Paul Trap Simulator Experiment (PTSX)*

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Outline

- Introduction
- Theory
  - Alternating Gradient Focusing Systems
  - Paul Traps
- PTSX Apparatus
- Initial Results
- Conclusions

Paul Trap Simulator Experiment

Objective:

- Simulate collective processes and transverse dynamics of intense charged particle beam propagation through an alternating-gradient quadrupole focusing field using a compact laboratory Paul trap.

Approach:

- Investigate dynamics and collective processes in a long one-component charge bunch confined in a Paul trap with oscillating wall voltage $V_0(t+T) = V_0(t)$.

References:

- "A Paul Trap Configuration to Simulate Intense Nonneutral Beam Propagation Over Large Distances Through a Periodic Focusing Quadrupole Magnetic Field," R. C. Davidson, H. Qin, and G. Shvets, Physics of Plasmas **7**, 1020 (2000).
- "Paul Trap Experiment for Simulating Intense Beam Propagation Through a Quadrupole Focusing Field," R. C. Davidson, P. Efthimion, R. Majeski, and H. Qin, Proceedings of the 2001 Particle Accelerator Conference, 2978 (2001).
- "Paul Trap Simulator Experiment to Simulate Intense Beam Propagation Through a Periodic Focusing Quadrupole Field," R. C. Davidson, P. C. Efthimion, E. Gilson, R. Majeski, and H. Qin, American Institute of Physics Conference Proceedings **606**, 576 (2002).

Introduction

- Purpose: simulate nonlinear dynamics of intense beam propagation over large distances through alternating-gradient transport systems.

  Okamoto and Tanaka, Nucl. Instrum. Methods A **437**, 178 (1999). Davidson, et al., Phys. Plasmas **7**, 1020 (2000).

- Applications: heavy ion fusion, spallation neutron sources, and nuclear waste treatment.

- PTSX will explore physics issues such as:

  - Beam mismatch and envelope instabilities,
  - Collective wave excitations,
  - Chaotic particle dynamics and production of halo particles,
  - Mechanisms for emittance growth,
  - Compression techniques,
  - Effects of distribution function on stability properties.

Introduction

Active area of research:

- PPPL

  PTSX

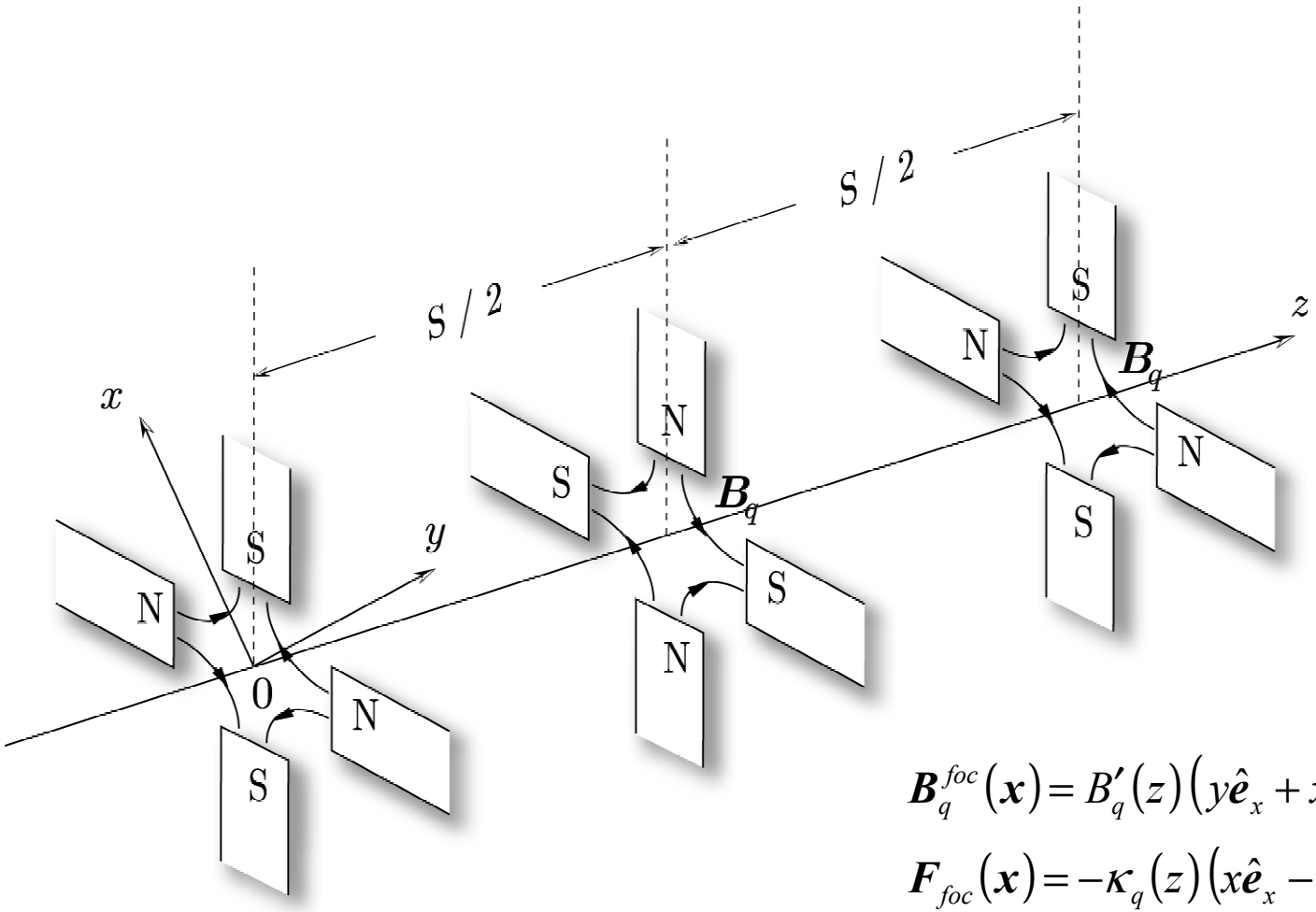
- University of Maryland Electron Ring (UMER) (M. Reiser, P. O'Shea, et al.)

  Simulate intense beams in storage rings

- University of Denmark, Aarhus (M. Drewsen, et al.)

  Beam crystallization

Alternating Gradient Focusing Systems



$$\boldsymbol{B}_q^{foc}(\boldsymbol{x}) = B_q'(z)\left(y\hat{\boldsymbol{e}}_x + x\hat{\boldsymbol{e}}_y\right)$$

$$\boldsymbol{F}_{foc}(\boldsymbol{x}) = -\kappa_q(z)\left(x\hat{\boldsymbol{e}}_x - y\hat{\boldsymbol{e}}_y\right)$$

Theoretical Model and Assumptions

- Consider a thin ($r_b << S$) intense nonneutral ion beam (ion charge = $+Z_b e$, rest mass = m_b) propagating in the z-direction through a periodic focusing quadrupole field with average axial momentum $\gamma_b m_b \beta_b c$, and axial periodicity length S.
- Here, r_b is the characteristic beam radius, $V_b = \beta_b c$ is the average axial velocity, and $(\gamma_b\text{-}1)m_b c^2$ is the directed kinetic energy, where $\gamma_b = (1\text{-}\beta_b^2)^{-1/2}$ is the relativistic mass factor.
- The particle motion in the beam frame is assumed to be nonrelativistic.

Theoretical Model and Assumptions

- Introduce the scaled "time" variable

$$s = \beta_b ct$$

and the (dimensionless) transverse velocities

$$x' = \frac{dx}{ds} \quad \text{and} \quad y' = \frac{dy}{ds}$$

- The beam particles propagate in the z-direction through an alternating-gradient quadrupole field

$$\boldsymbol{B}_q^{foc}(\boldsymbol{x}) = B_q'(s)\left(y\hat{\boldsymbol{e}}_x + x\hat{\boldsymbol{e}}_y\right)$$

with lattice coupling coefficient defined by

$$\kappa_q(s) \equiv \frac{Z_b e B_q'(s)}{\gamma_b m_b \beta_b c^2}$$

Here, $B_q'(s) \equiv \left(\partial B_x^q / \partial y\right)_{(0,0)} = \left(\partial B_y^q / \partial x\right)_{(0,0)}$ and $x\hat{\boldsymbol{e}}_x + y\hat{\boldsymbol{e}}_y$ is the transverse displacement of a particle from the beam axis.

- Here,

$$\kappa_q(s+S) = \kappa_q(s)$$

where $S = const.$ is the axial periodicity length.

Theoretical Model and Assumptions

- Neglecting the axial velocity spread, and approximating $v_z \cong \beta_b c$, the applied transverse focusing force on a beam particle is (inverse length units)

$$\boldsymbol{F}_{foc}(\boldsymbol{x}) = -\kappa_q(s)\left(x\hat{\boldsymbol{e}}_x - y\hat{\boldsymbol{e}}_y\right)$$

over the transverse dimensions of the beam ($r_b << S$).

- The (dimensionless) self-field potential experienced by a beam ion is

$$\psi(x,y,s) = \frac{Z_b e}{\gamma_b m_b \beta_b^2 c^2}\left[\phi(x,y,s) - \beta_b A_z(x,y,s)\right]$$

where $\phi(x, y, s)$ is the space-charge potential, and $A_z(x,y,s) \cong \beta_b \phi(x,y,s)$ is the axial component of the vector potential.

- The corresponding self-field force on a beam particle is (inverse length units)

$$\boldsymbol{F}_{self}(\boldsymbol{x}) = -\left[\frac{\partial \psi}{\partial x}\hat{\boldsymbol{e}}_x + \frac{\partial \psi}{\partial y}\hat{\boldsymbol{e}}_y\right]$$

Theoretical Model and Assumptions

- Transverse particle orbits $x(s)$ and $y(s)$ in the laboratory frame are determined from

$$\frac{d^2}{ds^2}x(s)+\kappa_q(s)x(s)=-\frac{\partial}{\partial x}\psi(x,y,s)$$

$$\frac{d^2}{ds^2}y(s)-\kappa_q(s)y(s)=-\frac{\partial}{\partial y}\psi(x,y,s)$$

- The characteristic axial wavelength λ_q of transverse particle oscillations induced by a quadrupole field with amplitude $\hat{\kappa}_q$ is

$$\lambda_q \sim \frac{2\pi}{\sqrt{\hat{\kappa}_q}}$$

- The dimensionless small parameter ε assumed in the present analysis is

$$\varepsilon \sim \left(\frac{S}{\lambda_q}\right)^2 \sim \frac{\hat{\kappa}_q S^2}{(2\pi)^2} < 1$$

which is proportional to the strength of the applied focusing field.

Theoretical Model and Assumptions

- The laboratory-frame Hamiltonian $\hat{H}(x, y, x', y', s)$ for single-particle motion in the transverse phase space (x, y, x', y') is

$$\hat{H}(x, y, x', y', s) = \frac{1}{2}(x'^2 + y'^2) + \frac{1}{2}\kappa_q(s)(x^2 - y^2) + \psi(x, y, s)$$

- The Vlasov equation describing the nonlinear evolution of the distribution function $f_b(x, y, x', y', s)$ in laboratory-frame variables is given by

$$\left\{\frac{\partial}{\partial s} + x'\frac{\partial}{\partial x} + y'\frac{\partial}{\partial y} - \left(\kappa_q(s)x + \frac{\partial \psi}{\partial x}\right)\frac{\partial}{\partial x'} - \left(-\kappa_q(s)y + \frac{\partial \psi}{\partial y}\right)\frac{\partial}{\partial y'}\right\} f_b = 0$$

where $\psi(x, y, s) = e_b \phi^s(x, y, s) / \gamma_b^3 m_b \beta_b^2 c^2$ is the dimensionless self - field potential.

Theoretical Model and Assumptions

- The self-field potential $\psi(x, y, s)$ is determined self-consistently in terms of the distribution function $f_b(x, y, x', y', \mathrm{s})$ from

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\psi=-\frac{2\pi K_b}{N_b}\int dx'dy'\, f_b$$

- Here, $n_b(x,y,s)=\int dx'dy'\, f_b(x,y,x',y',s)$ is the number density of the beam ions, and the constants K_b and N_b are the self-field perveance and the number of beam ions per unit axial length, respectively, defined by

$$K_b=\frac{2N_b Z_b^2 e^2}{\gamma_b^3 m_b \beta_b^2 c^2}=const.$$

$$N_b=\int dx\, dy\, dx'dy'\, f_b(x,y,x',y',s)=const.$$

Theoretical Model and Assumptions

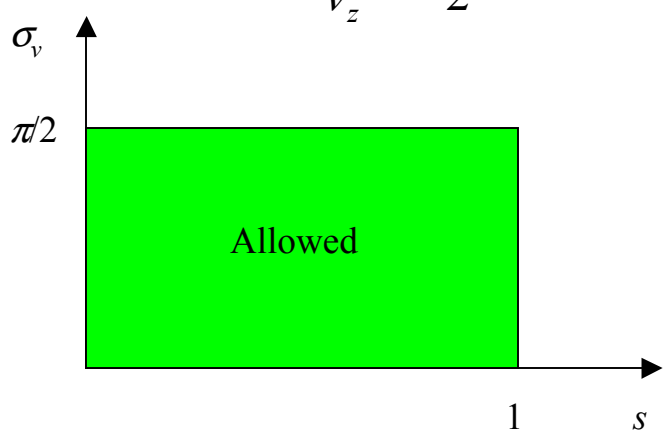
- To ensure the transverse confinement of the beam, the confining force must overcome the repulsive self-force of the beam.

$$s \equiv \frac{\omega_p^2}{2\gamma^2\omega_\beta^2} < 1$$

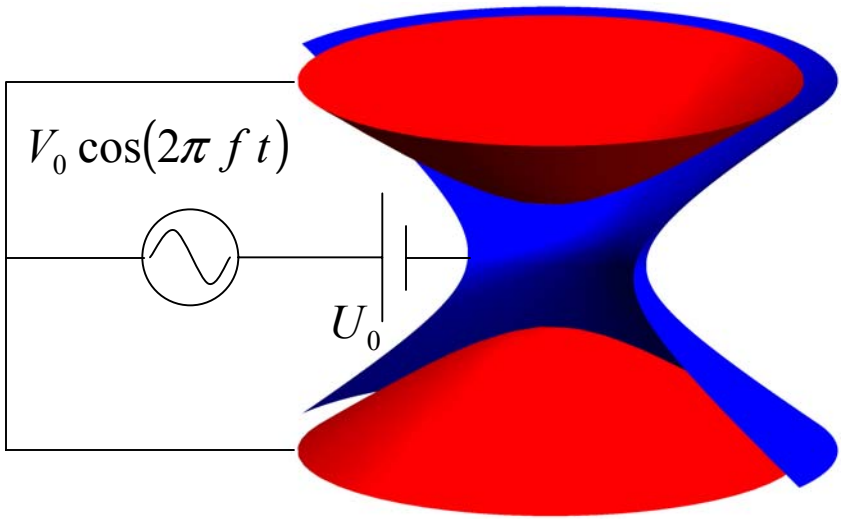
(s ~ 0.2 for SNS storage ring, and s ~ 0.15 Fermilab's Tevatron accelerator at the injection phase.)

- To prevent instabilities, the vacuum phase advance per lattice period, S, ought to be less than 90 degrees.

$$\sigma_v = \frac{\omega_\beta S}{v_z} < \frac{\pi}{2}$$



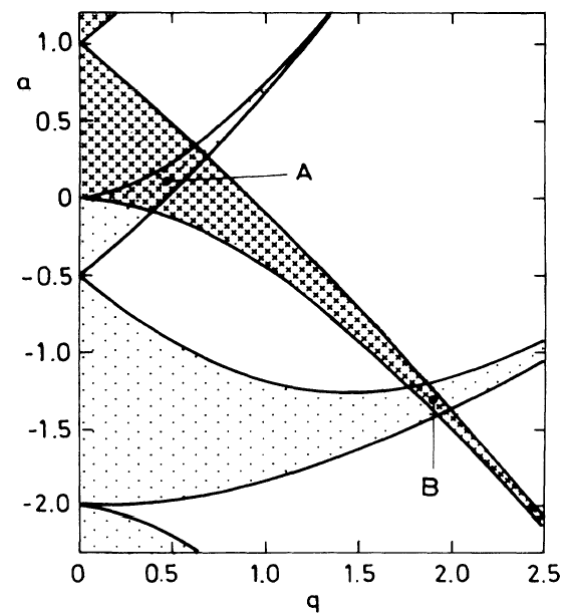
Paul Traps



$$\phi_{trap}(\boldsymbol{x}) = \frac{U_0 + V_0 \cos(2\pi f t)}{r_0^2 + 2z_0^2}\left(x^2 + y^2 - 2z^2\right)$$

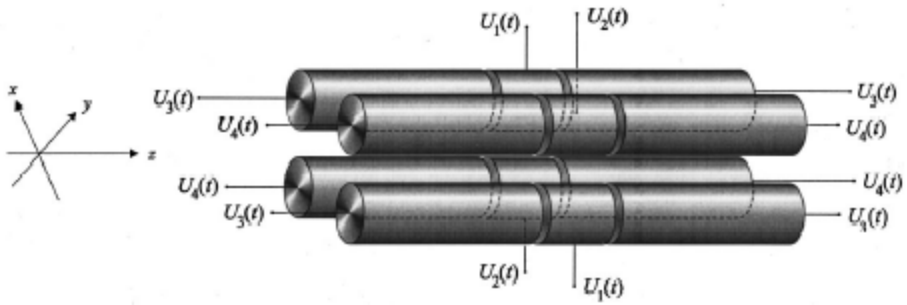
$$\frac{d^2}{d\tau^2}\begin{pmatrix} x \\ y \\ z \end{pmatrix} + \left[a + 2q\cos(2\tau)\right]\begin{pmatrix} x \\ y \\ -2z \end{pmatrix} = 0$$

$$a = \frac{8eU_0}{m(2\pi f)^2\left(r_0^2 + 2z_0^2\right)} \quad q = \frac{4eV_0}{m(2\pi f)^2\left(r_0^2 + 2z_0^2\right)} \quad \tau = \pi f t$$



R. Blümel, et al., Phys Rev A, **40** 808 (1989)

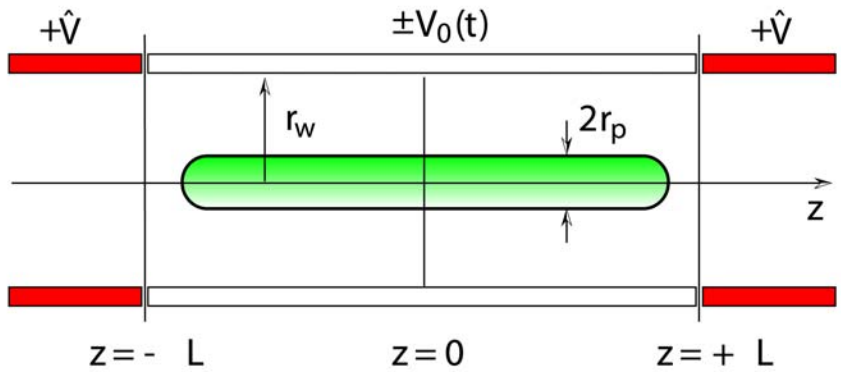
Cylindrical Paul Traps



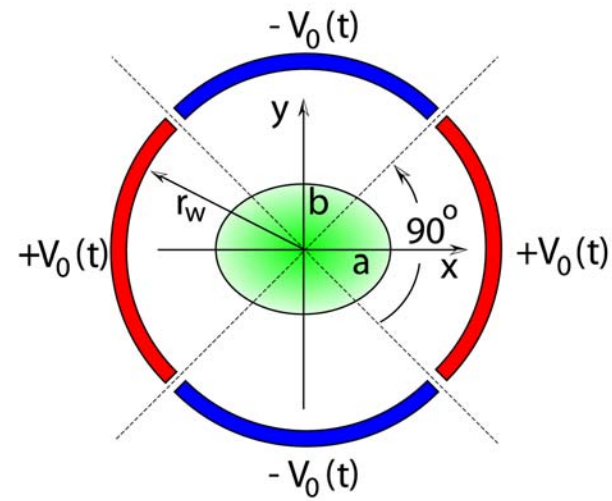
Drewsen and Brøner, Phys. Rev. A, **62**, 045401, 2000.

- r_0 = 3.5 mm
- z_0 = 5.4 mm
- ~ 1000 $^{24}Mg^+$ ions

Paul Trap Simulator Configuration



(a)



(b)

Plasma column length	2 m
Wall electrode radius	10 cm
Plasma column radius	1 cm
Maximum wall voltage	400 V
End electrode voltage	400 V
Voltage oscillation frequency	100 kHz

Transverse Hamiltonian for Particle Motion in a Paul Trap

Transverse Hamiltonian (dimensional units) for a long charge bunch in a Paul trap with time periodic wall voltages $V_0(t+T)=V_0(t)$ is given by

$$H_\perp(x,y,\dot{x},\dot{y},s)=\frac{1}{2}m_b(\dot{x}^2+\dot{y}^2)+e_b\phi_{ap}(x,y,t)+e_b\phi^s(x,y,t)$$

where the applied potential $(0\le r\le r_w)$

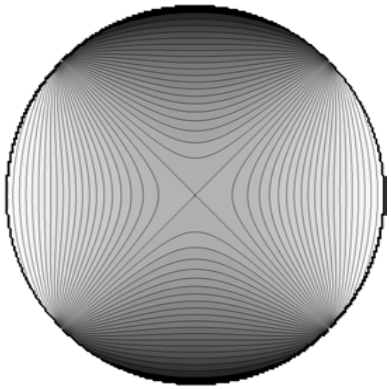
$$\phi_{ap}(x,y,t)=\frac{4V_0(t)}{\pi}\sum_{\ell=1}^{\infty}\frac{\sin(\ell\pi/2)}{\ell}\left(\frac{r}{r_w}\right)^{2\ell}\cos(2\ell\theta)$$

can be approximated by (for $r_p << r_w$)

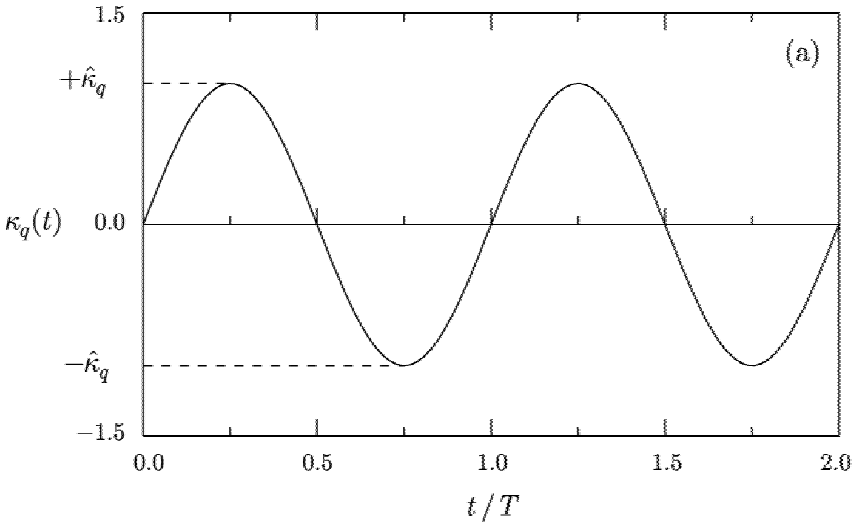
$$e_b\phi_{ap}(x,y,t)=\frac{1}{2}\kappa_q(t)(x^2-y^2),\ \text{ where }\ \kappa_q(t)=\frac{8e_bV_0(t)}{m_b\pi\, r_w^2}$$

with corrections of order $(r_p/r_w)^4$.

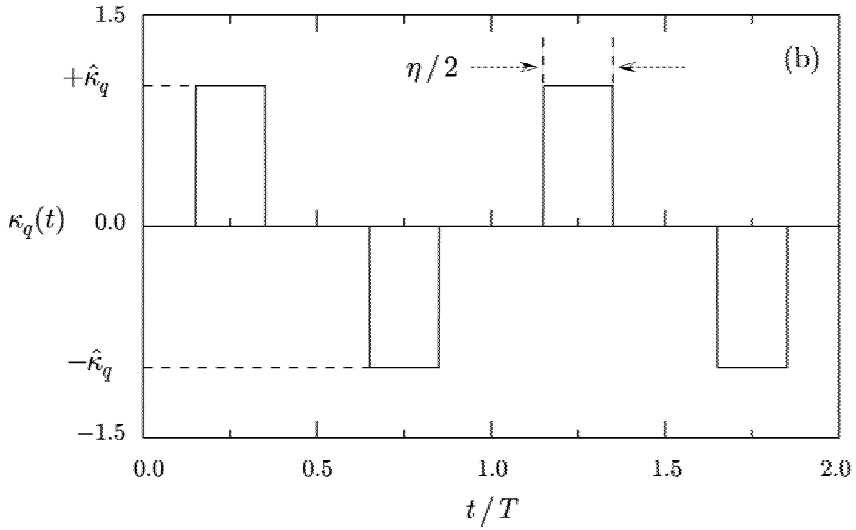
$\phi_{ap}(x,y,t_0)$



Waveform Examples



$$\kappa_q(t) = \hat{\kappa}_q \sin\left(\frac{2\pi t}{T}\right)$$



$$\kappa_q(t) = \hat{\kappa}_q \begin{cases} 1, & (0.25-\eta/4) < t/T < (0.25+\eta/4) \\ -1, & (0.75-\eta/4) < t/T < (0.75+\eta/4) \\ 0, & \text{otherwise} \end{cases}$$

Constraints on Parameters

- The radial confining force is characterized by the average oscillation frequency, ω_q, of a particle in the confining field defined by (smooth focusing approximation)

$$\omega_q = \frac{8 e_b V_{0\,\max}}{m_b \pi r_w^2 f} \xi$$

- Here, $V_{0\,\max}$ is the maximum value of V_0(t) and $f = 1/T$ is the frequency.
- The quantity ξ is defined by

$$\xi = \frac{1}{2\sqrt{2}\pi} \text{ for a sinusoidal waveform, and}$$

$$\xi = \frac{\eta\sqrt{3-2\eta}}{4\sqrt{3}} \text{ for a periodic step - function waveform with fill factor } \eta.$$

Constraints on Parameters

- Requirement for radial confinement

$$\frac{\omega_p}{\sqrt{2}} < \omega_q \qquad (s < 1)$$

- For validity of smooth focusing approximation and to avoid the envelope instability, choose

$$\omega_q < \frac{1}{5} 2\pi f$$

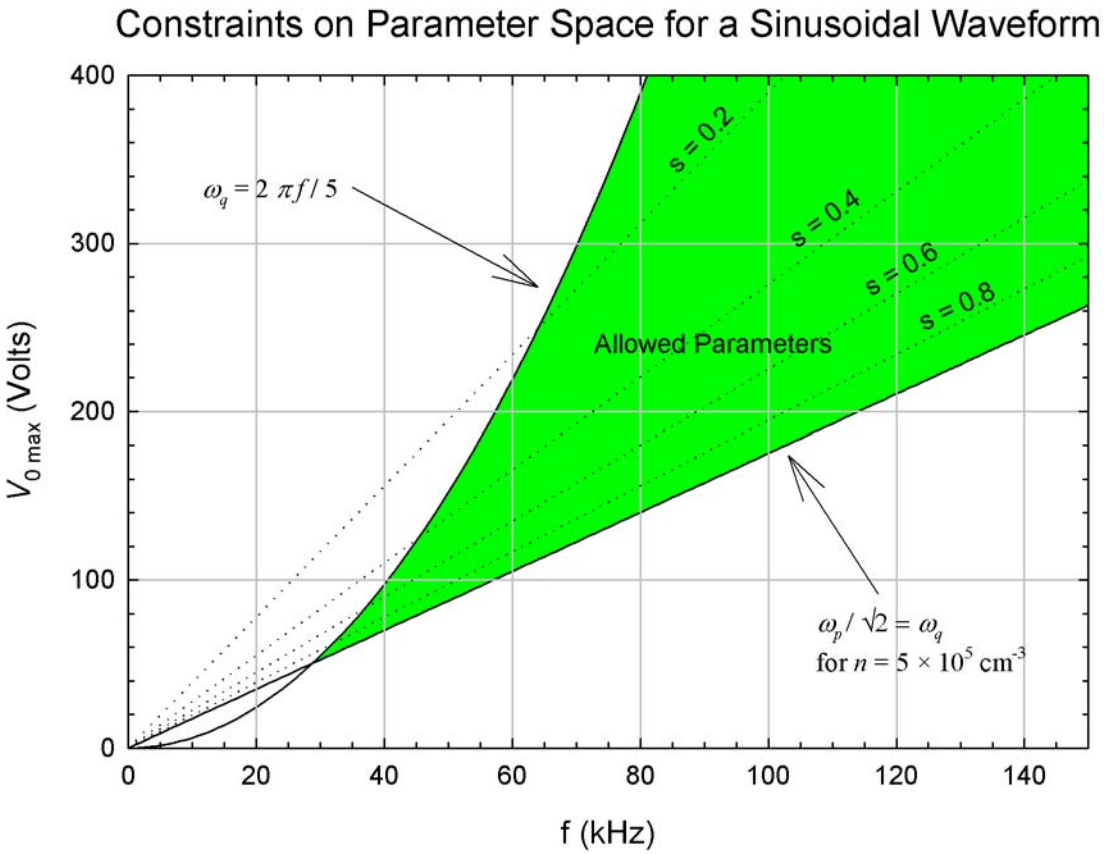
which corresponds to a vacuum phase advance $\sigma_v < 72°$.

- Combining the inequalities gives (for cesium)

$$64.4\, n^{1/2} < 1.46 \times 10^8\, \xi \frac{V_{0\,\max}}{f} < f$$

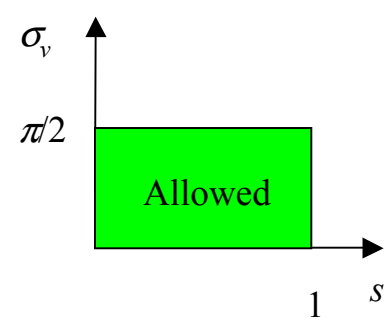
where n is in cm^{-3}, $V_{0\,\max}$ is in Volts, and f is in Hz.

Constraints on Parameter Space

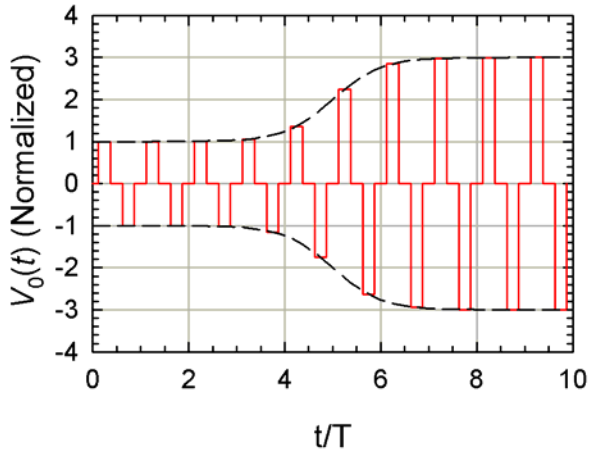


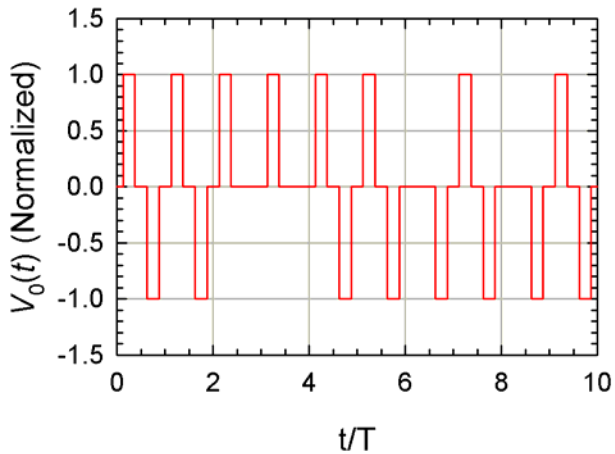
- Here, $s = \omega_p^2/2\omega_q^2$
- $s << 1$ implies "emittance-dominated" beams.
- $s \sim 1$ implies "space-charge-dominated" beams.

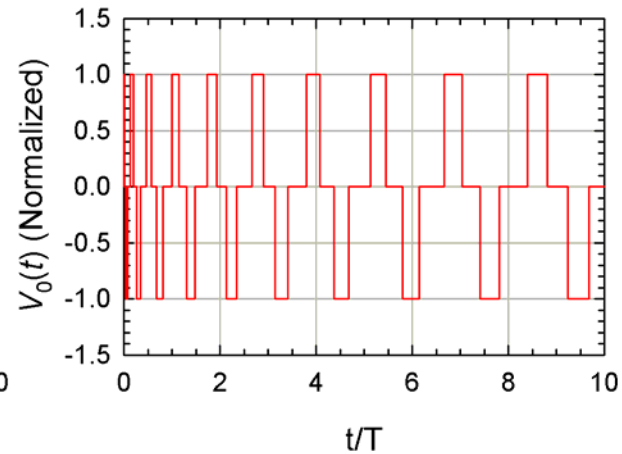
$$64.4\, n^{1/2} < 1.46\times10^8\, \xi \frac{V_{0\,\max}}{f} < f$$



Waveform Examples







- "Carrier" waveform is arbitrary.
- Individual electrodes will eventually be allowed to have different waveforms.

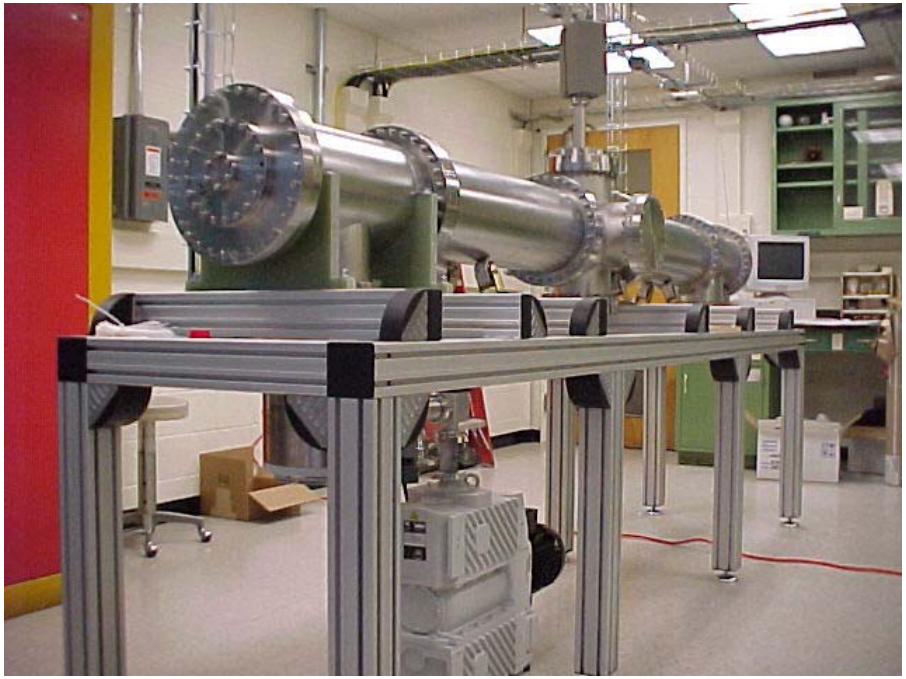
Paul Trap Simulator Experiment

Planned experimental studies include:

- Beam mismatch and envelope instabilities.
- Collective wave excitations.
- Chaotic particle dynamics and production of halo particles.
- Mechanisms for emittance growth.
- Effects of distribution function on stability properties.

Plasma is formed using a cesium source or a barium coated platinum or rhenium filament. Plasma microstate will be determined using laser-induced fluorescence (Levinton, FP&T).

PTSX Apparatus



Paul Trap Simulator Experiment vacuum chamber.

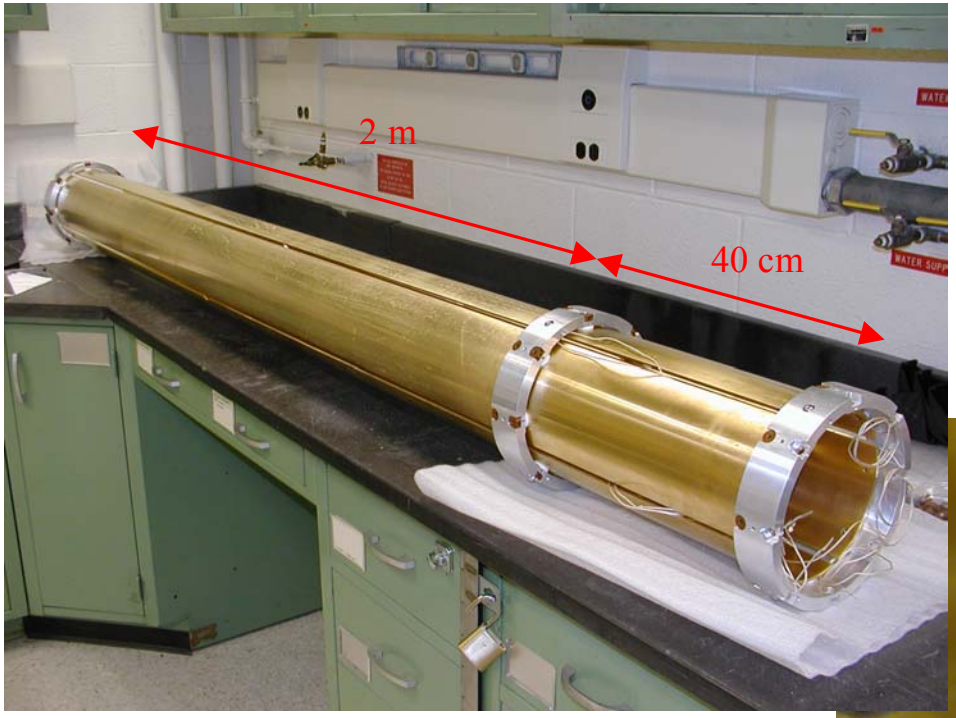
- Laboratory preparation, procurement, assembly, bakeout, and pumpdown of PTSX vacuum chamber to 5.25×10^{-10} Torr (May, 2002).

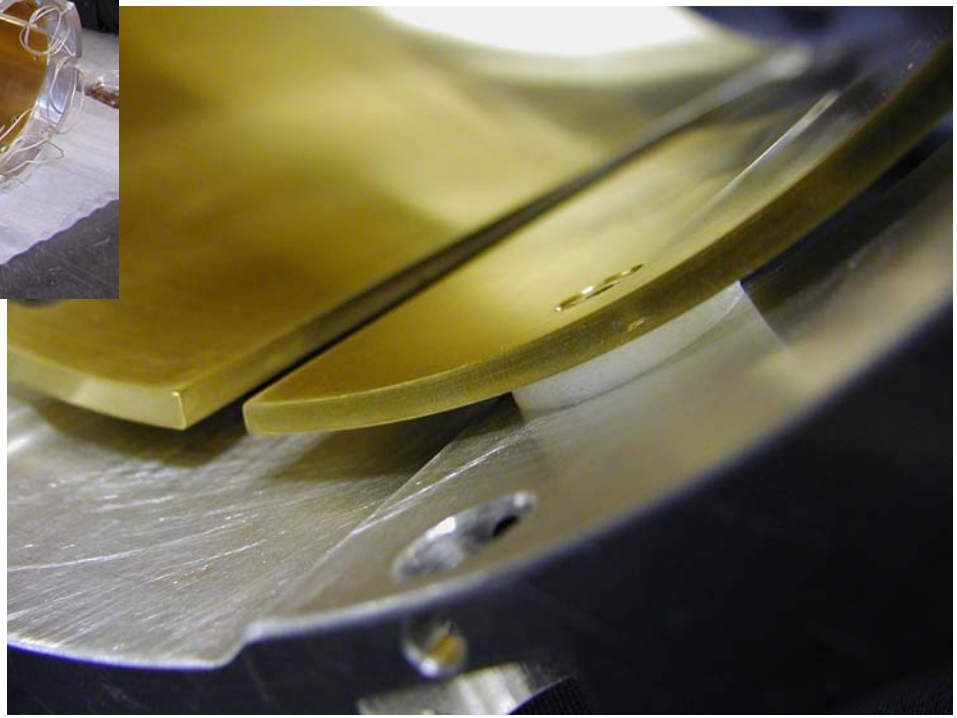
PTSX Dump Electrode

Paul Trap Simulator Experiment electrodes.

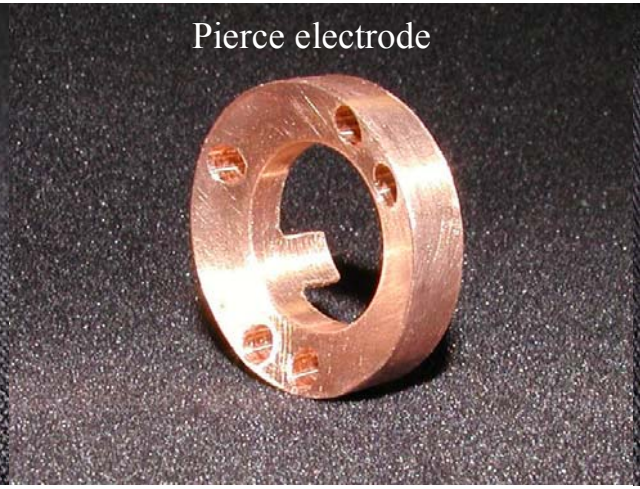
- 8 inch diameter stainless steel gold-plated electrodes are supported by aluminum rings, teflon, and vespel spacers.

PTSX Inject and Trap Electrodes





PTSX Ion Source



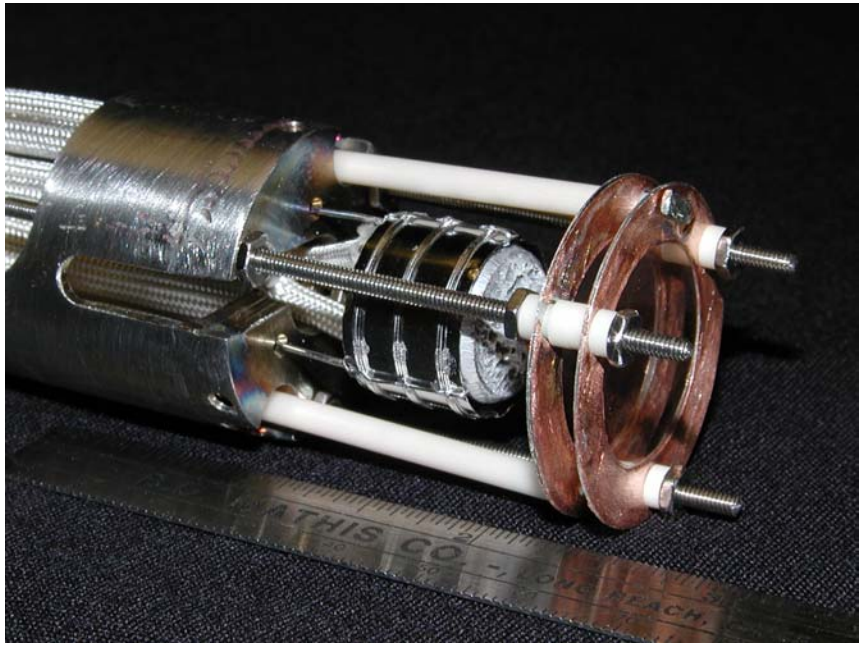
11 A

1100 C

67.5 opening angle

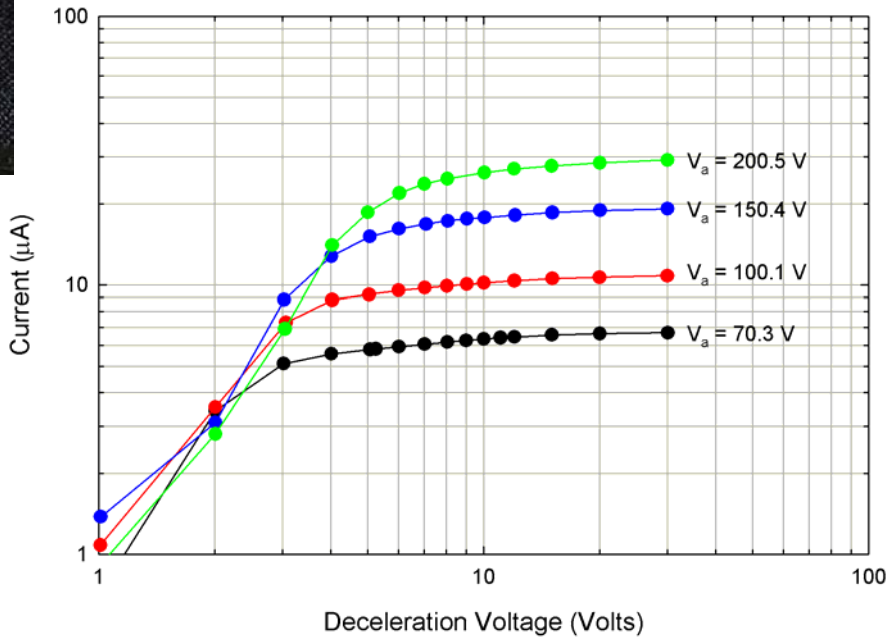
85% transparent electroformed copper mesh

PTSX Ion Source

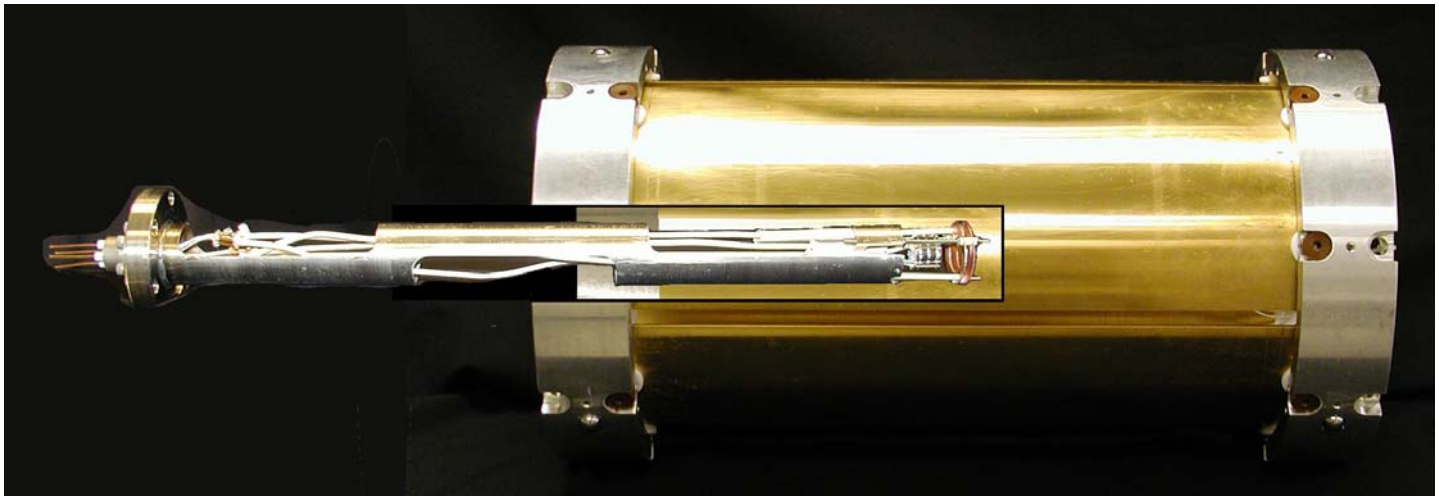


Paul Trap Simulator Experiment cesium source.

- Aluminosilicate cesium source produces up to 30 μA of ion current when a 200 V acceleration voltage is used.



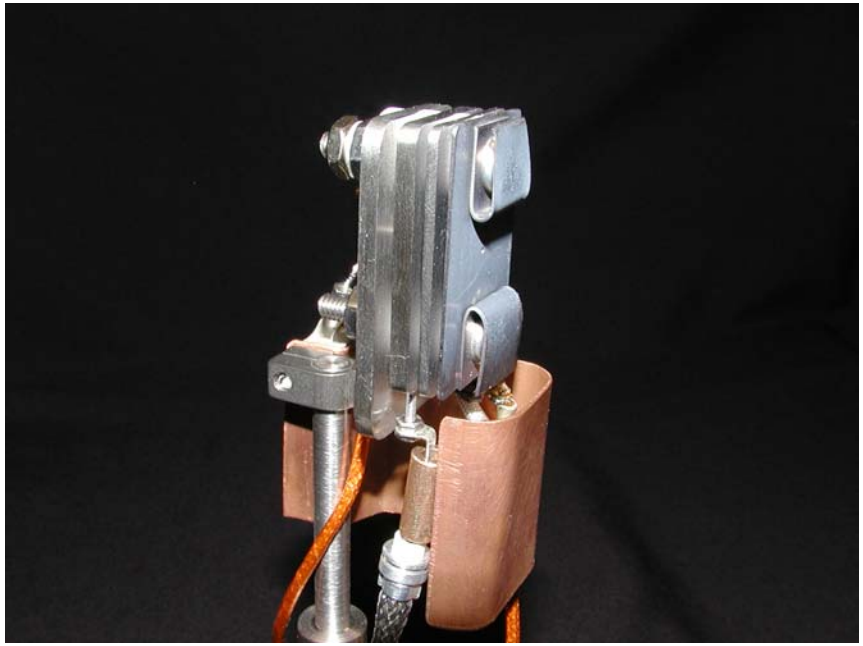
PTSX Ion Injection



Computer generated composite image of cut-away view of electrodes surrounding ion source.

- The electrodes oscillate with the voltage $\pm V_0(t)$ during ion injection.
- The ion source acceleration voltage is turned off as the electrodes are switched to a constant voltage to axially trap the ions.
- The 40 cm long electrodes at the far end of the trap are held at a constant voltage during injection to prevent ions from leaving the far end of the trap.

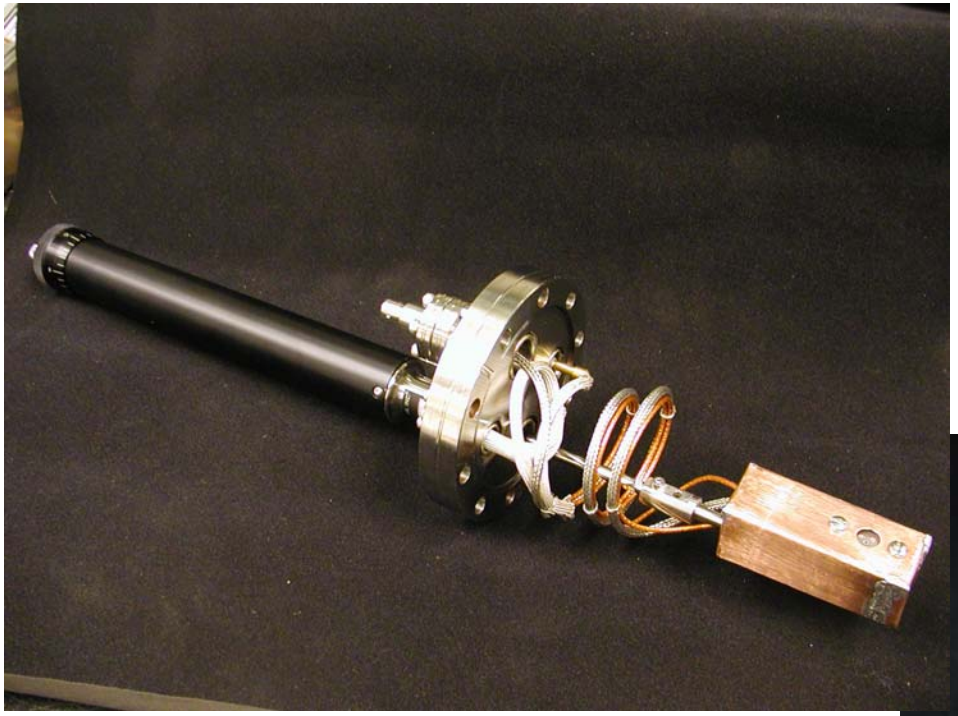
PTSX Faraday Cup Diagnostic

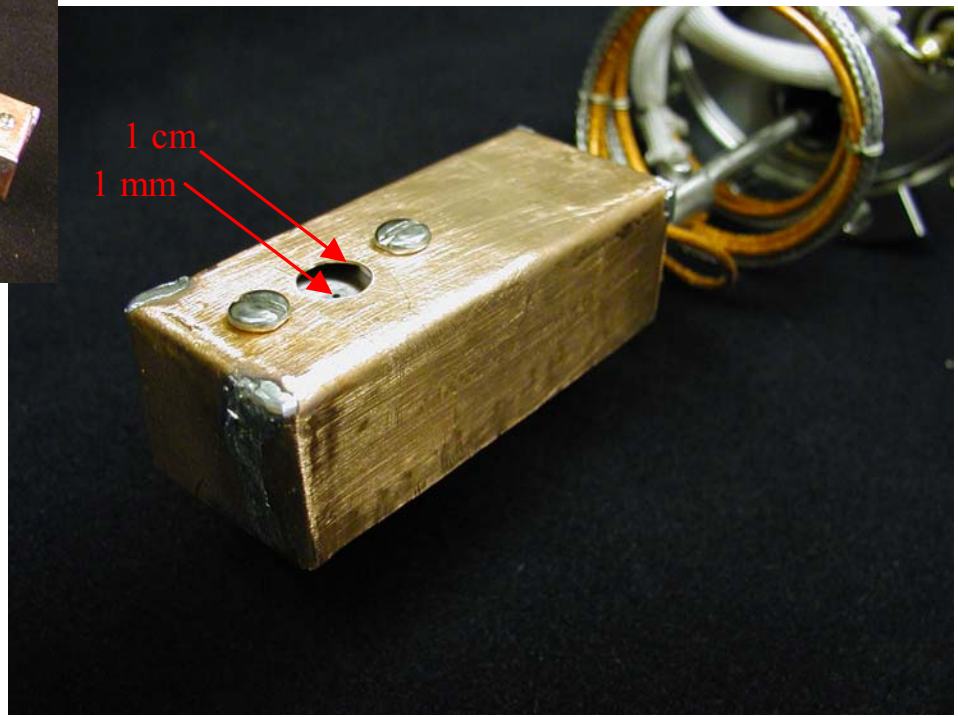


Paul Trap Simulator Experiment Faraday cup.

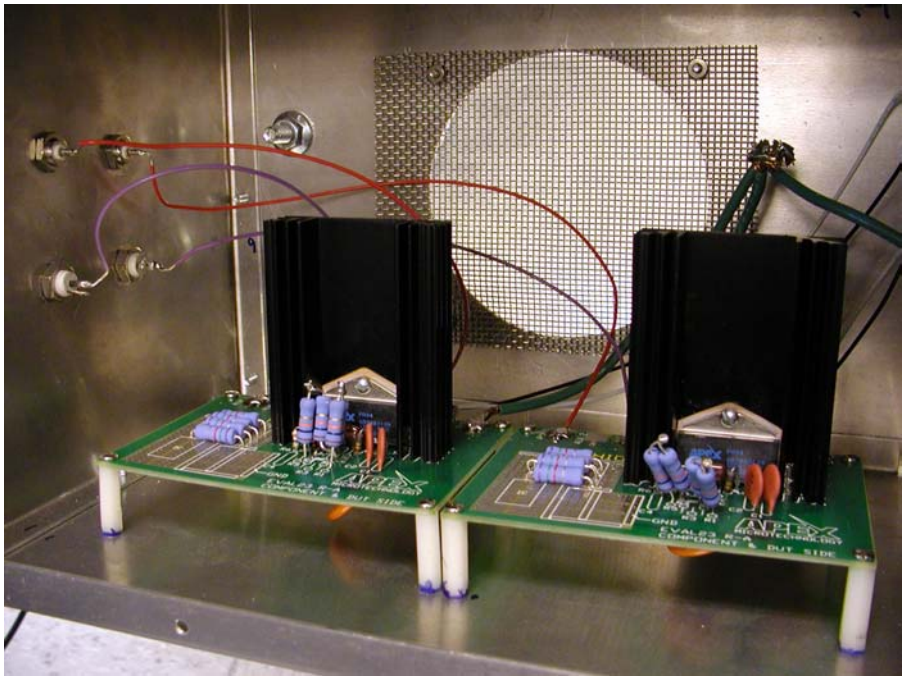
- Faraday cup with sensitive electrometer allows 20 fC resolution.
- Linear motion feedthrough with 6" stroke allows measurement of radial density dependence. 1 mm diameter aperture gives fine spatial resolution.
- Copper shield has been modified to further reduce impact of stray ions.

PTSX Faraday Cup Diagnostic





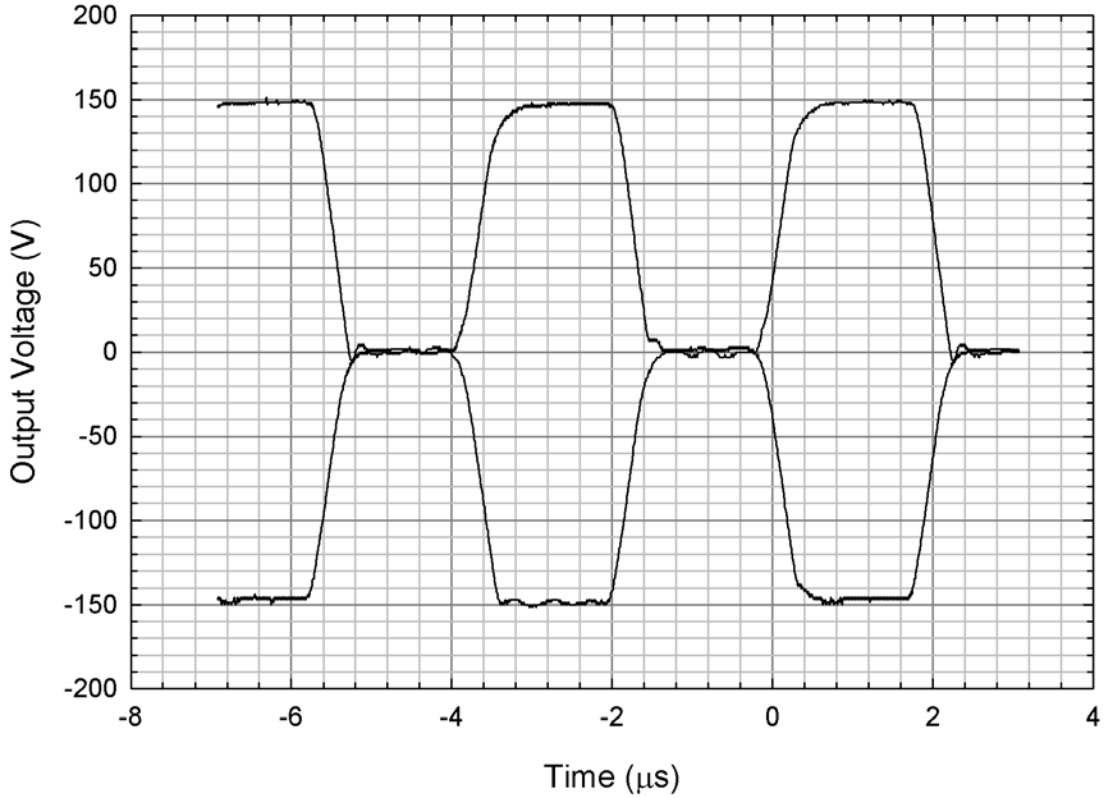
PTSX Electrode Driver



Paul Trap Simulator Experiment electrode driver circuit.

- Electrode driver development using high voltage power op-amp to apply 400 V, 100 kHz signals to electrodes (February, 2002).
- 8 op-amps are used to drive the 12 electrodes.

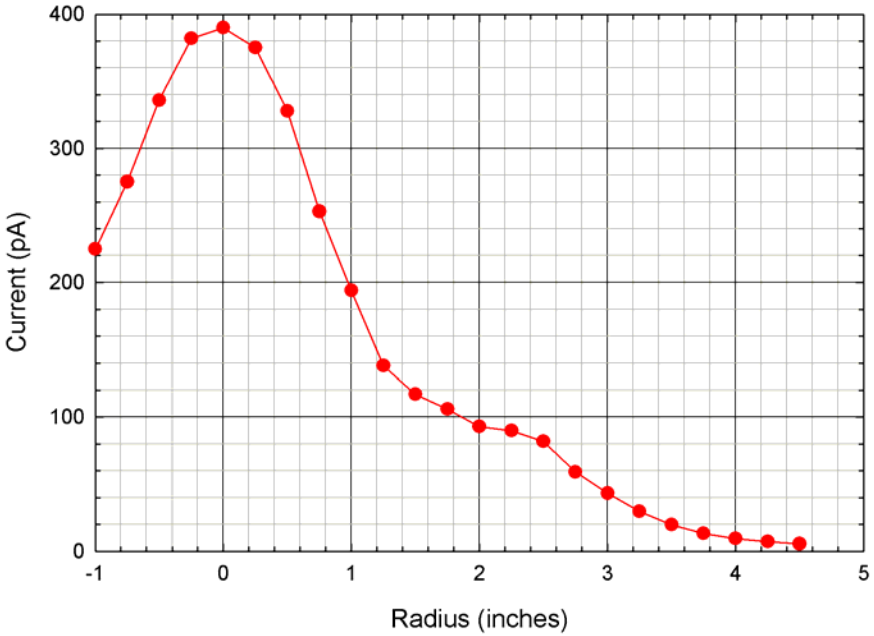
Sample Applied Waveforms



$f = 133$ kHz

$\eta = 0.5$

Initial Results: Radial Profile



Current collected on Faraday cup versus radius.

- Experiment - stream Cs^+ ions from source to collector without axial trapping of the plasma.

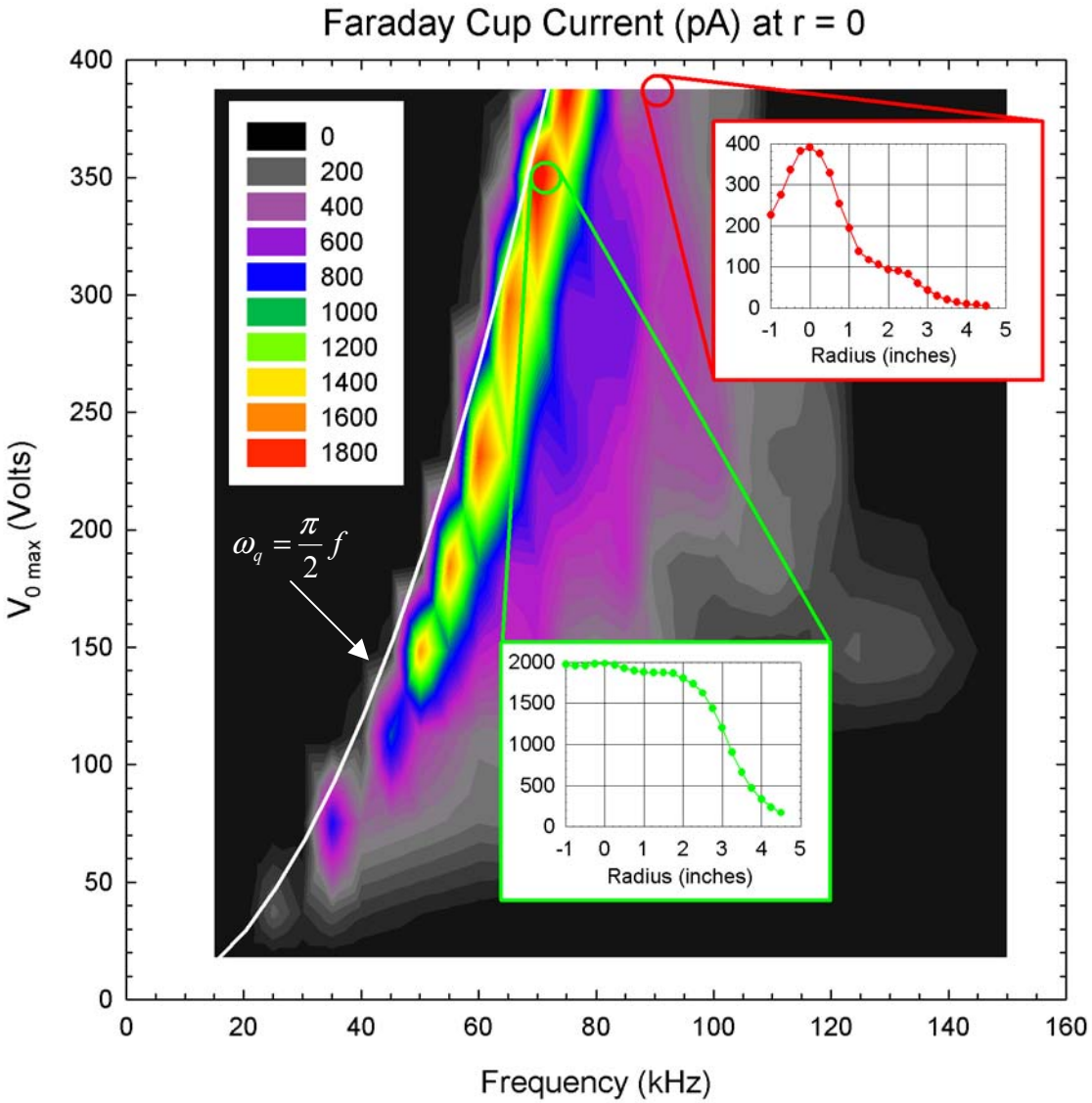
Electrode parameters:

- $V_0(t) = V_{0\,max} \sin(2\pi f t)$
- $V_{0\,max} = 387.5$ V
- $f = 90$ kHz

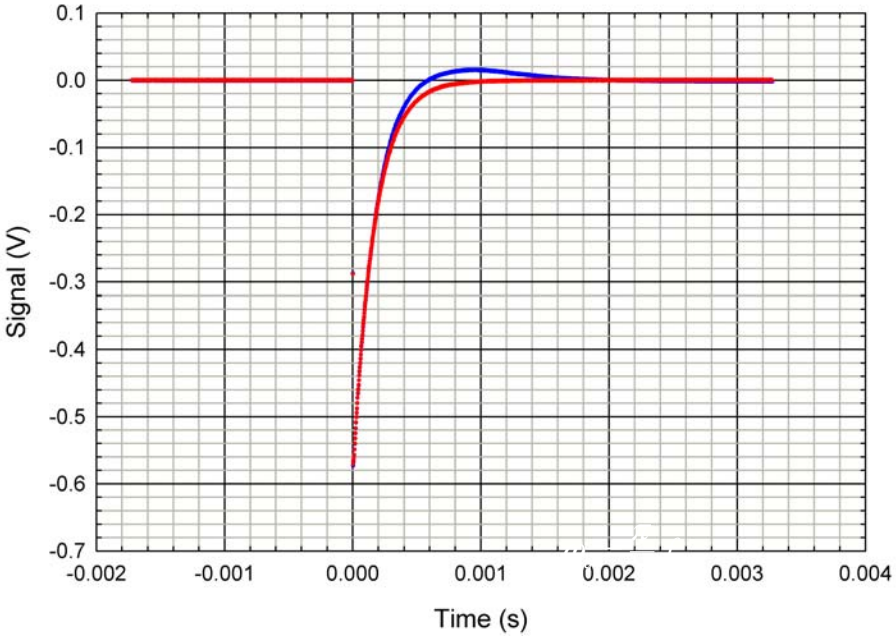
Ion source parameters:

- $V_{accel} = -183.3$ V
- $V_{decel} = -5.0$ V

Initial Results: Instability of Single Particle Orbits

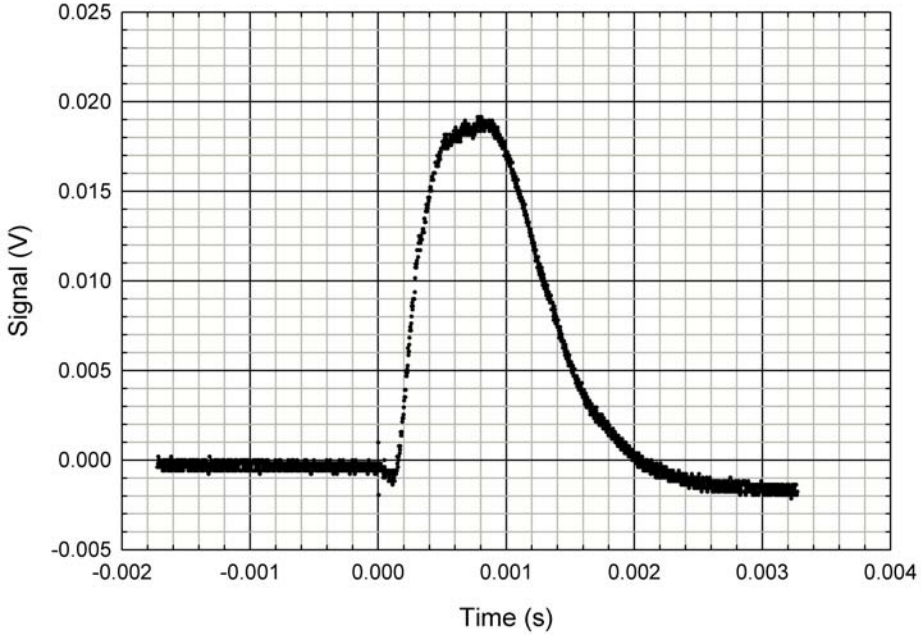


Initial Results: Lifetime



- Plasma signal is extracted from background coupling to short electrodes.

- Extracted plasma signal.



Plasma lives for ~45 seconds.

→ 4500 km beamline

Conclusions

Future Plans:

- Axially trap ions.
- Characterize trapped plasma properties such as density profile and lifetime.
- Optimize injection for well-behaved plasmas.
- Optimize hardware and software systems for precise control.
- Develop barium ion source and laser system for use in a Laser-Induced-Fluorescence diagnostic system.
- Computer simulation of injection, trapping, and dumping.