Paul Trap Simulator Experiment (PTSX)*

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Fusion Summer Study

Snowmass, Colorado

July 8-19, 2002

* Research supported by the U.S. Department of Energy

Paul Trap Simulator Experiment

Summary:

- The assembly of the Paul Trap Simulator Experiment (PTSX) is now complete and experimental operations have begun.
- The purpose of PTSX, a compact laboratory facility, is to simulate the nonlinear dynamics of intense charged particle beam propagation over a large distance through an alternating-gradient transport system.
- The simulation is possible because the quadrupole electric fields of the cylindrical Paul trap exert radial forces on the charged particles that are analogous to the radial forces that a periodic focusing quadrupole magnetic field exert on the beam particles in the beam frame.
- By controlling the waveform applied to the walls of the trap, PTSX will explore physics issues such as beam mismatch, envelope instabilities, halo particle production, compression techniques, collective wave excitations, and beam profile effects.

Paul Trap Simulator Experiment

Objective:

- Simulate collective processes and transverse dynamics of intense charged particle beam propagation through an alternating-gradient quadrupole focusing field using a compact laboratory Paul trap.

Approach:

- Investigate dynamics and collective processes in a long one-component charge bunch confined in a Paul trap with oscillating wall voltage $V_0(t+T)=V_0(t)$.

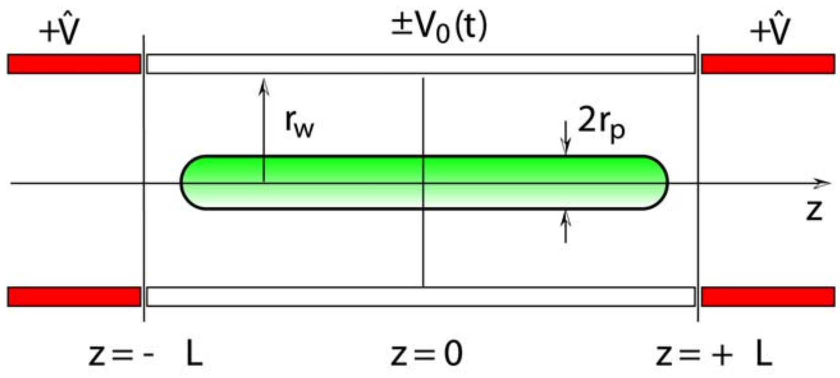
References:

- "A Paul Trap Configuration to Simulate Intense Nonneutral Beam Propagation Over Large Distances Through a Periodic Focusing Quadrupole Magnetic Field," R. C. Davidson, H. Qin, and G. Shvets, Physics of Plasmas **7**, 1020 (2000).

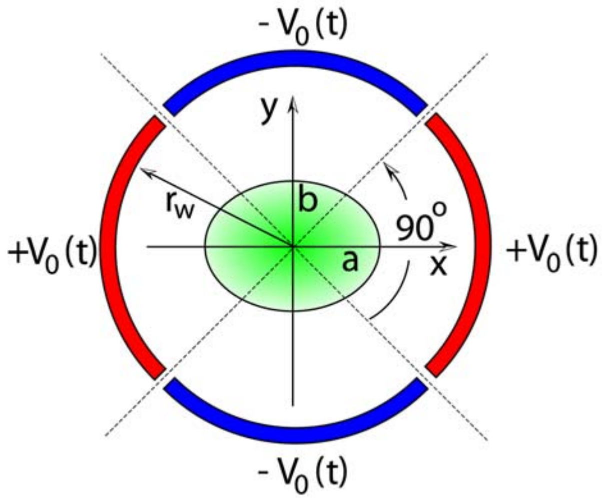
- "Paul Trap Experiment for Simulating Intense Beam Propagation Through a Quadrupole Focusing Field," R. C. Davidson, P. Efthimion, R. Majeski, and H. Qin, Proceedings of the 2001 Particle Accelerator Conference, 2978 (2001).

- "Paul Trap Simulator Experiment to Simulate Intense Beam Propagation Through a Periodic Focusing Quadrupole Field," R. C. Davidson, P. C. Efthimion, E. Gilson, R. Majeski, and H. Qin, American Institute of Physics Conference Proceedings **606**, 576 (2002).

Paul Trap Simulator Configuration



(a)



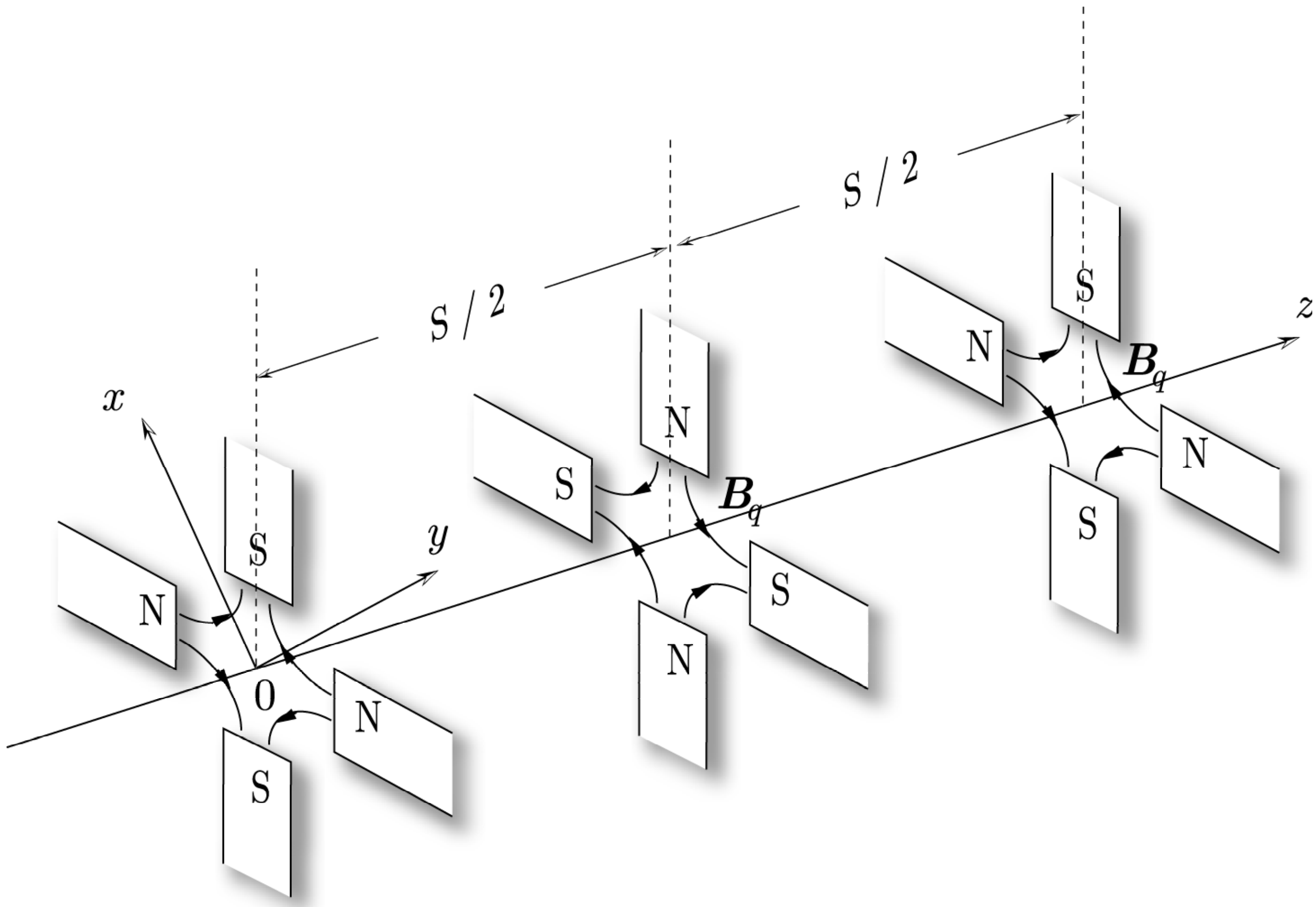
(b)

Paul Trap Simulator Experiment

Nominal Operating Parameters

Plasma column length	2 m
Wall electrode radius	10 cm
Plasma column radius	1 cm
Maximum wall voltage	400 V
End electrode voltage	400 V
Voltage oscillation frequency	100 kHz

Beam Propagation Through Periodic Quadrupole Magnetic Field



Theoretical Model and Assumptions

- Consider a thin ($r_b << S$) intense nonneutral ion beam (ion charge = $+Z_b e$, rest mass = m_b) propagating in the z-direction through a periodic focusing quadrupole field with average axial momentum $\gamma_b m_b \beta_b c$, and axial periodicity length S.
- Here, r_b is the characteristic beam radius, $V_b = \beta_b c$ is the average axial velocity, and $(\gamma_b\text{-}1)m_b c^2$ is the directed kinetic energy, where $\gamma_b = (1\text{-}\beta_b{}^2)^{-1/2}$ is the relativistic mass factor.
- The particle motion in the beam frame is assumed to be nonrelativistic.

Theoretical Model and Assumptions

- Introduce the scaled "time" variable

$$s = \beta_b ct$$

and the (dimensionless) transverse velocities

$$x' = \frac{dx}{ds} \quad \text{and} \quad y' = \frac{dy}{ds}$$

- The beam particles propagate in the z-direction through an alternating-gradient quadrupole field

$$\boldsymbol{B}_q^{foc}(\boldsymbol{x}) = B_q'(s)\left(y\hat{\boldsymbol{e}}_x + x\hat{\boldsymbol{e}}_y\right)$$

with lattice coupling coefficient defined by

$$\kappa_q(s) \equiv \frac{Z_b e B_q'(s)}{\gamma_b m_b \beta_b c^2}$$

Here, $B_q'(s) \equiv \left(\partial B_x^q / \partial y\right)_{(0,0)} = \left(\partial B_y^q / \partial x\right)_{(0,0)}$ and $x\hat{\boldsymbol{e}}_x + y\hat{\boldsymbol{e}}_y$ is the transverse displacement of a particle from the beam axis.

- Here,

$$\kappa_q(s+S) = \kappa_q(s)$$

where $S = const.$ is the axial periodicity length.

Theoretical Model and Assumptions

- Neglecting the axial velocity spread, and approximating $v_z \cong \beta_b c$, the applied transverse focusing force on a beam particle is (inverse length units)

$$\boldsymbol{F}_{foc}(\boldsymbol{x}) = -\kappa_q(s)\left(x\hat{\boldsymbol{e}}_x - y\hat{\boldsymbol{e}}_y\right)$$

over the transverse dimensions of the beam ($r_b << S$).

- The (dimensionless) self-field potential experienced by a beam ion is

$$\psi(x,y,s) = \frac{Z_b e}{\gamma_b m_b \beta_b^2 c^2}\left[\phi(x,y,s) - \beta_b A_z(x,y,s)\right]$$

where $\phi(x, y, s)$ is the space-charge potential, and $A_z(x,y,s) \cong \beta_b \phi(x,y,s)$ is the axial component of the vector potential.

- The corresponding self-field force on a beam particle is (inverse length units)

$$\boldsymbol{F}_{self}(\boldsymbol{x}) = -\left[\frac{\partial\psi}{\partial x}\hat{\boldsymbol{e}}_x + \frac{\partial\psi}{\partial y}\hat{\boldsymbol{e}}_y\right]$$

Theoretical Model and Assumptions

- Transverse particle orbits $x(s)$ and $y(s)$ in the laboratory frame are determined from

$$\frac{d^2}{ds^2}x(s)+\kappa_q(s)x(s)=-\frac{\partial}{\partial x}\psi(x,y,s)$$
$$\frac{d^2}{ds^2}y(s)-\kappa_q(s)y(s)=-\frac{\partial}{\partial y}\psi(x,y,s)$$

- The characteristic axial wavelength λ_q of transverse particle oscillations induced by a quadrupole field with amplitude $\hat{\kappa}_q$ is

$$\lambda_q \sim \frac{2\pi}{\sqrt{\hat{\kappa}_q}}$$

- The dimensionless small parameter ε assumed in the present analysis is

$$\varepsilon \sim \left(\frac{S}{\lambda_q}\right)^2 \sim \frac{\hat{\kappa}_q S^2}{(2\pi)^2} < 1$$

which is proportional to the strength of the applied focusing field.

Theoretical Model and Assumptions

- The laboratory-frame Hamiltonian $\hat{H}(x, y, x', y', s)$ for single-particle motion in the transverse phase space (x, y, x', y') is

$$\hat{H}(x, y, x', y', s) = \frac{1}{2}(x'^2 + y'^2) + \frac{1}{2}\kappa_q(s)(x^2 - y^2) + \psi(x, y, s)$$

- The Vlasov equation describing the nonlinear evolution of the distribution function $f_b(x, y, x', y', \mathrm{s})$ in laboratory-frame variables is given by

$$\left\{\frac{\partial}{\partial s} + x'\frac{\partial}{\partial x} + y'\frac{\partial}{\partial y} - \left(\kappa_q(s)x + \frac{\partial\psi}{\partial x}\right)\frac{\partial}{\partial x'} - \left(-\kappa_q(s)y + \frac{\partial\psi}{\partial y}\right)\frac{\partial}{\partial y'}\right\} f_b = 0$$

where $\psi(x, y, s) = e_b\phi^s(x, y, s)/\gamma_b^3 m_b \beta_b^2 c^2$ is the dimensionless self-field potential.

Theoretical Model and Assumptions

- The self-field potential $\psi(x, y, s)$ is determined self-consistently in terms of the distribution function $f_b(x, y, x', y', s)$ from

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\psi=-\frac{2\pi K_b}{N_b}\int dx'dy'\, f_b$$

- Here, $n_b(x,y,s)=\int dx'dy'\, f_b(x,y,x',y',s)$ is the number density of the beam ions, and the constants K_b and N_b are the self-field perveance and the number of beam ions per unit axial length, respectively, defined by

$$K_b=\frac{2N_b Z_b^2 e^2}{\gamma_b^3 m_b \beta_b^2 c^2}=const.$$

$$N_b=\int dx\,dy\,dx'dy'\, f_b(x,y,x',y',s)=const.$$

Transverse Hamiltonian for Intense Beam Propagation

Transverse Hamiltonian (dimensionless units) for intense beam propagation through a periodic focusing quadrupole magnetic field is given by

$$H_\perp(x, y, x', y', s) = \frac{1}{2}(x'^2 + y'^2) + \frac{1}{2}\kappa_q(s)(x^2 - y^2) + \psi(x, y, s)$$

where $x' = dx/ds,\ \ y' = dy/ds,\ \ \psi(x, y, s) = e_b\phi^s(x, y, s)/\gamma_b^3 m_b \beta_b^2 c^2$

and

$$\kappa_q(s) = \frac{e_b B_q'(s)}{\gamma_b m_b \beta_b c^2}$$

with

$$\kappa_q(s + S) = \kappa_q(s)$$

Transverse Hamiltonian for Particle Motion in a Paul Trap

Transverse Hamiltonian (dimensional units) for a long charge bunch in a Paul trap with time periodic wall voltages $V_0(t+T)=V_0(t)$ is given by

$$H_{\perp}(x,y,\dot{x},\dot{y},s)=\frac{1}{2}m_b(\dot{x}^2+\dot{y}^2)+e_b\phi_{ap}(x,y,t)+e_b\phi^s(x,y,t)$$

where the applied potential $(0\le r\le r_w)$

$$\phi_{ap}(x,y,t)=\frac{4V_0(t)}{\pi}\sum_{\ell=1}^{\infty}\frac{\sin(\ell\pi/2)}{\ell}\left(\frac{r}{r_w}\right)^{2\ell}\cos(2\ell\theta)$$

can be approximated by (for $r_p << r_w$)

$$e_b\phi_{ap}(x,y,t)=\frac{1}{2}\kappa_q(t)(x^2-y^2),\ \text{ where }\ \kappa_q(t)=\frac{8e_bV_0(t)}{m_b\pi\, r_w^2}$$

with corrections of order $(r_p/r_w)^4$.

Constraints on Parameters

- The radial confining force is characterized by the average oscillation frequency, ω_q, of a particle in the confining field defined by (smooth focusing approximation)

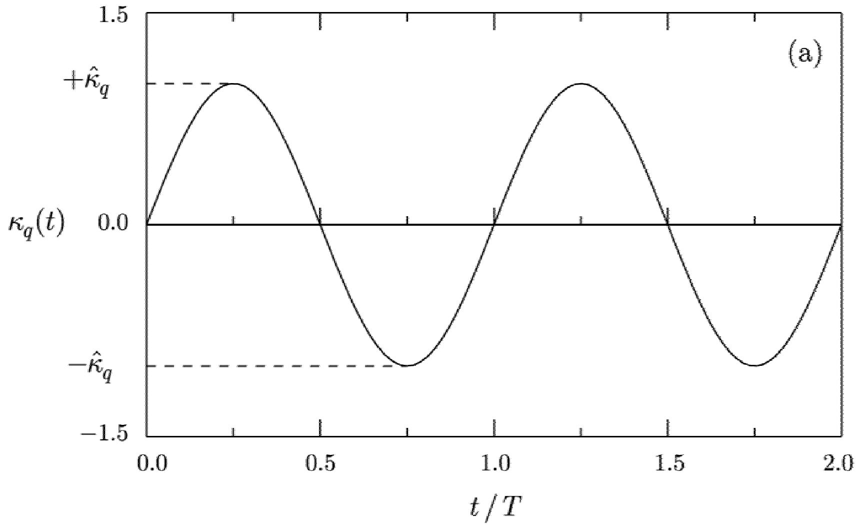
$$\omega_q = \frac{8 e_b V_{0\,\max}}{m_b \pi r_w^2 f} \xi$$

- Here, $V_{0\,\max}$ is the maximum value of V_0(t) and $f = 1/T$ is the frequency.
- The quantity ξ is defined by

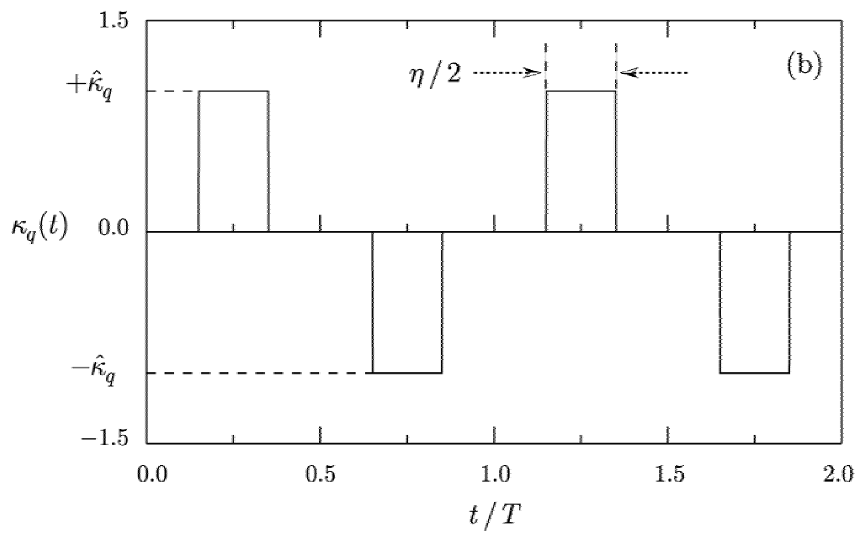
$$\xi = \frac{1}{2\sqrt{2}\pi} \text{ for a sinusoidal waveform, and}$$

$$\xi = \frac{\eta\sqrt{3-2\eta}}{4\sqrt{3}} \text{ for a periodic step - function waveform with fill factor } \eta.$$

Waveform Examples



$$\kappa_q(t) = \hat{\kappa}_q \sin\left(\frac{2\pi t}{T}\right)$$



$$\kappa_q(t) = \hat{\kappa}_q \begin{cases} 1, & (0.25-\eta/4) < t/T < (0.25+\eta/4) \\ -1, & (0.75-\eta/4) < t/T < (0.75+\eta/4) \\ 0, & \text{otherwise} \end{cases}$$

Constraints on Parameters

- Requirement for radial confinement

$$\frac{\omega_p}{\sqrt{2}} < \omega_q$$

- For validity of smooth focusing approximation and to avoid the envelope instability, choose

$$\omega_q < \frac{1}{5} 2\pi f$$

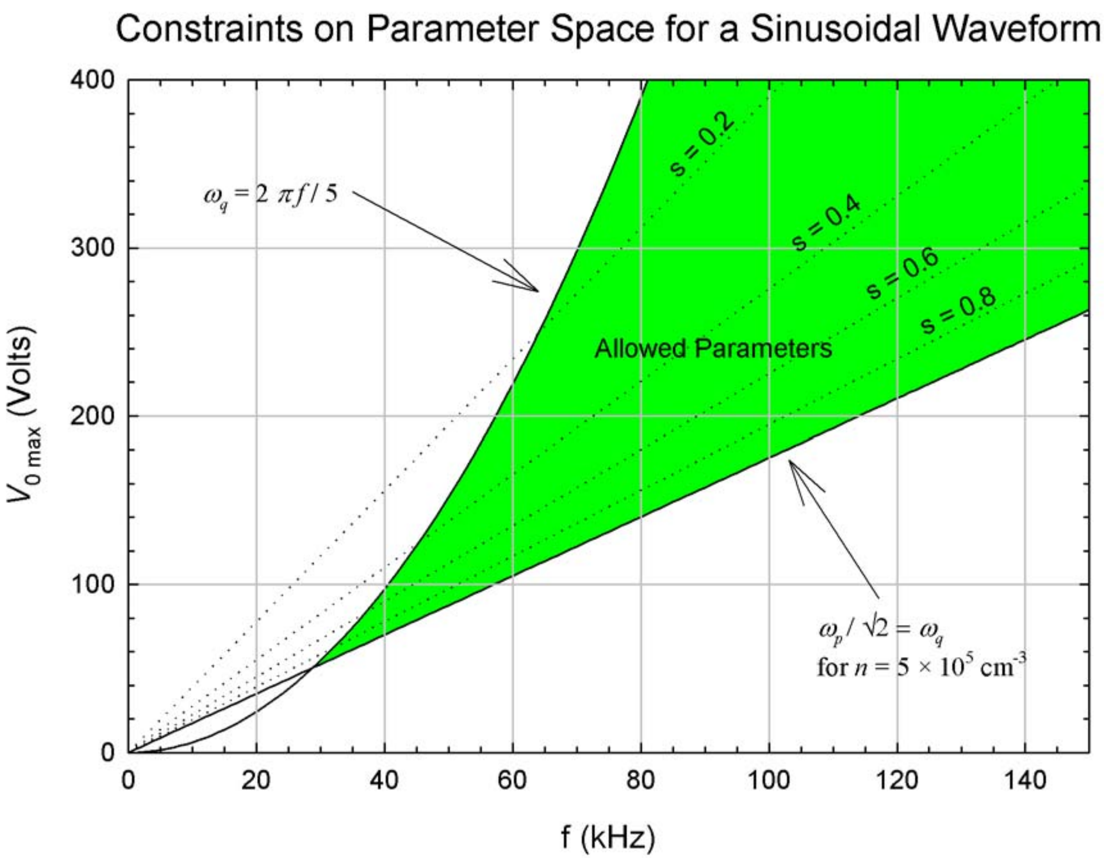
which corresponds to a vacuum phase advance $\sigma_v < 72°$.

- Combining the inequalities gives (for cesium)

$$64.4\, n^{1/2} < 1.46 \times 10^8 \xi \frac{V_{0\,\max}}{f} < f$$

where n is in cm^{-3}, $V_{0\,\max}$ is in Volts, and f is in Hz.

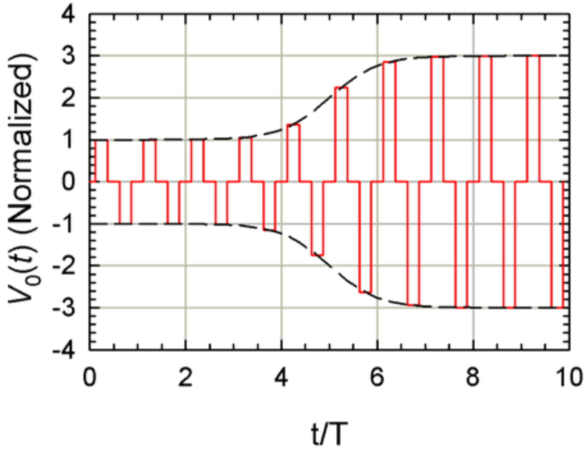
Constraints on Parameter Space

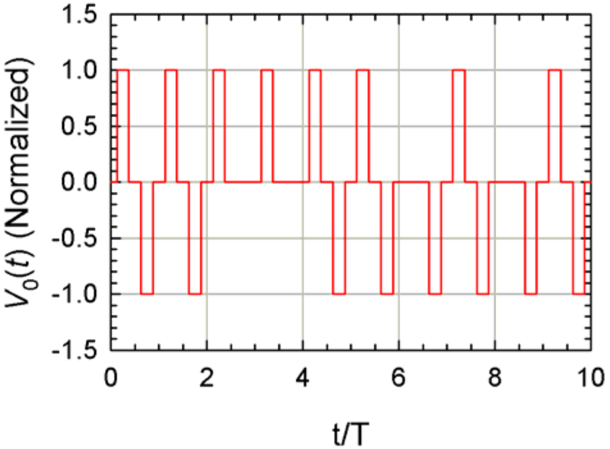


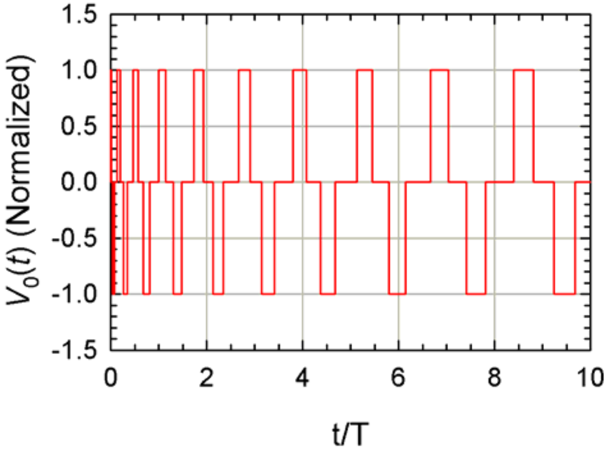
- Here, $s = \omega_p^2/2\omega_q^2$
- $s << 1$ implies "emittance-dominated" beams.
- $s \sim 1$ implies "space-charge-dominated" beams.

$$64.4\, n^{1/2} < 1.46\times 10^8 \xi \frac{V_{0\,\max}}{f} < f$$

Waveform Examples







- "Carrier" waveform is arbitrary.
- Individual electrodes will eventually be allowed to have different waveforms.

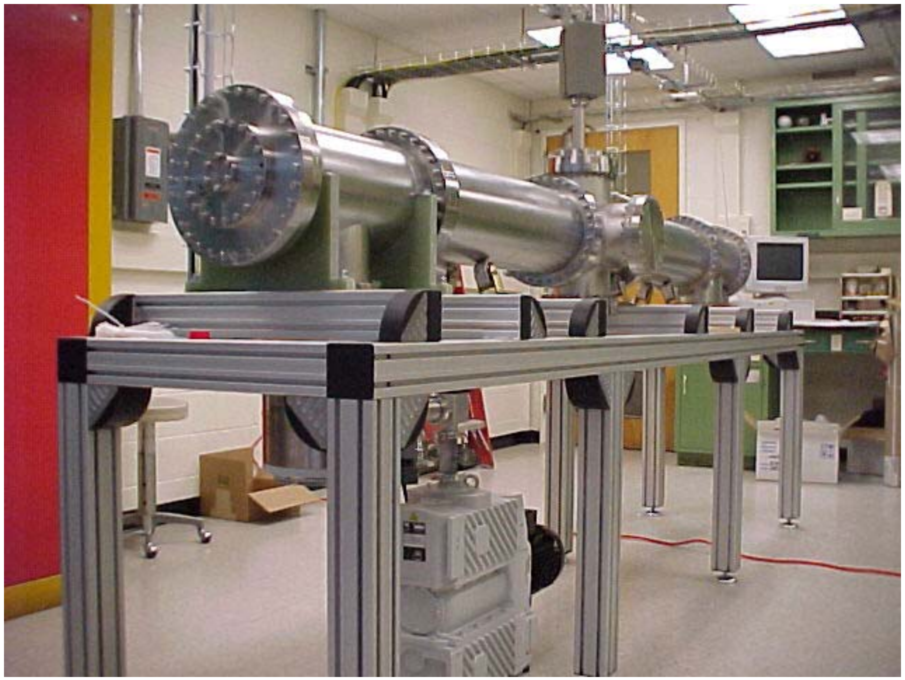
Paul Trap Simulator Experiment

Planned experimental studies include:

- Beam mismatch and envelope instabilities.
- Collective wave excitations.
- Chaotic particle dynamics and production of halo particles.
- Mechanisms for emittance growth.
- Effects of distribution function on stability properties.

Plasma is formed using a cesium source or a barium coated platinum or rhenium filament. Plasma microstate will be determined using laser-induced fluorescence (Levinton, FP&T).

Paul Trap Simulator Experiment



Paul Trap Simulator Experiment vacuum chamber.

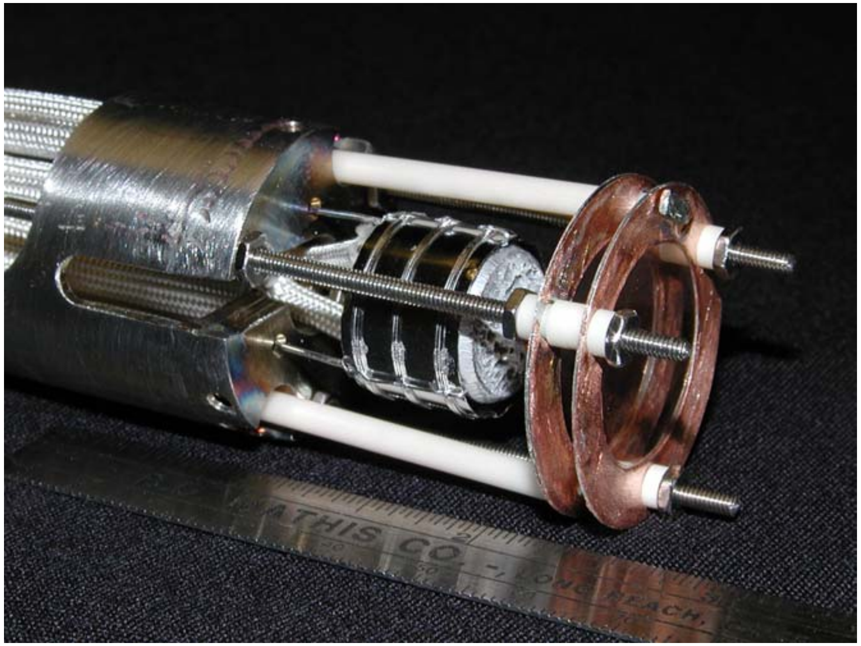
- Laboratory preparation, procurement, assembly, bakeout, and pumpdown of PTSX vacuum chamber to 5.25×10^{-10} Torr (May, 2002).

Paul Trap Simulator Experiment

Paul Trap Simulator Experiment electrodes.

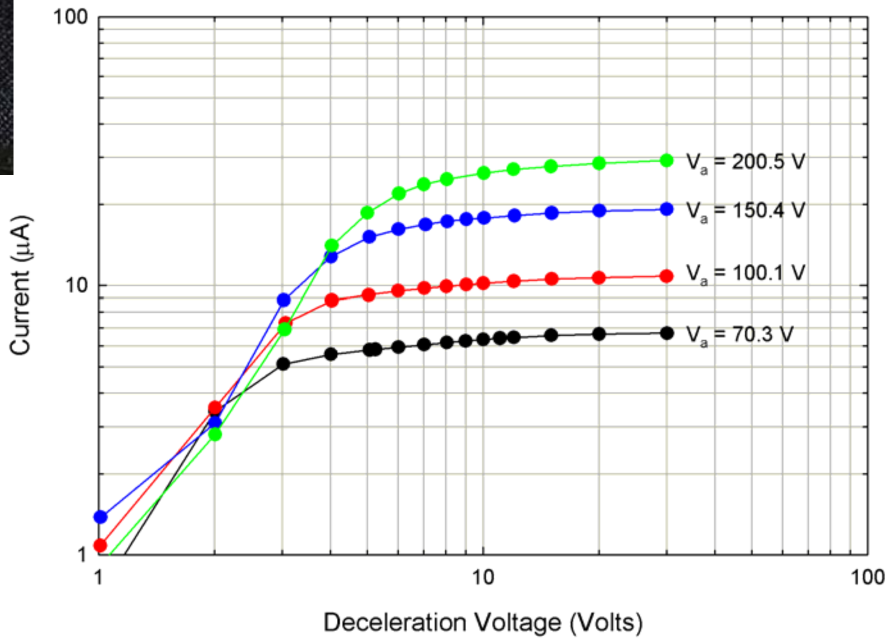
- 8 inch diameter stainless steel gold-plated electrodes are supported by aluminum rings, teflon, and vespel spacers.

Paul Trap Simulator Experiment

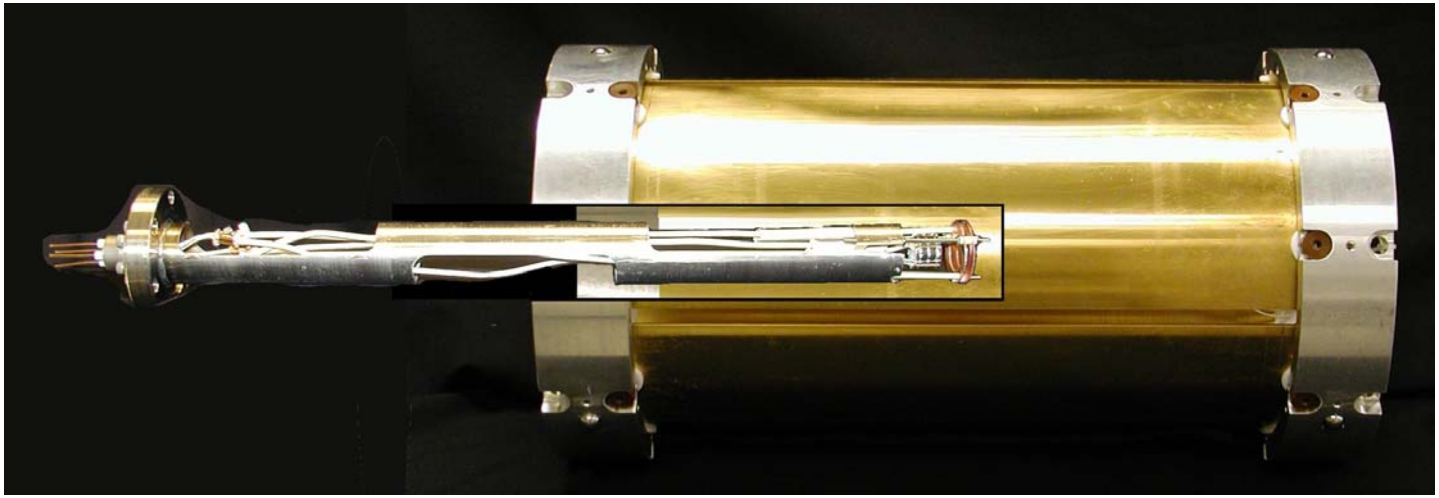


Paul Trap Simulator Experiment cesium source.

- Aluminosilicate cesium source produces up to 30 μA of ion current when a 200 V acceleration voltage is used.



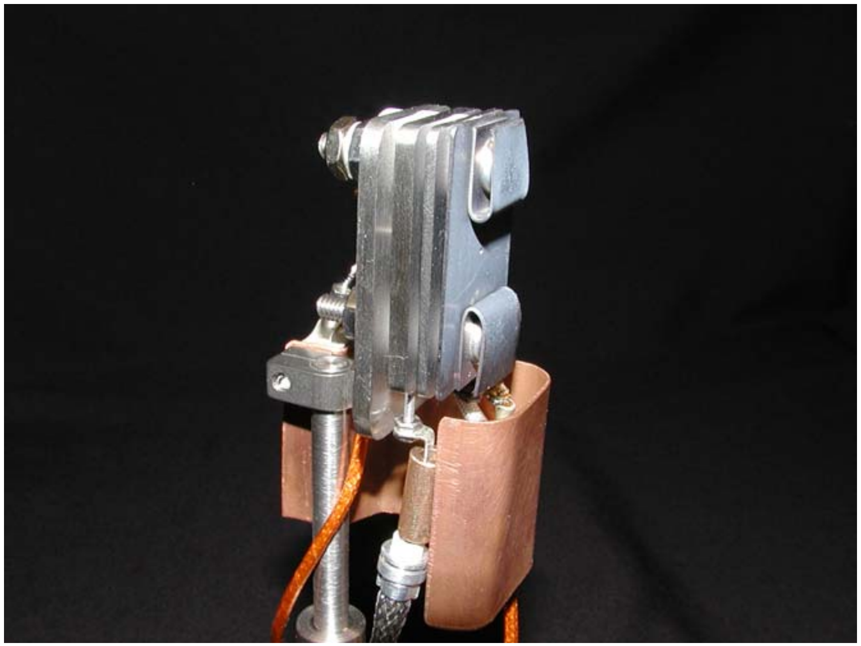
Ion Injection



Computer generated composite image of cut-away view of electrodes surrounding ion source.

- The electrodes oscillate with the voltage $\pm V_0(t)$ during ion injection.
- The ion source acceleration voltage is turned off as the electrodes are switched to a constant voltage to axially trap the ions.
- The 40 cm long electrodes at the far end of the trap are held at a constant voltage during injection to prevent ions from leaving the far end of the trap.

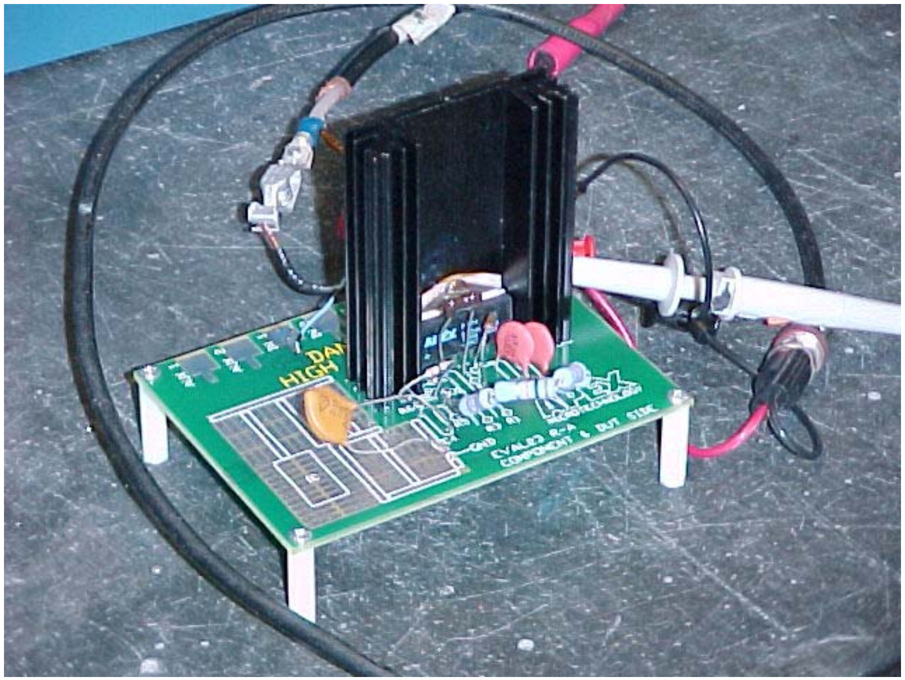
Paul Trap Simulator Experiment



Paul Trap Simulator Experiment Faraday cup.

- Faraday cup with sensitive electrometer allows 20 fC resolution.
- Linear motion feedthrough with 6" stroke allows measurement of radial density dependence. 1 mm diameter aperture gives fine spatial resolution.
- Copper shield is to be modified to further reduce impact of stray ions.

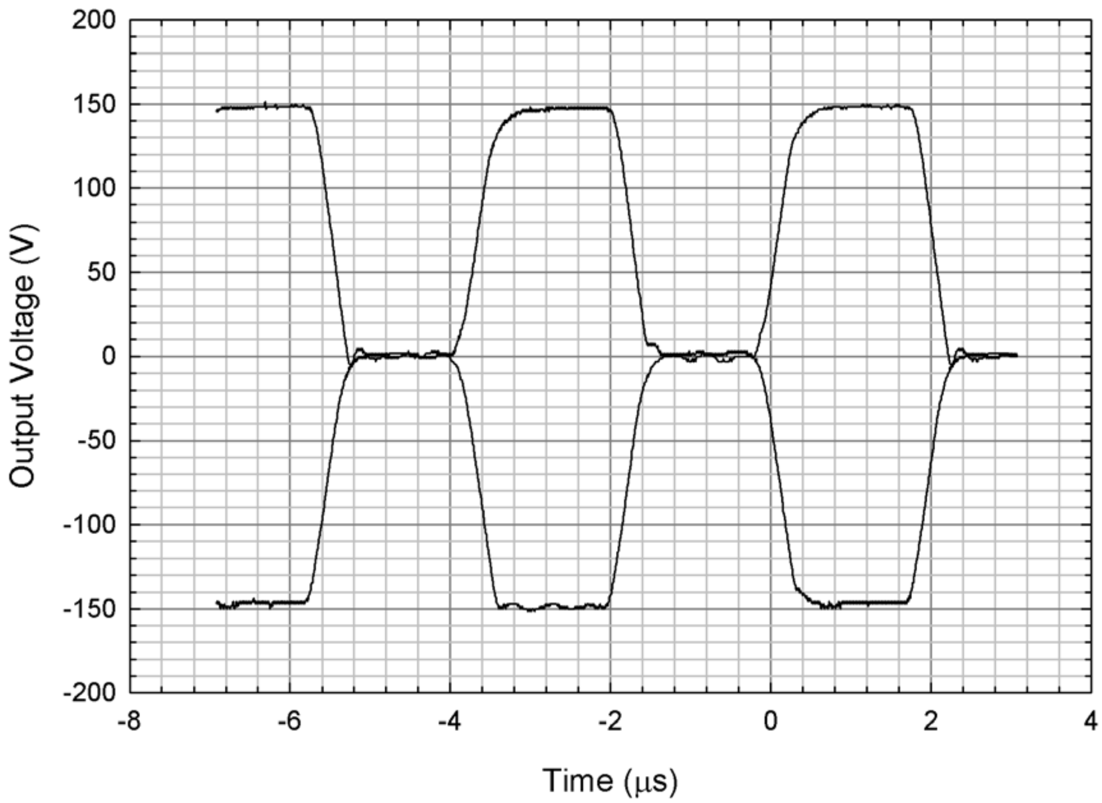
Paul Trap Simulator Experiment



Paul Trap Simulator Experiment electrode driver test circuit.

- Electrode driver development using high voltage power op-amp to apply 400 V, 100 kHz signals to electrodes (February, 2002).
- 8 op-amps are used to drive the 12 electrodes.

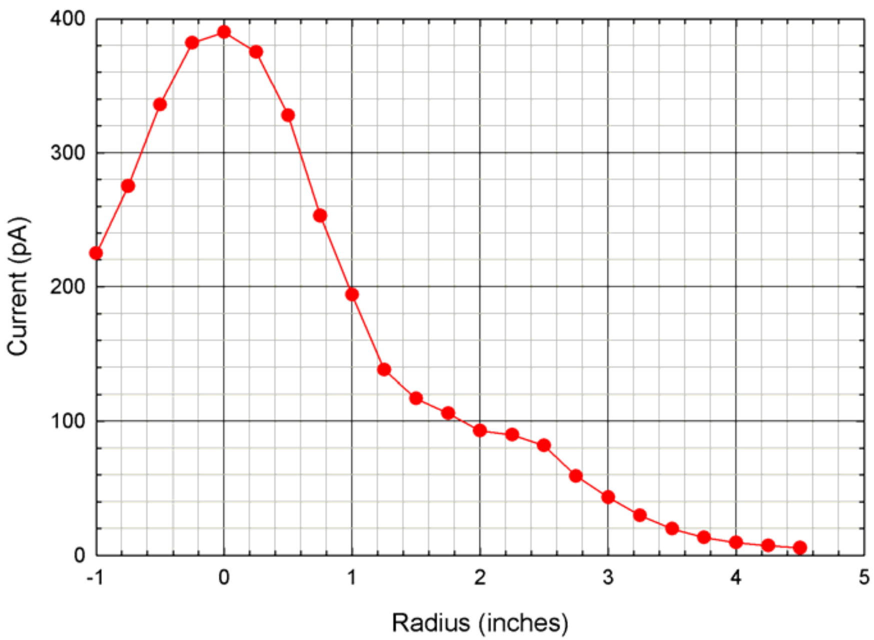
Applied Waveforms



$f = 133$ kHz

$\eta = 0.5$

Paul Trap Simulator Experiment Initial Results



Current collected on Faraday cup versus radius.

- Experiment - stream Cs^+ ions from source to collector without axial trapping of the plasma.

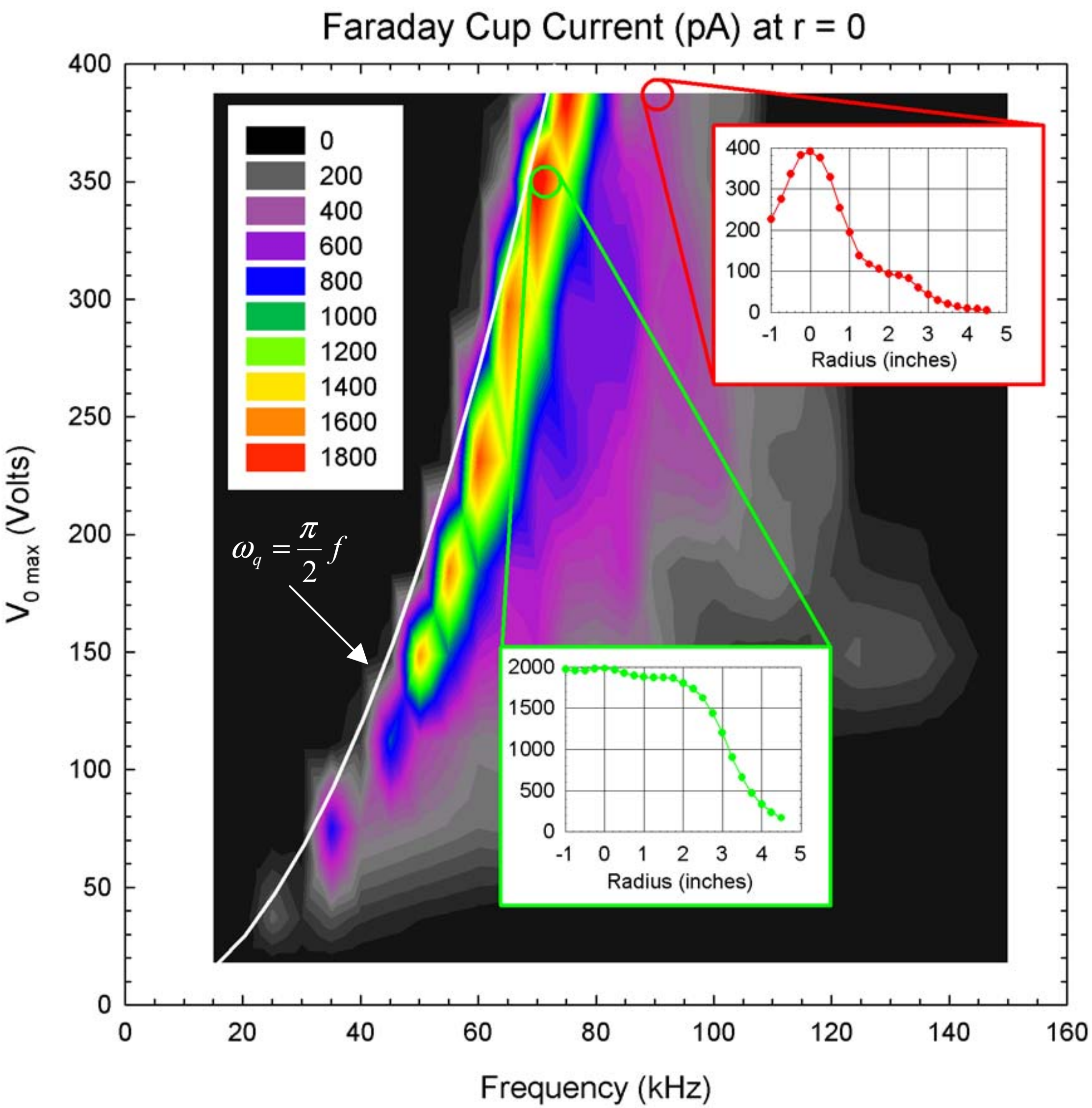
Electrode parameters:

- $V_0(t) = V_{0\,\max} \sin(2\pi f t)$
- $V_{0\,\max} = 387.5$ V
- $f = 90$ kHz

Ion source parameters:

- $V_{accel} = -183.3$ V
- $V_{decel} = -5.0$ V

Instability of Single Particle Orbits



Paul Trap Simulator Experiment

Future Plans:

- Axially trap ions.
- Characterize trapped plasma properties such as density profile and lifetime.
- Optimize injection for well-behaved plasmas.
- Modify Faraday cup shielding to reduce pick-up of stray ions.
- Optimize hardware and software systems for precise control.
- Develop barium ion source and laser system for use in a Laser-Induced-Fluorescence diagnostic system.
- Computer simulation of injection, trapping, and dumping.