Experiments on Transverse Bunch Compression on the Princeton Paul Trap Simulator Experiment*

Erik P. Gilson
Princeton Plasma Physics Laboratory

2007 Particle Accelerator Conference

Albuquerque, NM

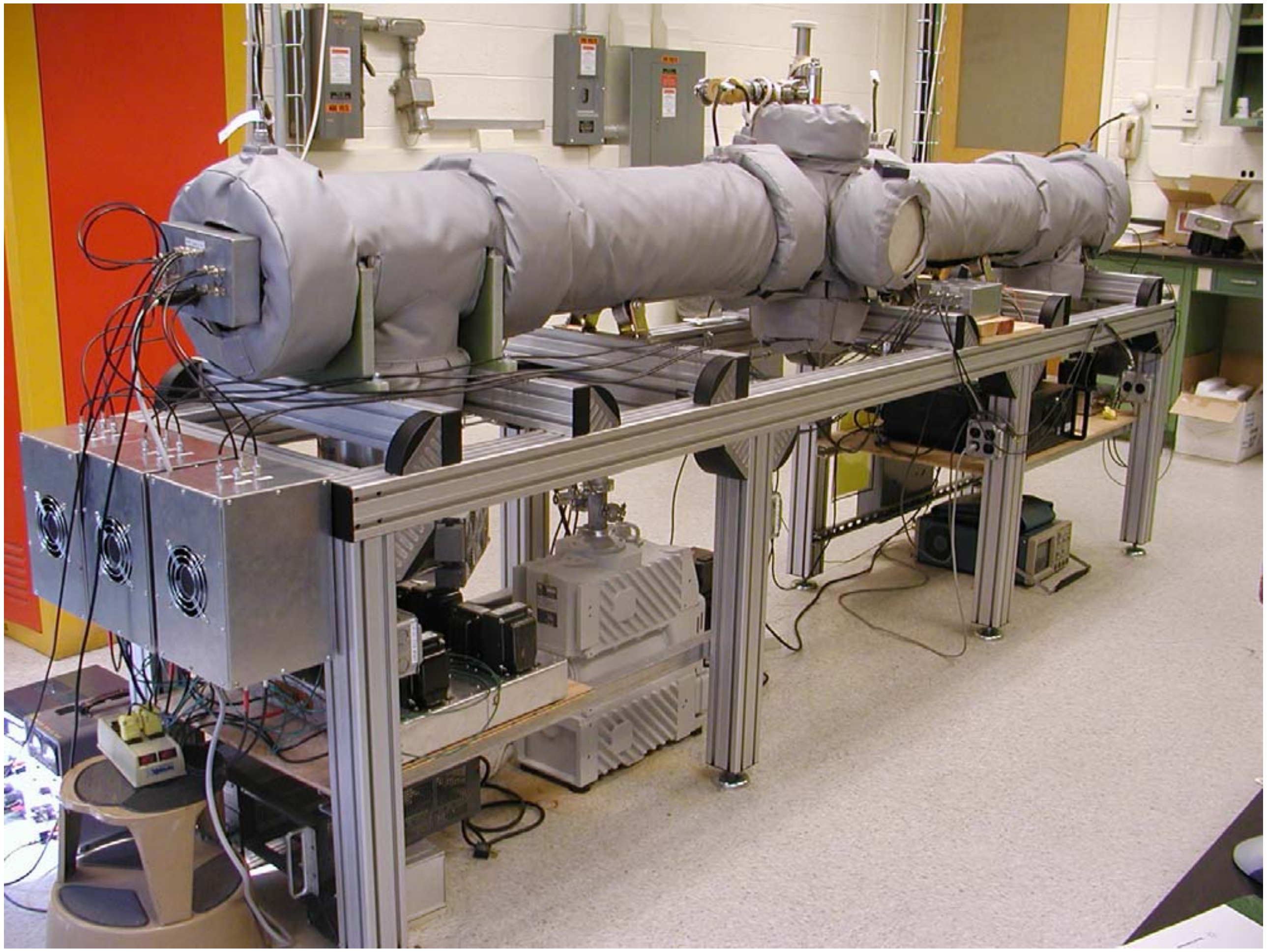
June 26th, 2007

In collaboration with: Moses Chung, Ronald C. Davidson, Mikhail Dorf, Philip C. Efthimion, Richard Majeski, and Edward A. Startsev

*This work is supported by the U.S. Department of Energy.

PTSX Simulates Nonlinear Beam Dynamics in Magnetic Alternating-Gradient Systems

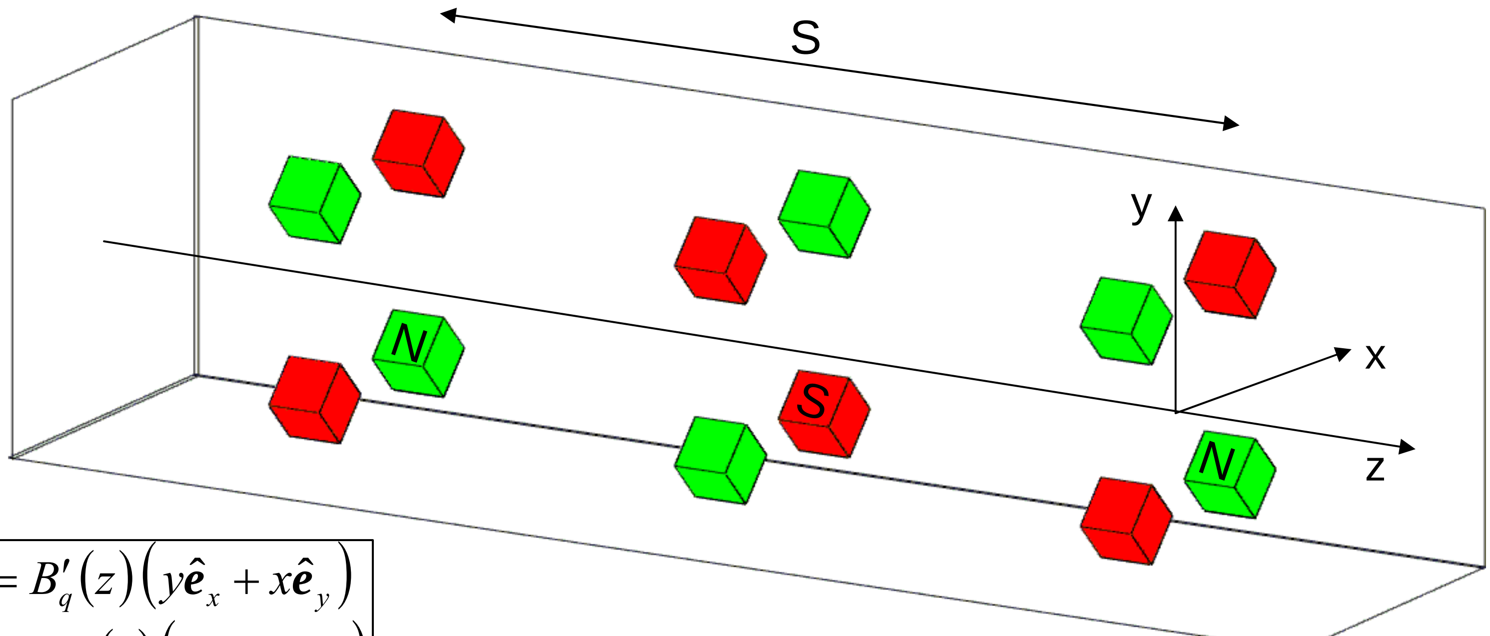
Purpose: Simulate the nonlinear transverse dynamics of intense beam propagation over large distances through magnetic alternating-gradient transport systems in *a compact experiment*.



Scientific Motivation

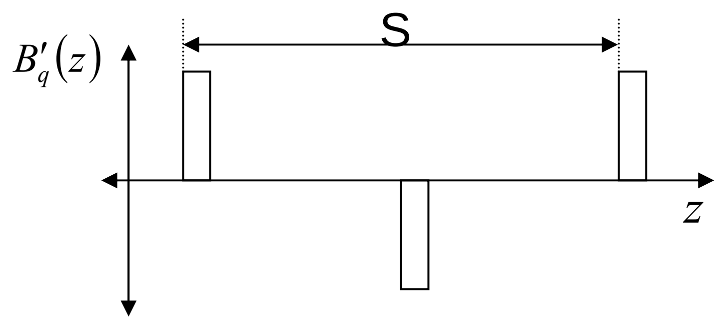
- Beam mismatch and envelope instabilities
- Collective wave excitations
- Chaotic particle dynamics and production of halo particles
- Mechanisms for emittance growth
- Effects of distribution function on stability properties
- Quiescent propagation over thousands of lattice periods
- **Transverse compression techniques**

Magnetic Alternating-Gradient Transport Systems

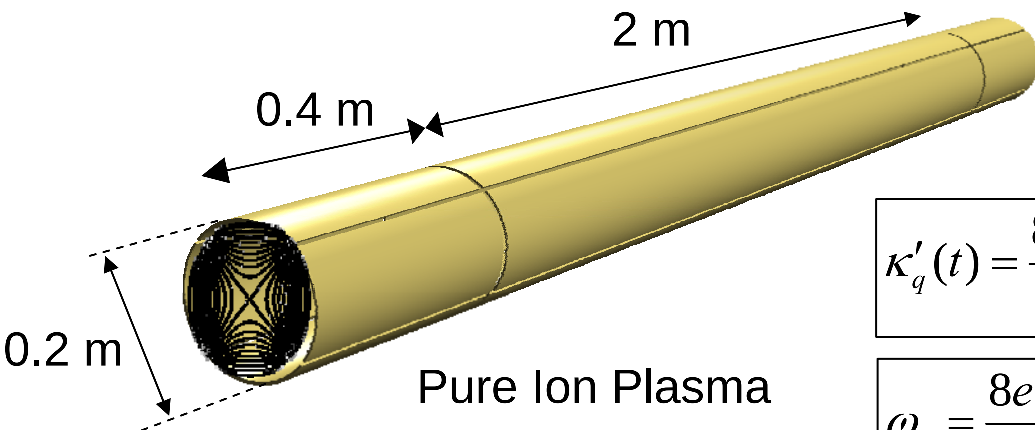


$$\boldsymbol{B}_q^{foc}(\boldsymbol{x}) = B_q'(z)\left(y\hat{\boldsymbol{e}}_x + x\hat{\boldsymbol{e}}_y\right)$$
$$\boldsymbol{F}_{foc}(\boldsymbol{x}) = -\kappa_q(z)\left(x\hat{\boldsymbol{e}}_x - y\hat{\boldsymbol{e}}_y\right)$$

$$\kappa_q(z) \equiv \frac{ZeB_q'(z)}{\gamma m \beta c^2}$$



PTSX Configuration – A Cylindrical Paul Trap



$$e\phi_{ap}(x,y,t)=\frac{1}{2}\kappa_q'(t)(x^2-y^2)$$

$$\kappa_q'(t)=\frac{8eV_0(t)}{m\pi r_w^2}$$

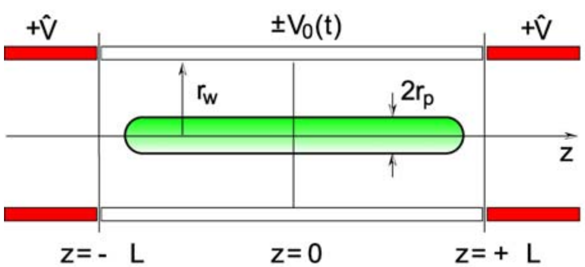
sinusoidal in this work

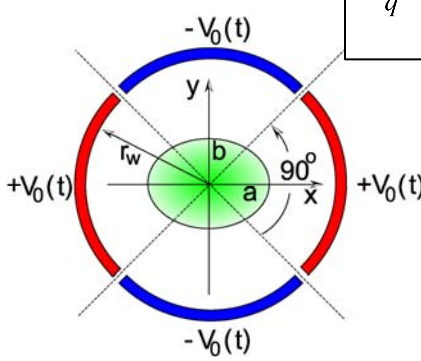
$$\omega_q=\frac{8eV_{0\,\max}}{m\pi r_w^2 f}\xi$$

$$\xi=\frac{1}{2\sqrt{2}\pi}$$ for a sinusoidal waveform $V(t)$.

$$\xi=\frac{\eta\sqrt{3-2\eta}}{4\sqrt{3}}$$ for a periodic step function waveform $V(t)$ with fill factor η.

When η = 0.572, they're equal.





Plasma length	2 m	Maximum wall voltage	~ 400 V
Wall radius	10 cm	End electrode voltage	< 150 V
Plasma radius	~ 1 cm	Frequency	< 100 kHz
Cesium ion mass	133 amu	Pressure	5×10^{-9} Torr
Ion source grid voltages	< 10 V		

Transverse Dynamics are the Same Including Self-Field Effects

If…

- Long coasting beams
- Beam radius << lattice period
- Motion in beam frame is nonrelativistic

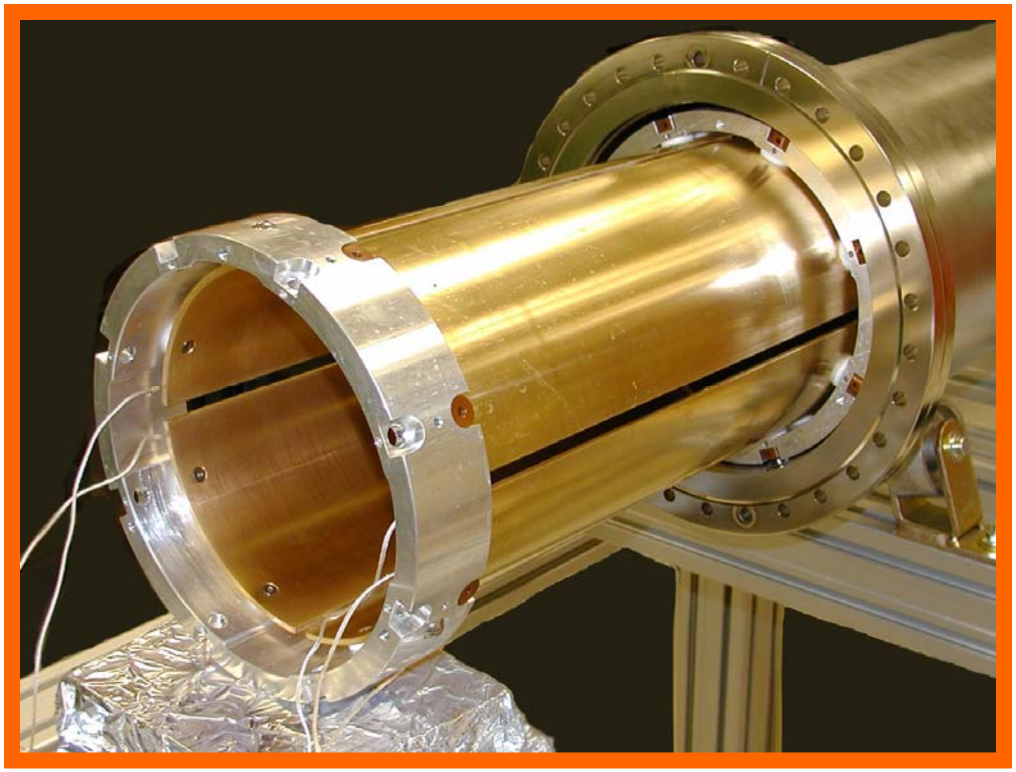
Then, when in the beam frame, both systems have…

- Quadrupolar external forces
- Self-forces governed by a Poisson-like equation
- Distributions evolve according to nonlinear Vlasov-Maxwell equation

Ions in PTSX have the same transverse equations of motion as ions in an alternating-gradient system ***in the beam frame***.

Electrodes, Ion Source, and Collector

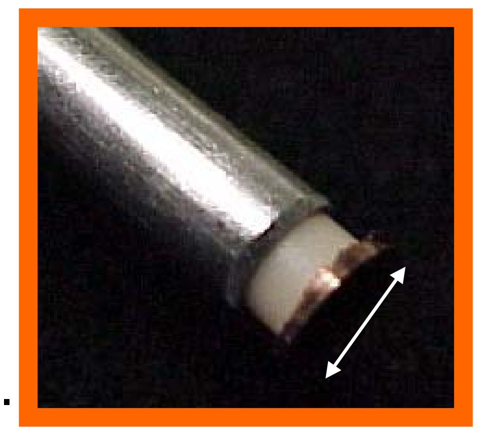
Broad flexibility in applying V(t) to electrodes with arbitrary function generator.



Increasing source current creates plasmas with intense space-charge.

1.25 in

Large dynamic range using sensitive electrometer.



5 mm

Measures average Q(r).

Force Balance and Normalized Intensity *s*

If $p = n\,kT$, then the statement of local force balance on a fluid element can be integrated over a radial density distribution such as,

$$\boxed{n(r) = n(0)\exp\left[-\frac{m\omega_q^2 r^2 + 2q\phi^s(r)}{2kT}\right]}$$

to give the global force balance equation,

$$\boxed{m\omega_q^2 R^2 = 2kT + \frac{Nq^2}{4\pi\varepsilon_o}}$$

$$\boxed{s \equiv \frac{\omega_p^2}{2\omega_q^2} < 1}$$

$$\boxed{\frac{\upsilon}{\upsilon_0} = (1-s)^{1/2}}$$

for a flat-top radial density distribution

$$\boxed{\omega_p^2 = \frac{n(0)q^2}{m\varepsilon_0}}$$

s	ν/ν_0
0.1	0.95
0.2	0.90
0.3	0.84
0.4	0.77
0.5	0.71
0.6	0.63
0.7	0.55
0.8	0.45
0.9	0.32
0.99	0.10
0.999	0.03

PTSX-accessible

Transverse Bunch Compression by Increasing ω_q

$$\boxed{m\omega_q^2 R^2 = 2kT + \frac{Nq^2}{4\pi\varepsilon_o}}$$

If line density N is constant and kT doesn't change too much, then increasing ω_q decreases R, and the bunch is compressed.

$$\boxed{\omega_q = \frac{8eV_{0\max}}{m\pi r_w^2 f}\xi}$$

Either

1.) increasing $V_{0\max}$ (increasing magnetic field strength) or

2.) decreasing f (increasing the magnet spacing)

increases ω_q

Instantaneous Changes in the Voltage and Frequency Do Not Compress the Bunch When ω_q is Fixed

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$$m\omega_q^2 R^2 = 2kT + \frac{Nq^2}{4\pi\varepsilon_o}$$

$s = 0.2$

$kT \sim 0.7$ eV

$N \sim$ constant

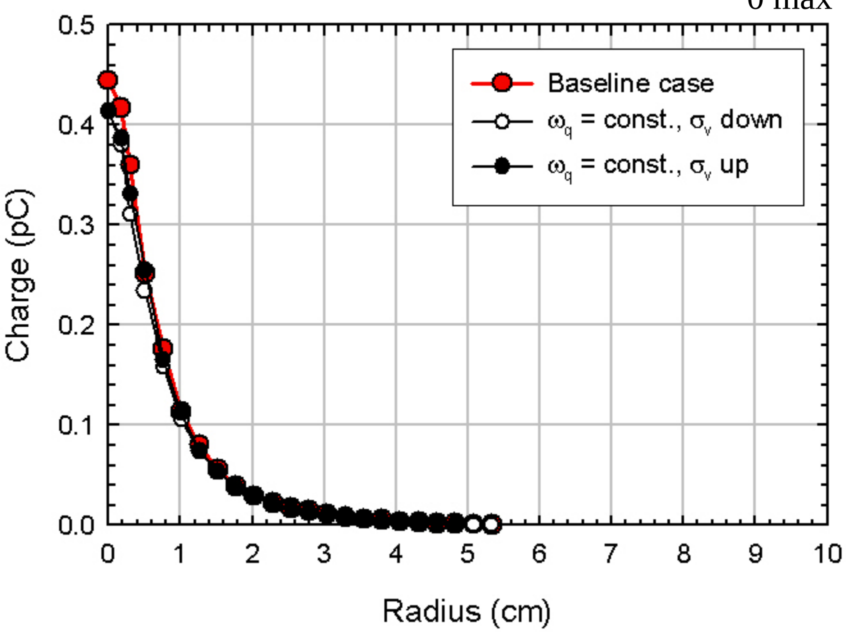
$$\omega_q = \frac{8eV_{0\,\max}}{m\pi r_w^2 f}\xi$$

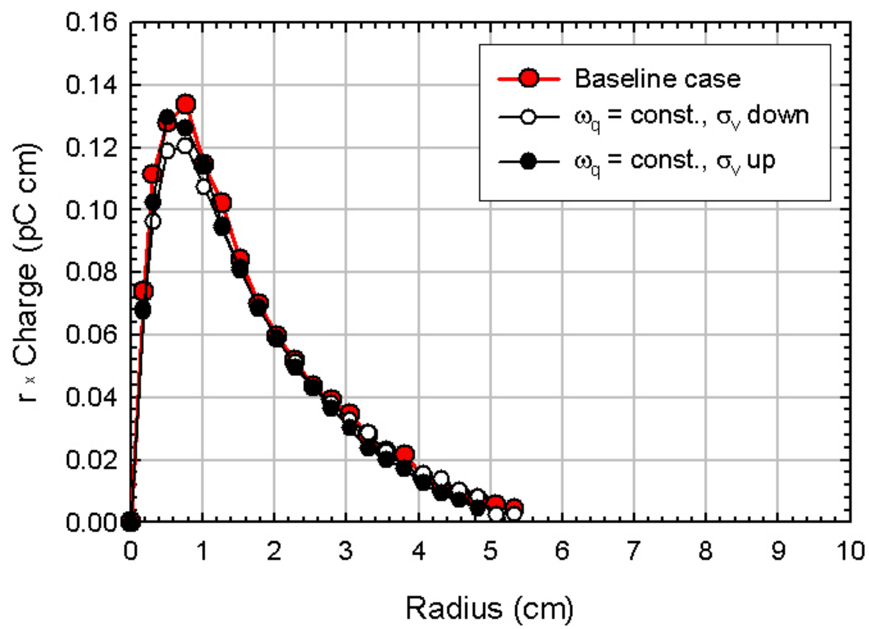
$V_{0\,\max}$ & f up to 1.5X — ○ $\sigma_v = 33^o$

Baseline case — ● (red) $\sigma_v = 50^o$

$V_{0\,\max}$ & f down to 0.66X — ● $\sigma_v = 75^o$

$$\sigma_v = \frac{\omega_q}{f}$$

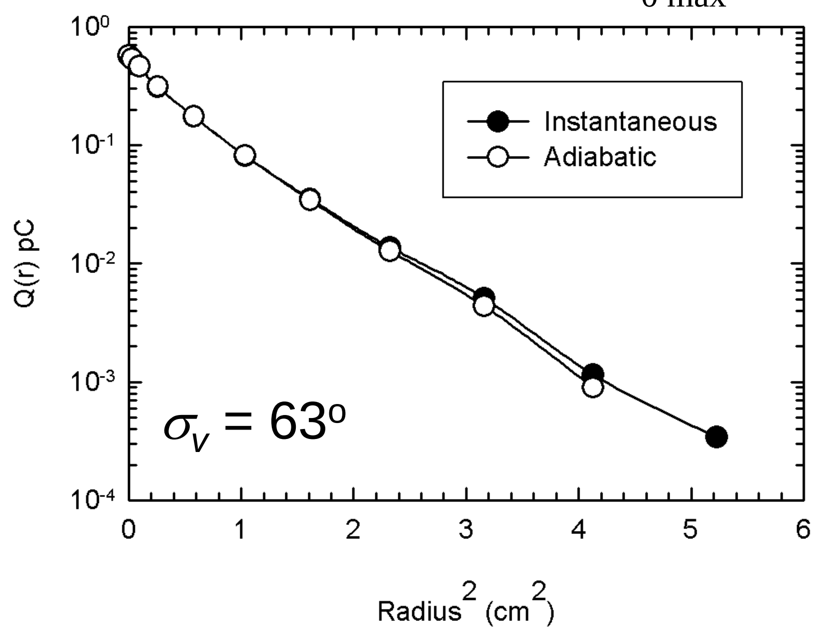




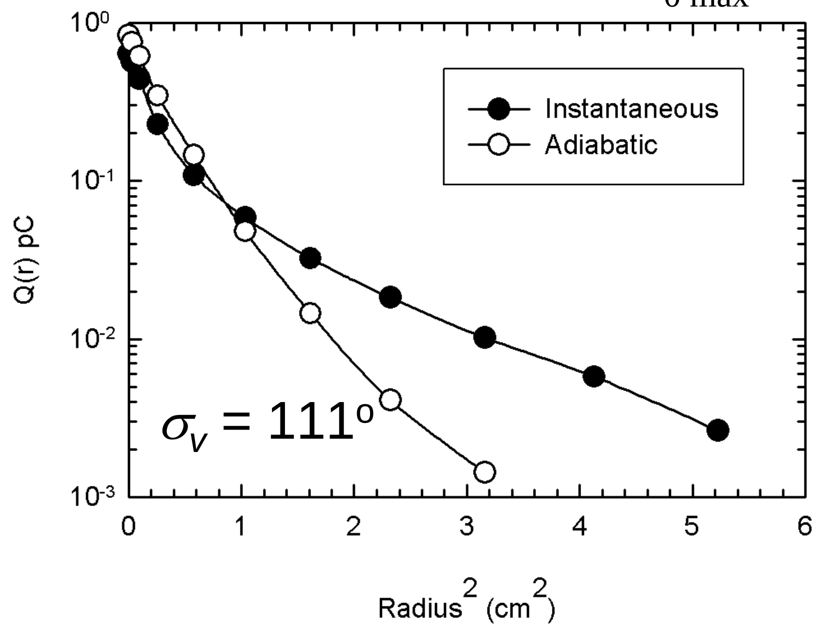
- $s = \omega_p^2/2\omega_q^2 = 0.20$
- $v/v_0 = 0.88$
- $V_{0\,\max} = 150$ V
- $f = 60$ kHz
- $\sigma_v = 49^o$

Adiabatic Amplitude Increases Transversely Compress the Bunch

20% increase in $V_{0\,\max}$



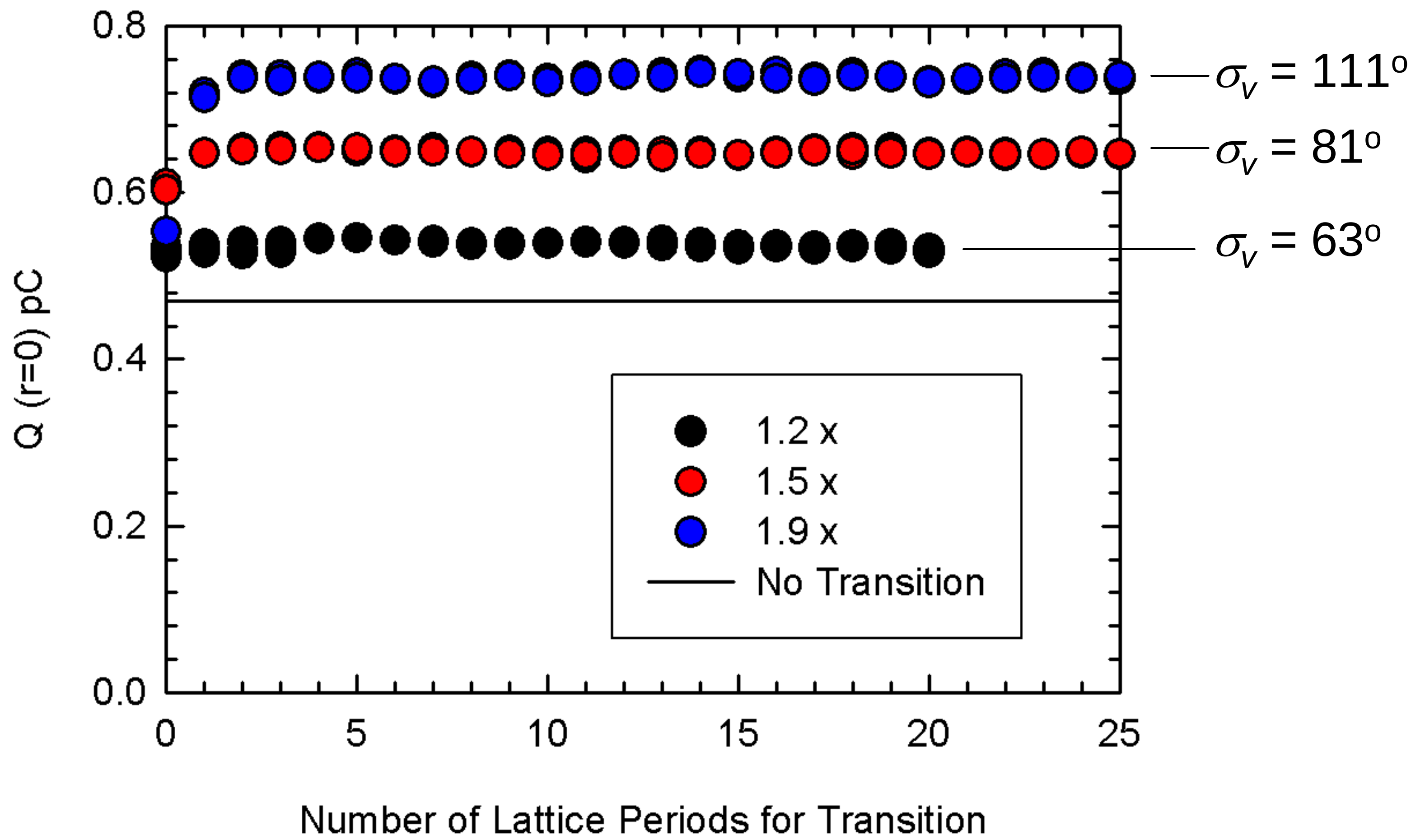
90% increase in $V_{0\,\max}$



Baseline		Adiabatic	Instantaneous
R = 0.83 cm	R = 0.79 cm	R = 0.63 cm	R = 0.93 cm
kT = 0.12 eV	kT = 0.16 eV	kT = 0.26 eV	kT = 0.58 eV
s = 0.20	s = 0.18	s = 0.10	s = 0.08
$\varepsilon \sim R\sqrt{kT} \rightarrow$	$\Delta\varepsilon$ = 10%	$\Delta\varepsilon$ = 10%	$\Delta\varepsilon$ = 140%

- $s = \omega_p^2/2\omega_q^2 = 0.20$
- $\nu/\nu_0 = 0.88$
- $V_{0\,\max}$ = 150 V
- f = 60 kHz
- σ_v = 49º

Less Than Four Lattice Periods Adiabatically Compress the Bunch



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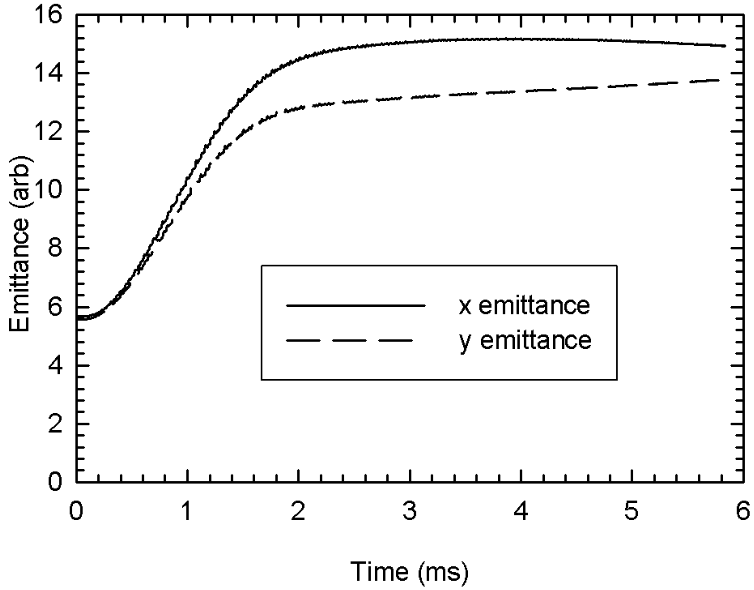
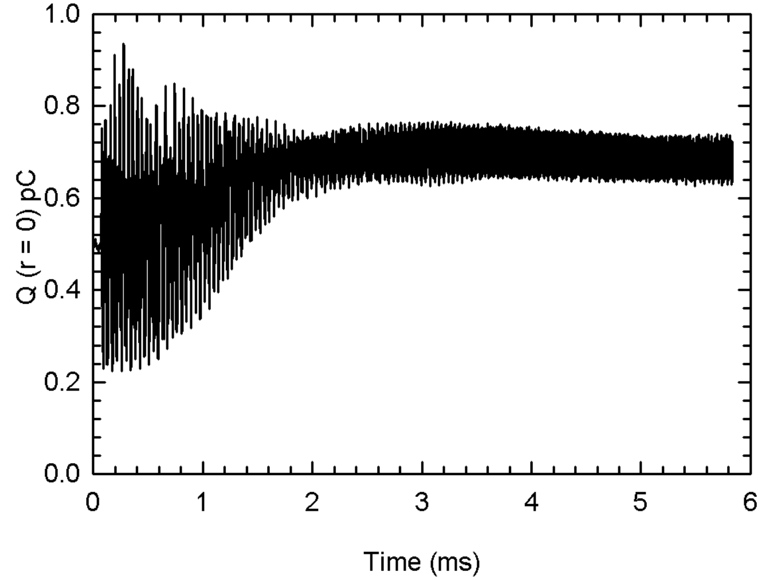
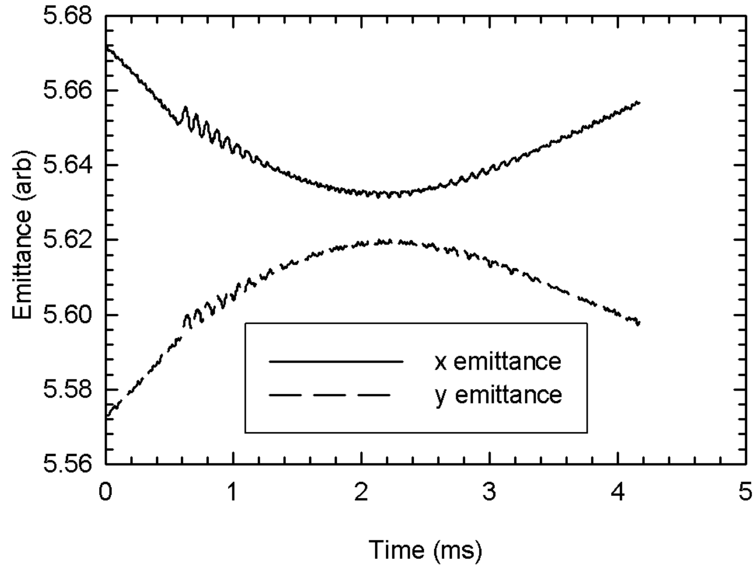
2D WARP PIC Simulations Corroborate Adiabatic Transitions in Only Four Lattice Periods

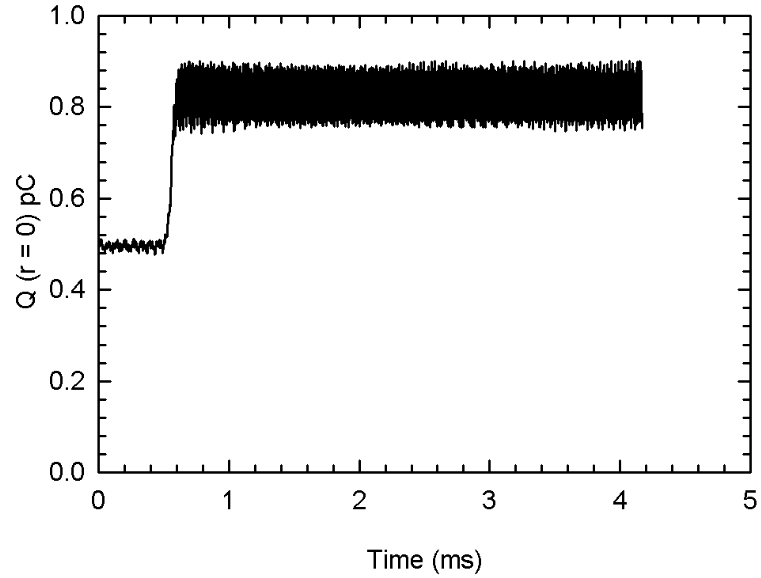
Instantaneous Change.





Change Over Four Lattice Periods.





Peak Density Scales Linearly With ω_q

$$m\omega_q^2 R^2 \sim 2kT$$

$$\varepsilon \sim R\sqrt{kT}$$

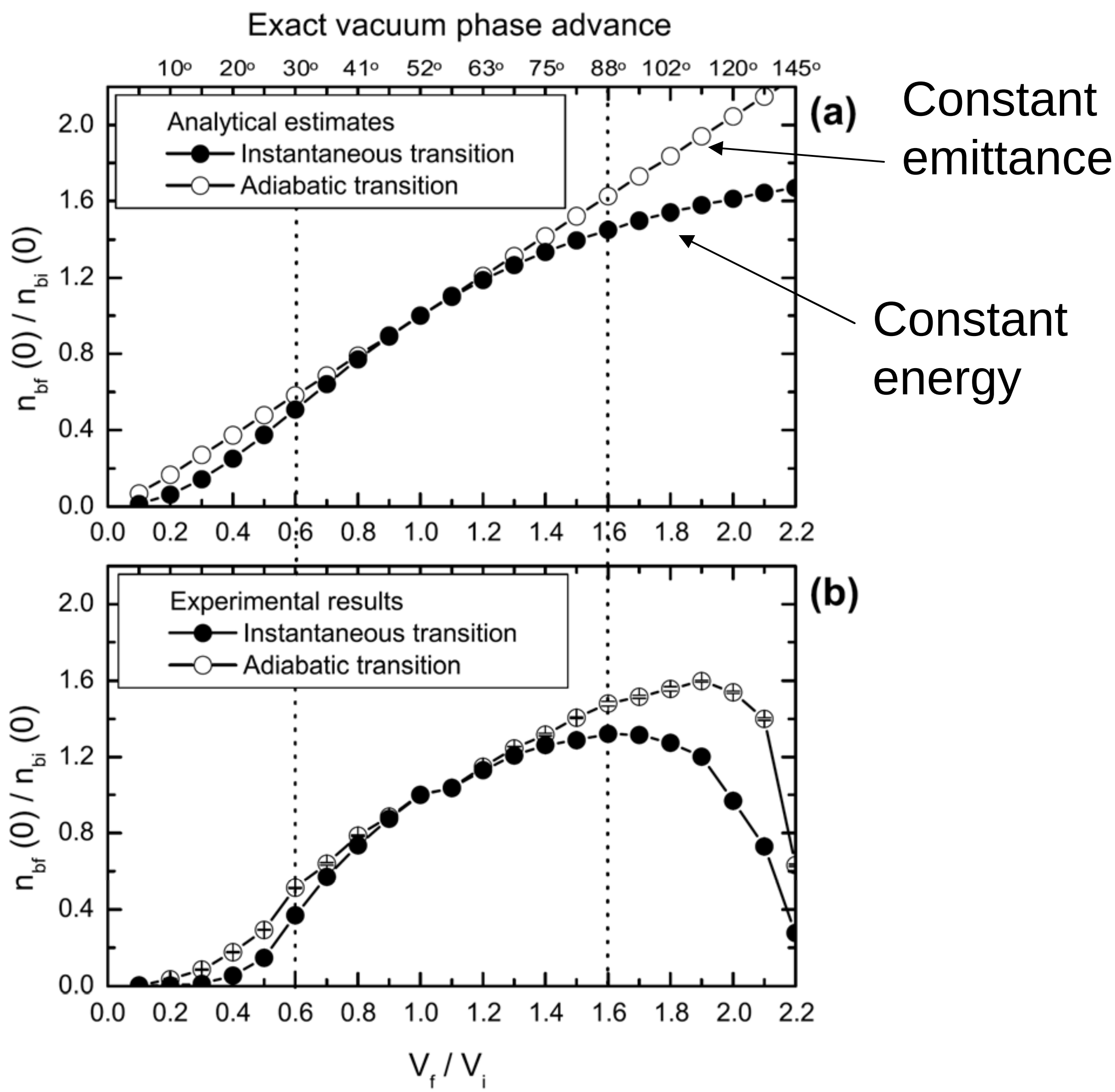
$$\Downarrow$$

$$\omega_q R^2 \sim const.$$

$$n(0)R^2 \sim N = const.$$

$$n(0) \sim \omega_q$$

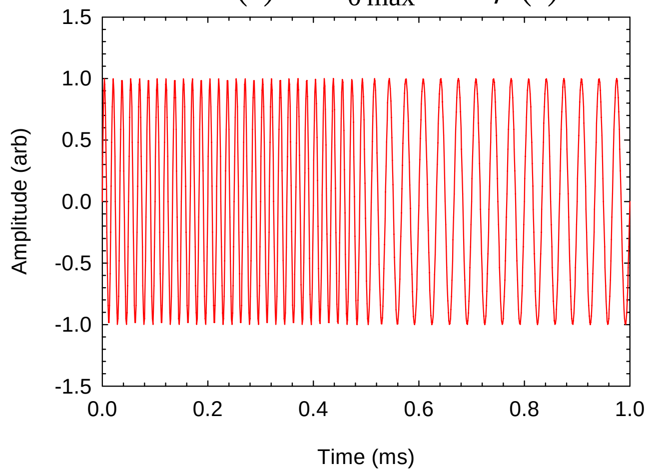
$$\omega_q = \frac{8eV_{0\max}}{m\pi r_w^2 f}\xi$$

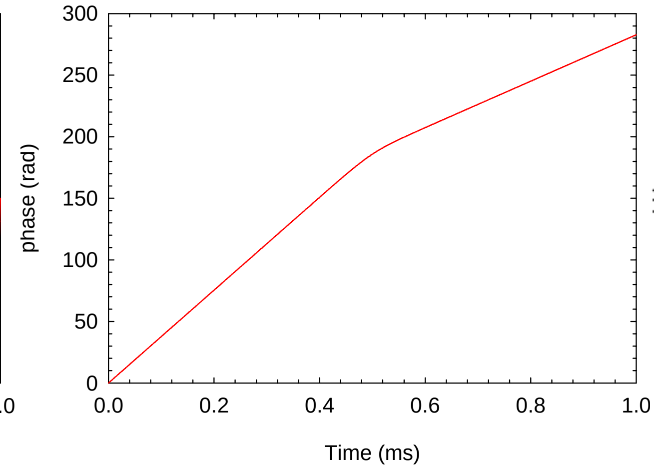


Increasing ω_q by Adiabatically Decreasing *f*

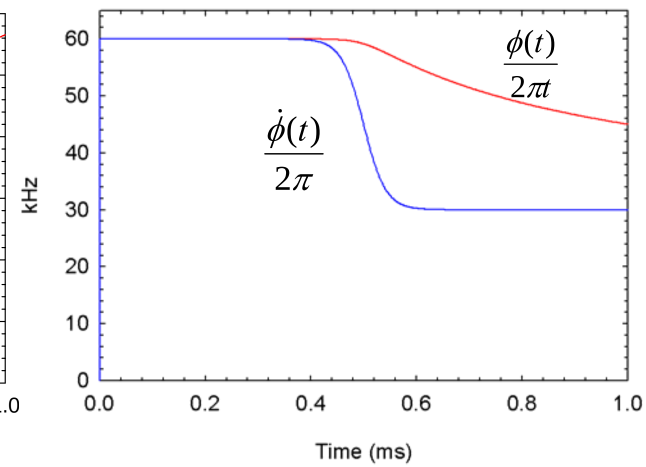
$$\boxed{\omega_q = \frac{8eV_{0\,\max}}{m\pi r_w^2 f}\xi}$$

$$V(t) = V_{0\,\max}\sin\phi(t)$$





$$\frac{\phi(t)}{2\pi} = \frac{f_i + f_f}{2}t + \frac{f_f - f_i}{4}\tau\ln\left[\cosh\frac{-(t - t_{1/2})}{\tau/2}\right] - f_0$$



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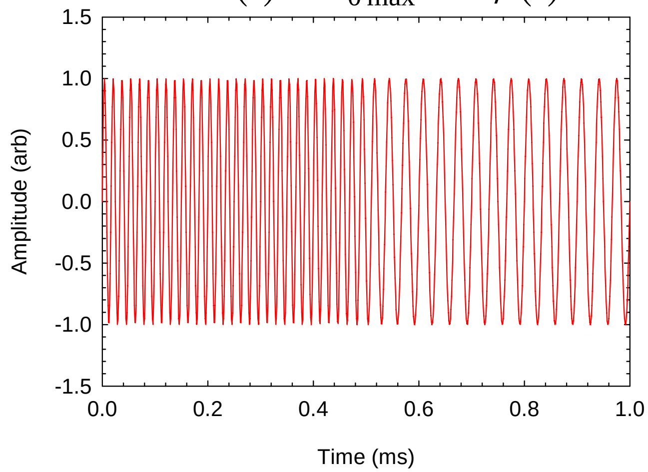
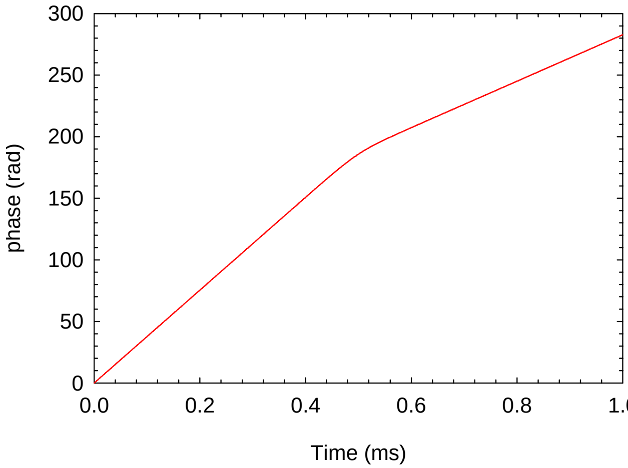
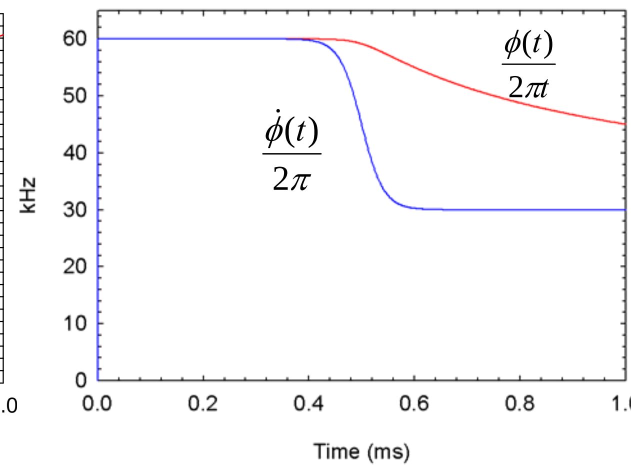
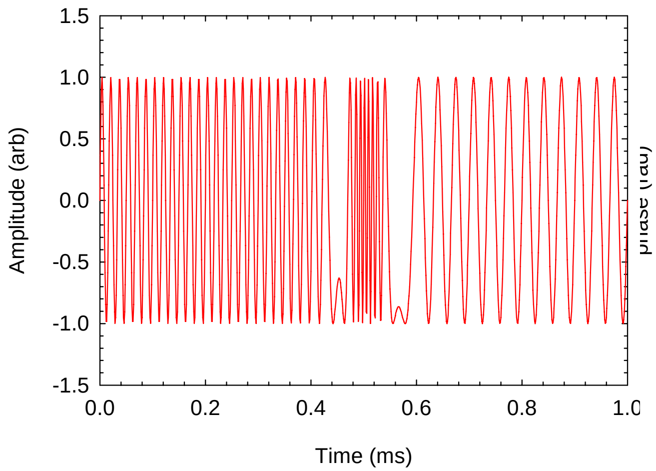


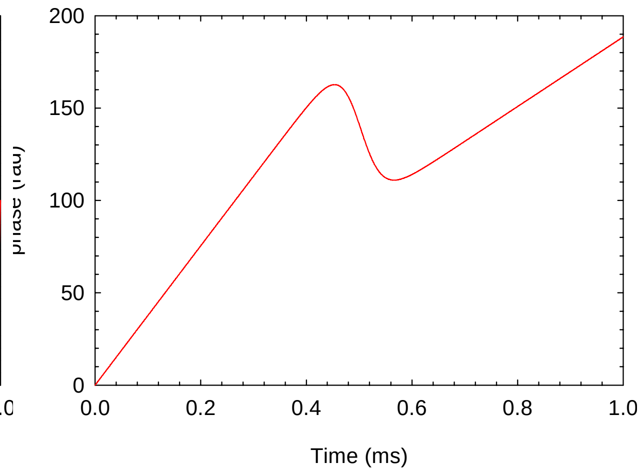


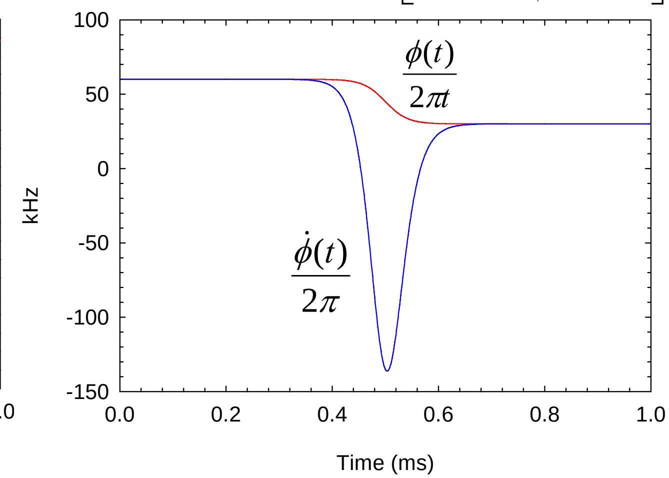


$$\frac{\phi(t)}{2\pi} = f_f t + \frac{f_i - f_f}{2}t\left[\tanh\frac{-(t - t_{1/2})}{\tau/2} + 1\right]$$

$$V(t) = V_{0\,\max}\sin\phi(t)$$

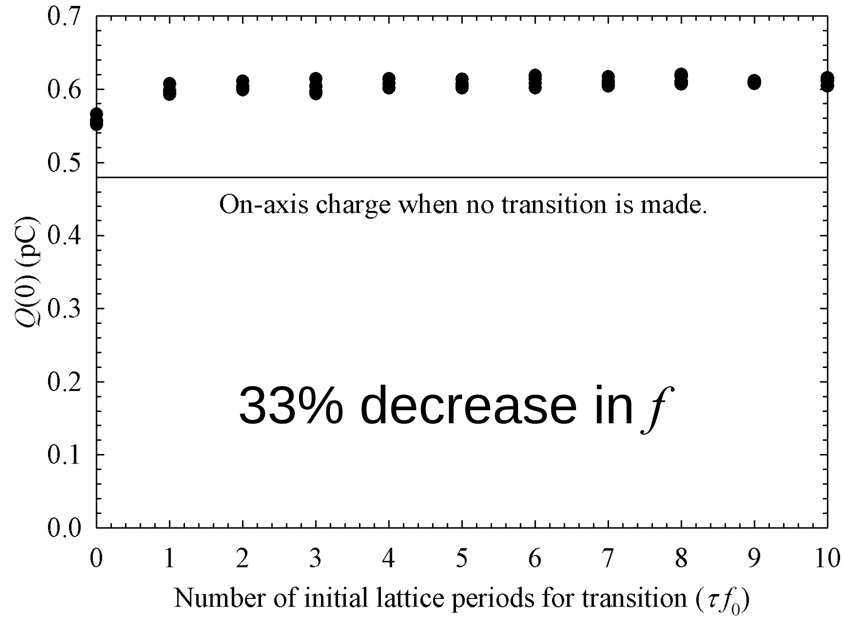




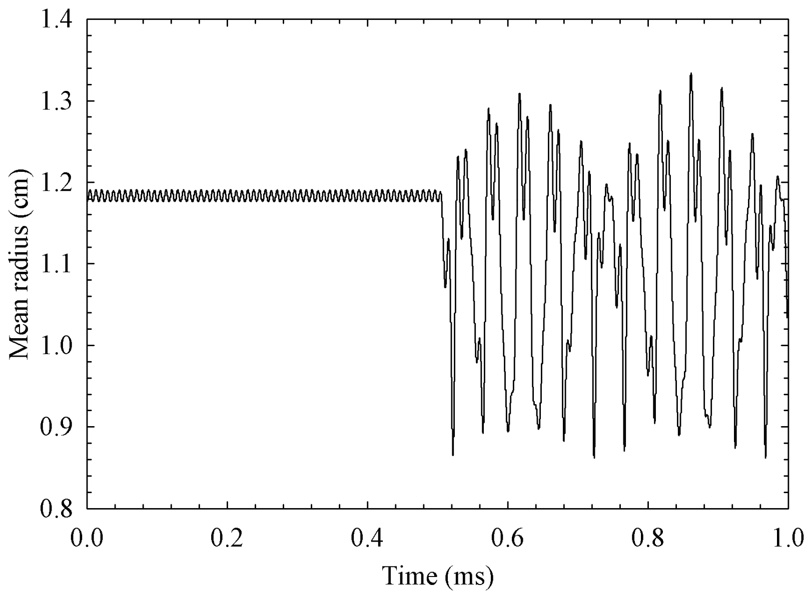


Adiabatically Decreasing *f* Compresses the Bunch

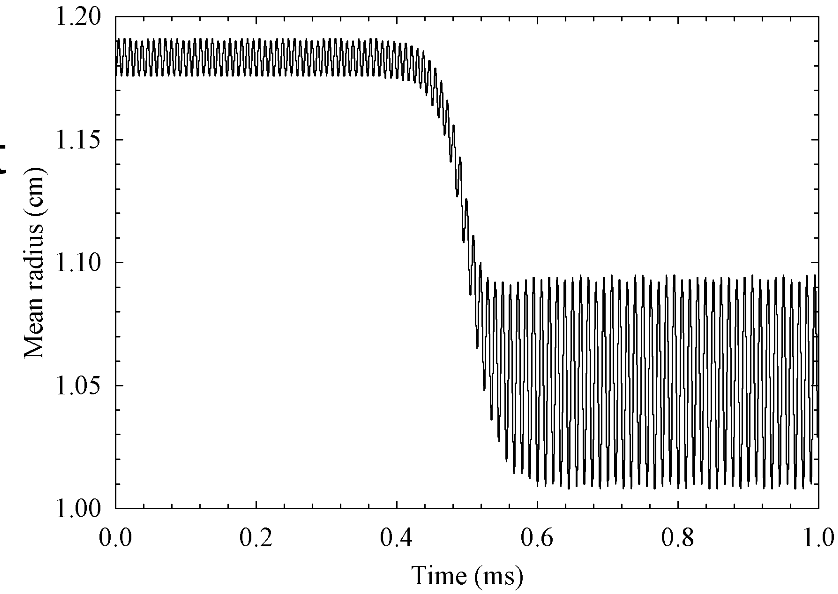
- $s = \omega_p^2/2\omega_q^2 = 0.2$.
- $\nu/\nu_0 = 0.88$
- $V_{0\,\max} = 150$ V
  $f = 60$ kHz
  $\sigma_v = 49^o$



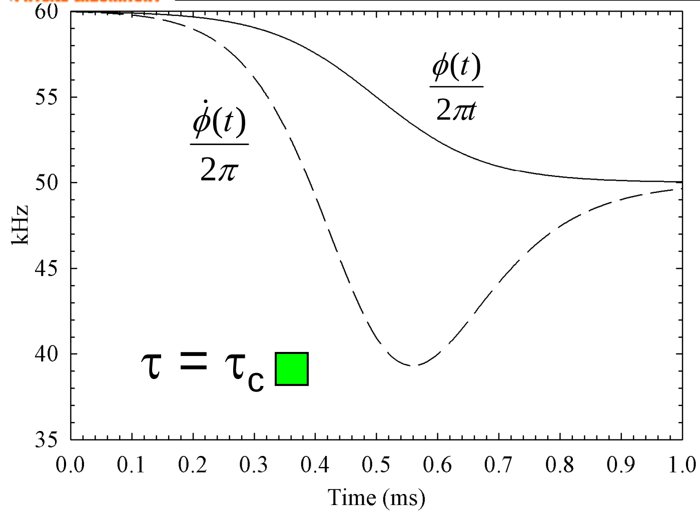
$$\omega_q = \frac{8eV_{0\,\max}}{m\pi r_w^2 f}\xi$$



Good agreement with KV-equivalent beam envelope solutions.

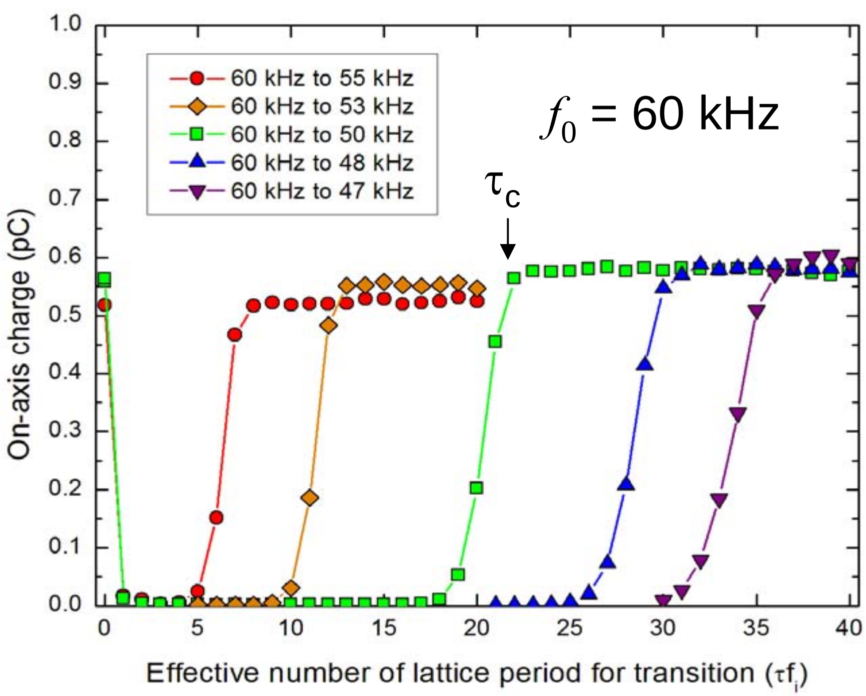


Transverse Confinement is Lost When Single-Particle Orbits are Unstable



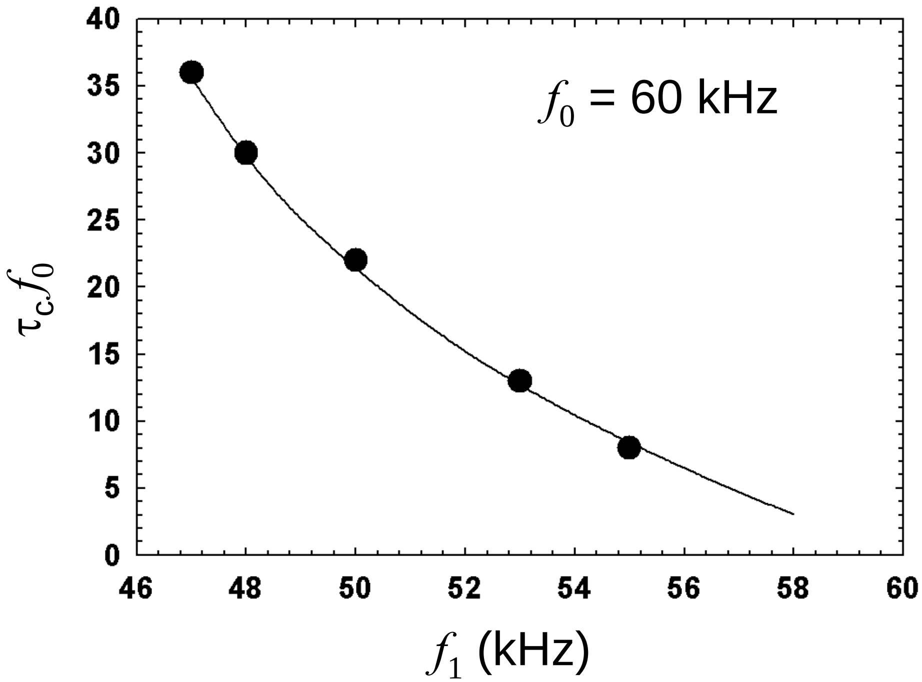
$$\frac{\phi(t)}{2\pi} = f_f t + \frac{f_i - f_f}{2} t \left[\tanh \frac{-(t - t_{1/2})}{\tau/2} + 1 \right]$$

$$\boxed{\sigma_v = \frac{\omega_q}{f} \propto \frac{1}{f^2}}$$

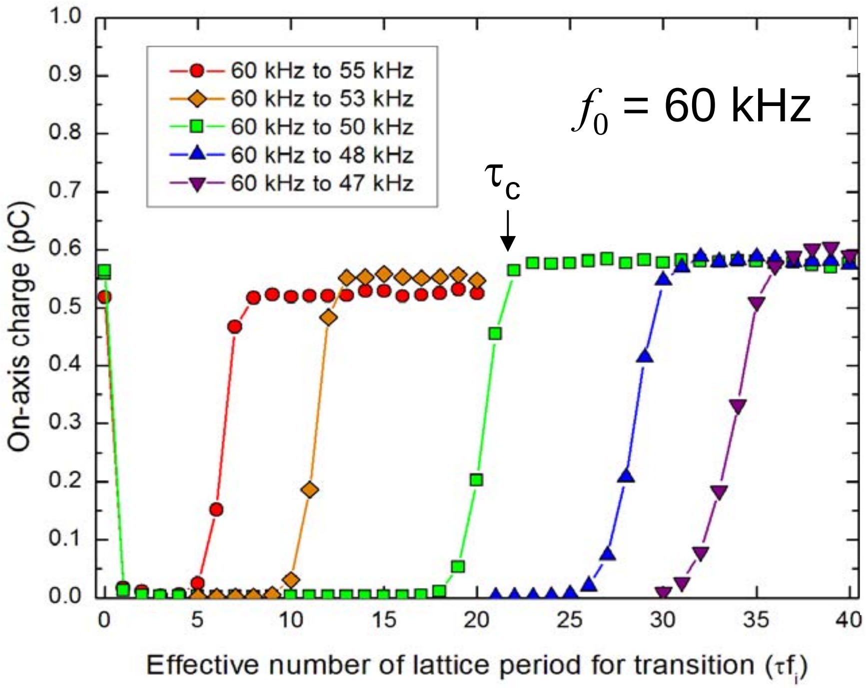


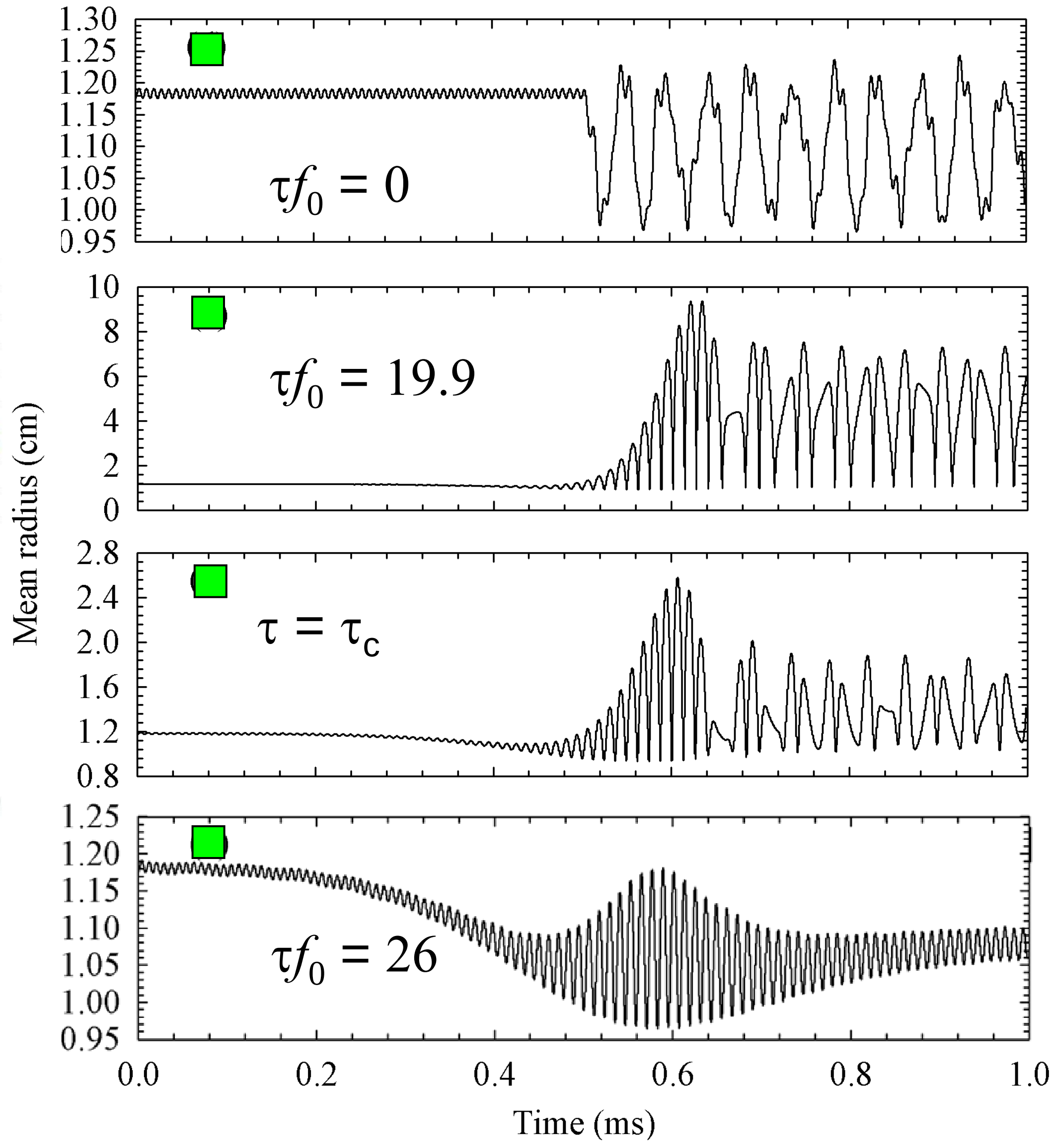
Measured τ_c (dots)

Set $\sigma_{v\,max}$ = 180° and solve for τ_c (line)



Good Agreement Between Data and KV-Equivalent Beam Envelope Solutions





Summary

- PTSX is a compact and flexible laboratory experiment.
- PTSX has performed experiments on plasmas with normalized intensity *s* up to 0.2.
- Instantaneous changes can cause significant emittance growth and lead to halo particle production.
- Adiabatic increases in ω_q approximately 100% can be applied over only four lattice periods.
- The charge bunch will "follow" even non-monotonic changes in ω_q.