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Controlled fusion with hot-ion mode in a degenerate plasma

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Abstract

In a Fermi-degenerate plasma, the rate of electron physical processes is much reduced from the classical prediction, possibly enabling new regimes for controlled nuclear fusion, including the hot-ion mode, a regime in which the ion temperature exceeds the electron temperature. Of great importance in the hot-ion mode operation are the ion–electron collision rate and the fraction of energy that goes from fusion by-products into electrons, the previous calculations of which are refined here by including the relativistic effect and partial degeneracy.

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1. Introduction

The fuel which can sustain thermonuclear reactions most easily is a mixture of deuterium and tritium (D–T). The D–T reaction, however, has serious drawbacks. First, tritium does not exist in substantial quantities. Second, the D–T reaction produces neutrons; neutrons activate other material and damage the first wall. Therefore, it would be desirable to utilize controlled thermonuclear reactions that produce the fewest neutrons or no neutrons. The most promising fuel with no neutron (sometimes called advanced fuel) is proton–boron-11 [$P + B^{11} \to 3\alpha(2.7\ \text{MeV})$] and deuterium–helium-3 [$D + He^3 \to p(14.7\ \text{MeV}) + \alpha(3.6\ \text{MeV})$]. But all advanced fuels require a much higher temperature than does the D–T mixture. There are many costs to maintaining the high temperature. In particular, the bremsstrahlung losses in this regime might be greater than the fusion power, which makes self-burning of advanced fuels unlikely [1].

In classical plasmas, both the fusion power and the radiation losses are proportional to the square of the density. Thus, the power balance is essentially a function only of the temperature and the ratio of the fuel concentration since the power balance is insensitive to the density, and self-sustained aneutronic fusion burning remains unlikely [2]. However, in Fermi degenerate plasmas, the prospect of the aneutronic fuel burning can be very different due to the reduction of electron collisions, which both allows the ion temperature to exceed the electron temperature and reduces the bremsstrahlung loss.

In previous work [3,4], we showed that certain properties of degenerate plasmas such as reduced i–e collisions enable an attractive fusion regime. We also showed that the fusion byproducts are primarily stopped not by electrons but by ions, even in the limit when the electron temperature goes to zero, thus allowing a regime of operation in which ions are hotter than electrons, the so-called "hot-ion mode" of operation. We estimated that the density should be more than $10^5\ \text{g/cm}^3$, at which the ion temperature is more than 100 keV and the electron temperature is 30 keV. This regime has more favorable energy balance than the equal temperature mode, and so can facilitate the self-sustained burning of aneutronic fuel. This reduction of i–e coupling also can affect the current drive efficiency [5].

However, in the previous Letter [3], we did not consider the effects of partial degeneracy and the relativistic effects on the i–e collisions (consequently, the fraction of energy that goes

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from fusion byproducts into the electrons) and the reduction of the bremsstrahlung. In this Letter, we show how the partial degeneracy and relativistic effects affects the i–e collision rate (consequently, the fraction of energy from fusion byproduct into electrons). While the effect of partial degeneracy on the i–e collision [6] has been worked out previously, we include brief derivation of it so that we can apply these results in the calculation of the fraction of energy that goes from fusion byproduct into electrons. As a consequence of these refinement along with the reduction of the bremsstrahlung [7], we propose that the possible self-sustained burning regime is larger than what we have predicted previously [3]. While the crude estimation is presented at the conclusion, more thorough estimation will be given in the companion Letter [8].

This Letter is organized as follows. In Section 2, we briefly discuss the power balance in classical plasmas and summarize prior work on fusing advanced fuel using inertial confinement. In Section 3, we explain essential differences between degenerate electron plasmas and classical plasmas. In Section 3.1, we show how the i–e collision rate in a degenerate plasma is greatly reduced from the classical prediction. In Section 3.2, we apply the result by Maynard and Deutsch [6] to obtain the partial degeneracy effects on the i–e collision rate. In Section 3.3, we calculate how the stopping power formula changes in a relativistic plasma. In Section 3.4, we calculate, based on the reduced stopping power formula, the fraction of energy that goes from the fusion byproducts to electrons or ions. In Section 4, our conclusions are summarized, giving the rough estimate of the expanded self-burning regime.

2. Self-burning requirement of aneutronic fuel

2.1. Self-burning requirements

Ignition requires that the fusing fuel be maintained at high ion-temperature long enough to produce enough fusion reactions [9]. However, for advanced fuel, the bremsstrahlung losses are severe because their fusion rates are appreciable in a very hot ion temperature. Thus, it is possible to self-burn advanced fuel only by maintaining a hot-ion mode. For the P–B-11, according to the recent reduced activity data [10], it seems impossible to sustain fusion reaction even in a hot-ion mode. In the D–He-3 case, due to the production of neutrons from deuteriums, it is desirable to have small deuterium density such as $n_{\mathrm{D}}/n_{\mathrm{He}} \leqslant 0.1$. However, at this fuel concentration, there exists no self-burning regime.

The hot ion mode is always desirable in fusion devices. For example, in the D–T reactor, it can enhance the performance and the confinement vastly [11]. In magnetic fusion devices, the hot-ion mode can be obtained, in principle, by catalyzing alpha particle power to ions using injected rf waves, i.e. alpha channeling [12]. Note, however, that the hot-ion mode is a necessary condition for advanced fuel while it is just advantageous for in the D–T fuel.

To achieve a hot-ion mode in inertial confinement fusion using P–B-11, there have been proposals to generate a detonation wave [13–16]. Eliezer [14] showed that compressed fuel can be burned by an expanding ion fusion-burning wave preceded by an electron-conduction heat detonation wave. A large gap between the electron temperature $T_{\mathrm{e}} \cong 80$ keV and the ion temperature $T_{\mathrm{i}} \cong 200$ keV might then be achievable. However, they withdrew this claim later because the activity data was revised lower [15]. More recently, however, the bremsstrahlung was predicted to be much reduced, giving brighter prospect for P–B-11 fusion [7]. Leon et al. [17] showed that plasma degeneracy lower the ignition temperature for D–T, and that for P–B-11, the ignition temperature can be lower than 20 keV when $\rho = 3.3 \times 10^7$ g/cm^3. They implied that the density is too large for economical fusion reactor.

In the D–He-3 ICF, Honda [18] pointed out that, due to the nuclear elastic scattering, there will be more energy transfer to ions from the 14 MeV proton. While this is still smaller than energy transfer to electrons, it nevertheless improves the fusion reactivity. However, the electron temperature is still the same as the ion temperature according to their scenario.

3. Degenerate plasma

In dense plasmas, the power balance becomes very different, in ways that favor the hot-ion mode. In quantum electron plasmas, the electron distribution satisfies Fermi–Dirac statistics, since electrons obey the exclusion principle. In the Fermi distribution, the occupation number g is defined as $g(E) = 1/(\exp[(E-\mu)/T_{\mathrm{e}}]+1)$, where μ is the chemical potential, $E = \hbar^2 k^2/2m_{\mathrm{e}}$ is the kinetic energy of an electron, and g is normalized as $n_{\mathrm{e}} = \int 2g\, d^3k/(8\pi^3)$. If the final state has the occupation number g_{s}, a transition from the initial state to the final state is forbidden with the probability $1 - g_{\mathrm{s}}$. If the electron De Broglie wave length is comparable to the inter-particle spacing, this exclusion principle becomes important to consider. This is usually when $\theta = T_{\mathrm{e}}/E_{\mathrm{F}} < 10$, where $E_{\mathrm{F}} = \hbar^2(3\pi^3 n_{\mathrm{e}})^{2/3}/2m_{\mathrm{e}}$ is the Fermi energy.

An example, consider metals at $\rho = 10$ g/cm^3, and $n_{\mathrm{e}} \cong 10^{22}$ cm^{-3} corresponding to the Fermi energy E_{F} of a few eV. At room temperature, we then obtain $\theta \cong 0.01$. Consider, as another example, a D–T ICF target with $\rho = 10^3$ g/cm^3 (E_{F} being a few keV) and the temperature $T_{\mathrm{e}} \cong 10$ keV. We obtain $\theta \cong 2$–5. As shown later, the relevant parameter regime for aneutronic burning will be $\rho \cong 10^5$ g/cm^3, at which the Fermi energy is a few tens of keV. For the electron temperature of a few tens of keV, the degeneracy parameter θ is order of unity.

3.1. Reduction of ion–electron collisions in a degenerate plasma with $T_{\mathrm{e}} = 0$

In a degenerate plasma, certain collisions are forbidden because of the exclusion principle, which reduces the total collision rate. If the velocity of an ion is slower than the electron Fermi-velocity, then as the ion moves in the plasma, it slows down giving its kinetic energy away to electrons. However, because the Fermi-sea is already occupied by electrons, only electrons on the Fermi-surface can take part in these collisions as shown in Fig. 1. The original calculation of the collision rate has been obtained by Fermi [19] and Lindhard [20]. Later on,

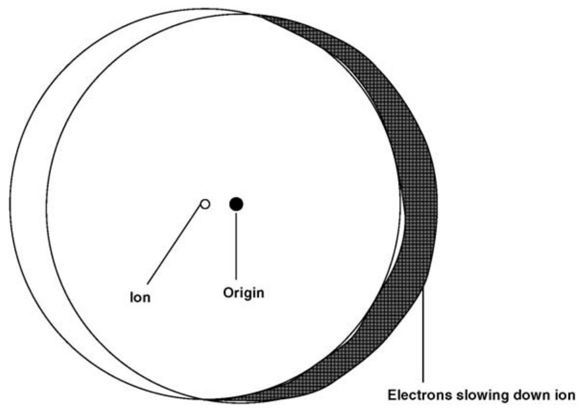

Fig. 1. Electron velocity space. Because the Fermi-sea (inside the circle) is already occupied, only electrons on the Fermi-surface (the area filled with black) can participate in slowing down the ion.

the electronic stopping power in an electron degenerate metal has been intensively studied theoretically [21–29,31] and experimentally [27,32–37], in the case when the velocity of an ion is smaller than the electron Fermi-velocity.

Lindhard derived the stopping power formula:

$$\frac{dK}{dl} = \frac{q^2}{2\pi^2}\int\left[\frac{\mathbf{k}\cdot\mathbf{v}}{k^2 v}\frac{\operatorname{Im} D(\mathbf{k},\mathbf{k}\cdot\mathbf{v})}{|D(\mathbf{k},\mathbf{k}\cdot\mathbf{v})|^2}\right]d^3\mathbf{k}, \tag{1}$$

where K is the energy of the ions, q is the charge of the ion, and D is the electron dielectric function. Note that the right-hand side is the kinetic energy loss of the ion per length. By using the dielectric function from the quantum random phase approximation, he obtained the stopping frequency,

$$\frac{dE}{dt} = C(\chi)\frac{8}{3\pi}\frac{m^2Z^2e^4}{\mu\hbar^3}E, \tag{2}$$

where μ is the ion mass, E is the ion energy, m is the electron mass, $\chi^2 = e^2/\pi\hbar v_{\mathrm{F}}$, v_{F} is the Fermi velocity, and $C(\chi) \cong (1/2)[\log(1+1/\chi^2) - 1/(1+\chi^2)]$ [21]. The above formula is valid if $v \ll v_{\mathrm{F}}$ and $r_{\mathrm{s}} \ll 1$, where v is the ion velocity, and $r_{\mathrm{s}} = (me^2/\hbar)(3/4\pi n_{\mathrm{e}})^{1/3}$ [19,20,22]. The collisions occur between the ion and the fastest electrons rather than, as in a weakly-coupled hot plasma, between the ion and the thermal electrons. The collisional cross-section decreases as $1/v_{\mathrm{F}}^4$. This strong dependence of the cross-section on v_{F} just suffices to cancel the effect of the greater electron density, the greater energy loss per collision, and the great relative velocity of the colliding particles. A surprising result is that the stopping frequency is almost independent of the electron density. From Eq. (2), the i–e collision frequency is given as

$$\nu_{\mathrm{ie}} = 3.47\times 10^{13}\left(Z^2/\mu\right)\frac{1}{\mathrm{s}}, \tag{3}$$

where μ is the nucleus mass in the unit of the proton mass, and $C(\chi) \cong 2$ when $n \cong 10^{28}$ $(1/\mathrm{cm}^3)$.

3.2. Reduction of ion–electron collision: $T_{\mathrm{e}} \neq 0$

The stopping power in a partially degenerate plasma has been obtained by Maynard and Deutsch [6]. We apply their formula to our regime of interest. The starting equations is Eq. (1)

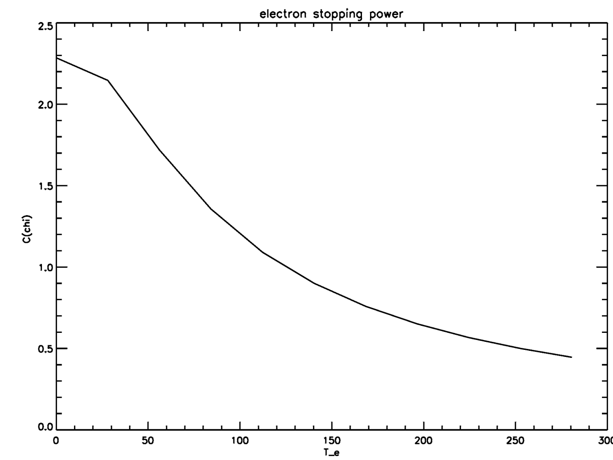

Fig. 2. $C(T_{\mathrm{e}})$ in Eq. (2) for $0 < T_{\mathrm{e}} < 250$ keV with $n_{\mathrm{e}} = 1.3\times 10^{29}$ cm^{-3} (the Fermi energy $E_{\mathrm{F}} = 93$ keV, Y-axis: $\chi(T_{\mathrm{e}})$, X-axis: electron temperature T_{e} in keV).

with the dielectric function,

$$\begin{aligned} D^l(\mathbf{k},\omega) &= 1 + \frac{2m^2\omega_{\mathrm{pe}}^2}{\hbar^2k^2}\Sigma_n\frac{g(E_n)}{N}\left(\frac{1}{k^2+2k\cdot k_n - \frac{2m}{\hbar}(\omega+i\gamma)}\right) \\ &\quad + \frac{2m^2\omega_{\mathrm{pe}}^2}{\hbar^2k^2}\Sigma_n\frac{g(E_n)}{N}\left(\frac{1}{k^2+2k\cdot k_n - \frac{2m}{\hbar}(\omega-i\gamma)}\right), \end{aligned} \tag{4}$$

where g is the occupation number and $N = \Sigma_n g(E_n)$ is the total number of electrons. We numerically integrate Eq. (1) for the density $n_{\mathrm{e}} = 10^{29}$ cm^{-3}, and obtain the stopping power as a function of the temperature in Fig. 2. As shown in the figure, for non-zero electron temperature, the i–e collision frequency decreases further with the electron temperature. See for details [6,38,39].

3.3. Relativistic effect and reduction of ion–electron collision

For a plasma with the density $n_{\mathrm{e}} \cong 10^{29}$ cm^{-3}, the Fermi energy is not negligible with respect to the rest-mass energy (500 keV), and the relativistic effect should be taken into account. While the exact dielectric function with full consideration of the relativistic effect can be found in the literature [30], a simpler approach with approximations is adopted here.

The stopping power formula for a degenerate plasma [20] can be written as

$$\frac{dK}{dl} = \frac{4\pi Z^2e^4}{m_{\mathrm{e}}v^2}n_{\mathrm{e}}L, \tag{5}$$

where K is the kinetic energy of the ion, and L is

$$L = \frac{6}{\pi}\int_0^{\frac{v}{v_{\mathrm{F}}}} u\,du\int_0^{\infty} z^3\,dz\frac{f_i(u,z)}{(z^2+\chi^2 f_r(u,z))^2+\chi^4 f_i(u,z)^2}, \tag{6}$$

where v_{F} is the Fermi velocity, $z = k/2k_{\mathrm{F}}$, $u = |\omega|/kv_{\mathrm{F}}$, $\chi^2 = e^2/\pi\hbar v_{\mathrm{F}}$, and f_i (f_r) is related to the longitudinal dielectric function as $D^l(k,\omega) = 1 + (3\omega_{\mathrm{pe}}^2/k^2v_{\mathrm{F}}^2)(f_r + \mathrm{i}f_i)$, where $\omega_{\mathrm{pe}} = \sqrt{4\pi ne^2/m_{\mathrm{e}}}$ is the plasma frequency.

For the relativistic dielectric function, we use Lindhard's except that the dispersion relation between the momentum and the energy is different from the classical one, i.e. $E(k) = \hbar\omega_k = \sqrt{m^2c^4 + \hbar^2k^2c^2}$ rather than $E_k = \hbar^2k^2/2m_e$. Then, the dielectric function is given as

$$D^l(\mathbf{k},\omega) = 1 + \frac{m\omega_{pe}^2}{k^2}\Sigma_n \frac{g(E_n)}{N}\frac{1}{E(k+k_n) - E(k_n) + \hbar(\omega + i\gamma)} + \frac{m\omega_{pe}^2}{k^2}\Sigma_n \frac{g(E_n)}{N}\frac{1}{E(-k+k_n) - E(k_n) - \hbar(\omega + i\gamma)}. \tag{7}$$

In Eq. (6), we need to integrate f_r and f_i with respect to u and z. Lindhard has shown firstly that when the velocity of the ion is much smaller than the electron Fermi velocity, the major contribution to L comes from the region in the integration over the region $z \ll 1$ and $u \cong 0$, and secondly that f_i is proportional to u when $u \ll 1$. Based on these observations, we can use two approximations. First, in the denominator of Eq. (6), we ignore f_i and consider $f_r(u,z)$ only when $u = 0$ and $z \ll 1$. Second, $f_i(z,u)$ in the numerator only needs to be obtained to the first order in u as a function of z. Therefore, we only need to evaluate $f_r(z,0)$ when $\omega = 0$ and $z \ll 1$, and $f_i(z,u)$ to the first order in u.

Firstly, let us evaluate f_r. We can write f_r from Eq. (7) as

$$f_r = \frac{3}{2}\frac{\hbar^2k_F^2}{2m_e}\Sigma_n \frac{f(E_n)}{N} \times \left[\frac{2E(k_n)}{E(k+k_n)^2 - E(k_n)^2} + \frac{2E(k_n)}{E(-k+k_n)^2 - E(k_n)^2}\right]. \tag{8}$$

After integration of the angle between k and k_n, Eq. (8) becomes

$$f_r = \frac{3}{2}k_f^2\frac{1}{2\pi^2 n_e}\int g(k_n)\frac{E(k_n)}{mc^2}\frac{k_n}{k}\log\left(\frac{k_n + k/2}{k_n - k/2}\right)dk_n, \tag{9}$$

where $g(k)$ is the occupation number. Assuming $k/2k_n \ll 1$, we can expand the logarithmic terms in terms of $k/2k_n$ and simplify Eq. (9) as

$$f_r \cong \frac{1}{k_F}\int g(k_n)\frac{E(k_n)}{mc^2}dk_n. \tag{10}$$

For a plasma with zero-temperature, we obtain $f_r \cong 1$ since $g(k_n) = 1$ if $k_n < k_F$, and $g(k_n) = 0$ if $k_n \leqslant k_F$. If we calculate Eq. (10) to the first order, we obtain $f_r(0,0) = 1 + (1/6)(\hbar^2k_F^2/m_e^2c^2)$. For a plasma with non-zero temperature, Eq. (10) should be used with appropriate $g(k)$.

Secondly, we consider f_i in the denominator of the right-hand side of Eq. (6). As mentioned, we only need to evaluate f_i to the first order in u. From Eq. (7), we can write f_i as

$$f_i = \frac{3}{2}\frac{\hbar^2k_F^2}{2m_e}\Sigma_n \frac{f(E_n)}{N}\pi\delta\big(E(k+k_n) - E(k_n) + \hbar\omega\big) - \frac{3}{2}\frac{\hbar^2k_F^2}{2m_e}\Sigma_n \frac{f(E_n)}{N}\pi\delta\big(E(-k+k_n) - E(k_n) - \hbar\omega\big).$$

After integrating out the angle between k and k_n, it becomes

$$f_i(z,u) = \frac{3}{2}\frac{k_F^2}{2m_ec^2}\frac{1}{2\pi^2n_e}\int_{R_1}\left[g(k_n)\frac{k_n}{2k}\big(E(k_n) + \hbar\omega\big)\right]dk_n + \frac{3}{2}\frac{k_F^2}{2m_ec^2}\frac{1}{2\pi^2n_e}\int_{R_2}\left[g(k_n)\frac{k_n}{2k}(-E(k_n) + \hbar\omega)\right]dk_n,$$

where R_1 is a set in real axis with $R_1 = \{k > 0\colon (E(|k|+|k_n|) > E(k_n) + \hbar\omega)\}$, and $R_2 = \{k > 0\colon (E(-|k| + |k_n|) > E(k_n) + \hbar\omega)\}$. Assuming $\hbar\omega/m_ec^2 \ll 1$, to the first order in u, it can be shown that

$$f_i(k,\omega) \cong \frac{\pi}{2}u\left[g(k)\frac{E(k)}{m_ec^2} - \frac{\hbar^2}{m_e^2c^2}\int g(k_n)k_n\,dk_n\right], \tag{11}$$

where the first term of the right-hand side in Eq. (11) is from $\int_{R_1} g(k_n)(k_n/2k)E(k_n)\,dk_n - \int_{R_2} g(k_n)(k_n/2k)E(k_n)\,dk_n$ and the second term is from $\int_{R_1} g(k_n)(k_n/2k)\hbar\omega\,dk_n + \int_{R_2} g(k_n) \times (k_n/2k)\hbar\omega\,dk_n$. For a plasma with zero-temperature, in the limit $z \ll 1$, we obtain f_i from Eq. (11) as

$$f_i(u,z) = \frac{\pi}{2}u\left(1 - \frac{1}{2}\frac{\hbar^2k_F^2}{m_e^2c^2}\right), \quad \text{if } z < 1,$$
$$f_i(u,z) = 0, \quad \text{if } z > 1, \tag{12}$$

which is the same as the classical formula in the limit $(\hbar k_F^2/m_e^2c^2) \ll 1$.

We now return to the original problem of the relativistic correction to the stopping power. If the temperature is zero, from Eqs. (6) and (12), we can conclude that the stopping power is smaller due the relativistic effect than the classical calculation by a factor:

$$\frac{L_{rel}}{L_{cla}} = \left[1 - \left(\frac{1}{2} - \frac{1}{12\log(\chi)}\right)\frac{\hbar^2k_F^2}{m_ec^2}\right]. \tag{13}$$

For a non-zero temperature, we should use Eqs. (6), (10) and (11). An approximate correction can be made by calculating the stopping power from classical mechanics [38], and then correcting for the relativistic effect, so that

$$\frac{L_{rel}}{L_{cla}} \cong \left[1 - \frac{\hbar^2}{m_e^2c^2g(0)}\int g(k_n)k_n\,dk_n + \frac{1}{2}\frac{\log(A)}{\log(\chi)}\right], \tag{14}$$

where

$$A = \frac{\int g(k_n)\frac{E(k_n)}{m_ec^2}dk_n}{\int g(k_n)\,dk_n}. \tag{15}$$

Eqs. (13) and (14) are the major results of this section. In Fig. 3, $R = L_{rel}/L_{classical}$ is plotted as a function of the electron temperature for $n_e = 10^{29}\ \mathrm{cm}^{-3}$. In short, the reduction of the stopping power due to relativistic effects are 10–20% of the classical result. Nagy [40] has obtained the exact stopping power formula for zero electron temperature:

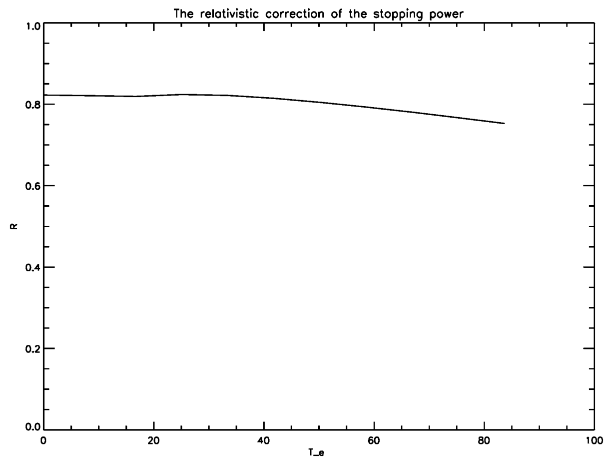

Fig. 3. The ratio of the relativistic stopping power to the classical stopping power for $0 < T_e < 80$ keV with $n_e = 10^{29}$ cm^{-3} (the Fermi energy $E_F = 78$ keV, Y-axis: $R = L_{\mathrm{rel}}/L_{\mathrm{classical}}$, X-axis: electron temperature T_e in keV).

$$C(\chi) = \frac{1}{2}\frac{1+a^2}{a^2}\left[\log\left(1+\frac{\pi}{\alpha(1+a^2)^{1/2}}\right) - \frac{\pi}{\pi+\alpha(1+a^2)^{1/2}}\right], \tag{16}$$

where $\alpha = e^2/\hbar c \simeq 1/137$ and $a = mc/p_F$. By comparing this formula with a classical formula in Eq. (2):

$$C(\chi) = \frac{1}{2}\left[\log\left(1+\frac{1}{\chi^2}\right) - \frac{1}{1+\chi^2}\right], \tag{17}$$

where $\chi^2 = \alpha a/\pi$, we can also predict 10–20% of reduction in the stopping due to the relativistic correction. Our result agrees well with Nagy's for zero temperature.

3.4. Fraction η of energy that goes from fusion by-products to electrons

In this section, the fraction η of energy that goes from fusion byproducts into the electrons is calculated, more exactly, using the result in Sections 3.1 3.2 and 3.3, as a function of the electron temperature, density and fuel concentrations.

First, note that the ion–ion collision frequency is given as

$$\nu_{i,j}^C = 1.8\times10^{-7}\left(\frac{\mu_i^{1/2}}{\mu_j}\right)\frac{1}{E^{3/2}} n_j Z_i^2 Z_j^2 \log\Lambda, \tag{18}$$

where $\log(\Lambda)$ is the Coulomb logarithm, i, j is the ion species, n_j is the density of the target ions, Z is the charge and all the unit in cgs and enery in ev. The Coulomb logarithm, $\log\Lambda$ can be obtained from the integration of the impact parameter [41]: $\log\Lambda = \int_0^{\rho_{\max}} d\rho\,\rho/(\rho_C^2+\rho^2)$, where $\rho_C = Z_i Z_j e^2/2E_0$ is the distance at the closest approach, $\rho_{\max}$ is the maximum impact parameter, and E_0 is the kinetic energy of the fusion byproduct. In our partially degenerate relativistic plasma, the maximum impact parameter can be estimated as the screening length D_s, which has been calculated, through the quantum random phase approximation [42]:

$$a/D_s = 0.1718, \tag{19}$$

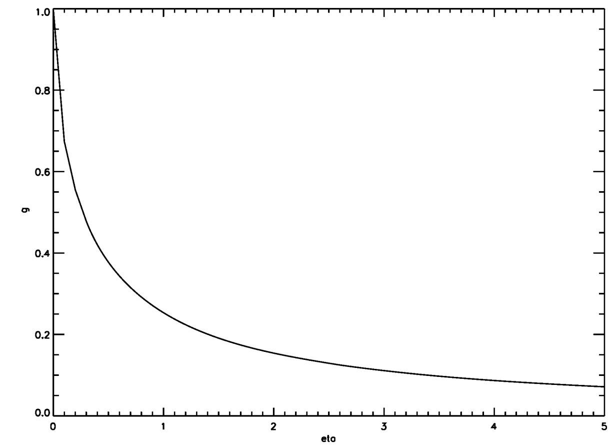

Fig. 4. The function g [Y-axis: $g(\zeta)$, X-axis: ζ].

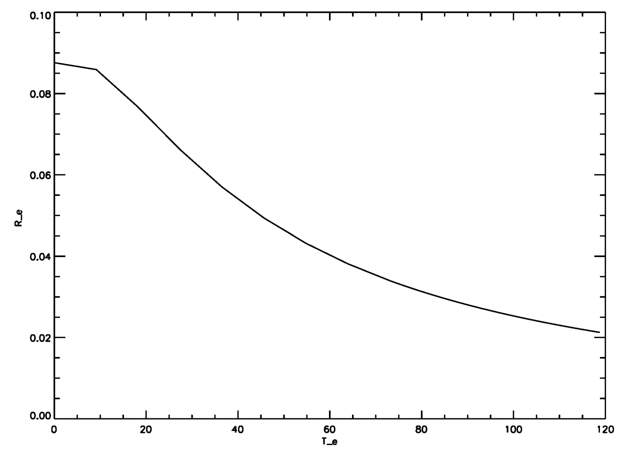

Fig. 5. The fraction of energy transfer from an alpha particle (2.7 MeV) to electrons as a function of electron temperature. The P–B-11 with density $n_e = 4\times10^{28}\,1/\mathrm{cm}^3$, $n_B/n_P = 0.3$, and the Fermi energy $E_F = 43$ keV, (Y-axis: r_e, X-axis : the electron temperature T_e in keV).

where a is the inter-particle spacing. Then, we can estimate

$$\log\Lambda \cong \log\left[(n_e)^{-1/3}/0.1718\rho_C\right]. \tag{20}$$

For an example, an alpha particle with $\epsilon = 3.7$ MeV and $n_e = 3\times10^{28}$ cm^{-3}, we obtain $\log(\Lambda) \cong 10.8$, which is larger than the Coulomb logarithm used in [3] by a factor 2. With the change of the Coulomb logarithm and the reduction of the electron stopping power in mind, we now obtain η-equation:

$$\eta = r_e(T_e) = \int_0^{E_0} \frac{dE}{E_0}\frac{\nu_{ie}(T_e)}{\nu_{ie}(T_e)+\sum_j \nu_{\alpha,j}(E)}, \tag{21}$$

where E_0 is the initial energy of the α particle, and ν_{ie} is given in Eq. (2). This equation can be simplified to

$$\eta(T_e) = \int_0^1 1/\left(1+\zeta(T_e)/s^{3/2}\right) ds, \tag{22}$$

where

$$\zeta(T_e) = \sum_j 1.8\times10^{-7}\frac{n_j Z_i^2 Z_j^2 \lambda}{\epsilon^{3/2}\nu_{ie}(T_e)}\left(\frac{m_i^{1/2}}{m_j}\right). \tag{23}$$

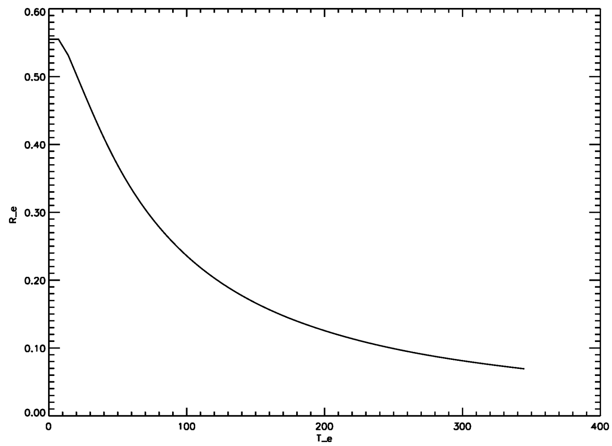

Fig. 6. The fraction of energy transfer from fusion by-products (a 3.6 MeV alpha particle and a 14.7 MeV proton) to electrons as a function of electron temperature. The D–He-3 with density $n_e = 3 \times 10^{28}\, 1/\mathrm{cm}^3$, $n_D/n_{he} = 0.3$, and the Fermi energy $E_F = 35$ keV, (Y-axis: r_e, X-axis: the electron temperature T_e in keV).

We plot $g(\zeta) = \eta$ as a function of ζ in Fig. 4. For an example, in the P–B-11 fuel with $n_B/n_P = 0.3$ and $n_e = 4 \times 10^{28}$ cm^{-3}, we plot $\eta(T_e)$ as a function of T_e in Fig. 5.

In the D–He-3, proton ($E_0 = 14.7$ MeV) and alpha particle ($E_0 = 3.6$ MeV) are fusion by-products. The fraction of the energy from an alpha particle to electrons, $r_{e,\alpha}$, can be obtained in the same way in the case of the P–B-11. For proton, the nuclear elastic collision (NEC) must be also taken into account. The NEC is an elastic collision between nuclei in which nuclei only exchange their kinetic energy. Especially, the NEC between proton and He-3 is quite large. Including the NEC, we can express $r_{e,p}$ as

$$r_{e,p}(T_e) = \int_0^{E_0} \frac{dE}{E_0}\left[\frac{\nu_{ie}(T_e)}{\nu_{ie}(T_e) + \sum_j \nu_{\alpha,j}(E) + \sigma_N(E)v(E)f(E)}\right], \tag{24}$$

where E_0 is the initial energy of the proton, $v(E)$ is the proton velocity, σ_N is the NEC cross-section, and $f(E)$ is the fraction of the proton energy per a NEC. By assuming $\sigma_N(E)v(E)f(E)$ as constant with respect to energy (this is a good approximation for the NEC between proton and He-3), we can use $r_{e,p}(T_e) = \gamma(T_e)g(\gamma(T_e)\zeta)$, where $\gamma(T_e) = \nu_{ie}(T_e)/(\nu_{ie}(T_e) + \sigma_N(E)v(E)f(E))$. Then, the total fraction can be obtained as $\eta = (14.7r_{e,p} + 3.6r_{e,\alpha})(14.7 + 3.6)$ with $r_{e,\alpha}$ is the fraction of energy from the alpha particle to electrons given in Eq. (21). For an example, we plot η as a function of T_e in Fig. 6 when $n_d/n_{he} = 0.2$ and $n_e = 3 \times 10^{28}$ cm^{-3}.

4. Conclusion

In degenerate plasmas, the electronic processes are much slower than the classical prediction due to Fermi–Dirac statistics. Especially, the i–e collision and bremsstrahlung, which are essential ingredients in the fusion power balance, are quite different from classical plasmas. This aspect of degenerate plasmas enhances the prospect of controlled fusion of advanced fuels, since the reduction of the ion–electron coupling and the bremsstrahlung losses eventually impede energy dissipation from hot fusing ions. In particular, the ion energy losses are no longer proportional to the square of the density. The power balance is then quite sensitive to density, which makes the advance fuel burning feasible.

We show here that the partial degeneracy and the relativistic effects reduces the ion–electron collision frequency considerably compared to the zero temperature result previously assumed [3]. This reduction means that the fraction of energy that goes from fusion byproducts into ions is larger than previously obtained. We also would like to point out Eliezer's result of the reduction of the electron bremsstrahlung. These new aspects of the electronic processes change the previous power balance [3] in many aspects.

For example, as shown in Section 3, the i–e collision rate can be several times less than the zero-temperature result in [3], which in turns reduces by several times the fraction of energy that goes from fusion byproduct into electrons. This in turn means that the density requirement of the self-burning regime can be several times less than otherwise thought. Since the bremsstrahlung is simultaneously much reduced compared to the classical result that was used in previous work, the electron temperature, which balances the bremsstrahlung losses and ion energy loss, will then be higher. The higher electron temperature without too much bremsstrahlung is further advantageous since it reduces ion-electron collisions further due to the partial degeneracy effects. This means that if we expect a 100% reduction of the i–e collision from the previous result, the density requirement will be reduced much more than 100%. Because these effects are coupled to each other, it is not straightforward to calculate them. The precise calculation of the self-sustained burning regime is treated in a companion Letter [8].

While these new effects tend to mitigate certain conditions required for advanced fuel burning [3], the compression of the fuel and the creation of the hot spot remain challenging [3,43]. Further improvements on these methodologies will be necessary to enter the advanced burning regimes suggested here.

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Further reading

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