Stochastic Extraction of Periodic Attosecond Bunches from Relativistic Electron Beams

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Intense laser waves can form a time-dependent gate, which transmits or reflects particles depending on their initial phases. When faced by a relativistic electron beam, such a barrier slices it by randomly scattering all but some particles, which nearly conserve their velocity. Subfemtosecond or attosecond periodic electron bunches are then formed downstream and can be used, for example, to generate coherent x rays via Thomson backscattering of the laser light.

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Significant progress has been made recently in producing ultrashort electron bunches for coherent terahertz and femtosecond x-ray generation [1], plasma accelerators [2], and radiation chemistry [3]. Picosecond and femtosecond bunches are created with radio frequency photoinjectors [4], magnetic [5] and laser [6] compression or ponderomotive deflection [7] of electron beams, beam slicing in storage rings [8], and plasma-based techniques [9]. The latest studies [10–12] also show the possibility of attosecond (1 as $= 10^{-18}$ s) bunches suitable for coherent x-ray generation, which is desired for numerous applications ranging from ultrafast time-resolved spectroscopy to *in vivo* imaging of biological structures [13]. To offer a novel, advantageous technique of producing attosecond electron bunches is the purpose of this Letter.

We propose that bunches are generated from a uniform electron beam, yet differently than in free electron lasers (FEL). In FELs, beams are folded smoothly, so the particle distribution remains continuous, and bunches are formed only in projection on a coordinate axis. Here, on the contrary, the beam is ripped apart, so it is the full phase space distribution that becomes abrupt now. The phase space topology can be altered using chaos, which allows diffusing out the unwanted parts of the phase volume while leaving intact the rest [14]. The particular implementation can be accomplished via Hamiltonian interaction of the beam with interfering laser waves in vacuum. The laser fields form a time-dependent gate, which transmits or reflects particles depending on their initial phases. When faced by a relativistic electron beam, such a barrier slices it by randomly scattering all but some particles, which nearly conserve their velocity. Subcycle electron bunches are then formed downstream and can be used, for example, to generate coherent x rays via Thomson backscattering of the laser light.

To explain the beam slicing effect, consider first a simplified nonrelativistic problem. Suppose that a uniform electron beam with the particle energy $\mathcal{E}_0 = p_0^2/2m$ is incident on a localized field $\mathbf{E}_\sim = \mathbf{E}(z)\cos\omega t$ of arbitrary polarization. It is well known that, if the field scale L is large enough, that is, $\varsigma \equiv p_0/m\omega L \ll 1$ and $\sigma \equiv e^2E/m\omega^2L \ll 1$, the average force on electrons is approximately conservative, with the effective "ponderomotive" potential given by $\Phi \approx e^2|E|^2/4m\omega^2$ [15,16]. (We neglect space charge effects at this point.) The barrier will then transmit all particles if $\mathcal{E}_0 > \Phi_{\rm max}$, and reflect all particles if $\mathcal{E}_0 < \Phi_{\rm max}$. Suppose that the above conditions are violated though. As $\epsilon \equiv \max\{\varsigma, \sigma\}$ is increased, the scattering exhibits dependence also on the electron initial phase $\theta_0 \equiv \omega t_0$, so the phase-averaged fraction of transmitted particles $\mathfrak{T}$ becomes a continuous function of $\mathcal{E}_0$ [16,17]. An analogy with quantum tunneling through a static potential can be drawn in this case [18]; yet $\mathfrak{T}(\mathcal{E}_0)$ is of algebraic form here rather than exponential. For a single-hump $E(z)$ with a small ϵ, the transmission coefficient was derived in Ref. [16]. It was shown that scattering is probabilistic only for trajectories near the separatrix corresponding to $\mathcal{E}_0 = \Phi_{\rm max}$ [Fig. 1(a)]. Transmission is impossible at $\mathcal{E}_0 < \mathcal{E}_{\rm min}$, and reflection is impossible at $\mathcal{E}_0 > \mathcal{E}_{\rm max}$, where $\mathcal{E}_{\rm max} - \mathcal{E}_{\rm min}$ is exponentially small with respect to ϵ.

Transmitted electrons have regularly distributed initial phases, with θ_0 (mod 2π) lying within a solid interval $\Delta\theta_0 = 2\pi\mathfrak{T}$ [16]. The barrier then acts like a time-dependent gate, which comes open for the time $\tau = T\mathfrak{T}$ on every field period T. Assuming an optical field ($T \sim 1$ fs) and $\mathfrak{T} \ll 1$, this allows redirecting the particle motion within attoseconds, through which, in principle, ultrashort electron bunches can be extracted. The latter, however, requires that transmitted particles travel synchronously past the top of the barrier. This does not occur in a single-hump field for it can only modify the particle phase space continuously. Electrons are therefore detained for different times depending on θ_0, and some particles get stuck at the field maximum, hence elongating the bunch tail [Fig. 1(a)].

To filter out the tail electrons, consider using additional time-dependent gates. A periodic chain of coherent ponderomotive barriers, or a standing wave might be employed then. In this case, the particle phase space is divided into energy zones, similar to those in a crystal. The lower energy domain is chaotic, hence yielding a "band gap": the corresponding electrons undergo phase space diffusion

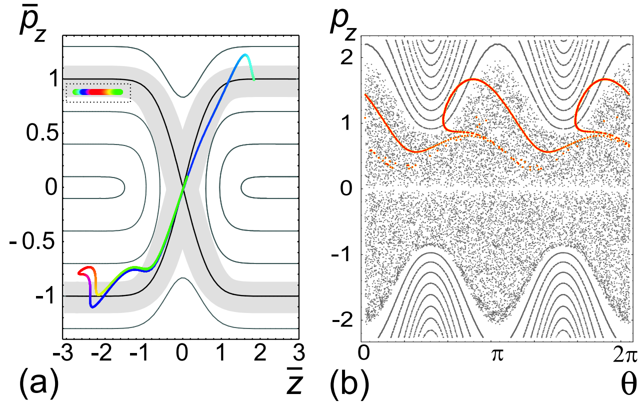

FIG. 1 (color online). (a) Phase plot of nonrelativistic electron average motion in the field $\mathbf{E}_\sim = \hat{\mathbf{z}}E_0 \exp(-z^2/L_0^2)\cos\omega t$, with $eE_0/m\omega^2 L_0 = 0.3$; $\bar{z}$ is the drift coordinate normalized to L_0, $\bar{p}_z$ is the drift momentum normalized to $p_c = eE_0/\omega\sqrt{2}$ [16]. Particles in shaded regions are reflected or transmitted depending on their initial phases θ_0. Colored line is a snapshot of electron distribution after scattering of a uniform beam with duration $T = 2\pi/\omega$ and initial momentum $p_0 = 0.88p_c$. Colors denote different θ_0; original beam (boxed) is shown separately in scale. (b) Poincaré spatial mapping for the relativistic electron motion in the field $\mathbf{E}_\sim = \hat{\mathbf{x}}E_0 \cos kz \cos\omega t$, $a_0 \equiv eE_0/mc\omega = 15$; $\theta \equiv \omega t \bmod 2\pi$ is the phase, p_z is the momentum normalized to mca_0. Colored line is the phase space mapping (transmitted particles only) produced by a single spatial period of the field when applied to a uniform beam with duration T and momentum $p_0 = 1.12mca_0$.

as they are randomly scattered by the field. However, the higher energy domain is regular; it then acts as a "conduction zone", where ballistic motion takes place. When a uniform electron beam enters the chain, the first barrier modulates the transmitted beam repeatedly so as to send particles to one of the two domains depending on θ_0 [Fig. 1(b)]. (It is not required that the first barrier reflect particles at all in this scheme.) Initially and within a few cycles [19], the chaotic and the regular populations are arranged in phase space such that the full distribution remains unbroken. However, as the beam propagates, chaotic electrons diffuse away and their phase space density becomes negligible compared to that of regular particles. After the chaotic debris is fully stripped off, localized bunches appear.

Since regular particles are not entirely monoenergetic, the bunches will spread as the chaotic population is filtered out. However, if the boundary between the regular and the chaotic domain corresponds to ultrarelativistic energies, all bunch electrons will have about the same speed, close to the speed of light c. Assuming that the direction of the particle instantaneous velocity is mainly preserved, the bunches will then retain their shape throughout the interaction. The bunch duration will hence be fixed at $\tau = T\mathfrak{T}$, where $\mathfrak{T}(\mathcal{E}_0)$ is the fraction of regular particles, so one can get $\tau \ll T$ by reducing $\mathfrak{T}$.

Consider now how to produce the effect with fields actually available in the laboratory. At particle energies $\mathcal{E}_0 \gg mc^2$, probabilistic scattering requires normalized amplitudes $a_0 \equiv eE_0/mc\omega \approx 0.85\lambda_{\mu\mathrm{m}}\sqrt{I_{18}}$ larger than 1. (Here the wavelength λ is measured in microns, and the intensity I is measured in 10^{18} W/cm^2.) Appropriate parameters ($a_0 \lesssim 10^2$) are accessible with optical pulses [20]; however, assuming one-dimensional (1D) geometry, electron interaction with a traveling wave is integrable [21], and hence cannot extract bunches from a uniform beam. Two or more waves must be employed then, so the standing wave in the interference region could produce chaos [21–24]. Even though electron injection into the regular domain is distributed over several field maxima in this case, the mechanism of bunch extraction remains qualitatively similar to that discussed above.

For clarity, suppose that a beam with the particle momentum $\mathbf{p} = \hat{\mathbf{z}}p_0$ is incident on a 1D standing wave $\mathbf{E}_\sim = \hat{\mathbf{x}}E(z)\cos kz \cos\omega t$, where $k = \omega/c$, and $a_0 \gg 1$. Assume $E(z)$ is localized, with the scale $L_0 \gg \lambda$. Like in the case of a uniform amplitude [22,25], the electron short-term dynamics is then regular at $p_z \gtrsim eE(z)/\omega$ and chaotic at $p_z \lesssim eE(z)/\omega$ [Fig. 1(b)], while the long-term dynamics is determined by particle transitions between those regions. At $p_0 \ll mca_0$, all electrons enter the chaotic domain at some point, whereas at $p_0 \gg mca_0$ all electrons continue regular motion throughout the interaction. Hence phase-dependent transmission occurs only at $p_0 \sim mca_0$, so the corresponding particles are ultrarelativistic, as required for our technique.

Numerical simulations confirm the possibility of producing ultrashort electron bunches in this case. As predicted, regular particles leave the interaction region coherently: they occupy a well-confined domain in the momentum space [Fig. 2(a)] and yield a sharply defined bunch of a subcycle duration τ. Because of the field transverse polarization, the bunch is oriented nearly perpendicularly to z, which brings τ further into the attosecond domain [Figs. 2(b) and 2(c)]. Chaotic particles exhibit a smooth random distribution in the coordinate space and a quasithermal momentum distribution, with an effective temperature $T_e \sim mc^2 a_0$ and a cutoff at the edge of the regular domain $p_z \sim T_e/c$, above which stochastic heating is impossible [Fig. 2(a)].

Ultrashort electron bunch formation persists as well for three dimensional, focused standing waves $\mathbf{E}_\sim = \mathrm{Re}[\bar{\mathbf{E}}(\mathbf{r})]\cos\omega t$, which we model using the paraxial approximation

$$\bar{E}_x = E_0 \frac{L_0}{L_0 + 2iz} \exp\left[-\frac{k(x^2+y^2)}{L_0 + 2iz} + ikz\right], \quad (1)$$

$\bar{E}_z = (i/k)\partial \bar{E}_x/\partial x$. (Here $L_0 = kw^2$, with w being the focal waist.) Regular particles are now pushed away from the z axis by the transverse ponderomotive force. As a result, bunches are formed along a narrow cone around

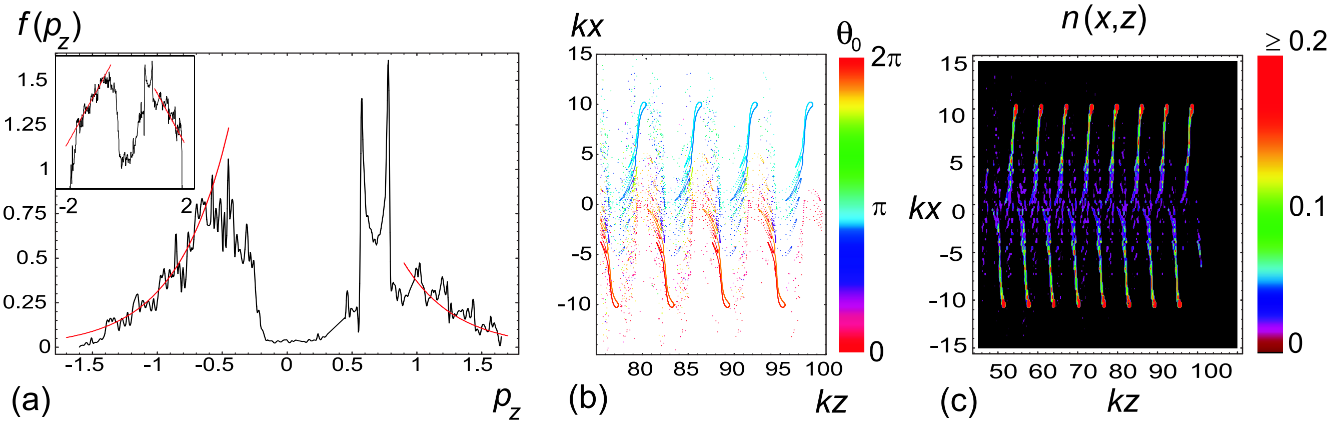

FIG. 2 (color online). Scattering of electrons initially of a uniform, zero-radius, 8λ-long beam off the field $\mathbf{E}_\sim = \hat{\mathbf{x}}E_0\exp(-z^2/L_0^2)\cos kz\cos\omega t$, $kL_0 = 20$, $p_0 = 0.6mca_0$, $a_0 = 15$. (a) Momentum distribution $f(p_z)$ (numerical, black) and Boltzmann fit $f(p_z) \propto \exp(-|p_z|c/T_e)$ with $T_e \approx 2.5mc^2a_0$ (red); inset with the same plot in the logarithmic scale emphasizes a cutoff at $p_z \sim T_e/c$. Regular trajectories correspond to $0.55 \lesssim p_z/mca_0 \lesssim 0.85$, where $f(p_z)$ is peaked. (b) Snapshot of the electron spatial distribution downstream from the wave barrier. The color denotes the particle initial phase θ_0. Only electrons with particular θ_0 are seen to contribute to regular structures. (c) Snapshot of the electron density profile downstream from the barrier, with $n(x, z)$ normalized to its maximum. The density peaks are primarily due to regular particles; compare (b).

$z = 0$ [Fig. 3(a)], while chaotic particles are scattered quasiuniformly at all angles.

The bunch parameters can be estimated as follows now. A bunch with a length L_b decorrelates on a distance $L \sim L_b a_0^2$; thus one can roughly take $L \sim L_0$ for the domain of interest. Space charge effects appear essentially when this distance is large enough, so the longitudinal electrostatic field can change the electron energy significantly compared to $\mathcal{E}_0$. Simulations show that a significant (of the order of unity) fraction of scattered electrons can be bunched; assuming it is the case, the maximum number of electrons per bunch is of the order of $a/2kr_e \sim 4 \times 10^8$. (We take $a \sim 15$ and $\lambda \sim 1$ μm; bunches are considered infinitely thin and transverse to the beam; $r_e = e^2/mc^2$.) This yields a charge of roughly 70 pC, comparable to those currently accessible via plasma accelerators on a femtosecond scale [26]. About the same limit follows also from the requirement that the beam transverse spreading must remain negligible during the interaction.

The technique can be applied for x-ray generation via Thomson backscattering of the same laser light used for beam slicing [27,28]. The maximum frequency $\omega_s = k_s c$ of incoherently scattered radiation is determined by the individual particle synchrotron spectrum having a cutoff at about $a_0^3\omega$ [27]. Bunching though also allows generating x-rays coherently, hence with higher intensity and, possibly, collimation. Assuming nearly backward scattering, one has $k_s' \approx \frac{1}{2}\kappa_n'$ for coherent waves, where κ_n is a regular density wave number, and the prime denotes the reference frame where the density $n(\mathbf{r}')$ is stationary. In the laboratory frame, the condition reads $k_s \approx \kappa_n$; thus, the minimum wavelength λ_s equals the smallest scale of $n(\mathbf{r})$ [11,29].

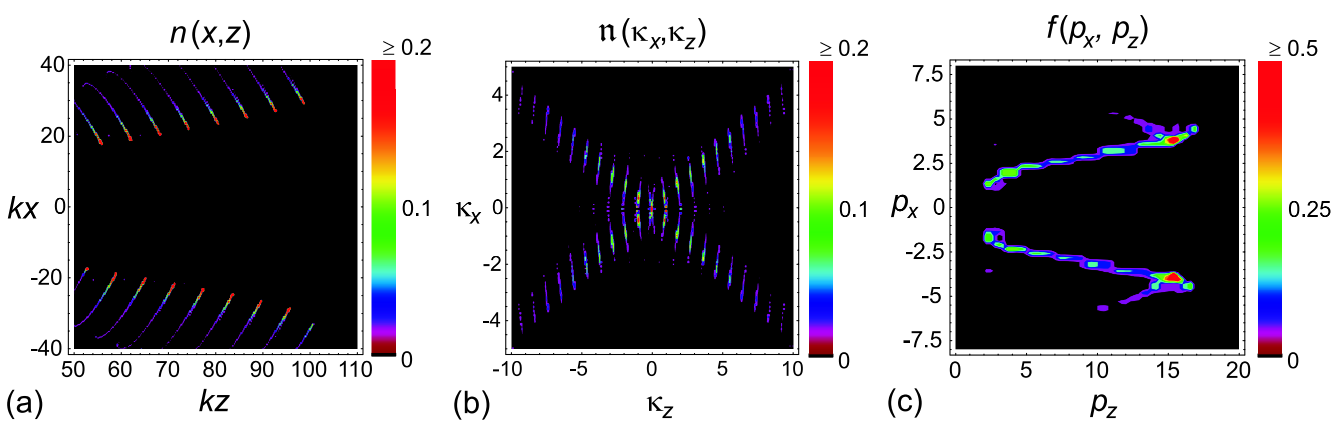

FIG. 3 (color online). Scattering of electrons initially of a uniform, zero-radius, 8λ-long beam off the focused field with $kL_0 = 20$, $p_0 = 0.6mca_0$, $a_0 = 15$. (a) Snapshot of the particle density profile $n(x, z)$ downstream from the barrier. (b) Density spectrum $\mathfrak{n}(\kappa_x, \kappa_z)$, with κ_i measured in units k. (c) Momentum distribution $f(p_x, p_z)$, with p_i measured in units mca_0. (All n, $\mathfrak{n}$, and f are normalized to their maximum values.) Particles remain in the (x, z) plane because of the chosen initial conditions.

Simulations predict a wide density spectrum when bunches are generated [Fig. 3(b)]; therefore substantial frequency upshift (by a factor of 10 or more) is possible. Moreover, since the bunch structure evolves in time, pulsed x rays can be produced.

In summary, we propose a novel mechanism of attosecond electron bunch generation based on relativistic electron beam scattering off interfering laser waves in vacuum. The laser fields form a chaotic gate, which transmits or reflects particles depending on their initial phases. Some of electrons are scattered randomly by the field, while others travel with a nearly fixed velocity. Subfemtosecond or attosecond periodic bunches are then formed downstream and can be used, for example, to produce coherent x rays via Thomson backscattering of the laser light. FEL-based techniques [10], the new scheme operates at considerably lower electron energies (few tens of MeV). Hence the input beam can be produced by a less expensive plasma accelerator, with parameters that match those for available laser fields [26]. Unlike in Refs. [11,12] though, the laser-plasma interface can now be detached from the operating region, hence eliminating plasma debris and enhancing the bunch focusability.

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