Nonlinearity in Magnetospheric Activity

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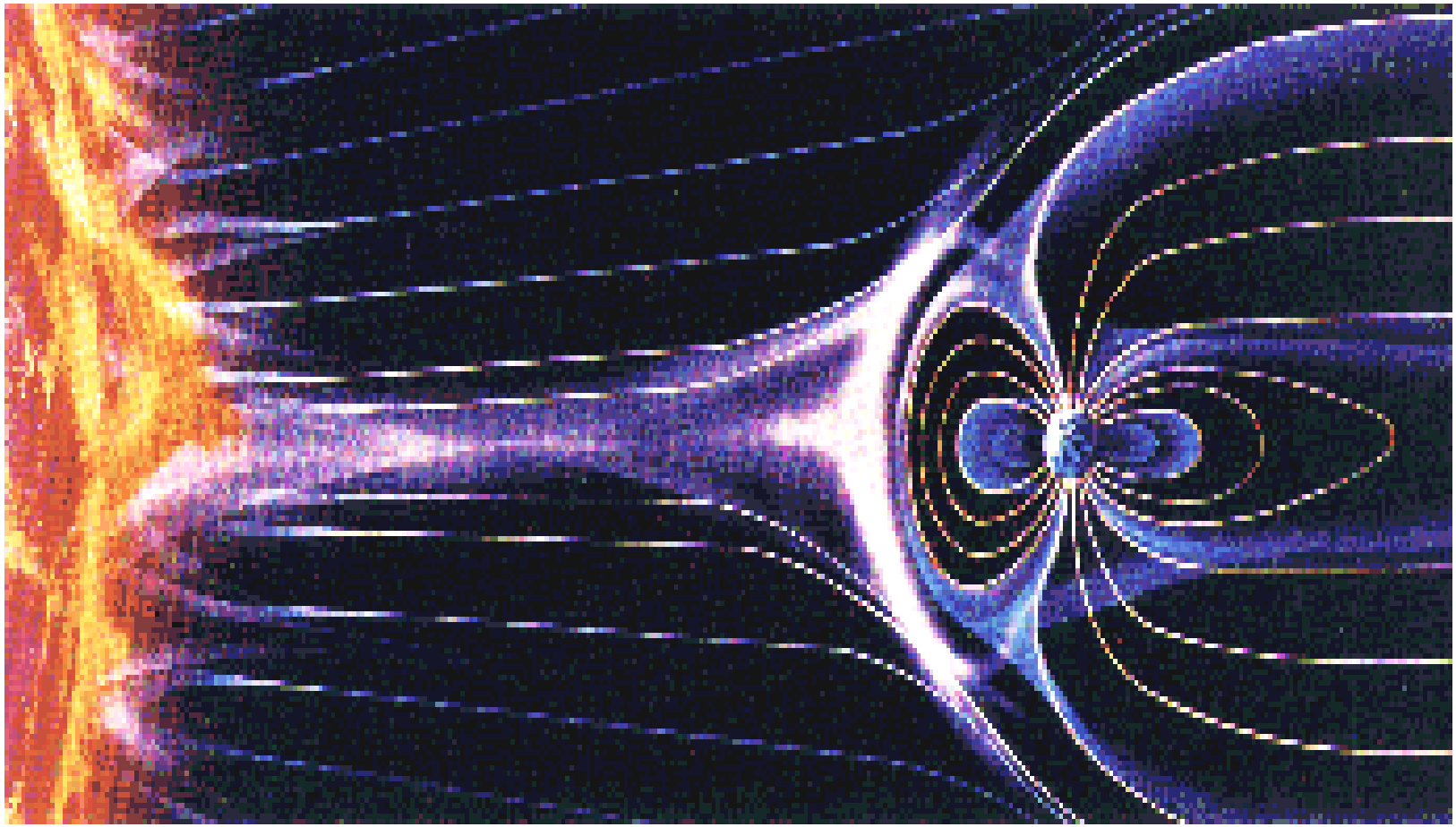
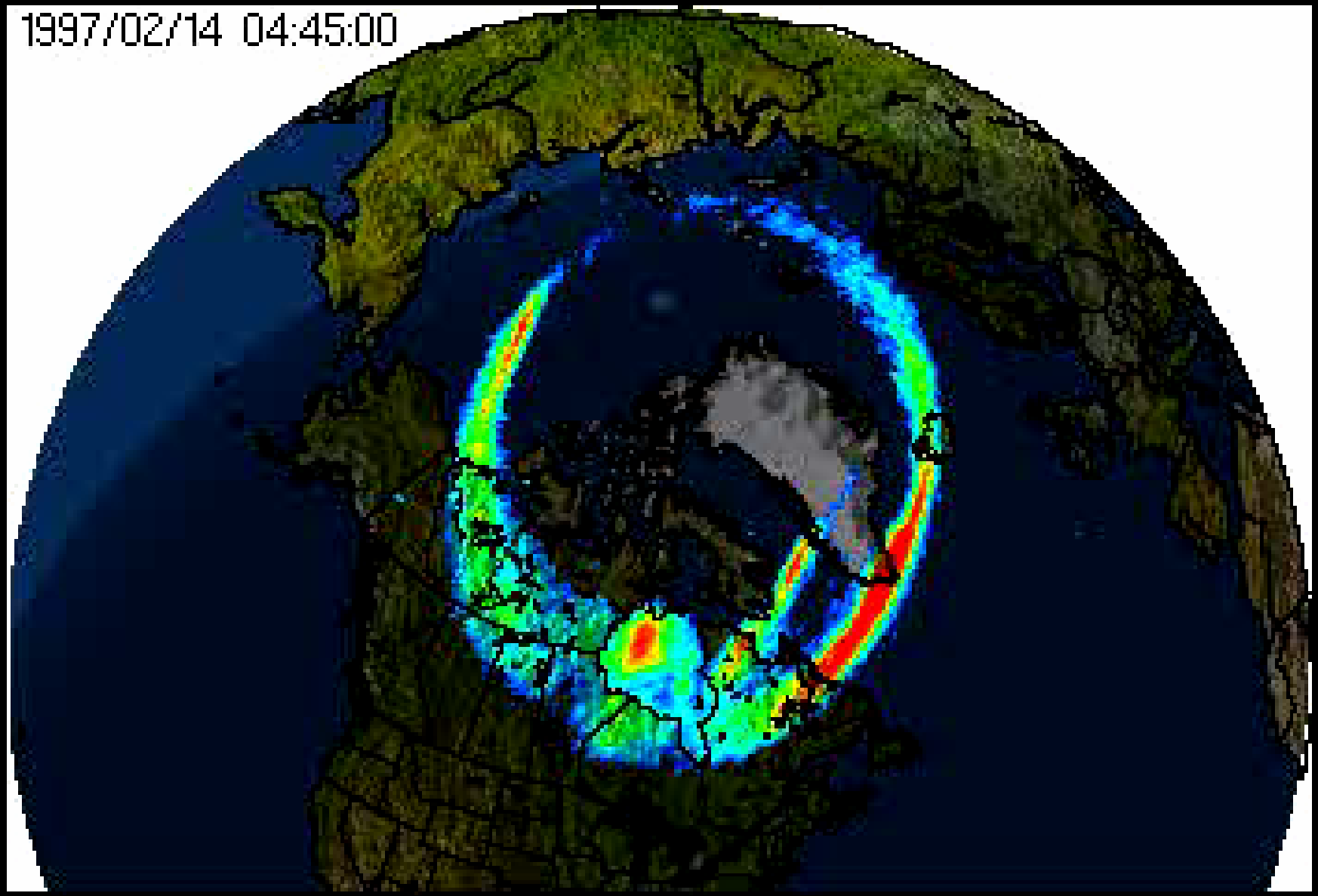
August 11, 2004

Solar Driver of Magnetospheric Activity





UVI from POLAR

Importance of Space Weather

- Power Grid Failure

The HydroQuebec Blackout of March 1989

On March 13, 1989, at 2:44 am, a transformer failure on one of the main power transmission lines in the HydroQuebec system precipitated a catastrophic collapse of the entire power grid. The string of events that produced the collapse took only 90 seconds from start to finish. There was no time for any meaningful intervention. The transformer failure was a direct consequence of ground induced currents from a space weather disturbance high in the atmosphere. 6 million people lost electrical power for 9 or more hours.

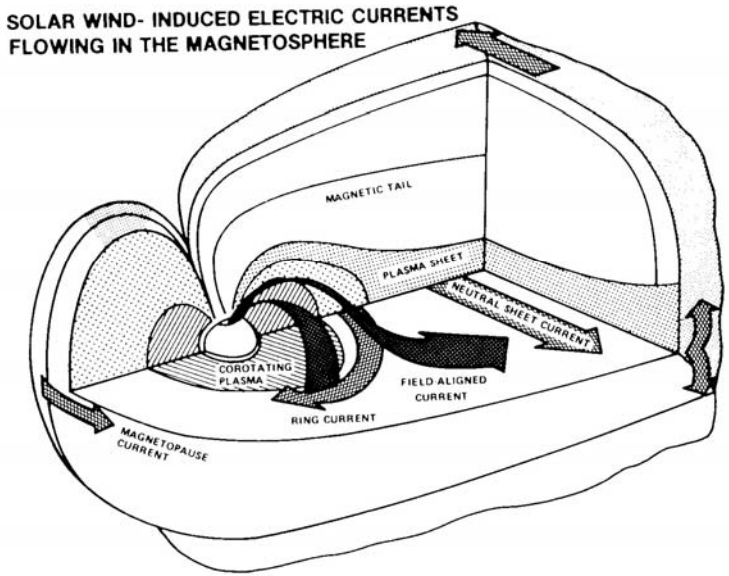
The space weather disturbance that produced this devastation was a great magnetic storm. Great magnetic storms are awesome disturbances in the near-Earth space environment that occur relatively rarely. The last five occurred in February 1986, March 1989, March 1991, November 1991 and May 1992. The frequency of large and great storms increases markedly as we enter the maximum in the solar activity cycle. The next predicted maximum is in the year 2000 (the last solar maximum was in 1989). Better warning of impending space weather events would allow power companies to take steps to reduce the load on sensitive circuits, delay maintenance and equipment replacement, prevent the development of large potential drops by selectively grounding sensitive devices and inteligently deal with systems designed to automatically protect the network during the duration of the event. This is the best way to prevent costly and dangerous black out situations triggered by space weather events.

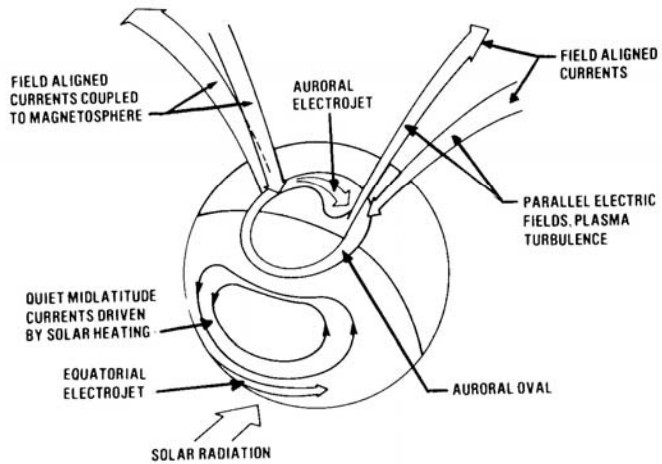
Importance of Space Weather

- Power Grid Failure
- Killer Electrons

MeV electrons have been linked to numerous satellite failures, such as the January 10, 1997, event that may have led to a complete failure 24 hours later of a Telstar 401 communication satellite. Even after insurance, the loss cost AT&T over $100M *SN97*. Although some have found that the satellite failures occur in the declining phase of an MeV event and therefore question the direct connection, everyone is in agreement that a threshold, given by the total fluence of these penetrating electrons, is crossed in nearly every instance of satellite upset.

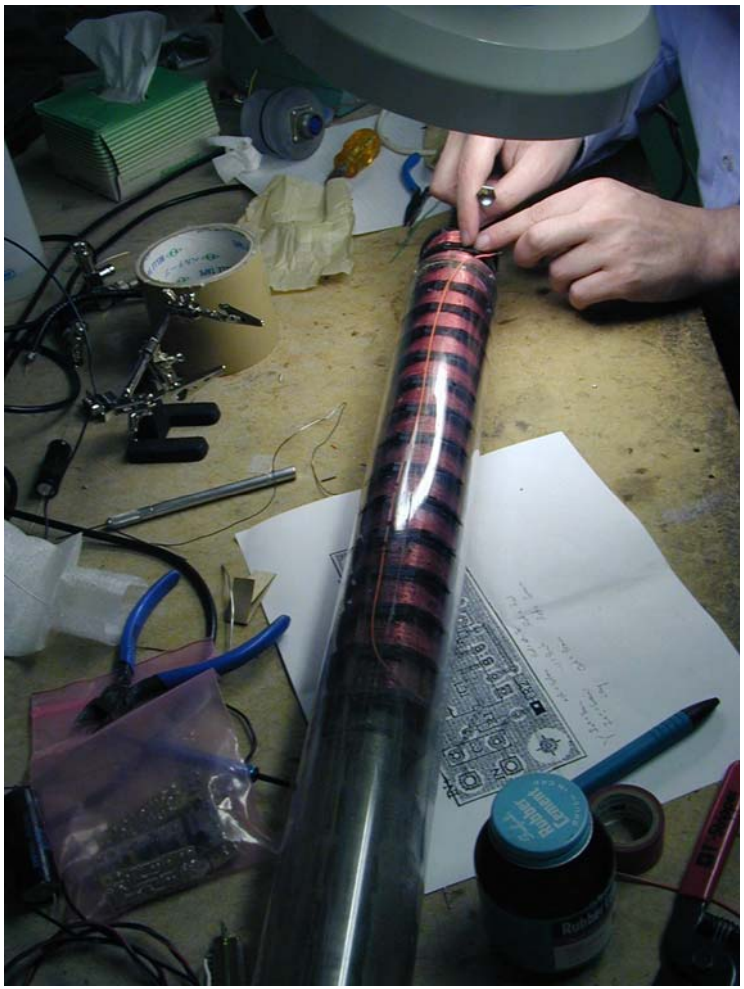
Magnetospheric Currents





- Boundary Current
- Ring Current
- Tail Current
- Birkeland Current
- Ionospheric Current

Ground Based Magnetometers



Search Coil Magnetometer

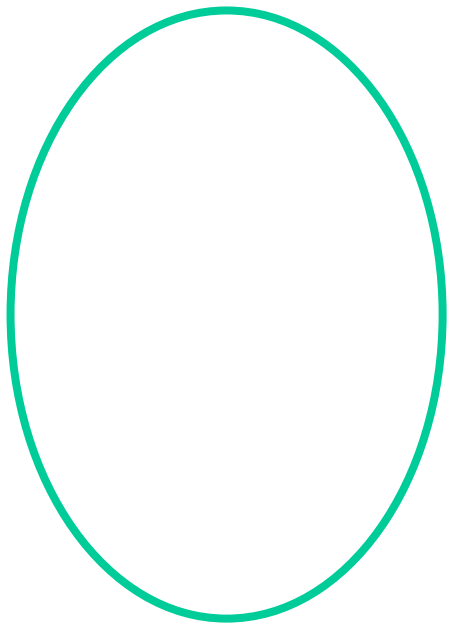
Fluxgate Magnetometer

Standard Indices of Geomagnetic Activity

- Range Index---K,K_p
  - (0, 1/3, 2/3, 1, … ,9)
  - Averaged over 13 standard observatories
  - Considered a measure of magnetospheric activity
- Storm Index---D_{st}
  - Ring Current
  - Measures average deviation of H (horizontal N-S) from a quiet day
- Substorm Index---AU,AL,AE,AO
  - Auroral Electroject uncontaminated by ring current
  - High latitude magnetometers---H componenGround Based Magnetometers

Dependencies between Solar Drivers and Magnetosphere

Observables



Dependencies

Observables

Kp
Dst
AE
Electron flux
Ion flux
…

Dependencies

Input Variables (x)

Kp
Dst
AE
Particle Fluxes

Output Variables (y)

Kp
Dst
AE
Electron flux
Ion flux
…

Internal/External

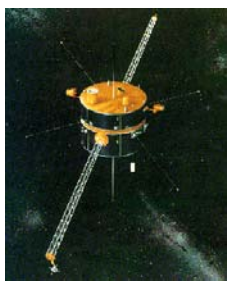
Dependencies

Input Variables (x)

Kp
Dst
AE
Particle Fluxes

VBs
Pdyn

Internal/External



Output Variables (y)

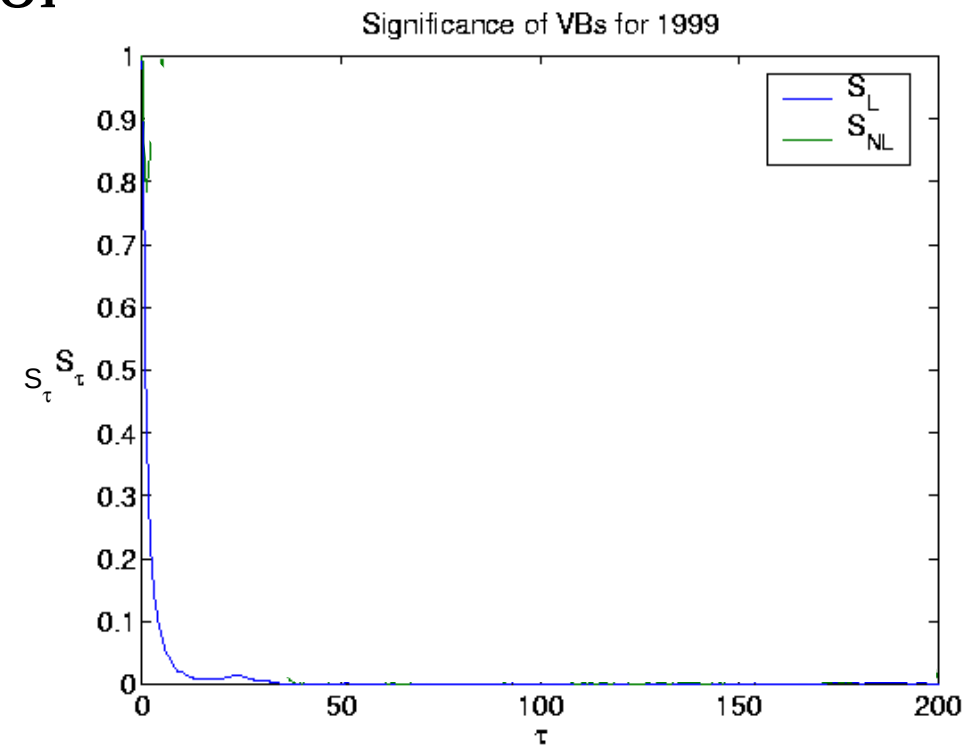
Kp
Dst
AE
Electron flux
Ion flux
…

Why Examine Dependencies?

- Identify the underlying dynamics and characteristic nonlinearities
- Assess system predictability
- Detect changes in underlying dynamics
- Basis for predictive model

Discriminating Statistical Measures

1. Correlation function



Correlation Function (Linear)

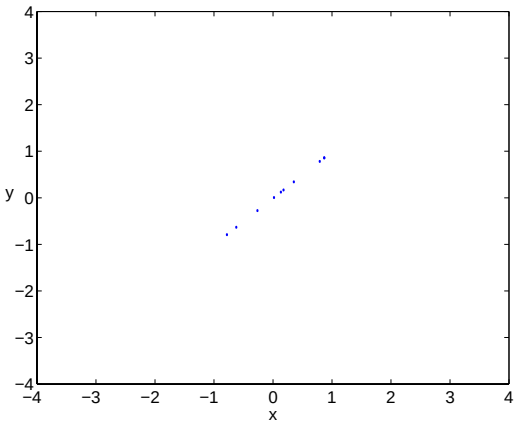
$$\lambda(\mathbf{X}, \mathbf{Y}) \equiv \sqrt{1 - \frac{\det C(\mathbf{X}, \mathbf{Y})}{\det C(\mathbf{X}) \det C(\mathbf{Y})}}$$

Input

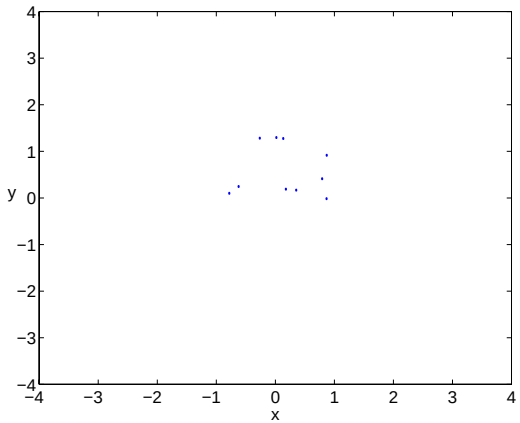
Output

Correlations

Correlated Data

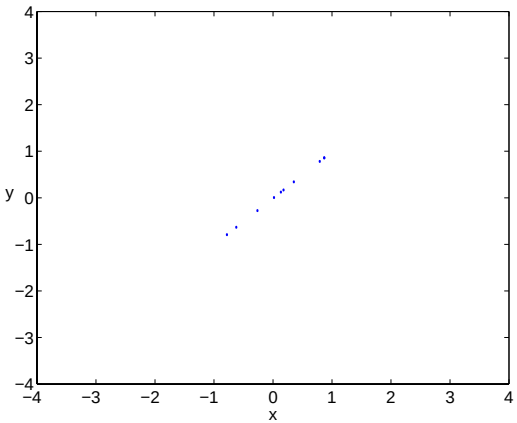


Uncorrelated Data



Higher Order Correlations

Correlated Data



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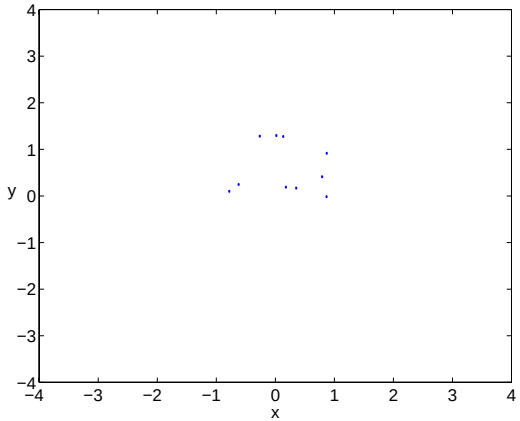
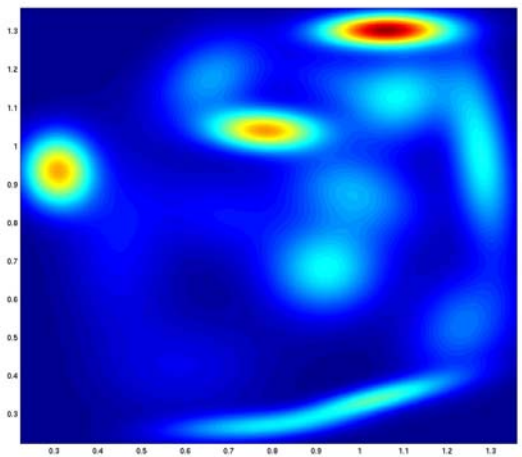


Higher-Order Correlations



Is this the same???

Discriminating Statistical Measures

1. Correlation function

$$P(x,y) = P(x)P(y) \ ?$$

Three Discriminating Statistical Measures

1. Correlation function
2. Mutual Information
3. Cumulants

Entropy and Mutual Information

$$H(x) = -\sum_{\aleph_1} p(x) \log p(x)$$

$$H(y) = -\sum_{\aleph_2} p(y) \log p(y)$$

$$H(x,y) = -\sum_{\aleph_1,\aleph_2} p(x,y) \log p(x,y)$$

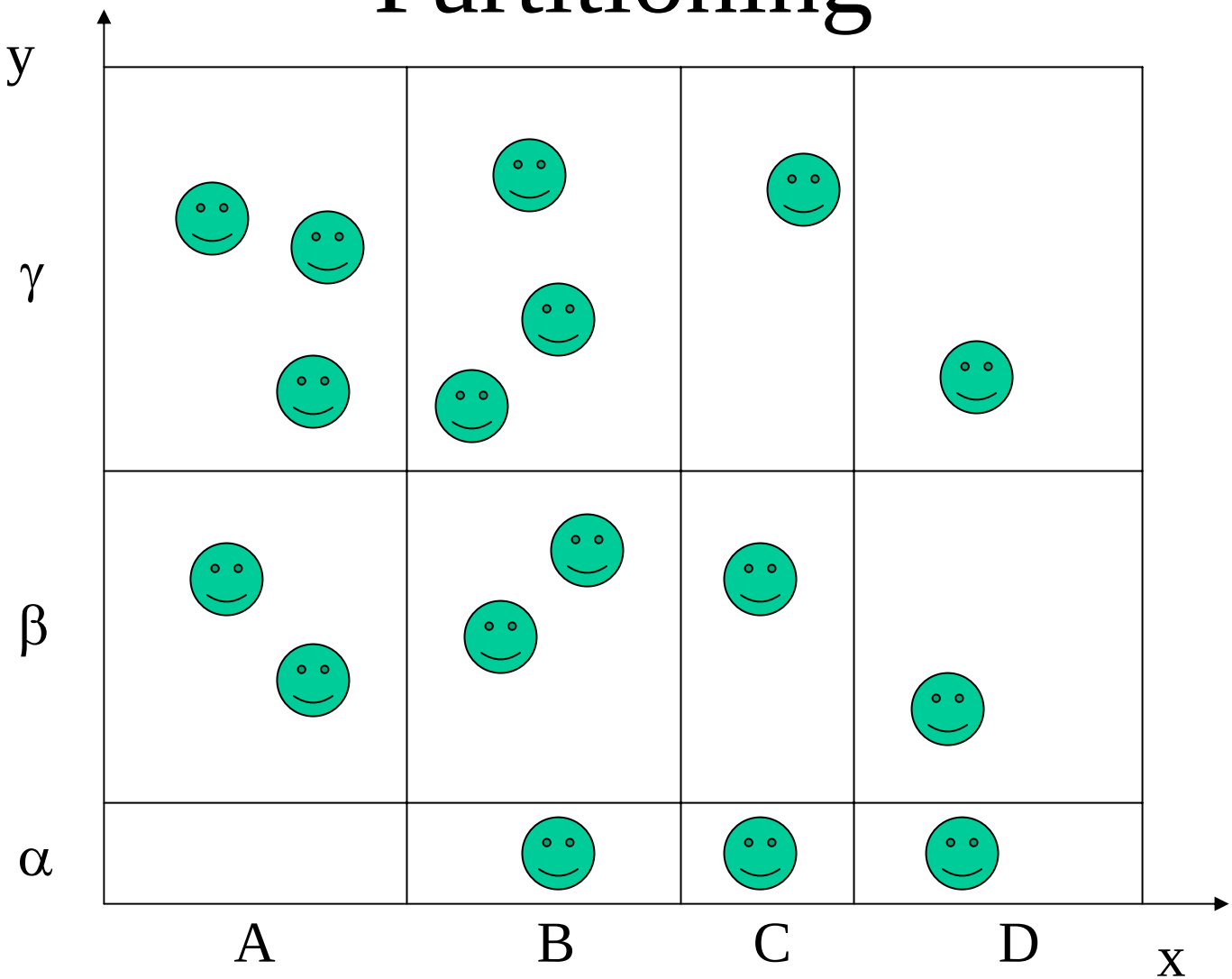
$$I(x,y) = H(x) + H(y) - H(x,y)$$

$$x \in \{1,\ldots,N\} \equiv \aleph_1$$

$$y \in \{1,\ldots,M\} \equiv \aleph_2$$

Mutual information is commonly used as an alternative to correlation functions which have limitations for nonlinear systems. Generalization to higher dimensions is called redundancy.

Partitioning



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Mutual information is commonly used as an alternative to correlation functions which have limitations for nonlinear systems. Generalization to higher dimensions is called redundancy.

Nonlinear Predictability (Mutual Information)

$$\Lambda(\mathbf{X}, \mathbf{Y}) \equiv \sqrt{1 - \exp(-2I(\mathbf{X}, \mathbf{Y}))}$$

Input

Output

$$\Lambda \Rightarrow \lambda$$

when Gaussian distributed joint PDF

Nonlinear Predictability (Mutual Information)

$$\Lambda(\mathbf{X},\mathbf{Y}) \equiv \sqrt{1-\exp(-2I(\mathbf{X},\mathbf{Y}))}$$

$$D_{MI} = \Lambda - \lambda$$

$$\Lambda \Rightarrow \lambda$$

when Gaussian distributed joint PDF

Cumulant Based Cost Function

$$P(x,y) = P(x)P(y) \ ?$$

Statistical Independence $\Rightarrow$ Cumulants vanish

Multivariate Cumulants

$$C_{i,\ldots,j} = \langle x_i \cdots x_j \rangle$$

Correlation Tensors

Cumulants

$$K_i = C_i = \langle x_i \rangle$$

$$K_{i,j} = C_{i,j} - C_i C_j = \langle x_i x_j \rangle - \langle x_i \rangle \langle x_j \rangle$$

$$K_{i,j,k} = C_{i,j,k} - C_{i,j} C_k - C_{j,k} C_i - C_{i,k} C_j + 2\, C_i C_j C_k$$

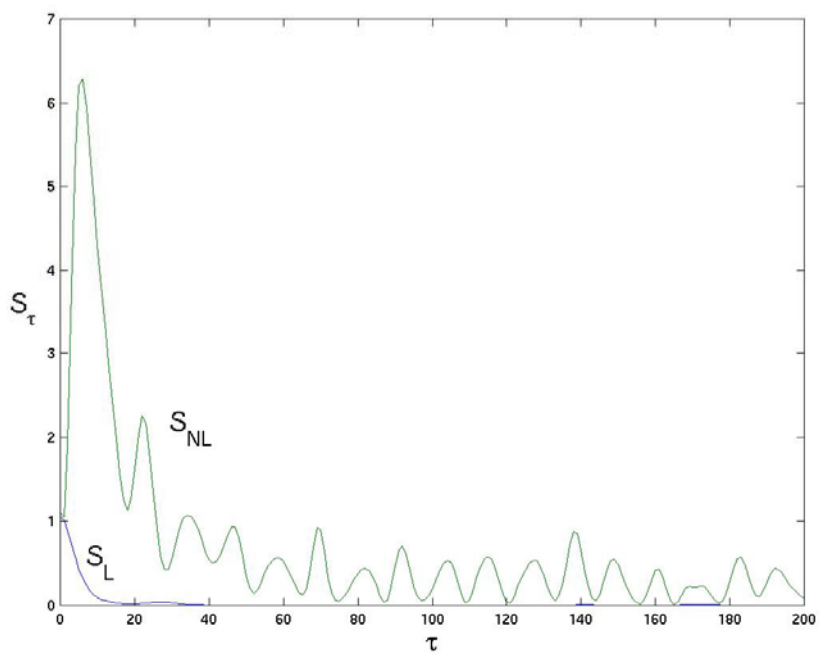
$$\begin{aligned} K_{i,j,k,l} = {} & C_{i,j,k,l} - C_{i,j,k} C_l - C_{i,j,l} C_k - C_{i,l,k} C_j - C_{l,j,k} C_i \\ & - C_{i,j} C_{k,l} - C_{i,l} C_{k,j} - C_{i,k} C_{j,l} \\ & + 2 \left(C_{i,j} C_k C_l + C_{i,k} C_j C_l + C_{i,l} C_j C_k + C_{j,k} C_i C_l + C_{j,l} C_i C_k + C_{k,l} C_i C_j \right) \\ & - 6\, C_i C_j C_k C_l \end{aligned}$$

$$\vdots$$

Cumulant Based Cost Function

$$P(x,y) = P(x)P(y) \ ?$$

Statistical Independence $\Rightarrow$ Cumulants vanish



Significance and Surrogate Data

$$S = \frac{|D_0 - \mu_s|}{\sigma_s}$$

Significance Measured
Relative to a Null Hypothesis

$$\mu_s = \frac{1}{N}\sum_{i=1}^{N} D_{S_i}$$

Surrogates Generated by
CAAFT process

$$\sigma_s^2 = \frac{1}{N-1}\sum_{i=1}^{N}(D_{S_i} - \mu_s)^2$$

Magnetospheric Response to Solar Activity

Papitashvili, 1999

Solar Cycle Variation

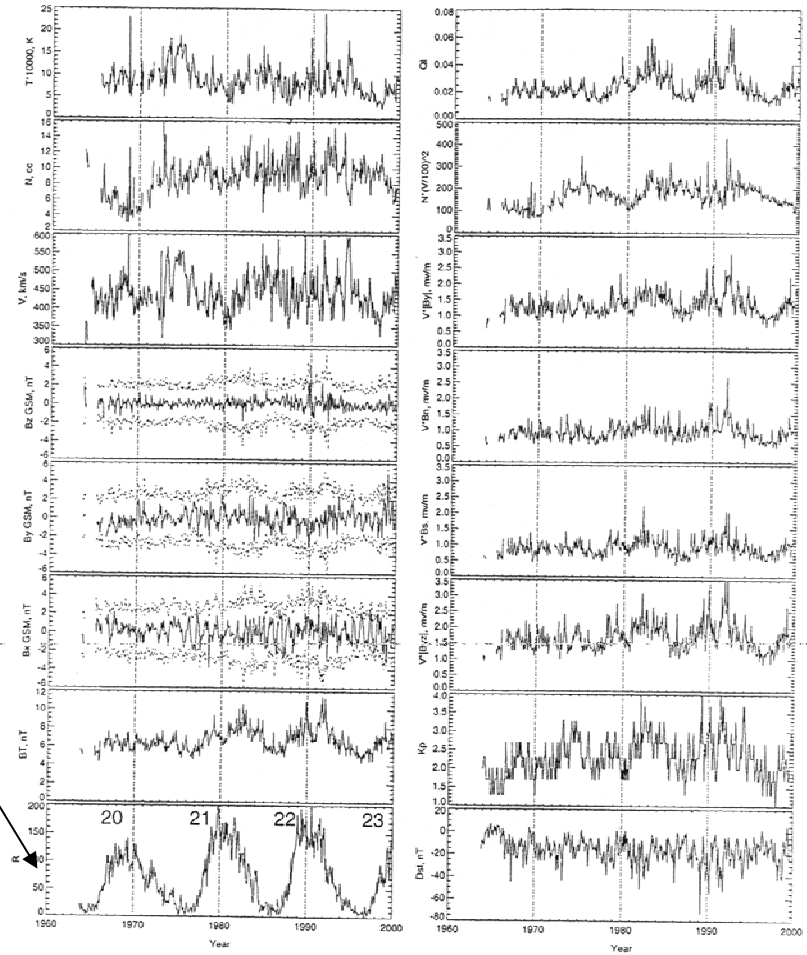


Figure 1. 27-day averages of some OMNI parameters from November 1963 through November 1999 in comparison with geomagnetic activity indices *Kp* and *Dst*, solar wind dynamic pressure, and various components of the interplanetary electric field. The solar cycle numbers (20, 21, 22, and 23) are printed at the bottom plot on the left panel.

P_{dyn}

VB_s

Solar Drivers

K_p

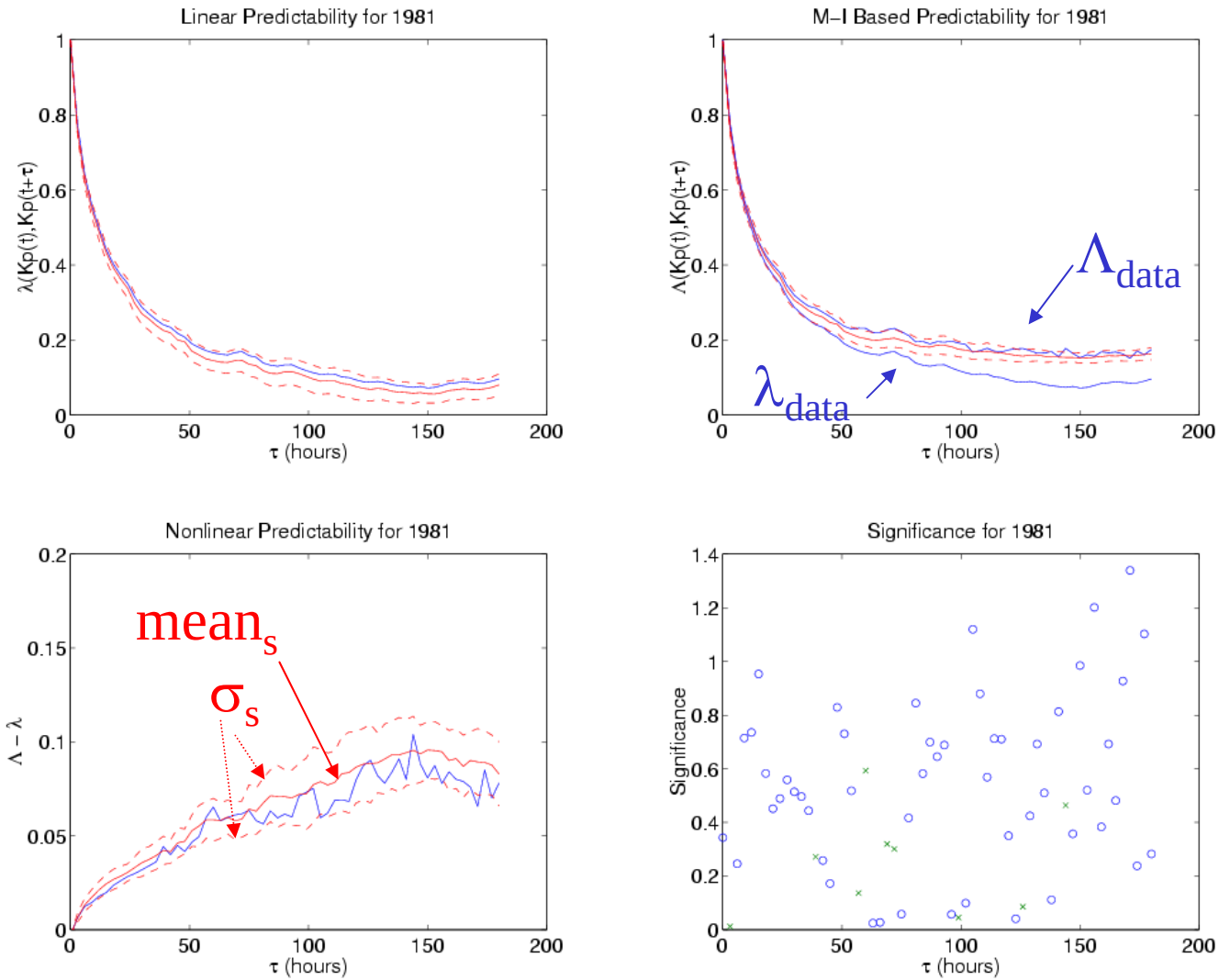
D_{st}

Msphere Response

Analyze Nonlinearity of Kp for All Historical Data

- Use discriminating statisics: $D_{MI}=\Lambda - \lambda$ and D_C to assess the original data
- Construct surrogate data having the same linear properties as the original data (λ)
  - CAAFT method
- Evaluate D_{MI} and D_C for the surrogate data
- Use the significance test to determine when the null hypothesis (linear dynamics) is invalid

Example---Solar Maximum (MI)



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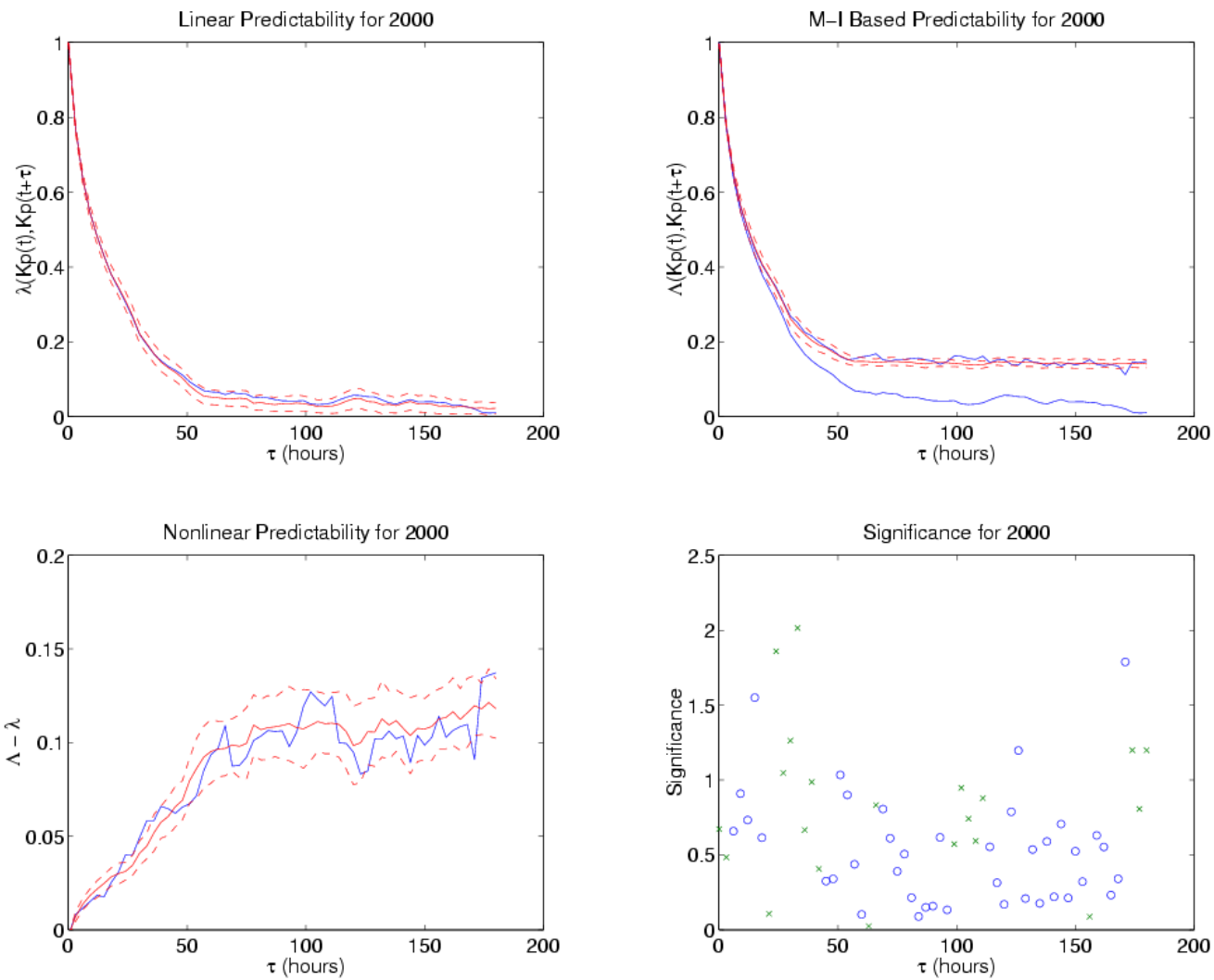

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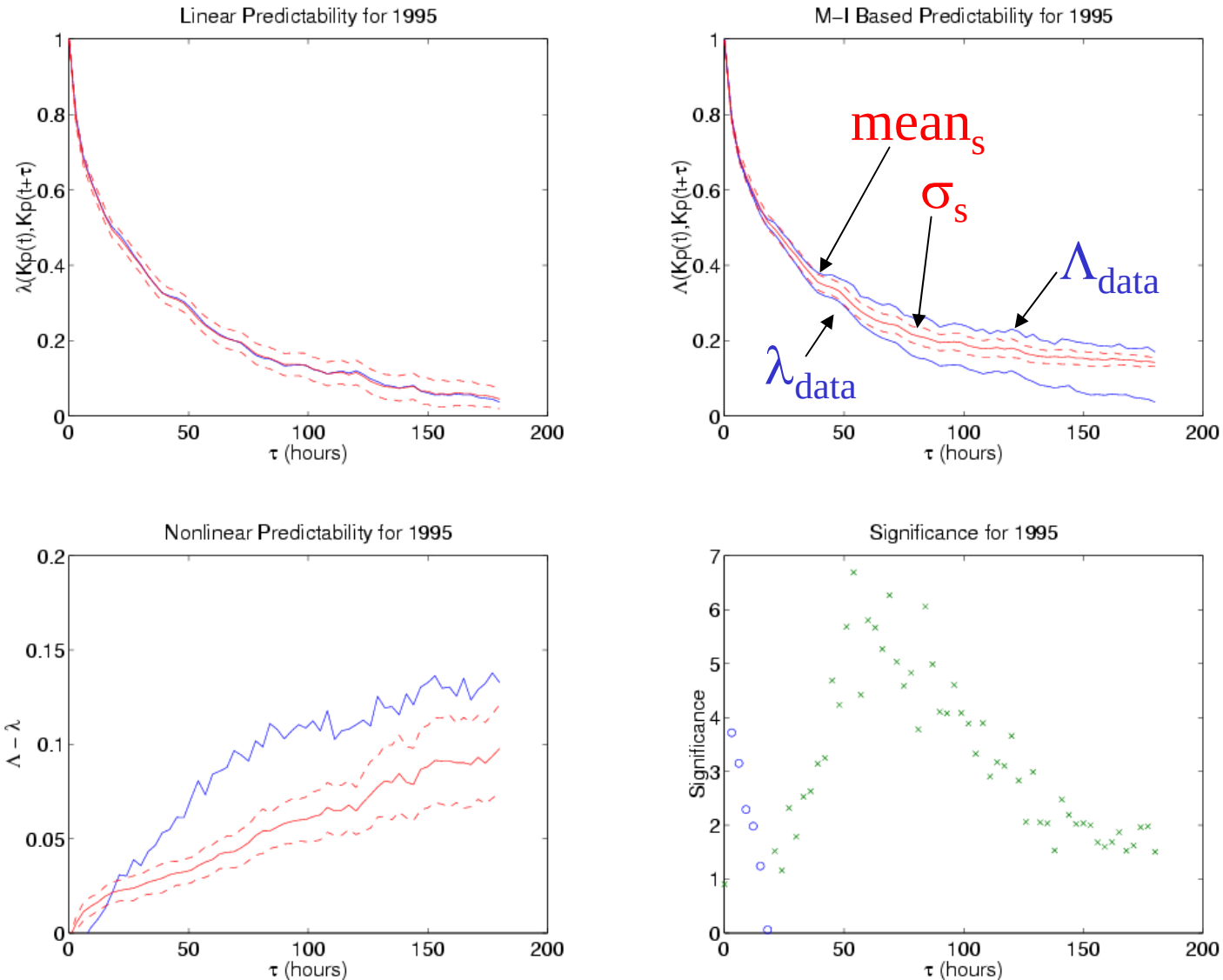

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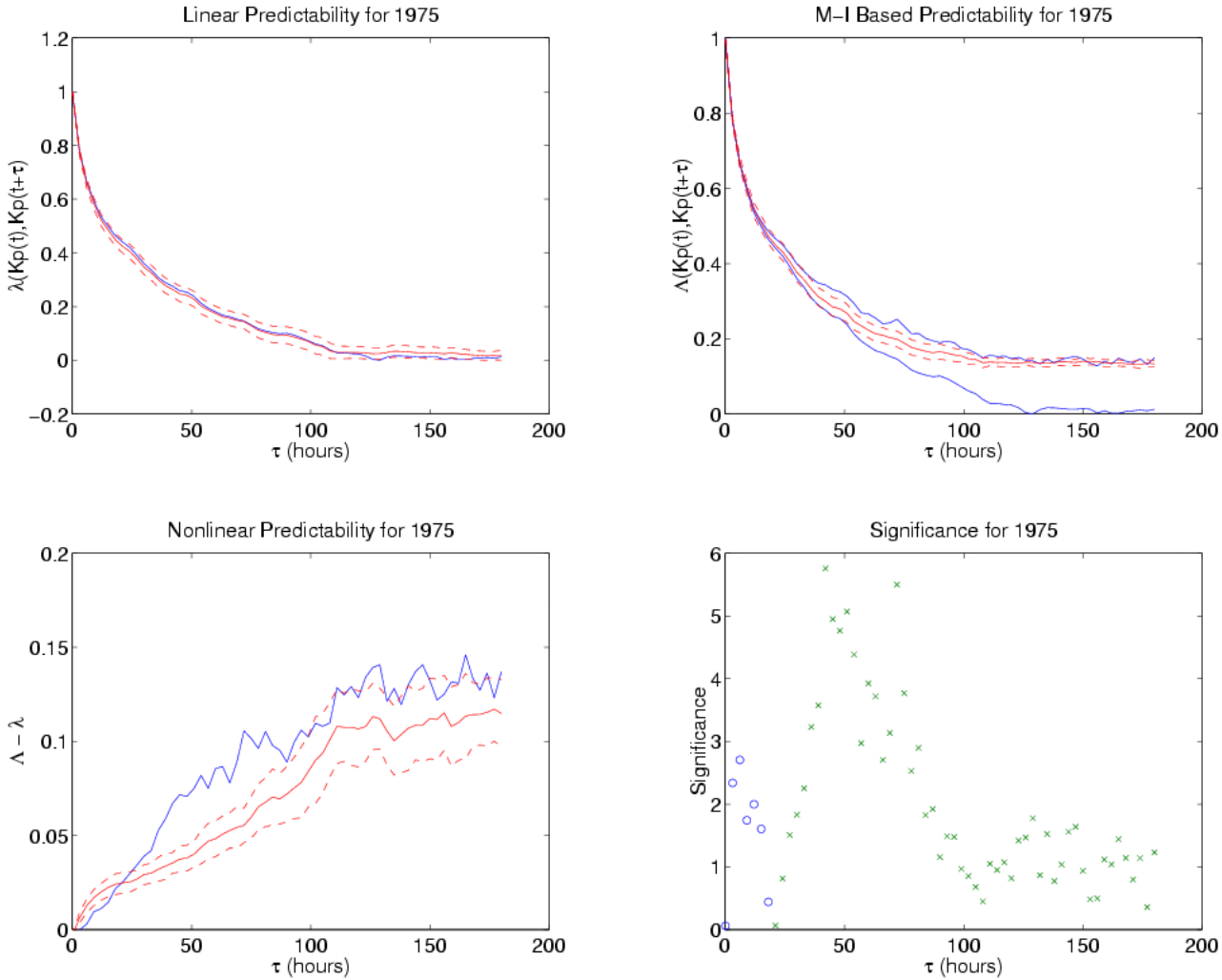
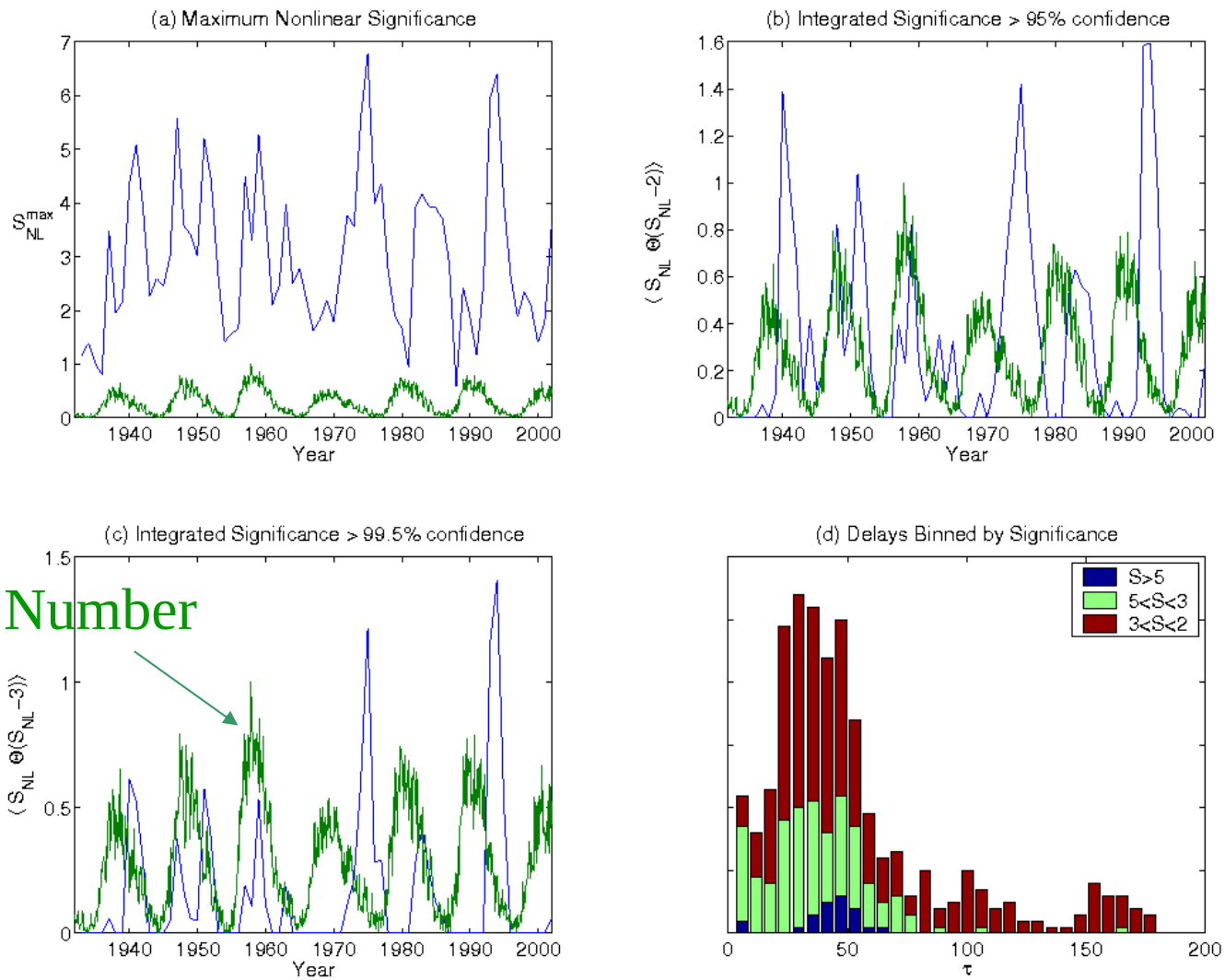
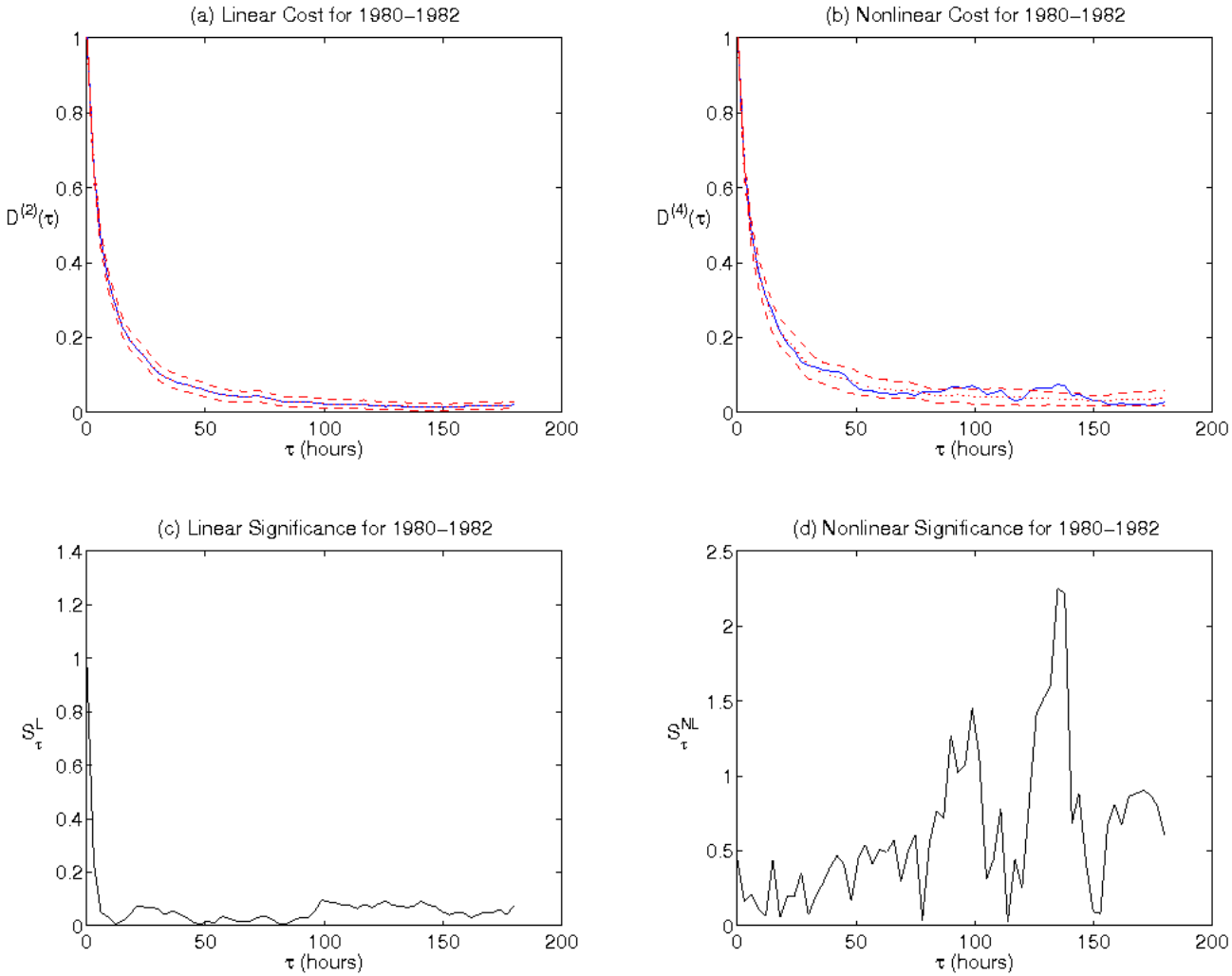


Discriminating Statistic---(Λ - λ)

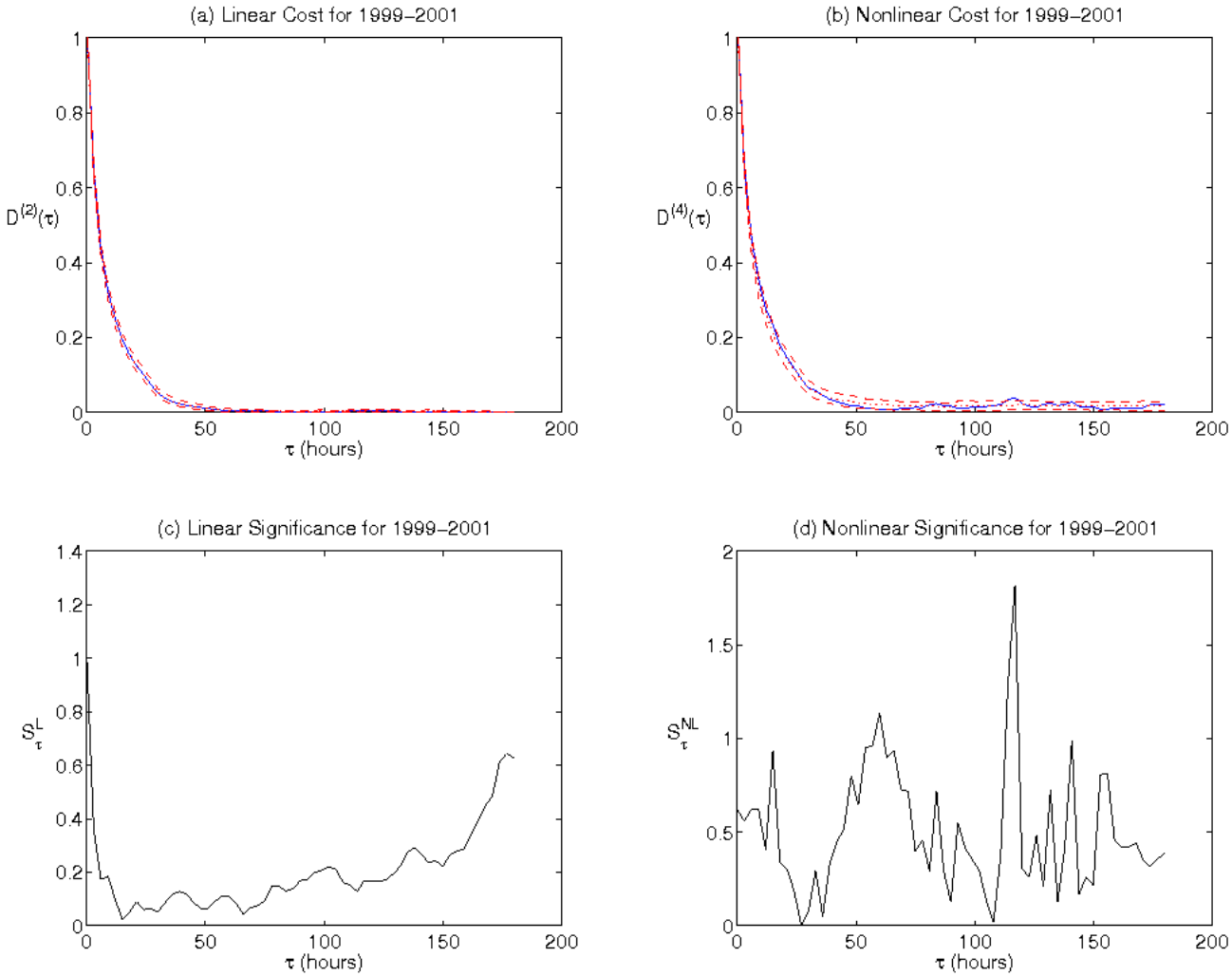


Sunspot Number

Example---Solar Maximum (C)



Example---Solar Maximum (C)



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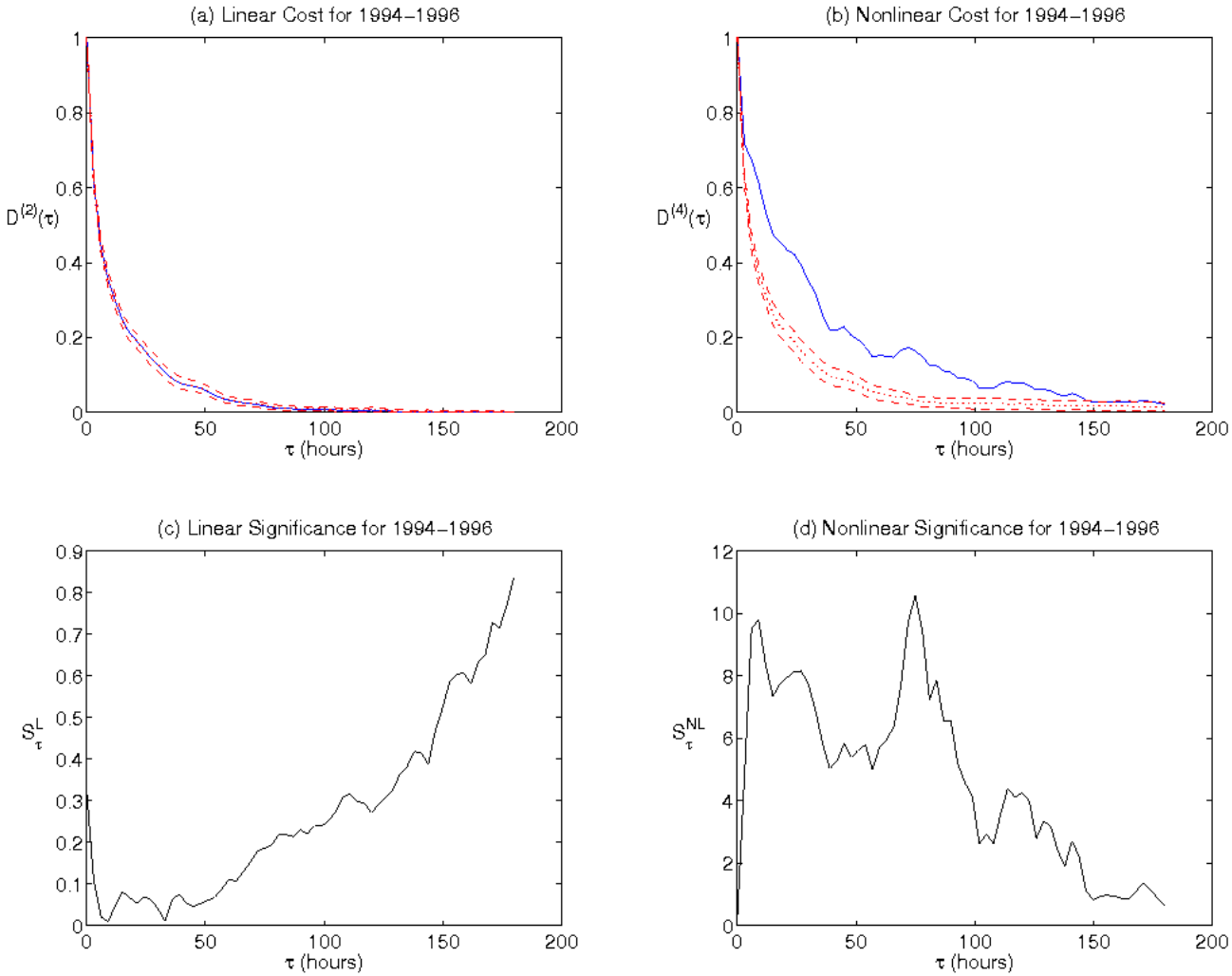

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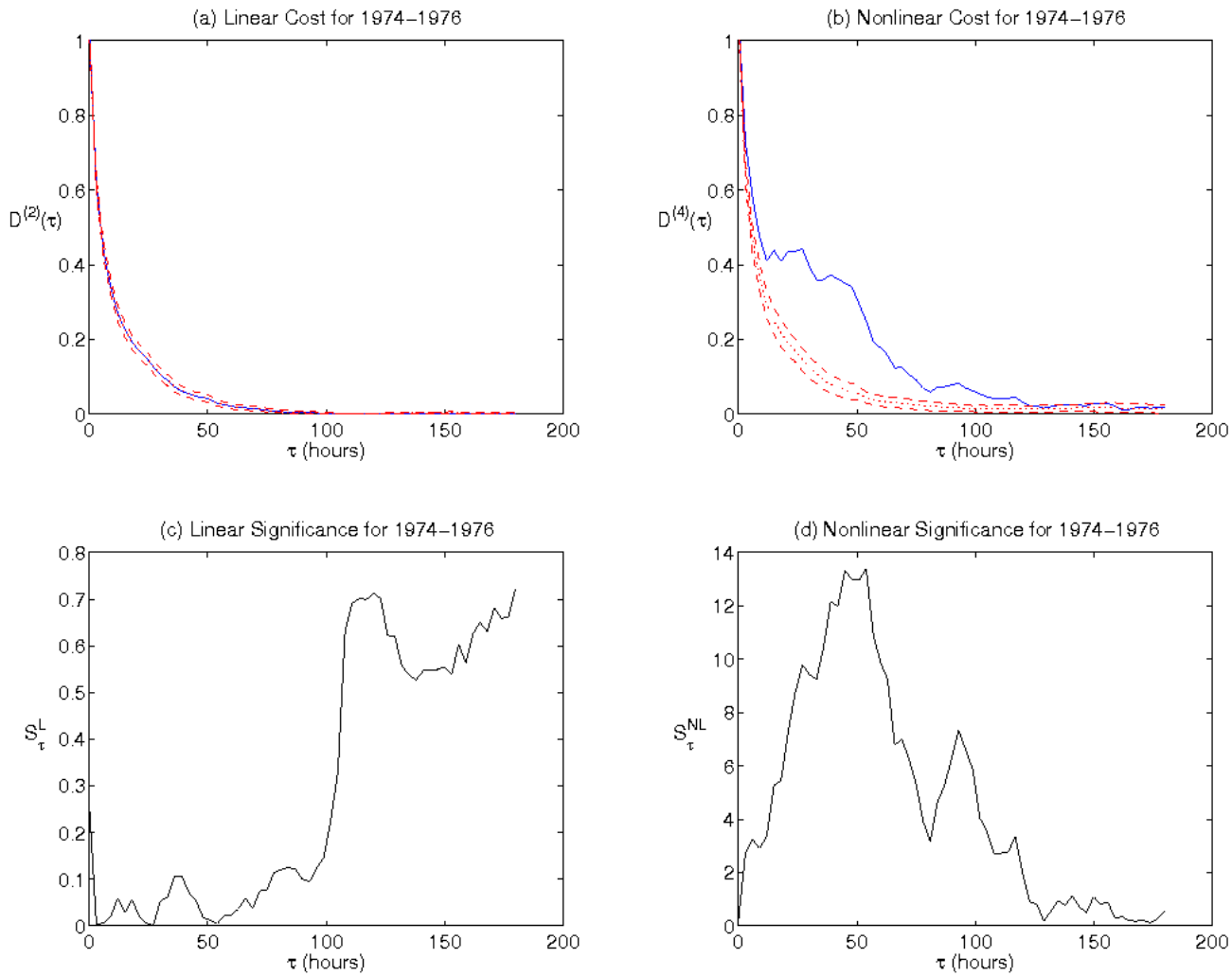
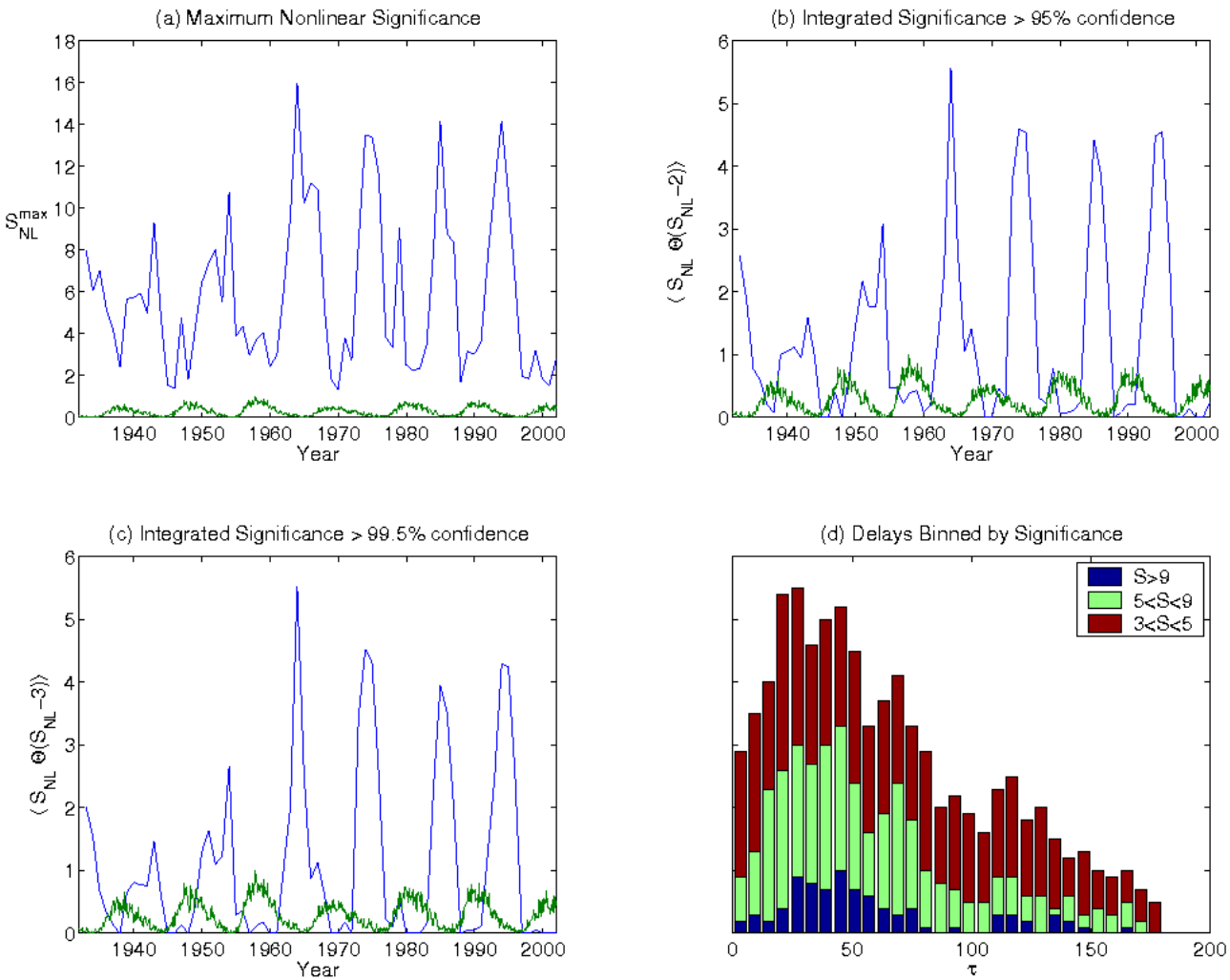


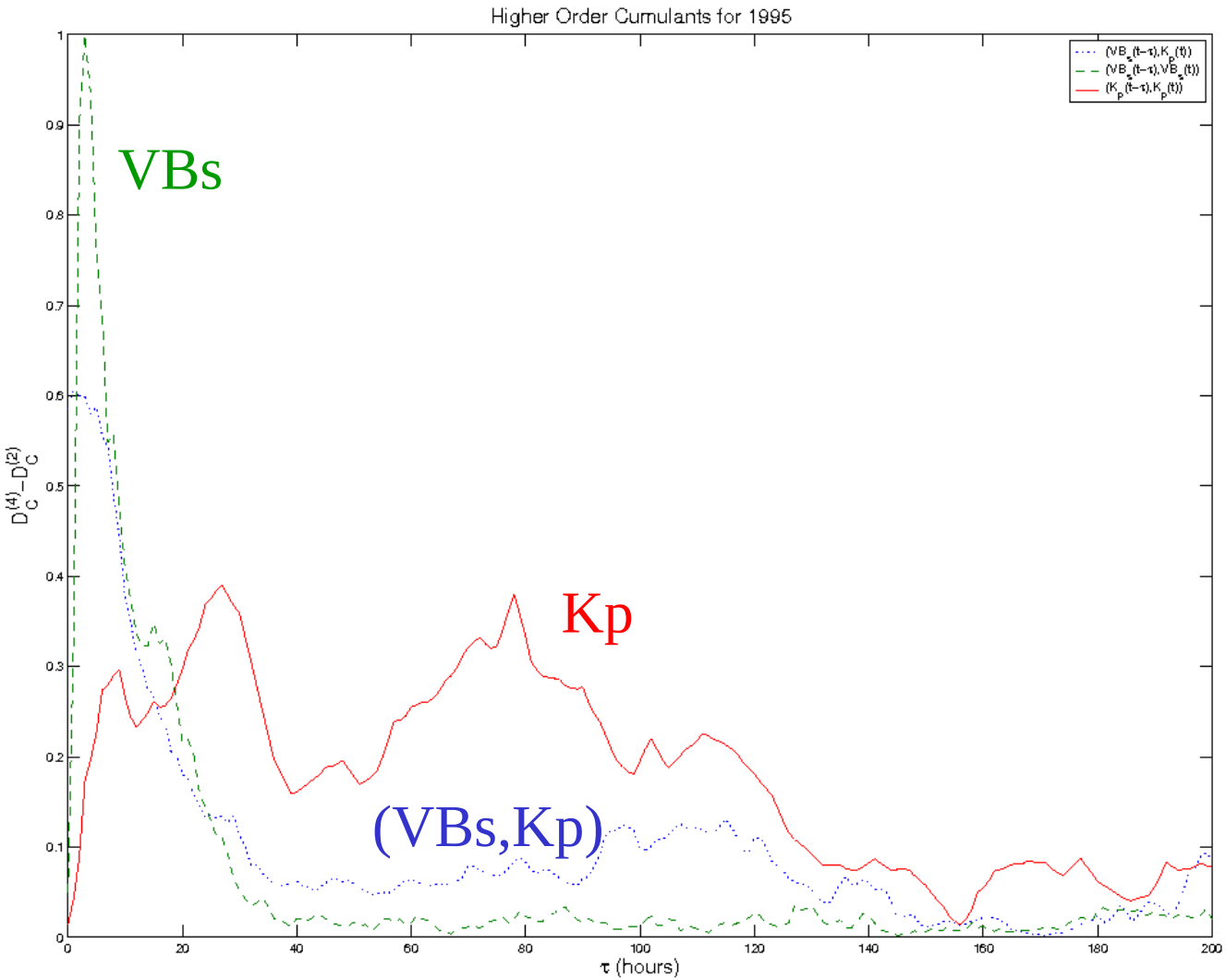
Discriminating Statistic---(C)



Conclusions on Kp Data

- There is a clear solar cycle dependence in both the MI and C discriminating statistics indicating the presence of nonlinearity
- Near Solar MAX the underlying dynamics can be described by a linear AR model.
- Near Solar MIN the underlying dynamics has nonlinearities that cannot be captured with a linear AR model
- Peaks in the nonlinearity occur roughly 2 years before minimum sunspot number.
- The timescale of the peak significance varies from solar cycle to solar cycle, but remains roughly the same for each cycle.
- The most significant timescale has a time delay around 40 hours ranging up to 1 week

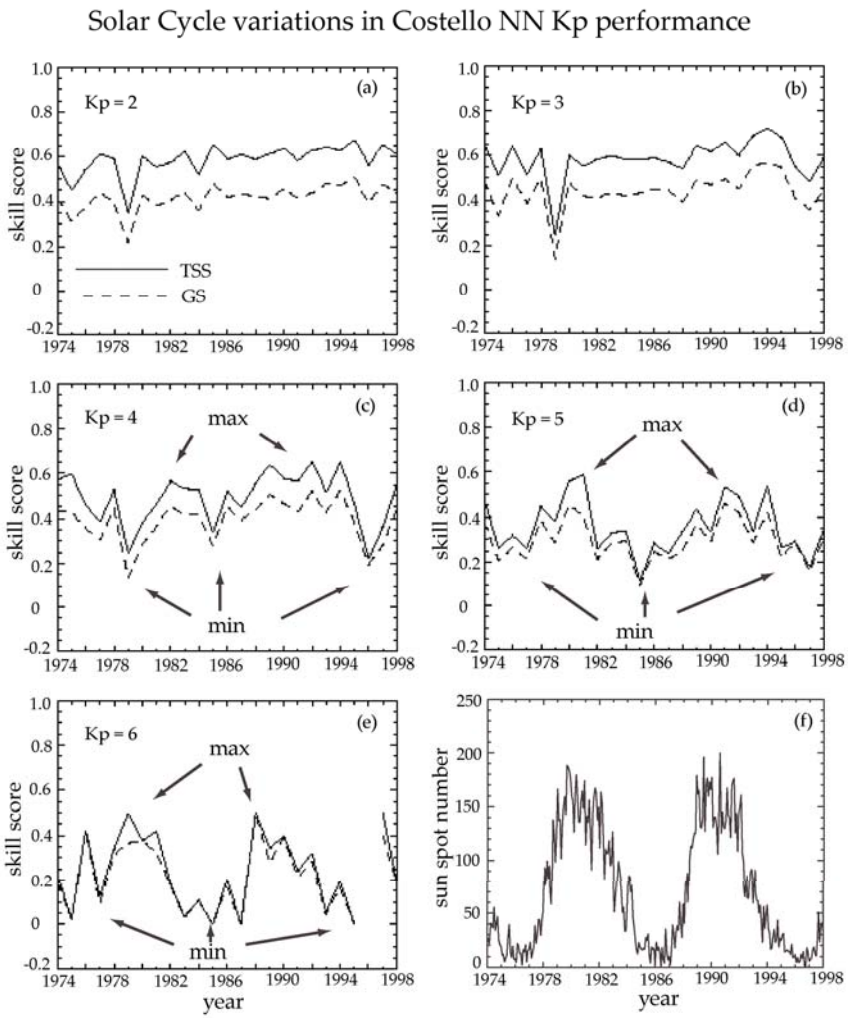
Nonlinearity and Solar Driver



Conclusions About Solar Wind Drivers and Internal Dynamics

- VBs contains significantly more nonlinearity than P_{dyn}
- Solar wind nonlinearity is much more peaked at shorter timescale than magnetospheric nonlinearity
- Cross correlation of VBs with Kp suggests that the nonlinearity of Kp is due to internal magnetospheric dynamics

Neural Network Predictability



- Costello
- Trained with only Solar Wind Drivers

Nonlinearity and Solar Max/Min

- Analogy: Driven, Damped Oscillator
  - Weakly driven---linear
  - Moderately driven---highly nonlinear
  - Strongly driven---linear

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  - Up to 150 hours (1 week)
  - Peaks around 40 hours

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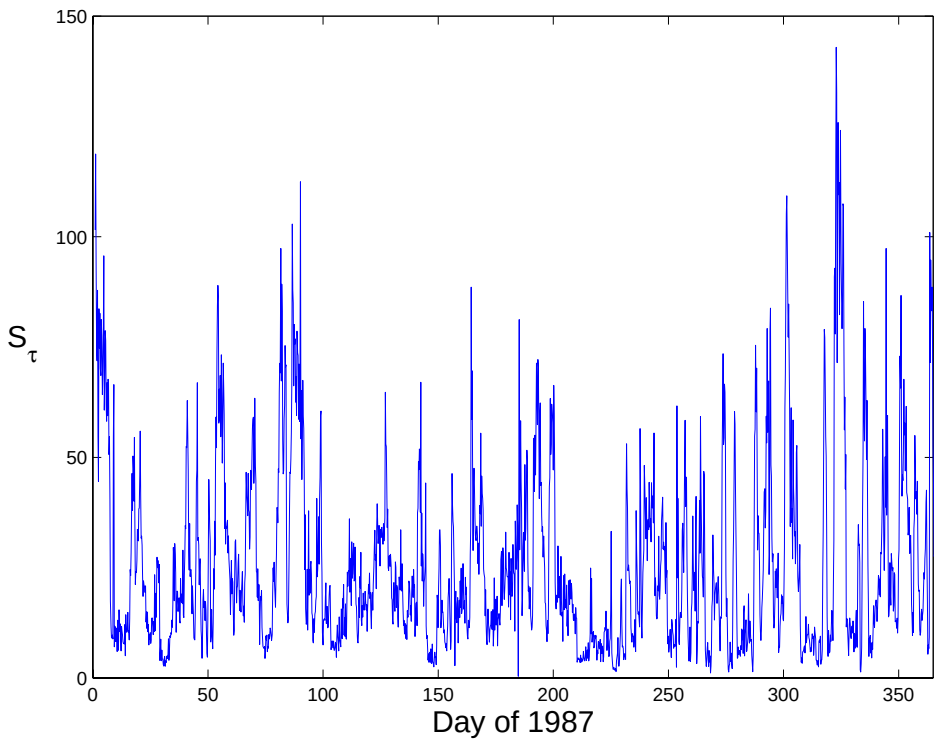
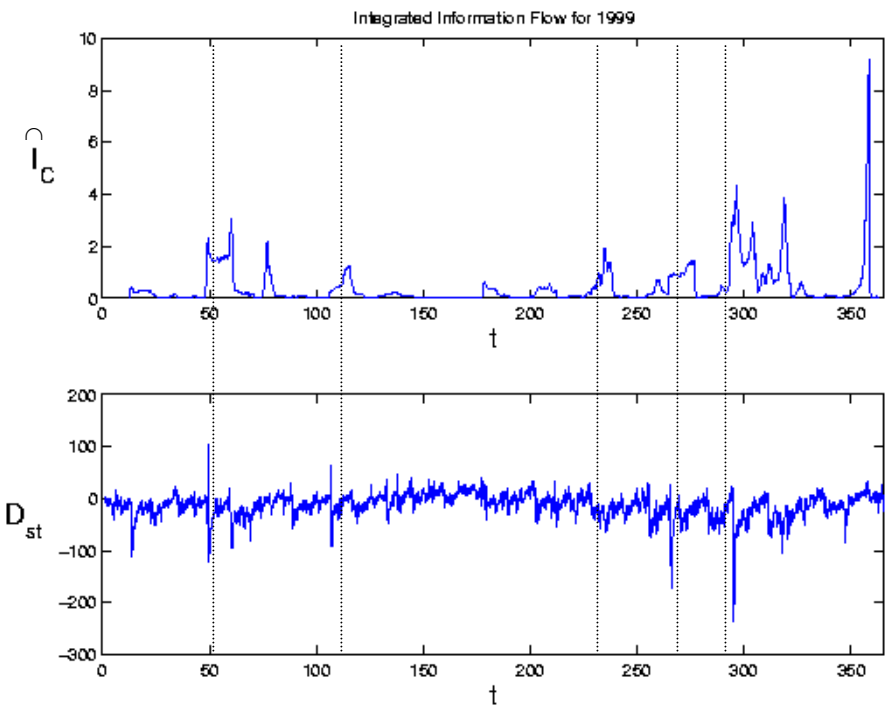
  - Up to 150 hours (1 week) Storms
  - Peaks around 40 hours Relaxation Processes
    - Charge exchange (hours to days)
    - Wave particle interaction (days)

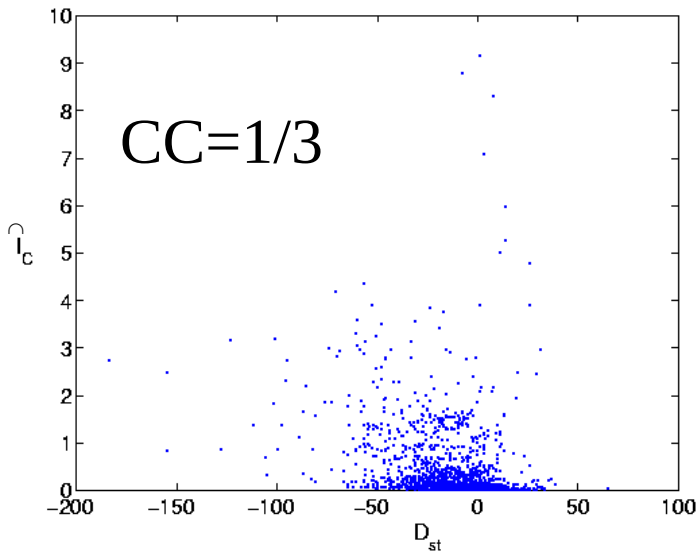
Variation in Underlying Magnetospheric Dynamics



- $m=3, \Delta=1hr, N_W=300hrs$
  $N_S=100$
- S >>1 means strong time ordering and good predictability over ~10 days
- Large Variations in S imply changes in underlying dynamics
- Training of Neural Network improved if data with high significance is used

Integrated Cumulant-Based Information and Predictability



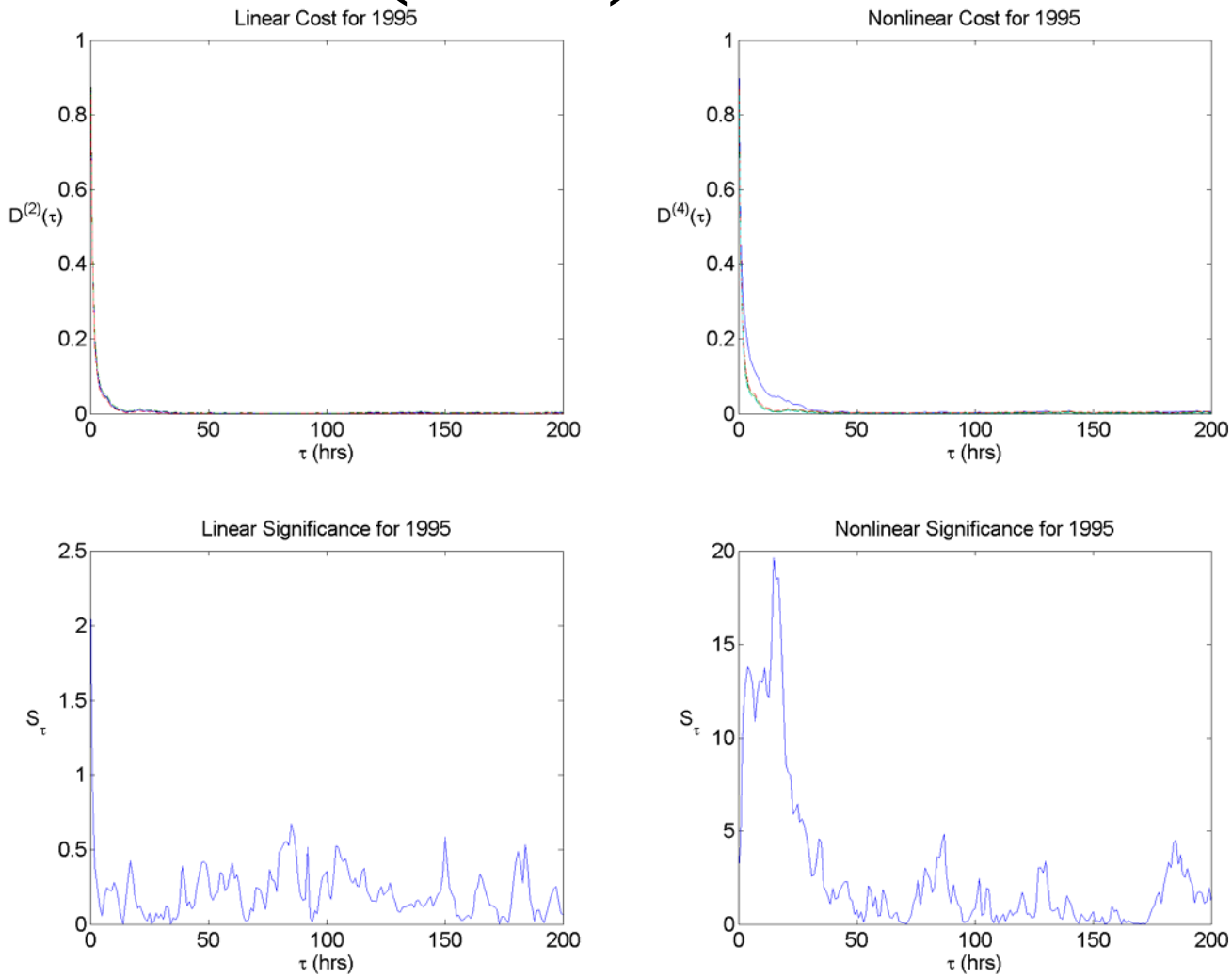


$\hat{I}_c(P)$ for P=20hrs,Δ=1hr

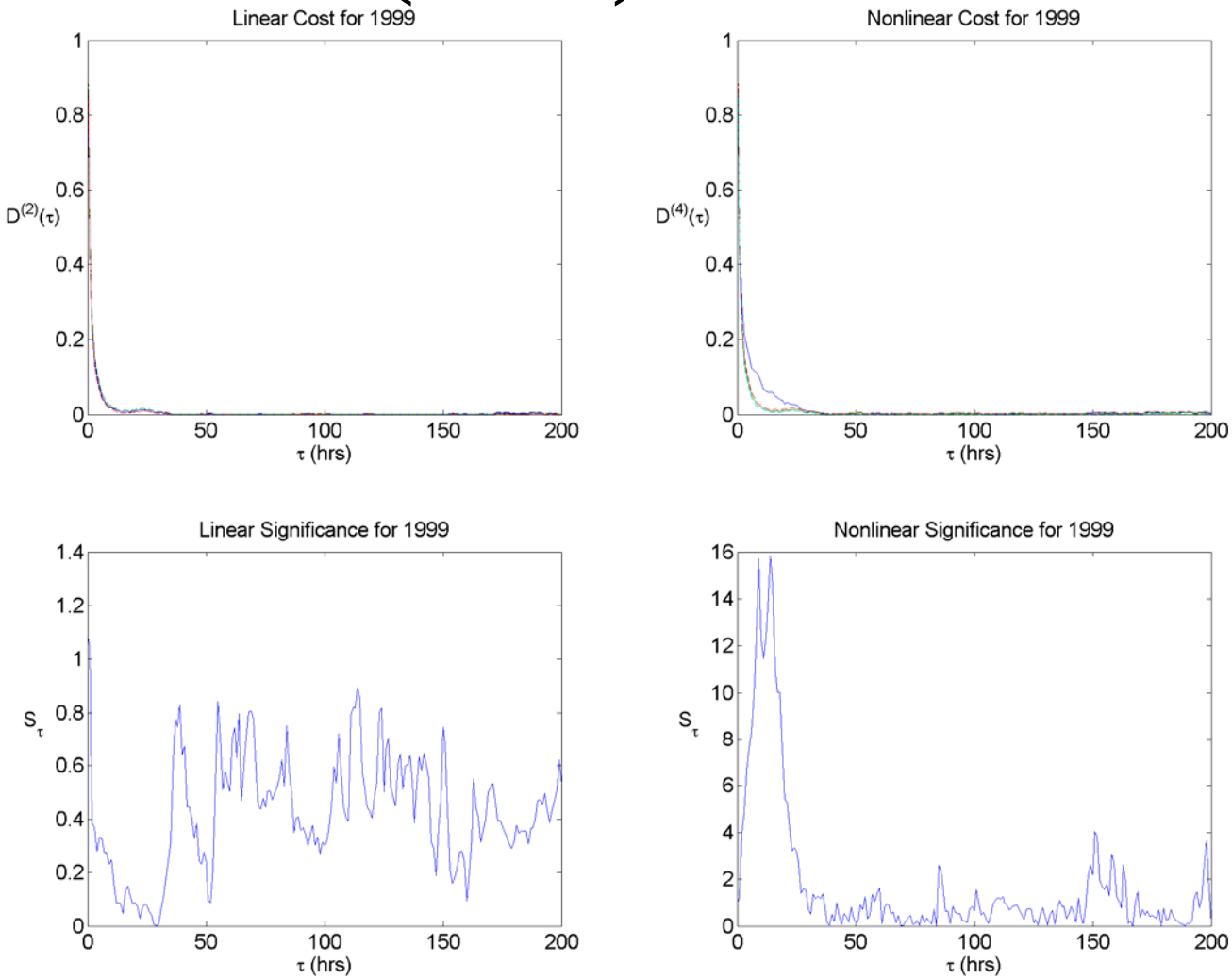
$\mathbf{x}(t) = (D_{st}(t+p), D_{st}(t), VB_s(t), VB_s(t-\Delta))$

Supplemental Material: Solar Nonlinearity

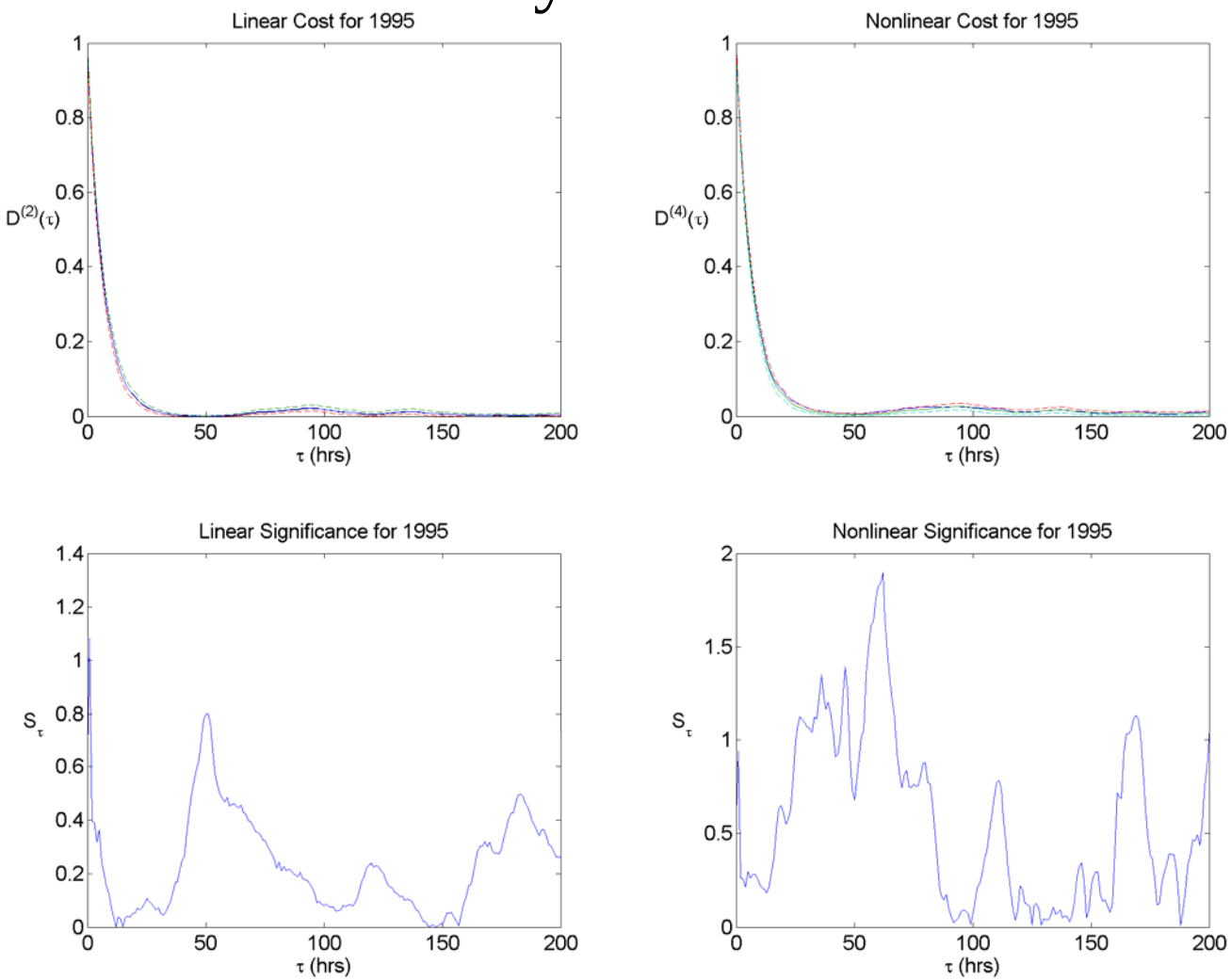
Nonlinearity in Solar Wind Driver (VBs) at Solar Min



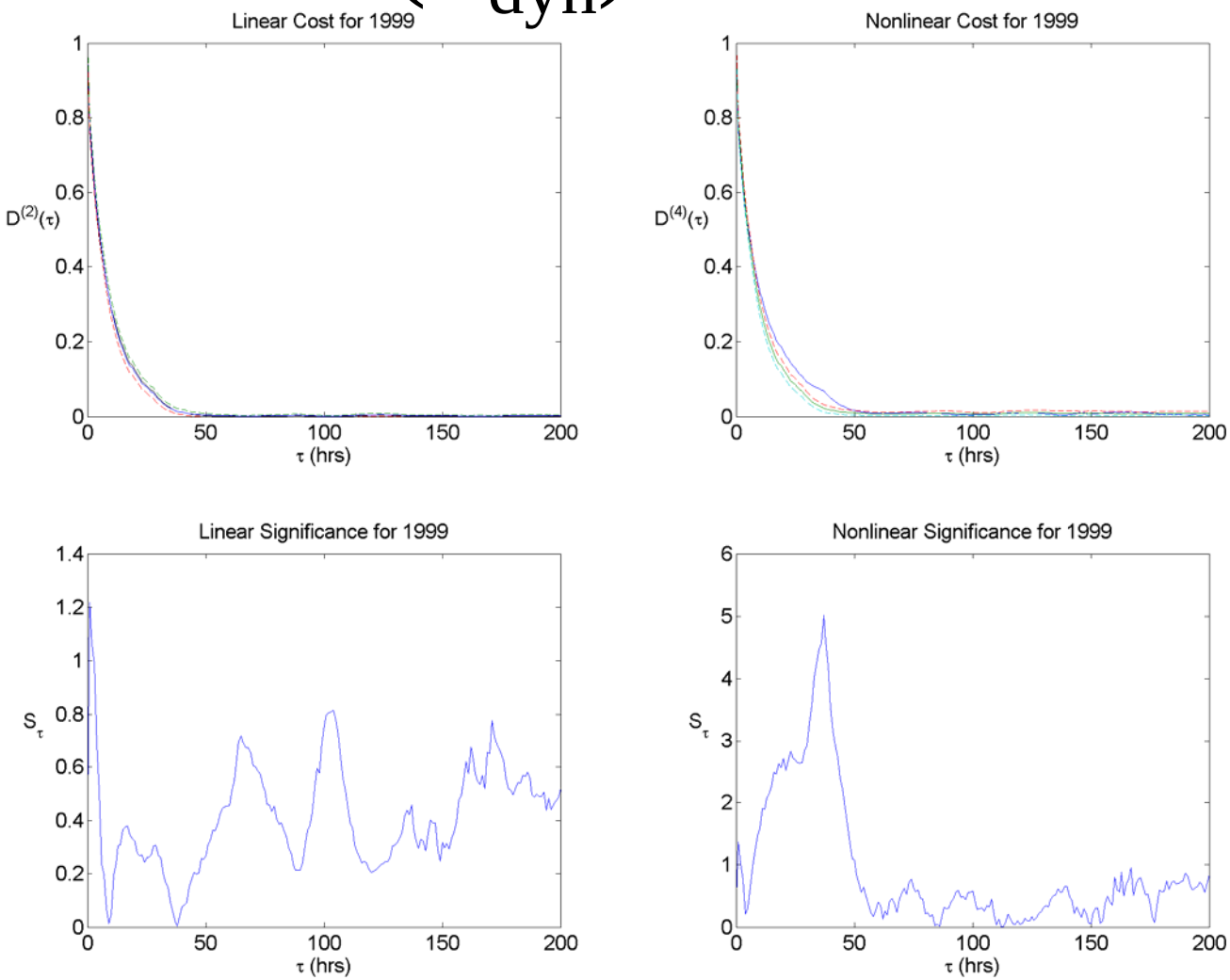
Nonlinearity in Solar Wind Driver (VBs) at Solar Max



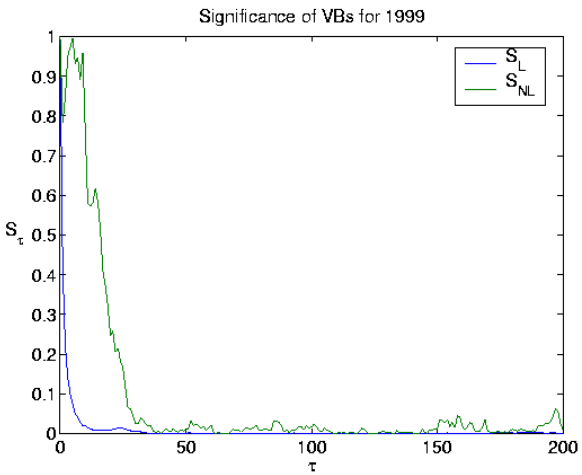
Nonlinearity in Solar Wind Driver (P_{dyn}) at Solar Min



Nonlinearity in Solar Wind Driver (P_{dyn}) at Solar Max

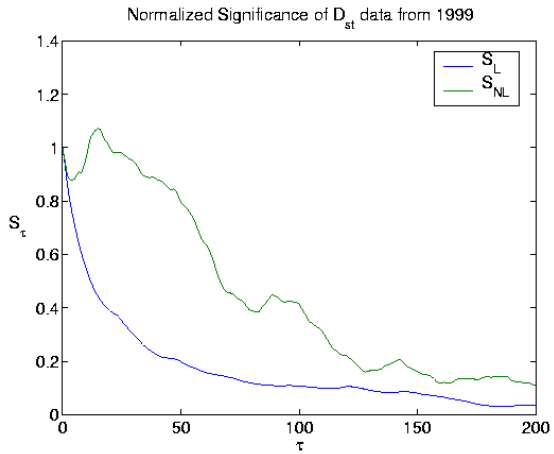


Solar Wind Correlations do not Exhibit Dst and Kp Behavior

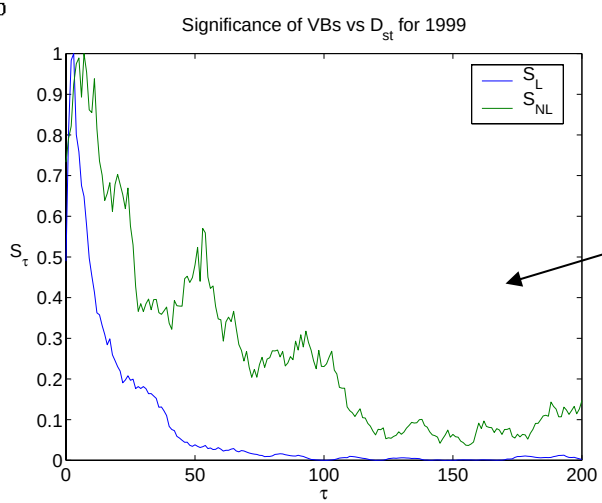


VBs Nonlinearity

VBs/Dst



Dst Nonlinearity



Looks like internal Dynamics

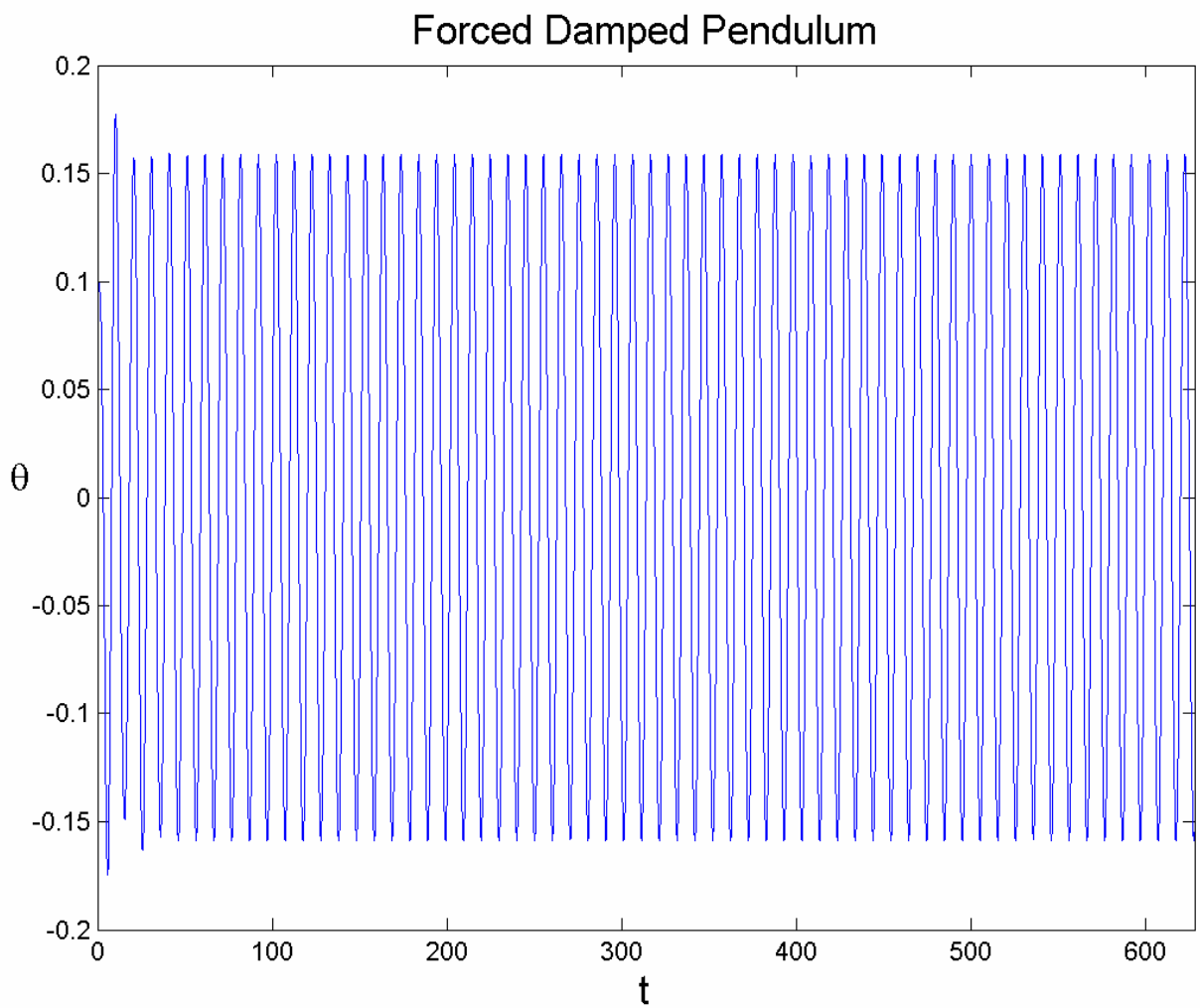
Supplemental Material: Driven Damped Oscillator

Forced Damped Oscillator

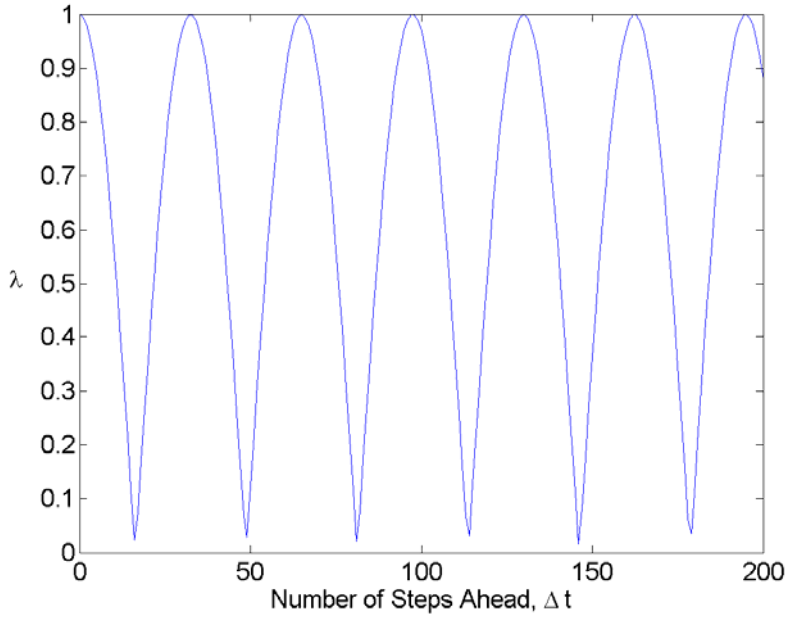
$$\frac{\partial^2\theta}{\partial t^2} + \nu\frac{\partial\theta}{\partial t} + \sin\theta = B\cos(2\pi f t)$$

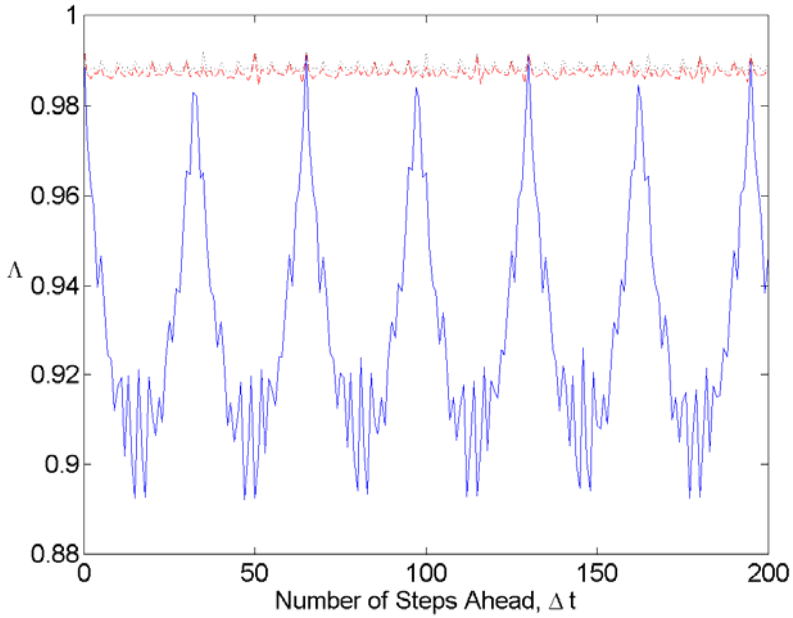
$$\nu = 0.5; f = 0.098; B = 0.1, 1.15, 3$$

Weakly Driven Oscillator

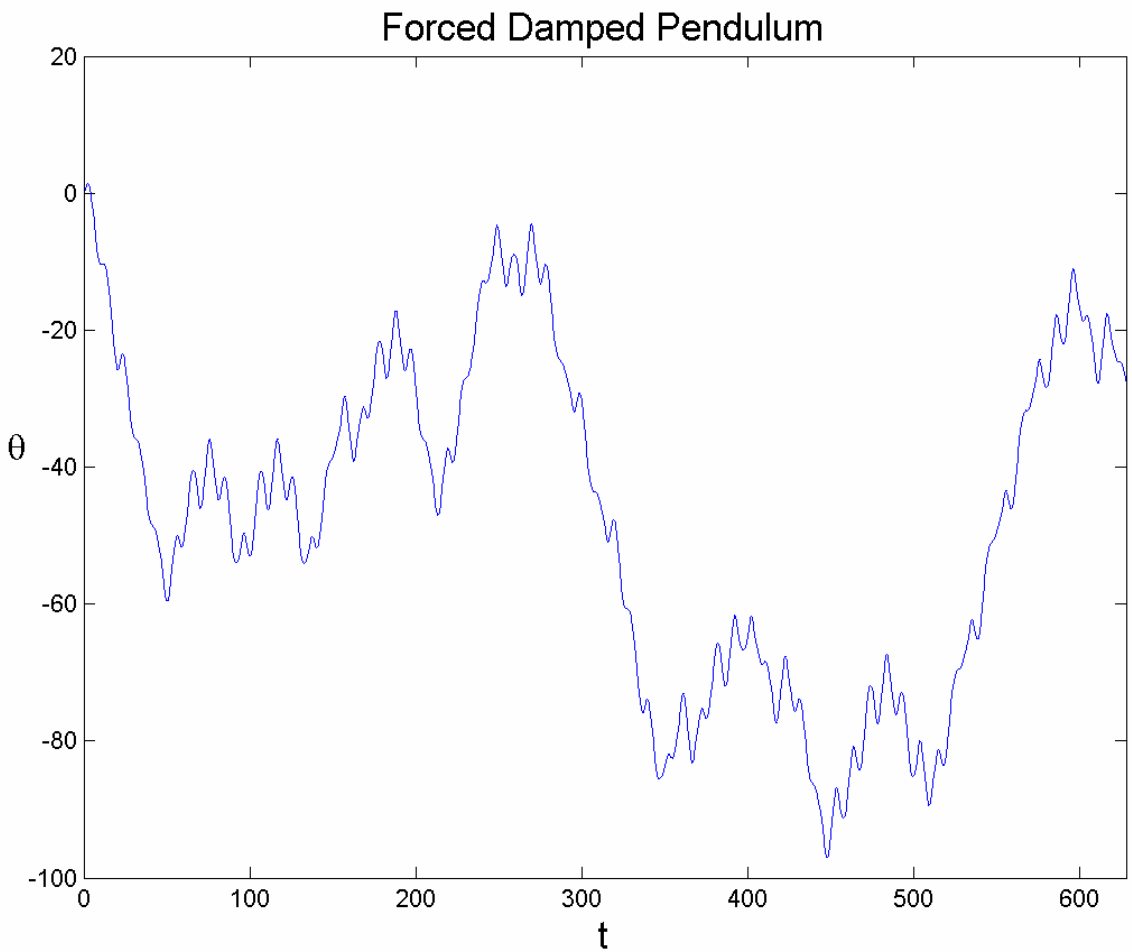


Linear/Nonlinear Predictability Weakly Driven Oscillator (B=0.01)

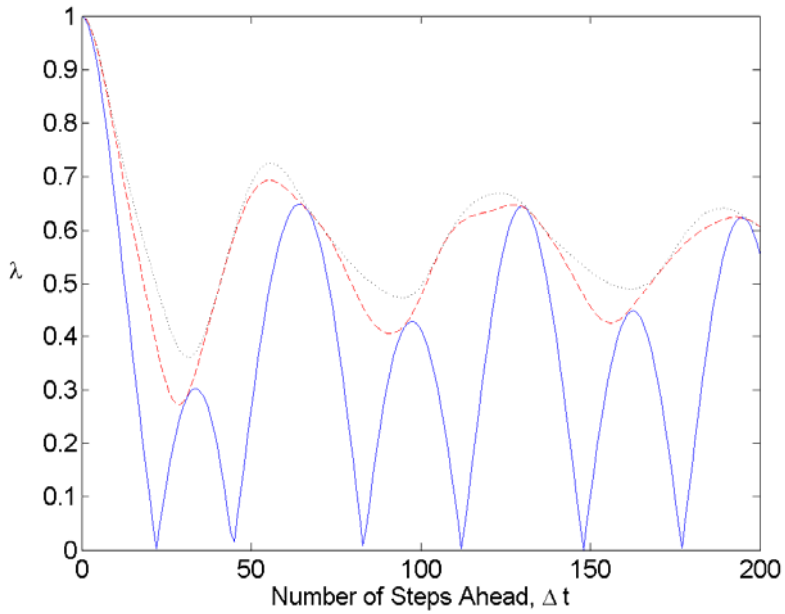


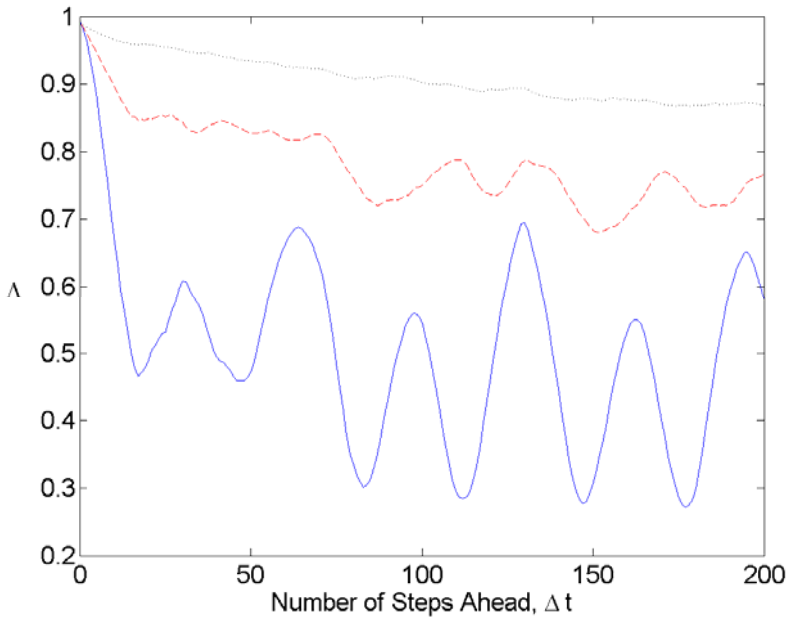


Chaotic Driven Oscillator

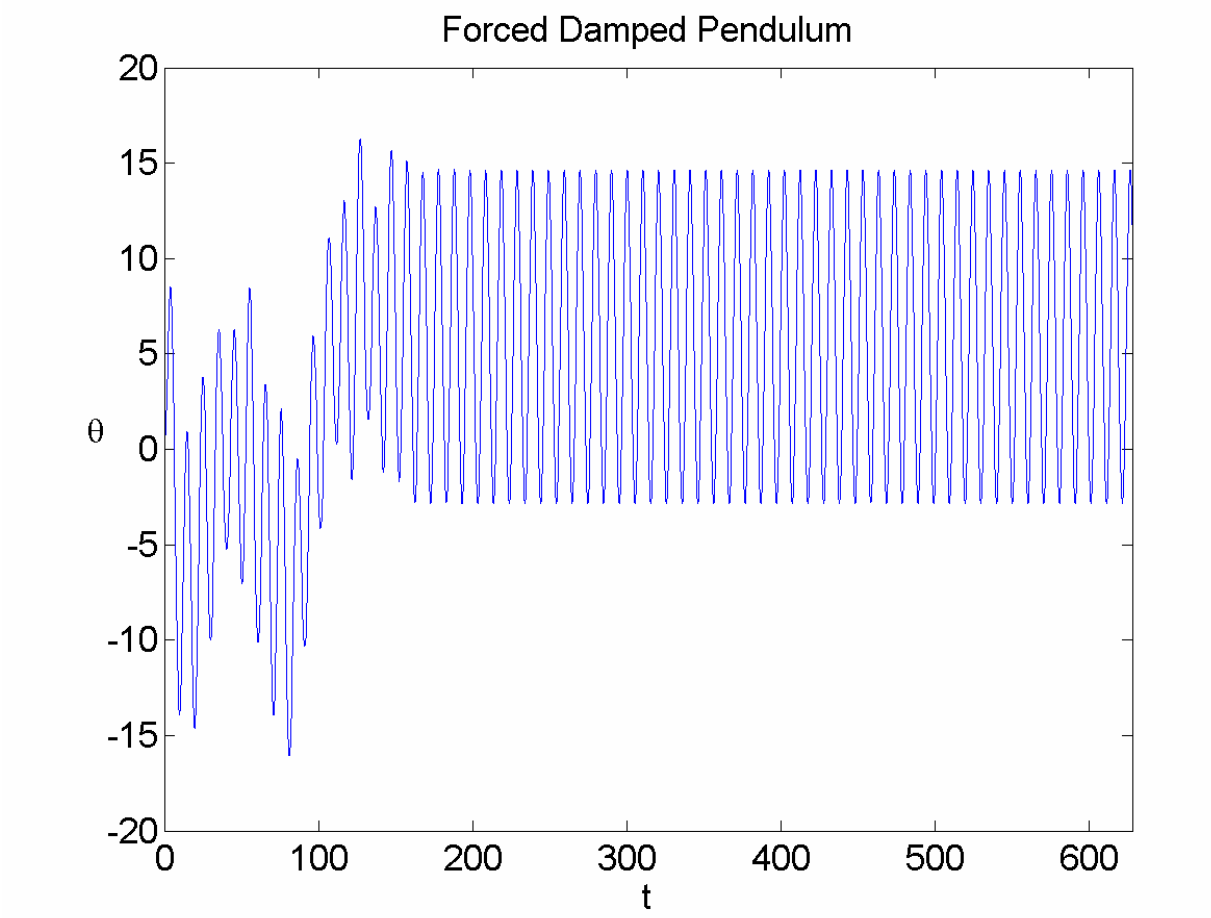
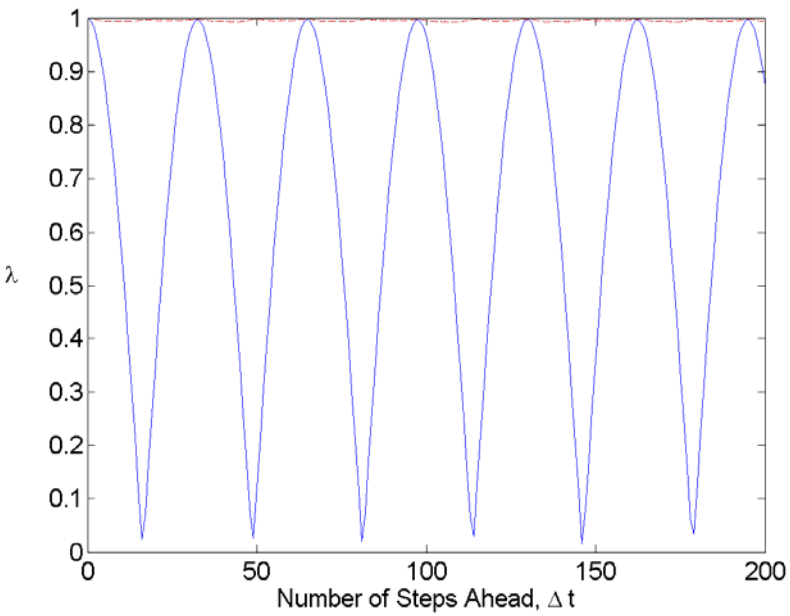
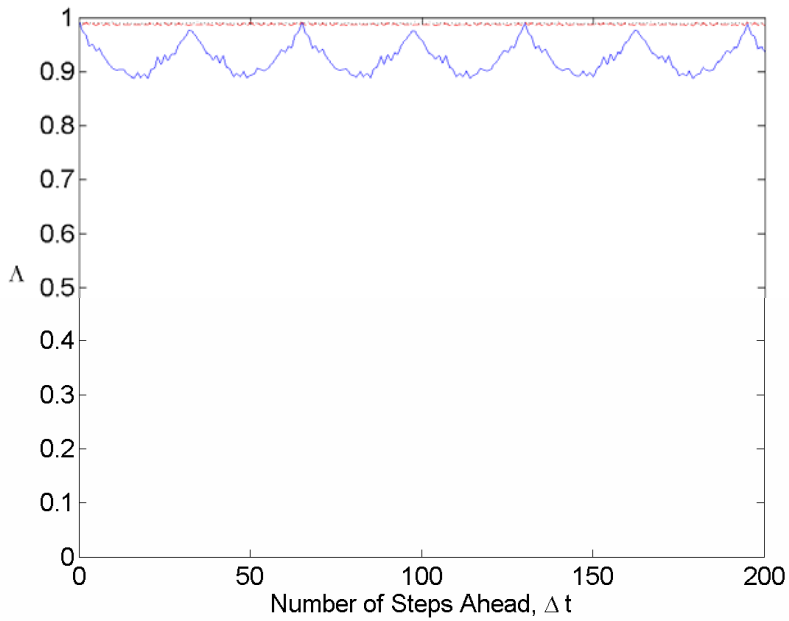


Linear/Nonlinear Predictability Chaotic Driven Oscillator (B=1.15)





Overdriven Oscillator



Linear/Nonlinear Predictability Overdriven Oscillator (B=3)