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ENERGY OF FORCE-FREE MAGNETIC FIELDS IN RELATION TO CORONAL MASS EJECTIONS

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ABSTRACT

In typical observations of coronal mass ejections (CMEs), a magnetic structure of a helmet-shaped closed configuration bulges out and eventually opens up. However, a spontaneous transition between these field configurations has been regarded to be energetically impossible in force-free fields according to the Aly-Sturrock theorem. The theorem states that the maximum energy state of force-free fields with a given boundary normal field distribution is the open field. The theorem implicitly assumes the existence of the maximum energy state, which may not be taken for granted. In this study, we have constructed force-free fields containing tangential discontinuities in multiple flux systems. These force-free fields can be generated from a potential field by footpoint motions that do not conserve the boundary normal field distribution. Some of these force-free fields are found to have more magnetic energy than the corresponding open fields. The constructed force-free configurations are compared with observational features of CME-bearing active regions. Possible mechanisms of CMEs are also discussed.

Subject headings: magnetic fields — MHD — Sun: coronal mass ejections (CMEs)

1. INTRODUCTION

A typical coronal mass ejection (CME) starts with the rising of a cavity and the formation of a CME loop, and it proceeds to an opening up of field lines, which reconnect in a flare and eventually recover closed configurations as before the eruption (e.g., Hundhausen 1988, 1999; Gosling 1993a, 1993b). These sequential processes are considered to occur spontaneously because the timescale of a CME (hours to a day) is much shorter than the timescale of energy buildup (days to a month). The free energy driving a CME is believed to be stored in the pre-eruption magnetic field, most of which has a closed configuration. Here the term "closed" means that both ends of field lines are connected to the solar surface. A spontaneous transition from a closed magnetic configuration to an open-field configuration requires that more magnetic energy should be contained in the closed magnetic field than in the open field.

The investigation of this energy hypothesis dates back to the early days of flare research (Barnes & Sturrock 1972). Due to the dominance of magnetic field pressure over plasma pressure in the corona, the research has been mostly concentrated on force-free fields (FFFs). Ground-breaking progress in this research was made by Aly (1984), who showed that there is an upper bound of magnetic energy of FFFs in an infinite half-space with the same boundary normal field (B_n) distribution. Aly (1984) further conjectured that the maximum energy of the closed FFFs should be the corresponding open-field energy. This conjecture was backed up by "physical proofs" provided by Aly (1991) and Sturrock (1991) and, since then, has been commonly called the "Aly-Sturrock theorem."

As Aly (1991) pointed out, the proofs by Aly (1991) and Sturrock (1991) are not complete because their validity is conditioned by two important assumptions. First, in the set of admissible FFFs, whose lines are connected to the bottom boundary and unknotted, an energy-maximizing sequence of FFFs should be able to converge to a field $\boldsymbol{B}+$, which may or may not belong to the set. Second, if the sequence converges, it should hold that $E[\boldsymbol{B}+] = E_m$, where $E[\boldsymbol{B}+]$ designates the magnetic energy of $\boldsymbol{B}+$ and E_m is the least upper bound of energy of the FFFs. Although these two conditions intuitively seem trivial, their validity cannot be taken for granted at all. Even if $\boldsymbol{B}+$ exists, there is a possibility that $E[\boldsymbol{B}+] < E_m$ (J. J. Aly 2002, private communication). In other words, no admissible FFF configuration may exist for the energy supremum E_m. The argument in § 4 of Sturrock (1991) does not consider such a possibility. In short, no rigorous proof for Aly's conjecture has been presented yet.

Aly's conjecture has been supported by quite a few numerical (Yang, Sturrock, & Antiochos 1986; Mikić & Linker 1994; Roumeliotis, Sturrock, & Antiochos 1994; Amari et al. 1996) and analytical (Lynden-Bell & Boily 1994; Aly 1994a; Wolfson 1995) studies of twisted or sheared magnetic fields. All these studies dealt with only those force-free states that are physically accessible under the ideal MHD conditions from potential fields by footpoint motions always conserving the boundary normal field distribution. In this Letter, we investigate FFFs numerically constructed in multiple flux systems. The FFFs in our study differ from the FFFs in the previous studies in that our FFFs contain singular current sheets (tangential discontinuities) and in that our FFFs cannot be generated from a potential field by B_n-conserving footpoint motions. We find that some of our FFFs have more energy than the corresponding open fields. In § 2, we describe our FFF model and the numerical procedure. Our computational results for field configurations and magnetic energy are presented in § 3. In § 4, a comparison is made between our results and observational features of eruptive active regions. A summary and discussions regarding possible mechanisms of CMEs are provided in § 5.

2. MODELING OF FORCE-FREE FIELDS IN TWO-FLUX SYSTEMS

Our study of multiple flux systems is motivated by the fact that magnetic fields in and below the solar photosphere are made of filamentary flux tubes. After a magnetic field emerges from below the solar surface, the high β-plasma barrier between elementary flux tubes will be drained, and only current sheets will separate individual flux tubes. Unless magnetic reconnection totally destroys the current sheets, the magnetic field will probably retain the cellular structure. The simplest model of such a cellular magnetic structure is two interwinding flux tubes. The formation of interwinding flux tubes of a much larger scale is expected when a new flux tube emerges under

a preexisting field. Since there is no reason for these two flux systems to have any connectivity, the magnetic field is discontinuous in the interface of two flux systems. Large-scale photospheric motions can interwind these flux tubes.

We consider a magnetic field system occupying the infinite half-space $\{z > 0\}$ above a plane. The magnetic field normal to the boundary plane $\{z = 0\}$ is concentrated on four separate circular patches of a finite area so that

$$B_n(x, y) = B_z(x, y, 0) = \sum_i B_{zi}(\rho_i), \tag{1}$$

where

$$B_{zi}(\rho_i) = \begin{cases} \pm\left[1 - (\rho_i/R)\right]^2 & \text{if } \rho_i \le R, \\ 0 & \text{if } \rho_i > R, \end{cases} \tag{2}$$

in which $\rho_i = |\boldsymbol{r} - \boldsymbol{r}_i|$ is the distance from the center position of the i-indexed patch, $\boldsymbol{r}_i$, to a point $\boldsymbol{r}$ in the $z = 0$ plane and R is the radius of the patches. We assume that for $x_i > 0$, $B_{zi} > 0$, and that for $x_i < 0$, $B_{zi} < 0$. Let us consider two flux tubes designated by a and b, whose intersections with the $z = 0$ plane are circular patches of radius $R = 0.7$ centered at

$$\boldsymbol{r}_{a\pm} = (\pm 1.7, 0, 0), \quad \boldsymbol{r}_{b\pm} = (\pm 3.3, 0, 0). \tag{3}$$

One possible configuration of the two-flux system (not in equilibrium) satisfying the above boundary conditions is shown in Figure 1*a*. The potential field satisfying the boundary condition given by equations (1)–(3) will have the field line connectivity as follows:

$$x_{0+} = -x_{0-}, \quad y_{0+} = y_{0-}, \tag{4}$$

where (x_{0+}, y_{0+}) and (x_{0-}, y_{0-}) are coordinates of two conjugate field line footpoints, one in $x > 0$ and the other in $x < 0$. Now we impose horizontal boundary motions, respectively, on each side of the polarity inversion line so that each pair of flux patches undergoes a rotational motion with a constant angular velocity centered at $\boldsymbol{r}_{C\pm} = (\boldsymbol{r}_{a\pm} + \boldsymbol{r}_{b\pm})/2 = (\pm 2.5, 0, 0)$. Beyond a distance wholly covering each pair of flux patches, the angular speed is assumed to be gradually tapered down to 0 with the increasing distance. Under ideal MHD conditions, two flux tubes are interwound by this rotational boundary motion, conserving the field line connectivity. The angle of twist (or interwinding) between two flux tubes, which will be denoted by Φ, is twice the rotation angle on each side of the polarity inversion line. Figure 1*b* shows one possible configuration of the two-flux system (not in equilibrium) for $\Phi = 2\pi$. If we relax the two-flux system to a force-free equilibrium after interwinding by any nonzero angle Φ, part of the separatrix between two flux systems will become a current sheet (tangential discontinuity). The field connectivity of an FFF with $\Phi \neq 0$ is definitely different from that of the potential field for the same $B_n(x, y)$ and cannot be created from the potential field by any footpoint motions conserving $B_n(x, y)$ under the ideal MHD conditions.

In this study, we have numerically constructed FFFs for different twist angles. Since our FFFs contain current sheets, it is important to prevent any spurious magnetic reconnection and maintain the specified field connectivity. To do this, we describe the magnetic field with "unmatched Euler potentials"

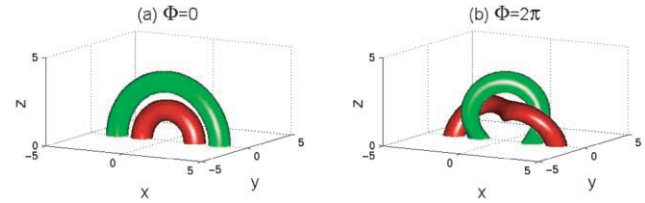

FIG. 1.—Possible configurations of a two-flux system (not in equilibrium). The configuration in (*a*) has the same field connectivity as the potential field. The configuration in (*b*) can be created by rotational boundary motions under the ideal MHD conditions.

(e.g., Stern 1970) as

$$\boldsymbol{B} = \sum_j F_j(\alpha_j, \beta_j)(\nabla\alpha_j \times \nabla\beta_j), \tag{5}$$

where index j denotes the flux system a or b. Either F_a or F_b is allowed to have a nonzero value at any location (x, y, z). The Euler potential description has the advantage of inducing far less spurious magnetic reconnection than employing the three components of a magnetic field $\boldsymbol{B}$ or a vector potential $\boldsymbol{A}$. To specify the location of field line footpoints easily, we select a functional form of Euler potentials as $\alpha = \alpha(x_0^2)$, $\beta = \beta(y_0)$, where x_0 and y_0 are the coordinates of field line footpoints at $z = 0$ when the angle of twist Φ is zero. With this setting, we have

$$B_z(x, y, 0) = \sum_j F_j(\alpha_j, \beta_j)\frac{d\alpha_j}{dx_0}\frac{d\beta_j}{dy_0}. \tag{6}$$

With the field description given by equation (5), the magnetic induction equation (Faraday's law) can simply be written as

$$\frac{\partial\alpha}{\partial t} = -\boldsymbol{v}\cdot\nabla\alpha, \quad \frac{\partial\beta}{\partial t} = -\boldsymbol{v}\cdot\nabla\beta, \tag{7}$$

where $\boldsymbol{v}$ is the bulk velocity of plasma. To construct force-free equilibria, we have employed a magnetofrictional method (e.g., Chodura & Schlüter 1981; Choe & Lee 1996), which drives the system to the minimum energy state by continuously removing kinetic energy with the field connectivity conserved. In this method, we also employ equation (7), in which t is now a relaxation parameter and $\boldsymbol{v}$ is a vector proportional to the Lorentz force.

In specifying boundary conditions, we have to consider two points. First, different from time-dependent problems (e.g., Mikić & Linker 1994; Amari et al. 1996), an equilibrium problem is well defined only if B_n is specified at all boundaries (Chodura & Schlüter 1981). Second, the stress exerted by the boundary should be minimized in order to simulate an open system. Thus, we specify B_n as given by equations (1)–(3) at the bottom boundary and set B_n to zero at all other boundaries. This boundary condition naturally allows field movement tangential to the boundary where $B_n = 0$ during the relaxation process.

3. CONSTRUCTED FORCE-FREE FIELDS AND THEIR ENERGY

We have constructed a total of seven FFFs with $\Phi = 0, 0.5\pi, \pi, 1.5\pi, 2\pi, 2.5\pi$, and 3π. Two flux surfaces containing a quarter of the total flux in each flux tube are shown in Figure 2 for

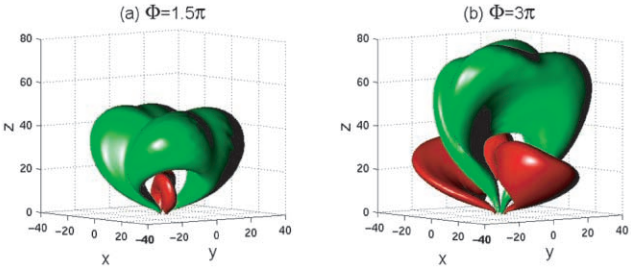

FIG. 2.—Flux surfaces of the numerically generated FFFs consisting of two interwinding flux systems. Each flux surface contains 25% of the total flux in each flux system.

$\Phi = 1.5\pi$ and 3π. It is interesting that the flux system occupying a finite flux volume at $\Phi = 0$ (the red one in Figs. 1 and 2) always takes a finite volume after interwinding, whereas the other system (the green one) always takes an infinite volume (the rest volume). The notable pleated structure of the flux surfaces can be attributed to the self-twist of the flux tubes generated by the rotational footpoint motion.

The magnetic energy of the FFFs that we have constructed increases with the twist angle, as shown in Figure 3. The energies of the potential fields and of the open fields can be obtained with a Green function method familiar in electrostatics and are plotted in the figure for reference. An upper bound (not the least upper bound) of energy of FFFs derived by Aly (1984),

$$E_{\rm UB} = \frac{1}{4\pi}\left(\int_{z=0} B_n^2\, dx\, dy\right)^{1/2}\left(\int_{z=0} r^2 B_n^2\, dx\, dy\right)^{1/2},$$

is $10.7E_{\rm pot}(\Phi = 0) = 8.69E_{\rm open}(\Phi = 0)$, far greater than the energies we have obtained. In the figure, we can find that the magnetic energy of FFFs exceeds the open-field energy for $\Phi \gtrsim 1.5\pi$.

Although we deal with systems in an infinite half-space, our computation is performed in a finite computational box. Thus, we need to make sure that the uncertainty in energy does not affect our conclusion. To do this, we first compared the energies of FFFs that we obtained in domains of different sizes. The energies obtained in a box of the size 160 × 160 × 160 (refer to eq. [3] for units) differ from those obtained in a box of the size 300 × 300 × 300 by not more than 5%. The latter differ from the energies obtained in a box of the size 400 × 400 × 400 by not more than 1.4%. The values presented in Figure 3 are obtained with the largest box. The energy of a magnetic field whose whole flux is confined in a box of a finite volume L^3 will approximately vary as $E_L \sim E_\infty + C/L$, as $L \to \infty$, because field strength roughly varies as L^{-2} far enough away from the boundary normal flux concentrations. The energy differences given above thus tell that the error due to the finite box size would not exceed 5%. The energy of the constructed FFF for $\Phi = 3\pi$ is about 30% more than the open-field energy. This difference is considered to be well above the error level because of the finite size of the domain. Another way of assessing the effect of the finite box size is to check integral relations that should be satisfied by FFFs in an infinite half-space above a plane. Aly (1984) derived the following equations from the tensor virial theorem:

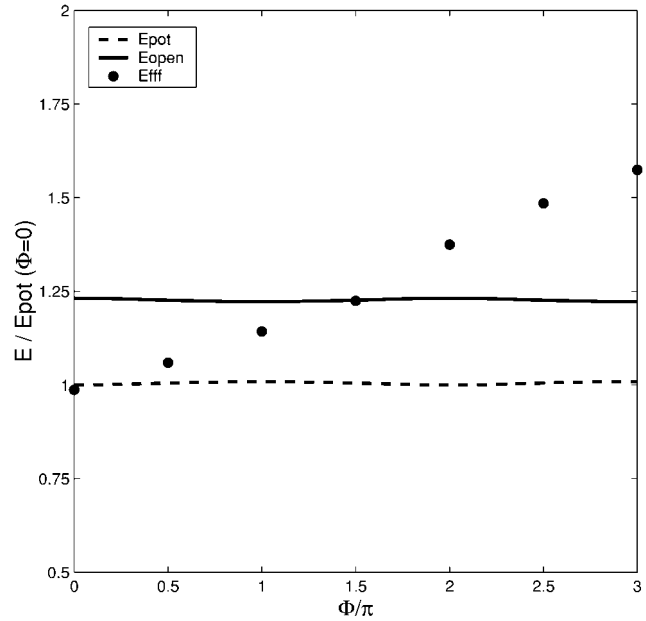

FIG. 3.—Energy of the numerically generated FFFs for different twist angles. The energies of the potential fields (*dashed curve*) and of the open fields (*solid curve*) are also shown as a function of the twist angle.

$$\int_z (B_x^2 + B_y^2)dx\, dy = \int_z B_z^2\, dx\, dy. \tag{8}$$

$$E(z>0) = \frac{1}{8\pi}\int_{z>0} (B_x^2 + B_y^2 + B_z^2)dx\, dy\, dz = \frac{1}{4\pi} \times \int_{z=0} (xB_x + yB_y)B_z\, dx\, dy. \tag{9}$$

For our FFF solutions, the discrepancy between the left-hand side and the right-hand side of the above equations is found to be less than 3% for equation (8) and of less than 9% for equation (9), respectively. For not too large twist angles, the discrepancies are even smaller than the above values. Therefore, we conclude that some of the FFFs we have constructed indeed have more energy than the open field.

What, then, makes our results on multiple flux systems different from the results of previous studies on single flux systems (Yang et al. 1986; Mikić & Linker 1994; Roumeliotis et al. 1994; Amari et al. 1996; Lynden-Bell & Boily 1994; Aly 1994a; Wolfson 1995)? The single flux systems treated in most previous studies have smooth, well-ordered field structures. Their field lines tend to partially open beyond a certain shear or twist so that further accumulation of magnetic energy in the system is hindered. On the contrary, we do not find any tendency of field opening in our interwinding two-flux system, although the system takes a larger flux volume with increasing Φ. By interwinding, each flux system seems to suppress free expansion of the other. This shackling behavior of twisted fields is also reported by Klimchuk, Antiochos, & Norton (2000) for a single flux system in a smooth equilibrium.

4. RELEVANCE TO OBSERVATIONS

The most well-known observational condition for solar eruption detected in the photosphere and chromosphere is the high magnetic shear (e.g., Krall et al. 1982). In addition, polarity inversion lines also show a tendency of distortion during the evolution of active regions toward eruption (e.g., Uddin, Pande, & Shelke 1986). Recent X-ray observations have revealed the appearance of an S- or inverse-S–shaped bundle of coronal loops before solar eruption (Acton et al. 1992; Canfield, Hudson, & McKenzie 1999). This structure is called a sigmoid (Rust & Kumar 1996).

To compare our FFF model with these observations, we have generated vector magnetograms at a height level of $z = 0.22$ with our FFF solutions. Also, images projected onto the bottom boundary are created for a flux surface containing 95% of the total flux of the inner flux tube. Figure 4 shows one set (for $\Phi = 2.5\pi$) of those plots. As much expected, the magnetic shear increases along the polarity inversion line with the twist angle. Moreover, the polarity inversion line becomes more and more tilted and distorted from the original straight line with the increase of the twist angle. The projected image of a flux surface takes an inverse-S shape and becomes more twisted and larger in size with increasing Φ. We note that the outermost flux surface of the inner flux tube, which is the separatrix between the two flux systems, also takes a shape similar to the flux surface shown in the figure, but with a little larger scale. If the emission from a sigmoid is due to the heating by magnetic reconnection in the pre-eruption stage, the resulting change in field topology may play an important role in the subsequent solar eruption.

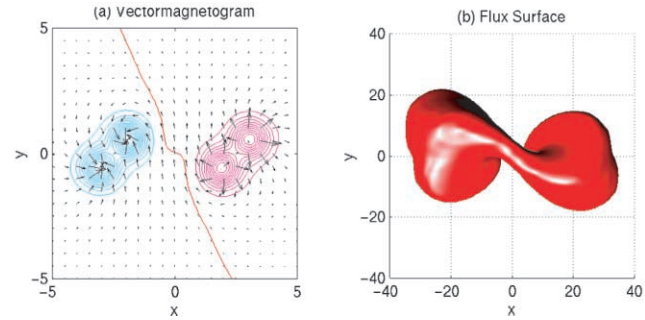

FIG. 4.—(*a*) Vector magnetogram generated from our FFF solution for $\Phi = 2.5\pi$ at the height $z = 0.22$. (*b*) Projected image of the flux surface containing 95% of the total flux of the inner flux tube (the red one in Figs. 1 and 2).

5. SUMMARY AND DISCUSSION

We have found that there is a class of FFFs of closed-field configuration having more energy than the open fields. The FFFs considered in our study consist of two interwinding flux systems with current sheets.

Although we have constructed closed FFFs having more energy than the open fields, we still do not know whether they can lead to a CME eruption and what kind of mechanism is involved if eruption can take place. For future studies, we can think of several possibilities leading to eruption. The eruption up to field opening may be solely an ideal MHD process or may somehow involve magnetic reconnection. In the former case, a global nonequilibrium (see Aly 1994b for the definition) may take place beyond a certain amount of twist. So far, however, our study does not hint at this possibility. If magnetic reconnection is involved, a variety of possibilities can arise. Magnetic reconnection results in a change of field topology. If no equilibrium of closed configuration is available in a field topology created during a reconnection process and if the field still retains more energy than the open field, an eruption with field opening can take place. If closed equilibrium states are always available in field topologies generated by a reconnection process and if flux volumes become much larger in the new equilibrium states, CMEs with apparent field opening to a finite distance can occur. If flux volumes in the new equilibrium states are not considerably larger than the original flux volume, a flare may take place without a CME. Investigation of these possibilities will be performed in our future studies.

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