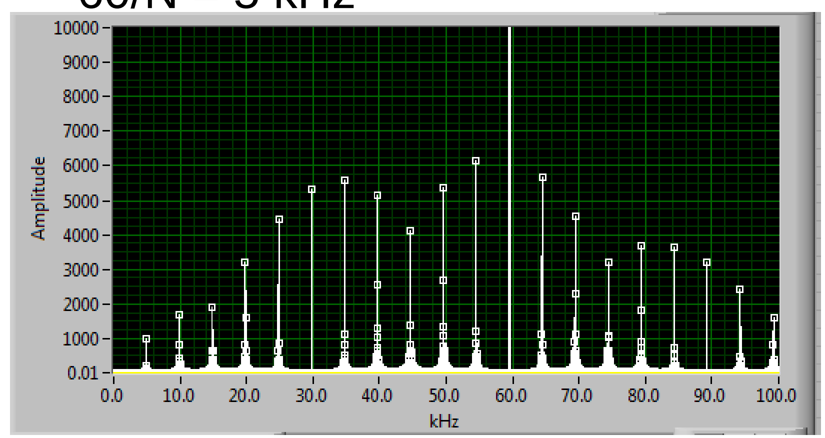


Tune $\nu = f_q/f_{ring}$

Fourier spectrum



First 100 kHz of Fourier spectrum. Waveform made of multiples of 60/N = 5 kHz

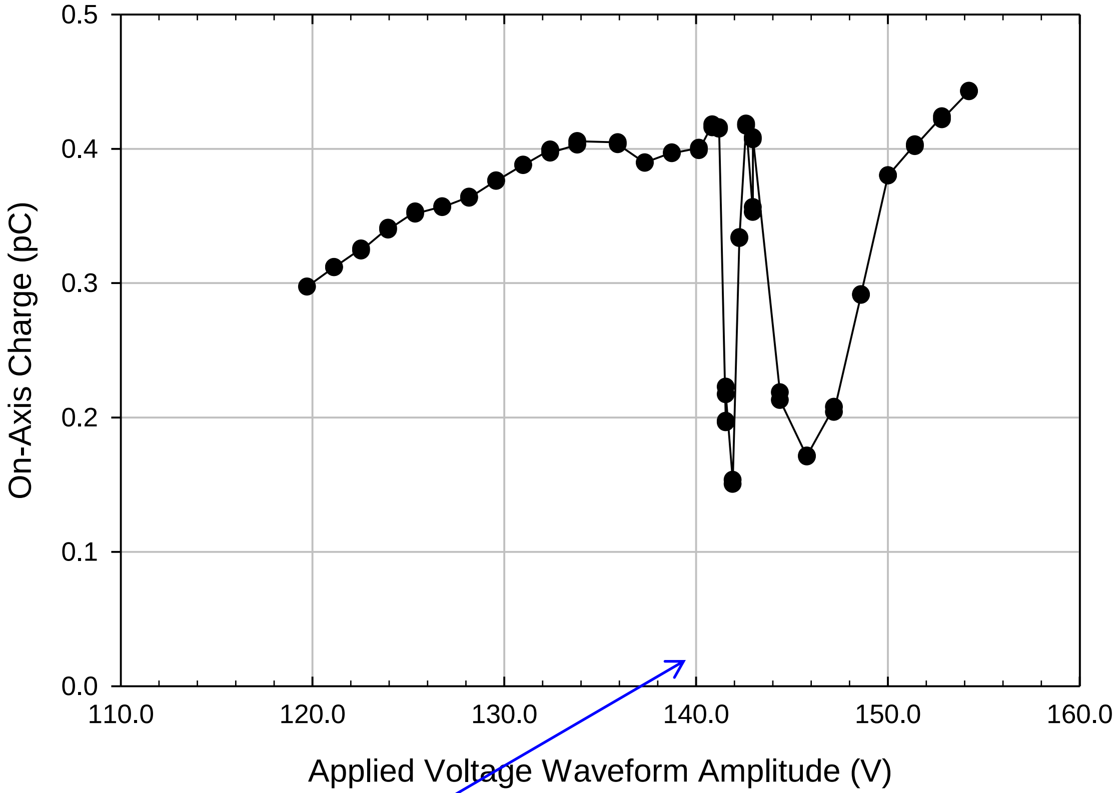


PTSX Usually Operates at 48 Degree Smooth Focusing Vacuum Phase Advance

$$\omega_q = \frac{8 e_b V_{0\,\max}}{m_b \pi r_w^2 f} \xi$$

$$\sigma_v = \omega_q / f$$

2% Amplitude Increase Every 7 Periods
($2\pi/7$ = 51.4 deg.)

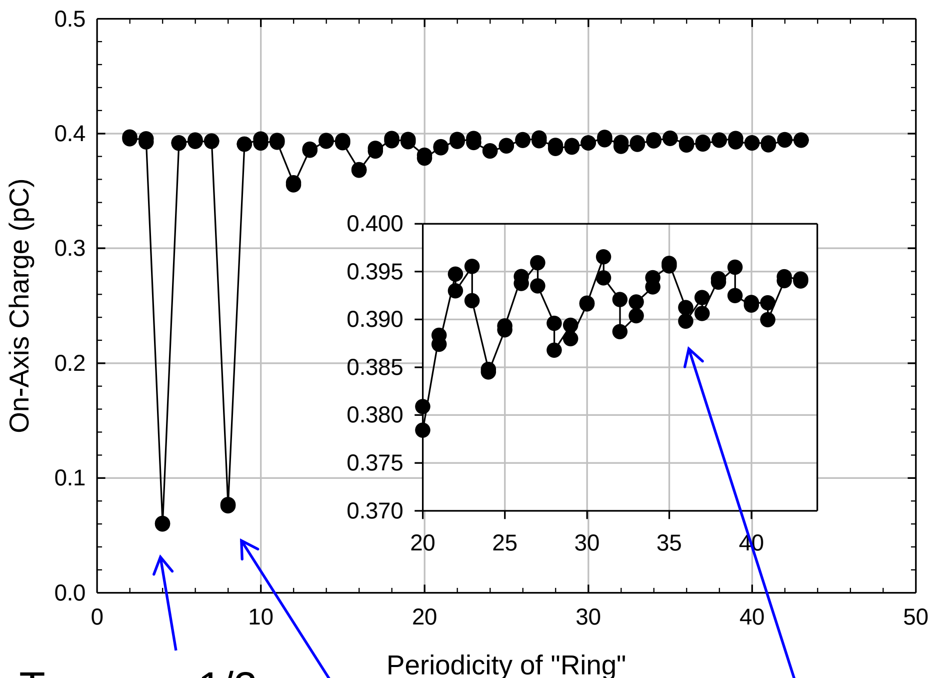


Normal Operating Point
V_0 = 140 V

Half Integer Resonances are Seen

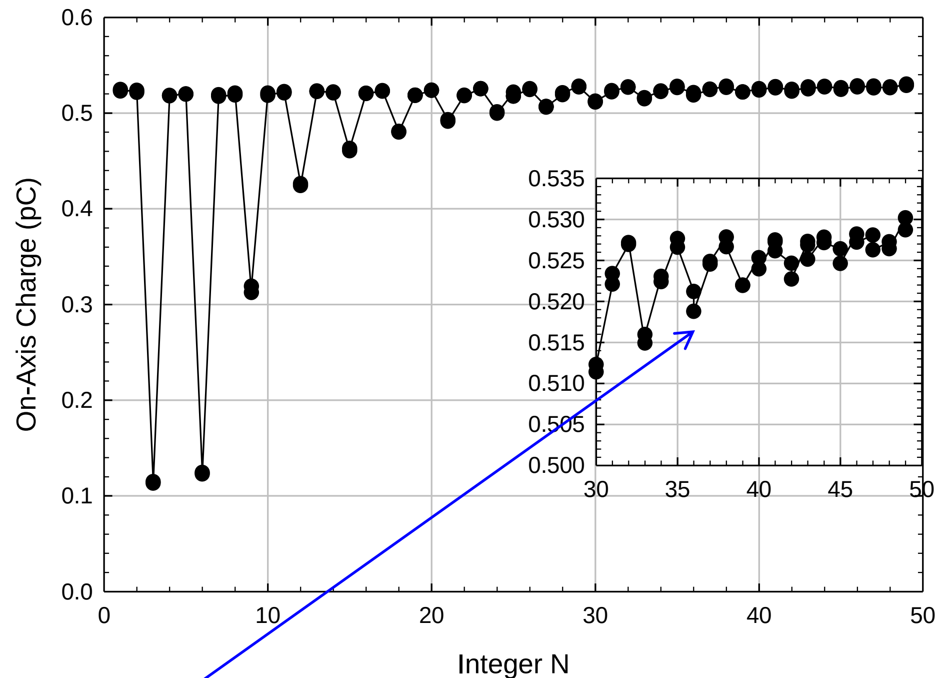
σ_v = 45 deg.

Resonance as a Function of "Ring Circumference"



σ_v = 60 deg.

Resonance as a Function of "Ring Circumference"



Tune $\nu = 1/2$

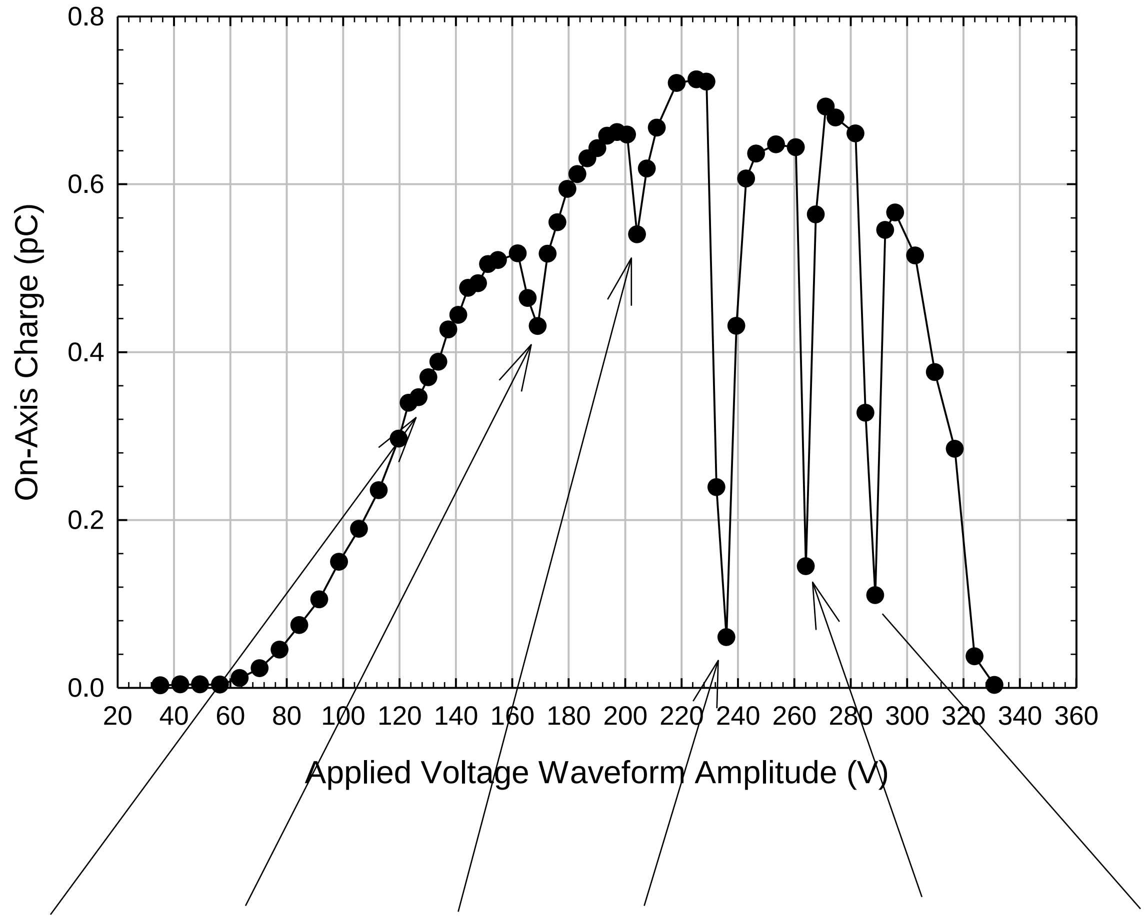
Tune $\nu = 1$

UMER: N = 36

Quadrupole Errors Lead to Half-Integer Resonances

2% Amplitude Increase Every 12 Periods
($2\pi/12$ = 30 deg.)

Similar to:
Ohtsubo et al., “Experimental Study of Coherent Betatron Resonances with a Paul Trap”, Phys. Rev. ST Accel. Beams, **13**, 044201 (2010).



σ_v^{sf} = 43.2 deg.
σ_v = 44.2 deg.
tune = 1.5

σ_v^{sf} = 57.6 deg.
σ_v = 60.1 deg.
tune = 2

σ_v^{sf} = 69.6 deg.
σ_v = 74.5 deg.
tune = 2.5

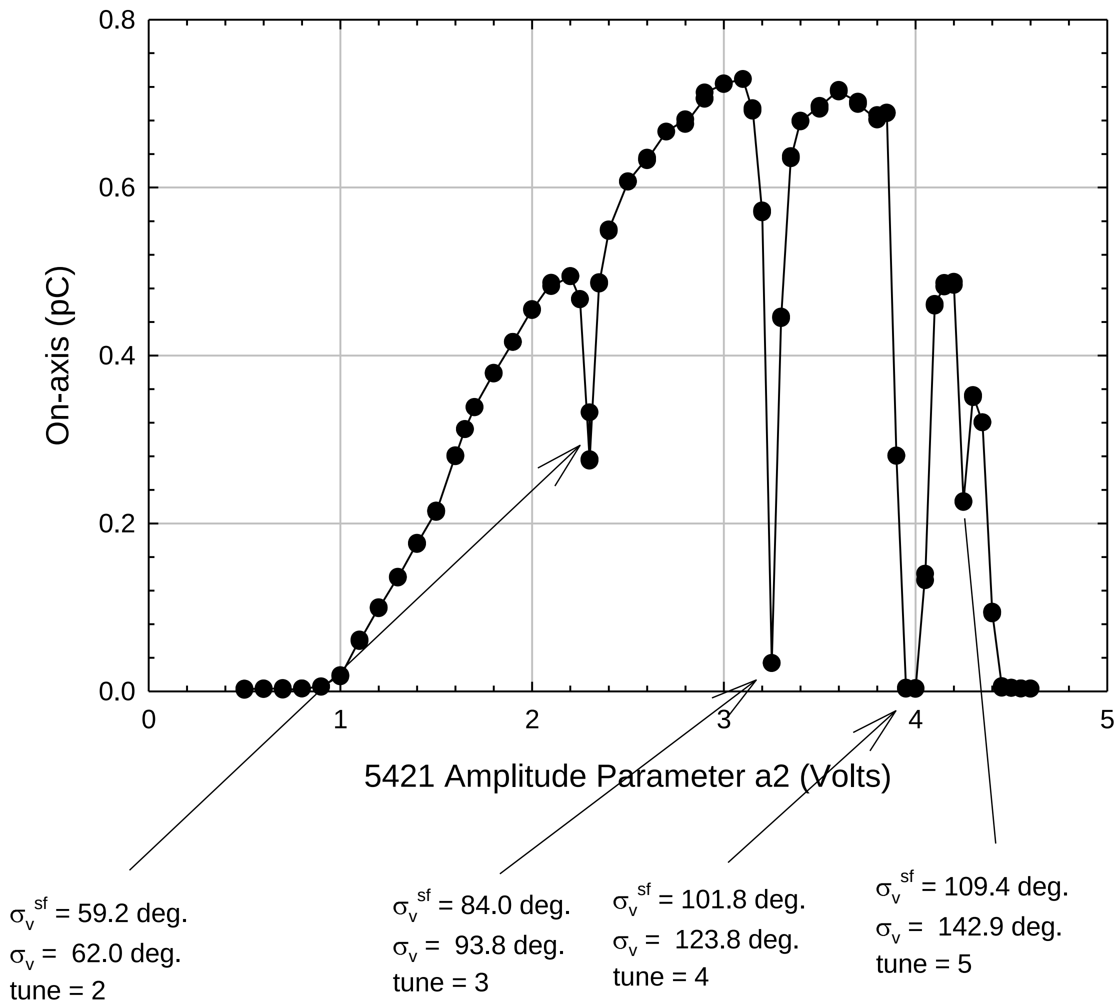
σ_v^{sf} = 80.4 deg.
σ_v = 88.7 deg.
tune = 3

σ_v^{sf} = 90.0 deg.
σ_v = 102.7 deg.
tune = 3.5

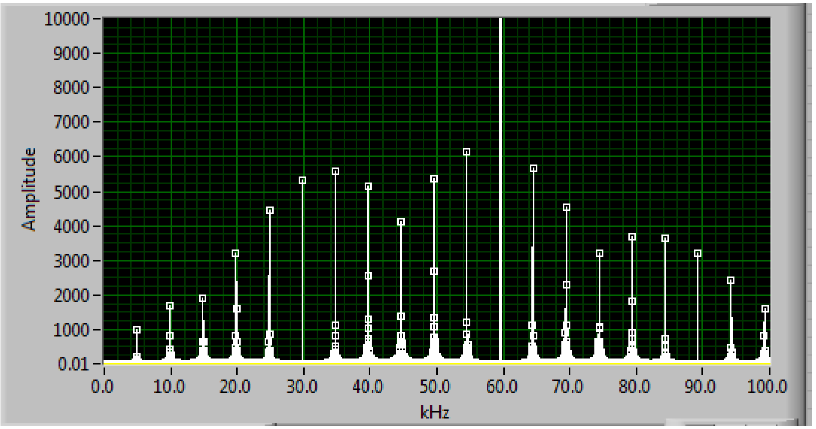
σ_v^{sf} = 98.5 deg.
σ_v = 117.3 deg.
tune = 4

Dipole Errors Lead to Integer Resonances

2% Perturbation with N = 12 on One Electrode



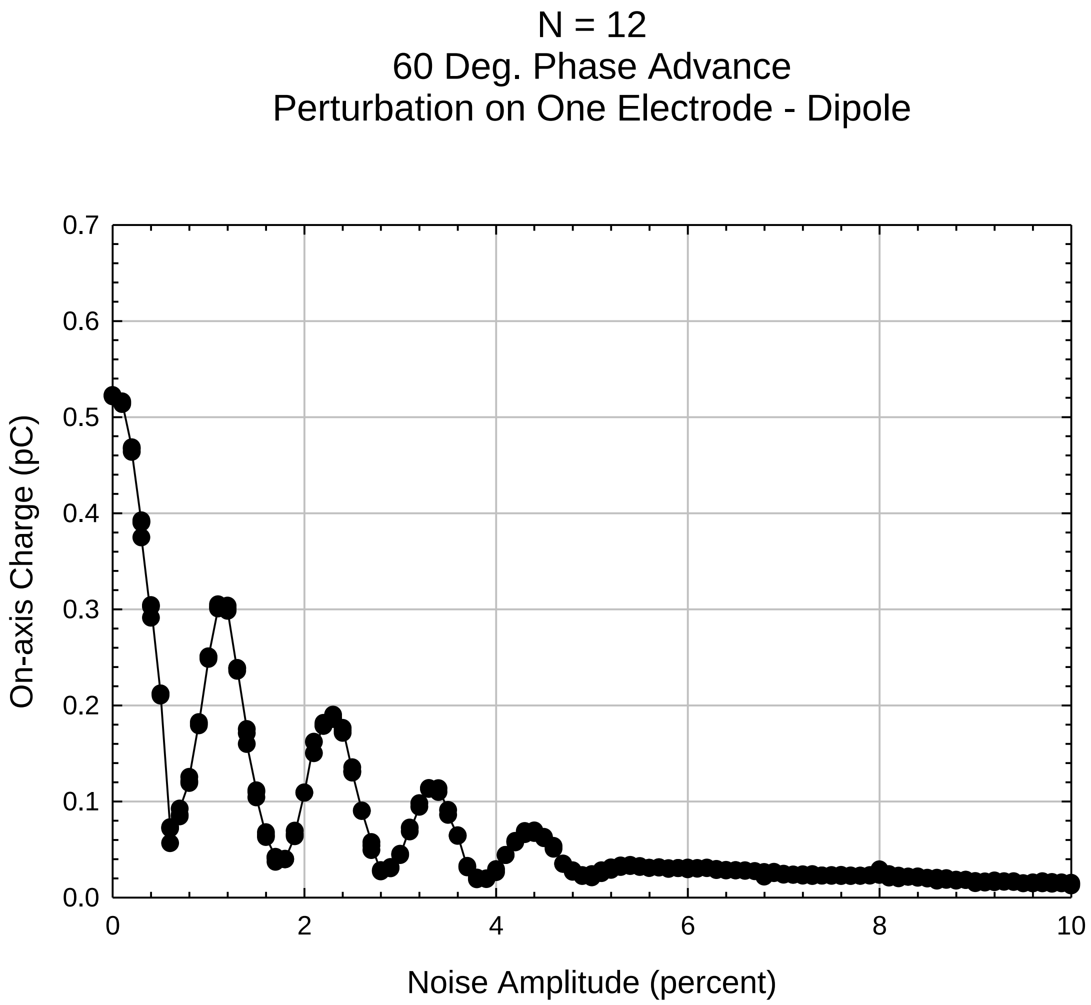
The Coherent Betatron Resonances Disappear When Components at the Mode Frequency are Removed



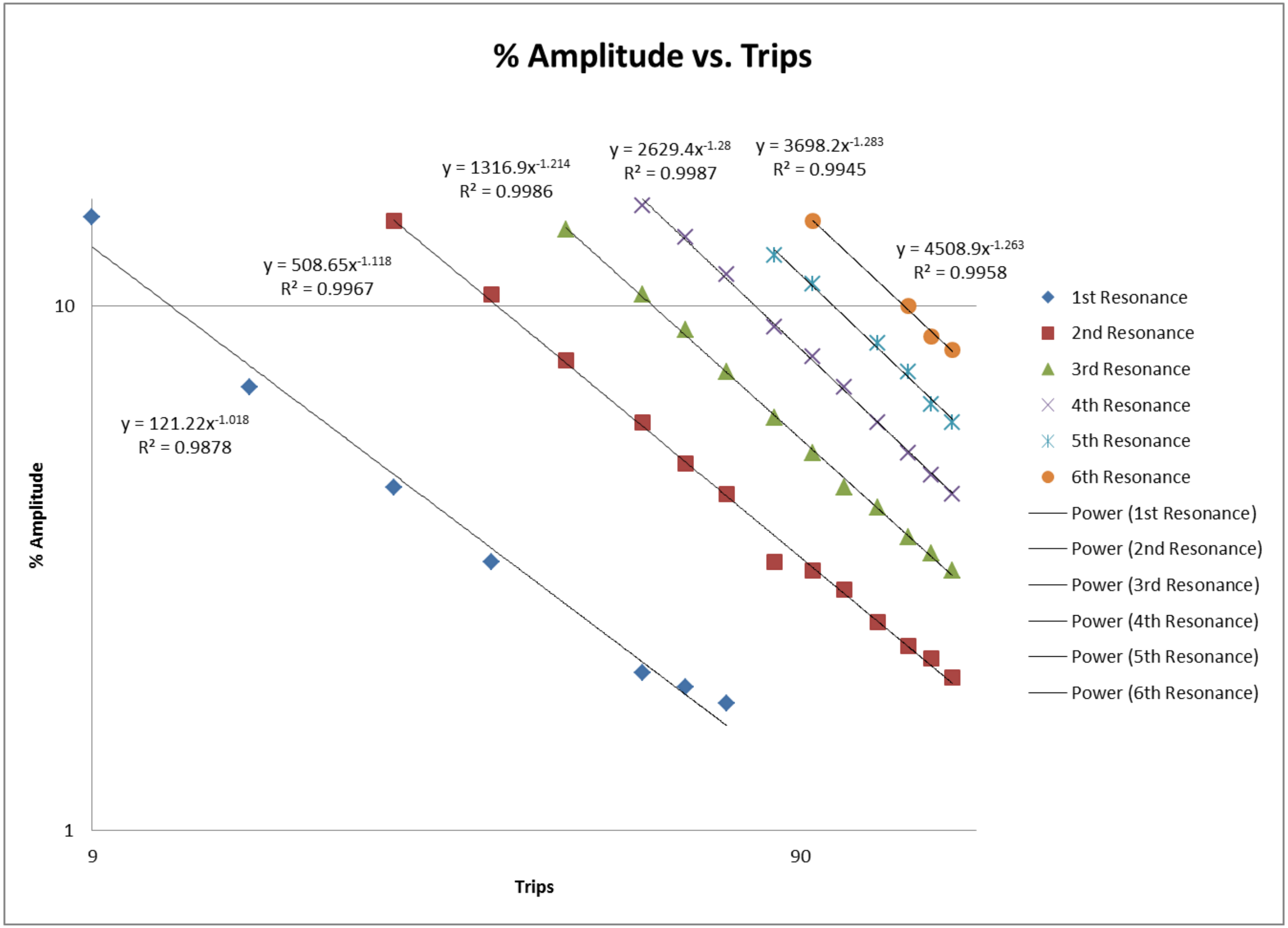
Example:
f_0 = 60 kHz
σ_v = 60 degrees
f_q = 10 kHz

Removing 10 kHz, 50 kHz, and 70 kHz make the resonance disappear for dipole perturbation.

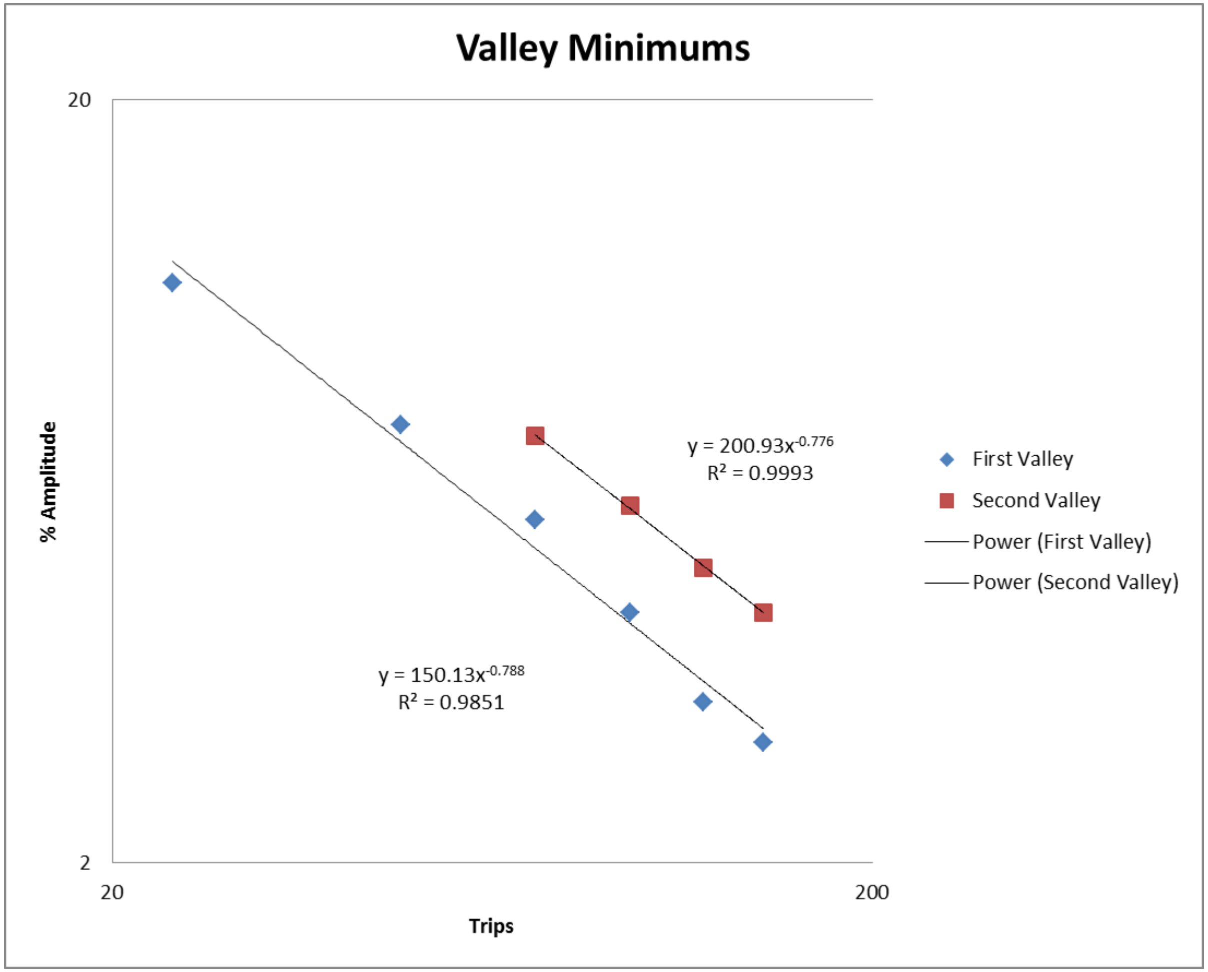
The Adverse Effect of the Resonance is Not a Monotonically Decreasing Function of Amplitude



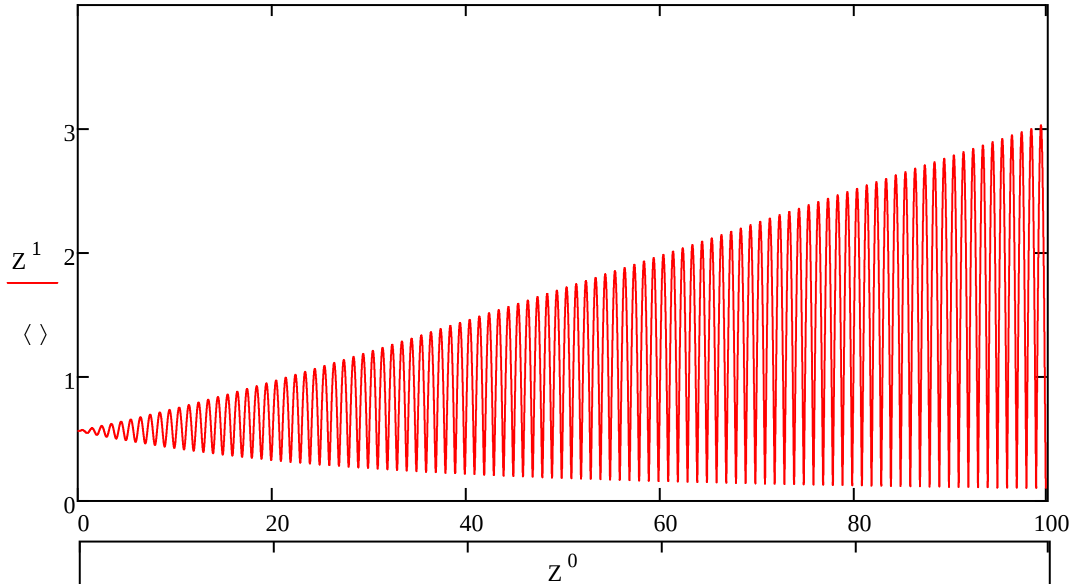
Dipole Perturbation – The Minima Follow a Scaling Law ~ (% #)$^{-5/4}$



Quadrupole Perturbation – Scaling Law ~ (% #)$^{-3/4}$

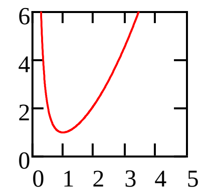


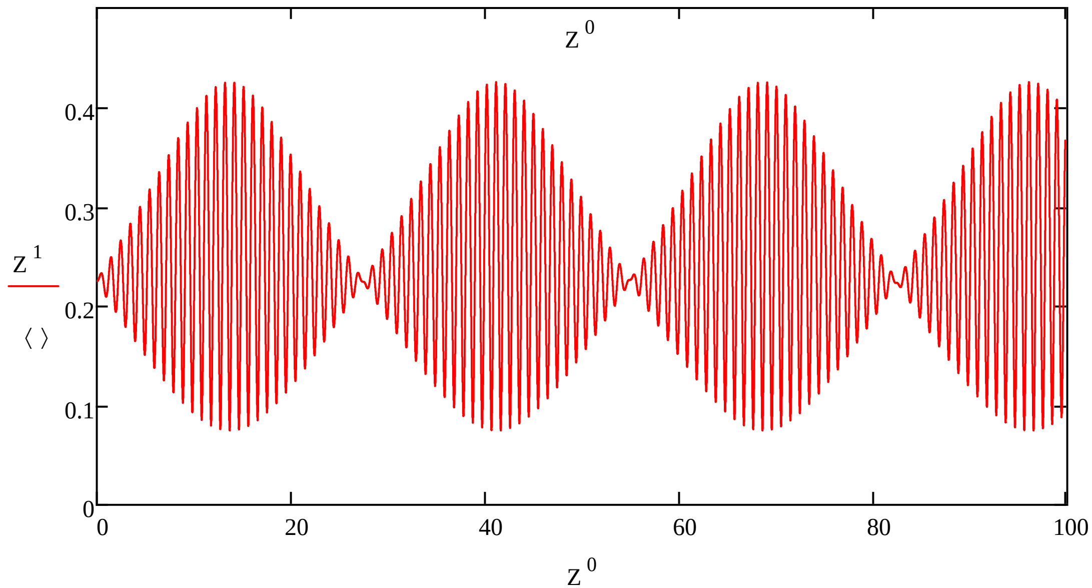
The Smooth-Focusing KV Envelope Equation is Linear for Small Space Charge and Nonlinear for Large Space Charge



$$\ddot{x}+\omega_0^2 x-\cancelto{0}{\frac{1}{x}}-\frac{1}{x^3}=\delta\sin\left(2\omega_0 t\right)$$

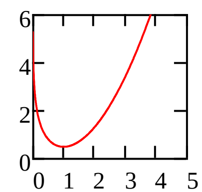
$$V\left(x\right)=\frac{1}{2}\left(x^2+\frac{1}{x^2}\right)$$





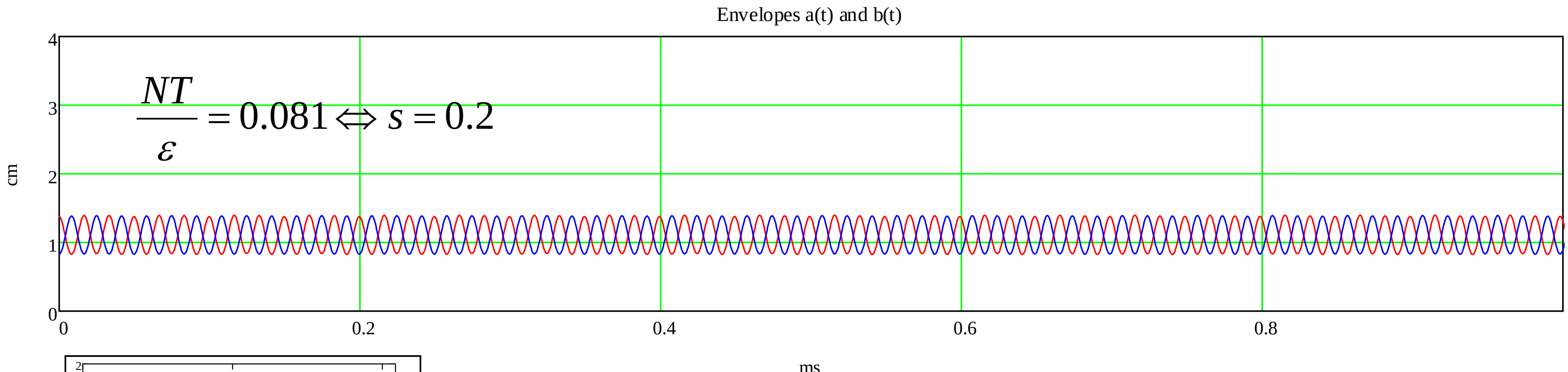
$$\ddot{x}+\omega_0^2 x-\frac{1}{x}-\cancelto{0}{\frac{1}{x^3}}=\delta\sin\left(\sqrt{2}\omega_0 t\right)$$

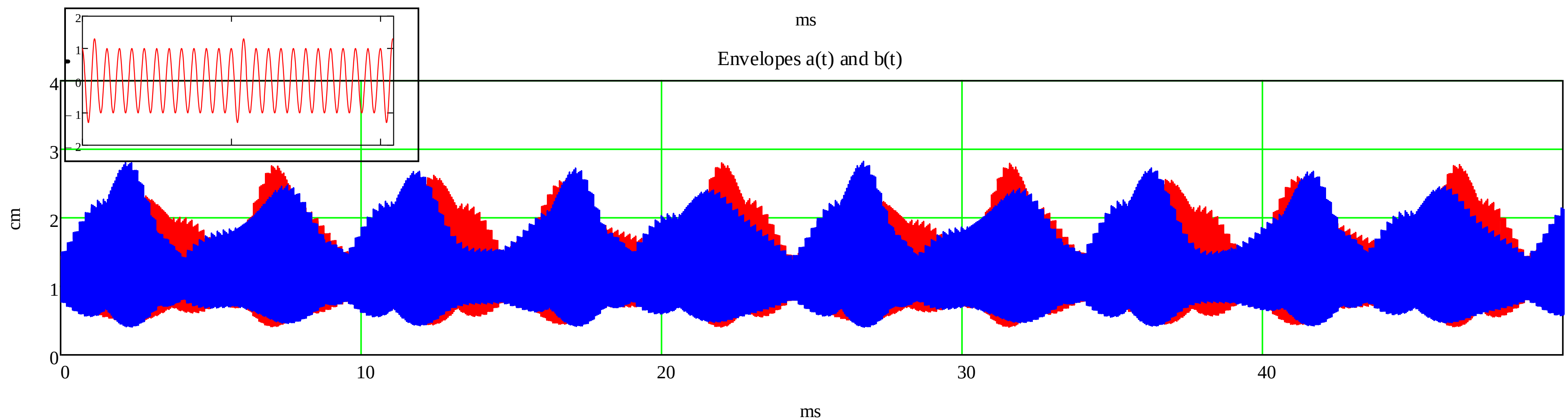
$$V\left(x\right)=\frac{1}{2}\left(x^2-2\ln(x)\right)$$



The Fully-Time-Dependent KV Envelope Equations Behave Similarly

$$\ddot{x} + \kappa(t)x - \frac{2N}{x+y} - \frac{\varepsilon}{x^3} = 0 \qquad \ddot{y} + \kappa(t)y - \frac{2N}{x+y} - \frac{\varepsilon}{y^3} = 0$$





PRINCETON PLASMA PHYSICS LABORATORY

Summary

- Transverse dipole and quadrupole modes have been excited in the Paul Trap Simulator Experiment.
- Lattice noise interacts with the plasma through its coupling to the plasma modes.
- Coherent periodic perturbations also couple to the plasma modes and have been shown to excite large-amplitude modes that appear to be nonlinear due to space-charge effects.