MHD Waves and Field Line Resonances in the Magnetosphere-Magnetopause

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Introduction and Motivation for the Study

This work develops a theoretical model for field-line resonances (FLRs), which are the most common low-frequency waves in the magnetosphere-magnetopause. The FLRs pulsate at the "natural" frequencies of the magnetic field lines and can be pictured as the vibrations of a tied string (representing the magnetic field line) with ends on the northern and southern ionospheres. The oscillation frequencies are determined by the length of the geomagnetic field lines and the shear wave propagation speed (Alfvén speed) along them. The eigenfrequencies of the shear Alfvén waves standing along the field lines vary spatially from line to line and constitute the so-called shear Alfvén continuum. While studied for over two decades, recently the connection between the FLRs and the discrete auroral arcs has become observationally and theoretically compelling. This connection is obvious through such observed features as the poleward motion of the auroral forms, the narrow frequency band of the luminosity power spectrum, the arc duration of many hours, the progressively lower frequency signal with increasing latitude, and the nearly 180° shift in the wave electric and magnetic field. The shear Alfvén resonances also exhibit potentially large field aligned currents (FACs) and electron inertia effects produce large parallel electric fields near to the polar ionospheres, thus possessing great potential for accelerating auroral particles.

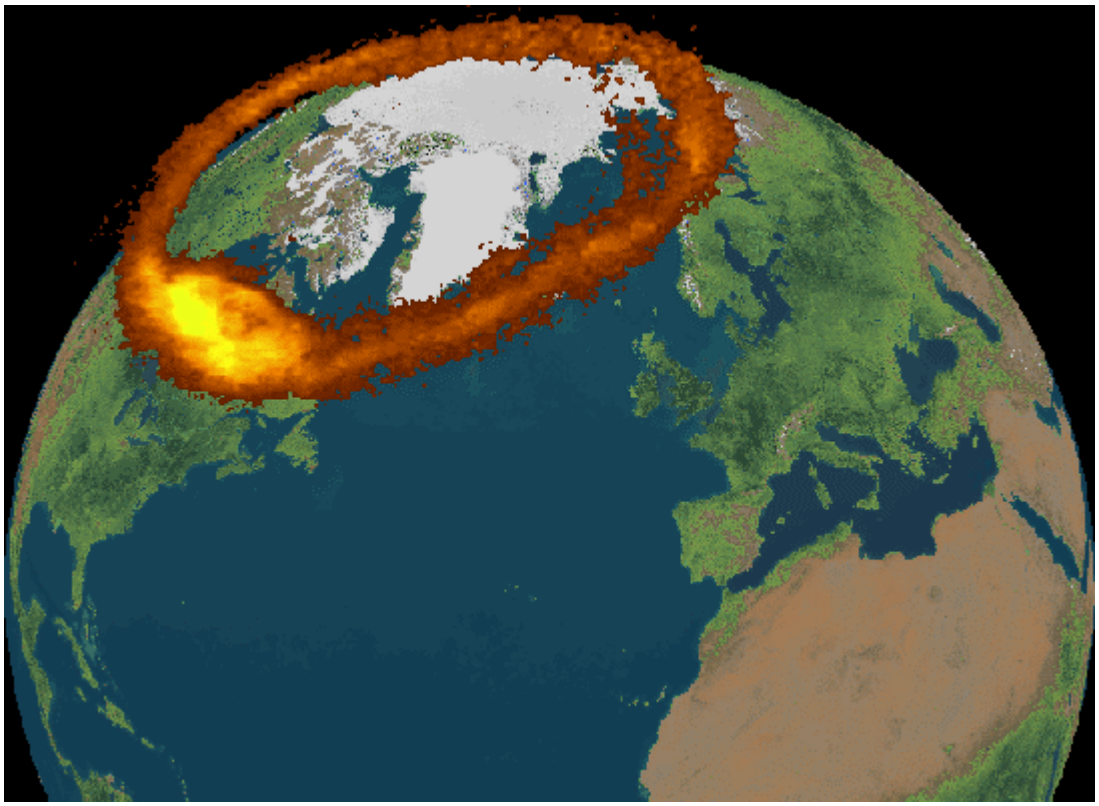

Figure 1. Auroral oval detected by the VIS instrument. Notice the concentration of auroral activity near local midnight.

Because FLR is a global feature of a magnetic field line, understanding its modes of stimulation offers a potentially powerful diagnostic for identifying the signatures of auroral arcs that may be detected by equatorial satellites, and, conversely, for using the aurora, especially satellite images, to remote sense magnetospheric dynamics. Ultimately, adequate understanding of the physics of FLR may prove useful for understanding specific features of magnetospheric storm and substorm phases responsible for auroral brightening, localization and dynamical behavior. Because of the numerous implications of these low-frequency waves (reported frequencies are 1 – 4 mHz at auroral latitudes in the nightside magnetosphere), it is of particular interest to the space physics community to develop more precise models for understanding the phenomenon.

There has been steady progress in understanding FLR over the past 40 years. As mentioned earlier, the FLR depends strongly on the magnetic field configuration and Alfvén velocity along the magnetic field lines. Early work assumed the simple magnetic field configuration of a dipole and was able to produce an Alfvén continuum. However, the dipole approximation breaks down beyond geosynchronous orbit region (6.6 R_E) on the nightside, and even closer to the Earth during disturbed magnetospheric periods. Consequently, due to short field line lengths, such studies generally produced frequencies too high to match observations. More recent work included the field line stretching by using so-called empirical magnetospheric field models, although those empirical models may not be self-consistent, i.e. the magnetic field forces and plasma pressure forces are not in equilibrium. Unfortunately, the departure from equilibrium in those models is sufficiently large to require significant adjustment to magnetic field curvature to be consistent with observed pressure profiles in the plasma sheet and therefore wave calculations based on those models would have a degree of inaccuracy. Another source of inaccuracy is that many of the past studies were limited to the treatment of a cold plasma model.

A qualitative improvement of the FLR treatment was recently made in the work of *Lui and Cheng, [2001],* who performed the study in a 2-D equilibrium magnetosphere. Considering a warm plasma model, they analyzed both the shear Alvfén waves, as well as a second branch of the FLR waves, the so-called slow modes. The resonance frequencies they obtained were consistent with observations; however the study was only limited to one local time due to the 2-D nature of the model.

Study Highlights

Our work presented here is qualitatively different from previous work, as we investigate the FLRs using a 3-D self-consistent background magnetospheric field, as obtained by our equilibrium code MAG-3D *[Cheng, 1995].* In our model, the equilibrium magnetic field is represented by two Euler potentials as $\mathbf{B} = \nabla\psi \times \nabla\alpha$, where ψ is the (poloidal) magnetic flux, while α is an angle-like variable; in this system, the plasma pressure can

be written as $P = P(\psi,\alpha)$. The eigenmode equations, reformulated for the 3-D environment, are:

Shear Alvfén Mode Branch:

$$[\mathbf{B} \cdot \nabla \left(\frac{|\nabla\psi|^2}{B^2} \mathbf{B} \cdot \nabla\xi_s\right) + \frac{\rho\omega^2 |\nabla\psi|^2}{B^2} \xi_s + \kappa_s \left(\frac{\nabla P \cdot \mathbf{B}\times\nabla\psi}{B^2}\right) \xi_s] + \Gamma_s P \, \kappa_s \, \Delta = 0 \qquad (1)$$

Slow Magnetosonic Mode Branch:

$$[\mathbf{B} \cdot \nabla \left(\frac{\Gamma_s P}{\rho\omega^2 B^2} \mathbf{B} \cdot \nabla\Delta\right) + \frac{B^2 + \Gamma_s P}{B^2} \Delta] + \kappa_s \xi_s = 0 \qquad (2)$$

A new term (the last term in the bracket in Eq.1) is found in the shear Alfvén wave equation, as compared to the 2-D case. This new term, proportional to the product of the local-time variation $\partial P/\partial\alpha$ and the geodesic magnetic field curvature κ_s, reduces the values of the resonant frequencies, bringing them more in line with observations. Also, as seen in both equations above, there is another new term (outside the brackets), which couples the shear Alfvén and slow resonances via the effect of the geodesic curvature κ_s and the plasma pressure P.

The numerical results for the resonant frequencies in a quiet-time 3-D magnetospheric case are highlighted in the figure below, which shows field line resonance harmonics versus the field line equatorial distance R (in R_E) for field lines with a constant α that corresponds to the local time 22:48:00 at the Earth.

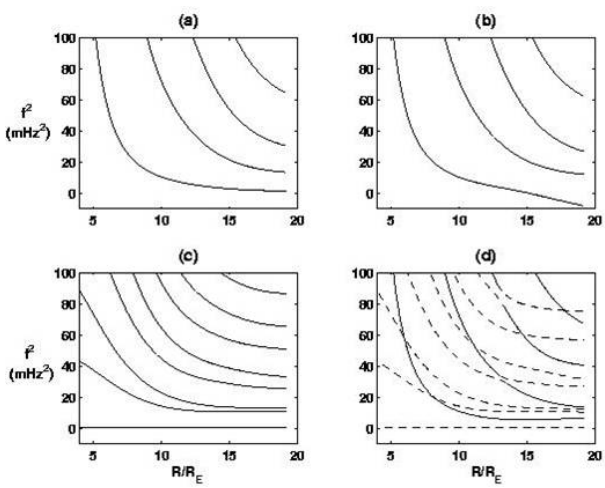

***Figure 2**. (a) the shear Alvfén waves in the cold plasma limit. (b) shear Alvfén waves in the warm plasma model, without the coupling terms. (c) un-coupled magnetosonic modes in the warm MHD plasma model. (d) coupled solutions in the warm plasma MHD model, the shear Alvfén waves (solid lines) and the slow modes (dashed curves).*

By comparing panels (a) and (b), it is clear that the new term proportional to $\kappa_s \, \partial P/\partial\alpha$ in Eq.(1) reduces the f^2 for all harmonics. The effect on the fundamental harmonic is most pronounced, and f^2 becomes negative (unstable) at large Rs. However, the instability is removed when the coupling between the shear Alvfén and the slow modes is considered, as seen in (d). The effect of the coupling results in an increase of the shear Alvfén FLR frequencies as compared to the cold-plasma model. [We note here that the slow magnetosonic waves are not detected by observations, as they are likely to suffer strong Landau damping if the ion temperature is equal or larger than the electron temperature, as is usually the case in the magnetosphere.]

It is also interesting to look at the local-time dependence of the FLR frequencies, as shown in the figures below, where constant fundamental FLR shear mode frequencies are presented, in both the equatorial plane and on the top of the ionosphere.

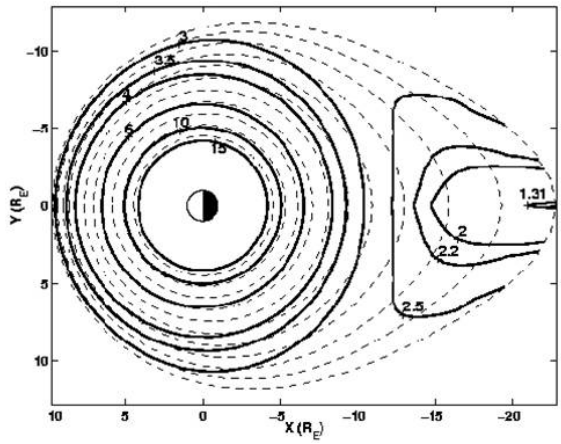

Figure. 3. Contours of the fundamental harmonic shear Alfvén resonance frequency (in mHz) are plotted in the equatorial plane for a quiet time magnetosphere. The dashed curves are the intersection of constant ψ surfaces with the equatorial plane.

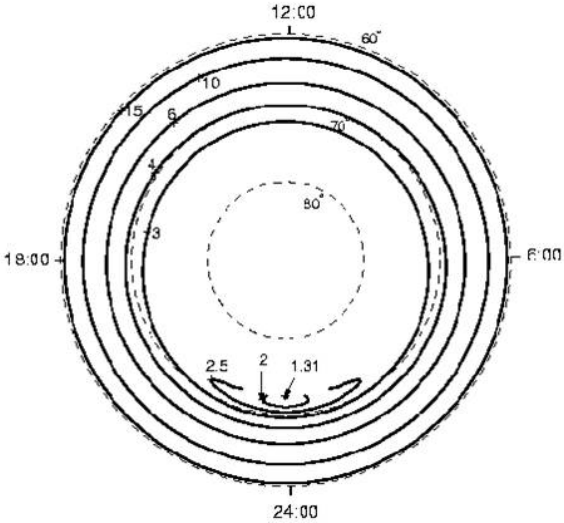

Figure 4. Contours of the fundamental harmonic shear Alfvén resonance frequency (in mHz) are plotted in the northern plar ionosphere for a quiet time magnetospheric equilibrium. The dashed circles represent constant magnetic latitude lines.

It is seen that while in the lower latitude region the frequency contours are only slightly different from the constant flux surfaces, at higher latitudes the constant FLR surfaces form closed contours localized near midnight. Notice that beyond 6.6 R_E that the contours of constant shear Alfvén resonance frequency depart from the flux surface and appear further from earth on the dayside equatorial plane. This results because the magnetic field strength is smaller near midnight due to increased curvature and the field line is stretched. Both effects lower the FLR frequency. Away from midnight (at the same distance), the magnetic field strength is larger due to smaller curvature and; therefore, the FLR frequency increases. Hence, constant FLR frequency will be at larger distance (higher latitude) away from midnight. Beyond a critical distance the field is so stretched that there is a local minimum in the magnetic field strength, which produces the topological change in the isofrequency contour. Because this behavior occurs at auroral latitudes it could have important implications for the formation of discrete auroral arcs.

Moreover, during conditions of increased magnetopheric activity, the tail is further stretched leading to a greater concentration of FLR surfaces near local midnight with limited azimuthal extent. This concentration of FLR surfaces near local midnight is strikingly similar to the pattern of intensity of auroral activity near local midnight as seen in Figure 1. Moreover, intensification of the aurora near local midnight has also been associated with the onset of substorms. This similarity suggests that further understanding of substorm onset could be provided by a full investigation of unstable solutions of the global kinetic-fluid equations which may be localized near midnight when the magnetotail is highly stretched.

Summary of Work

We performed the following tasks:

- Included a 3-D force-balanced magnetospheric model as background configuration for the FLR study; the equilibrium magnetic field is represented by two Euler potentials as $\mathbf{B} = \nabla\psi \times \nabla\alpha$, where ψ is the magnetic flux, while α is an angle-like variable; in this system, the plasma pressure can be written as $P = P(\psi,\alpha)$.

- Reformulated our ideal MHD eigenmode equations for isotropic pressure distributions $P(\psi,\alpha)$ for general 3-D magnetic field geometries; the 3-D solutions consist in a set of two coupled modes, the shear waves and the slow magnetosonic modes. The two are coupled via the combined effect of plasma pressure and geodesic curvature.

It is important to note that the particular dependence of mass density on the distance employed is idealized, and an empirical mass distribution can be used to adjust the results.

Tasks for the Future

Several important improvements are considered for the future. Among them, the relaxation of the ideal MHD approximation with isotropic pressure would involve extending the present theory to include anisotropy and kinetic effects. Secondly, we plan to solve the global MHD modes. The global magnetospheric ULF (ultra-low frequency) pulsations have been observed for decades in space and on the Earth. These pulsations have been recently explained as drived directly by fluctuations in the solar wind *[Kepko et al., 2002]*. If the frequencies of these global modes are inside the FLR continuous spectrum, the compressional MHD waves will be absorbed at the FLR locations and cannot propagate closer to Earth. In order to study these fluctuations, numerical solutions of the (already developed) global MHD eigenmode equations will be pursued. By imposing a source disturbance at the magnetopause boundary as a boundary condition, we will obtain the L-shell dependence of the FLR power spectrum. Thus, a global

solution will not only provide the information of radial wave structures, but will also improve our understanding of the azimuthal variation of the FLRs.