Motion of charged particles near magnetic-field discontinuities

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Charged particles near discontinuities in magnetic fields, so-called ''boundary particles,'' can be constrained to remain near the discontinuity, even an arbitrarily fractured discontinuity, as the particle drifts along the fractured boundary. These particles are shown to exhibit new and interesting effects along broken and branching surfaces, including the wetting of fractured surfaces.

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The classical description of particle behavior in nonuniform magnetic field assumes a circular orbit that drifts in a slowly changing field (see, e.g., [1,2]). The magnetic flux through the particle Larmor orbit is then an adiabatic invariant of the motion. Both the conventional guiding-center formalism [3–8] and its high-order corrections [9–11] describe the dynamics of guiding centers when the particle gyroradius r_g is much smaller than the characteristic spatial scale L of the magnetic field. Large gyroradii particles, or particles undergoing periodic motion other than circular, also exhibit adiabatic invariants, so long as the magnetic structure seen by the particle varies slowly compared to an orbit period [12–14].

What we show here is that particle motion near a magnetic discontinuity is constrained even as the drift motion encounters sharp discontinuities in magnetic structure over an orbit period. We may identify what we call ''boundary particles,'' with guiding centers within a gyroradius of the magnetic field discontinuities. The present work identifies and describes new and unusual properties of the motion of boundary particles along plane, broken, and branching boundaries.

Consider motion along a smooth boundary, say, a magnetic field $\vec{B}=\vec{z}^0 B(x)$, with the simple discontinuity

$$B(x)=\begin{cases} B_1, & x<0 \\ B_2, & x>0. \end{cases} \tag{1}$$

The motion of a boundary particle in the magnetic field given by Eq. (1) is shown in Fig. 1. The particle crosses the magnetic field discontinuity with different gyroradii on different sides of the magnetic boundary. After one period of transverse oscillations (i.e., after two crossings of the boundary), it is displaced along the boundary, in complete analogy to the classical ∇B drift in smooth nonuniform fields. The one-dimensional (1D) particle oscillation perpendicular to the boundary can be described by the Hamiltonian

$$H=\frac{p_x^2}{2m}+\frac{1}{2m}\left(p_y-\frac{q}{c}xB(x)\right)^2=\frac{mV^2}{2}=\text{const},$$

$$p_y=\text{const},$$

where x is the direction perpendicular to the boundary. For slowly varying parameters, the area confined inside the phase-space particle trajectory is an adiabatic invariant of the particle motion [15–17]. Figure 2 shows such a closed trajectory. A new invariant of the particle motion can then be written as

$$\mu=\frac{mV^2}{2}\psi(\alpha)=\text{const}, \tag{2}$$

$$\psi(\alpha)=\frac{1}{2\pi}\begin{cases} \pi(1/\omega_1+1/\omega_2)+(1/\omega_1-1/\omega_2)(\pi-2\alpha+\sin 2\alpha), & B_1B_2>0 \\ (1/\omega_1+1/\omega_2)(2\pi-2\alpha+\sin 2\alpha), & B_1B_2<0, \end{cases}$$

where $\omega_k=|qB_k|/mc$ are the gyrofrequencies in the corresponding regions. Here $\alpha=\arccos(p_y/mV)$ is the angle at which the particle crosses the boundary; it determines the guiding center transverse displacement relative to the boundary (see Fig. 1). Note that μ reduces to the well-known magnetic moment ($mV^2/2\omega=\text{const}$) for uniform magnetic fields ($B_1=B_2$).

Consider first some of the interesting features of boundary particles even along simple boundaries: For example, suppose one of the magnetic fields in Eq. (1) varies in space along the boundary, smoothly enough that $r_g d\ln|B|/dy \ll 1$, assuring adiabaticity. For simplicity, assume $B_1=\text{const}$ so that the invariance of both μ and energy implies

$$[1/B_1-1/B_2(y)]\Theta(\alpha)=\text{const},$$

where $\Theta(\alpha)=2\alpha-\sin 2\alpha$ is a monotonically increasing function of α. While remaining on the boundary, for $B_2>B_1$, α increases with the decrease of B_2, so that a positively charged particle will drift in the positive x direction, with $\alpha<\pi$ (Fig. 3). The direction of this drift coincides with the direction given by the classic ∇B drift for smooth mag-

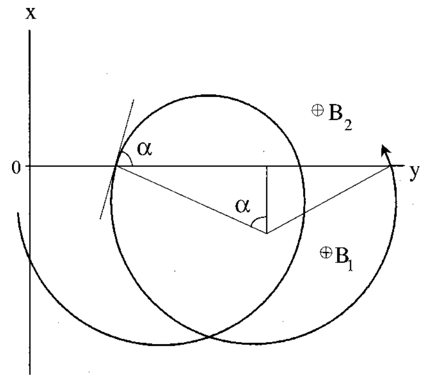

FIG. 1. Boundary particle motion along a plane magnetic boundary.

netic fields. Note that for ∇B_2 in the positive x direction, ions in the region $x > 0$ drift toward the boundary at $x = 0$, at which point they attach to the boundary in such a way that where $B_2 > B_1$, the ions drift along the boundary to regions of higher B_2, whereas if $B_2 < B_1$, the ions drift along the boundary to regions of lower B_2. In either case, the drift along the boundary is to regions of highest magnetic disparity, where the ion will eventually adhere to the boundary from the low-field side, as either $\alpha \to \pi$ or $\alpha \to 0$.

Consider now motion along abruptly changing magnetic boundaries. More complex 2D magnetic field profiles give rise to complicated motion, yet under certain conditions boundary particles retain an important property, namely, that they remain boundary particles even for branched, abrupt, or fractured boundaries, so long as the boundaries separate regions of uniform magnetic fields. Interior particles then execute closed orbits that do not intersect the boundary, and so cannot move to or from the boundary. Thus, boundary particles cannot turn into interior particles and vice versa. This remains true even if the boundary particle encounters sharp discontinuities in magnetic structure over an orbit period.

Though the focusing property of the straight magnetic boundary is well known [18], new and interesting dynamics appear in fields with broken and branching boundaries, which can lead to new ways of manipulating beams. For example, we show in Fig. 4 how boundary particles turn sharp corners. Because the change in field is abrupt over the particle period, the validity conditions for the adiabatic invariance of μ are violated. As a result of ''scattering on the corner,'' neither μ nor oscillation phase are conserved. Yet the particle cannot live outside the boundary, and so it must ''turn the corner'' with the boundary. For example, if a region in the x-y plane is permeated by a uniform z-directed magnetic field of value different than the uniform z-directed magnetic field exterior to that region, boundary particles would necessarily circle the region. We thus have the surprising result that boundary particles essentially ''wet'' a surface of field discontinuity, even as that surface itself changes abruptly, drifting along the surface like liquid follows a wetting surface due to surface tension.

The wetting effect results in some interesting possibilities, possibly with practical consequences. For example, consider the situation when the boundary particle comes to a point where the boundary branches into two or more new boundaries. If the drift velocities corresponding to all boundary branches are directed away from the branching point, the particle will choose one of new boundaries to drift along. Which path is picked, and what the adiabatic invariant and phase will be along that path, depend in detail on initial conditions. Thus, stochastic guiding center motion can be found in relatively simple magnetic field configurations.

An example exhibiting stochastic guiding center motion is the four-field configuration given in Fig. 5. This configuration produces a separation of a particle stream. The magnitudes of magnetic field on different sides of the boundaries are chosen so that a particle approaches the scattering region from the B_1-B_3 boundary only and leaves it either through channels B_1-B_2 or B_2-B_3. For $B_1 > B_2 > B_3 > B_4$, and positive particle charge, particles drift only counter-clockwise along the central boundary loop.

After a particle has come to the central loop, it has a probability to ''leak out'' along the boundary B_1-B_2, but it can also scatter on the triple point $B_1B_2B_4$ and remain on the loop. The same applies to the next triple point $B_1B_3B_4$. Therefore, although a particle may leak out from the central part of the system at the first or second scattering events, it also may stay on the loop for more than one period of drift

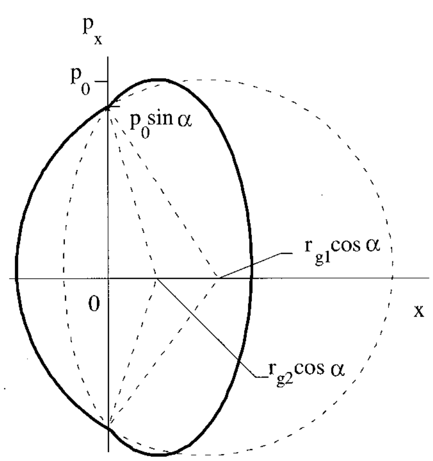

FIG. 2. Phase-space trajectory of boundary particle's transverse oscillations.

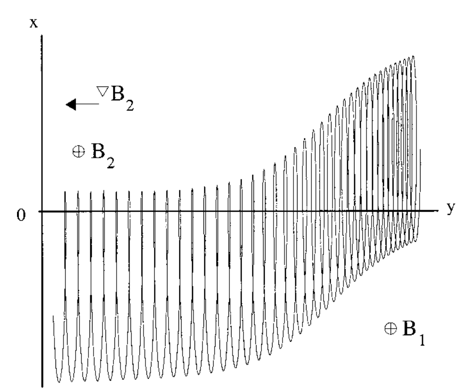

FIG. 3. Transverse adiabatic drift of a boundary particle (scales of the x and y axes are not equal).

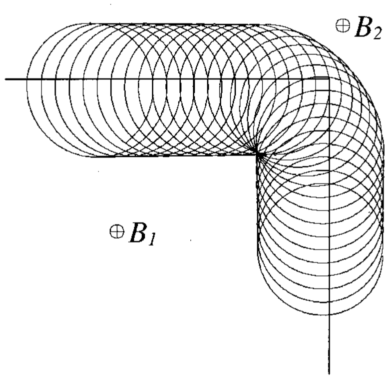

FIG. 4. Boundary particle scattering on a straight corner of a magnetic boundary: the ''wetting effect'' on a broken boundary.

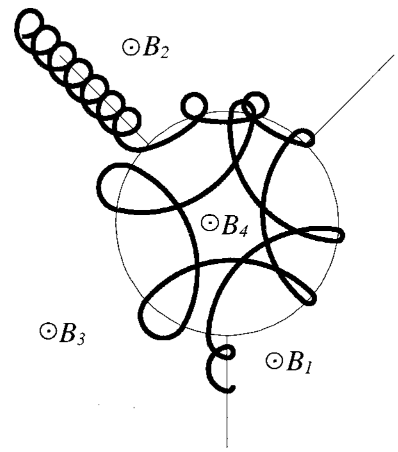

FIG. 5. Boundary particle trajectories along branching magnetic boundaries.

rotation, depending on the initial parameters of the particle motion. Since the probability of leaking out strongly depends on the particle's initial phase, which abruptly changes in each scattering event, the exit choice is strongly sensitive to initial conditions. Detailed numerical simulations show that the characteristic initial phase step needed to distinguish alternative paths is less than 0.01 rad for $B_1 \sim B_2 \sim B_3 \sim B_4$ and loop radius close to r_{g4}. The latter property provides two output streams along B_1-B_2 and B_2-B_3, with uniform distribution over particle initial phases.

Certain practical devices might be envisioned utilizing these unusual properties of boundary particles. While the four-field configuration has been offered as a boundary-particle beam separator, the same, but field-reversed, system can be used for the merging of streams, with input from channels B_1-B_2 and B_2-B_3, into the single output stream B_1-B_3. Effective mixing caused by stochastic rotational drift along the central loop could provide the output beam of particles with uniform initial phase distribution as well. A beam merger could be of some practical use in the transport and transverse combination of energetic heavy ion beams for applications to inertial confinement fusion [17,19]. For high-energy beams, the preponderance of velocity could be directed along the magnetic field.

Note for drift motion in vacuum, that the six-dimensional phase space of the ions is conserved either in beam combining or in beam separation. To reduce the phase space, dissipation must be introduced, for example, by passing the ions through an ionizing gas or plasma. The dissipation can be used for beam focusing of boundary particles.

To see how focusing can occur, consider boundary particle motion in crossed static magnetic and electric fields, with an external uniform electrostatic field directed along the boundary $\vec{E}=\vec{y}^0 E$, and with an abrupt magnetic-field configuration of the form of Eq. (1). Assume oppositely directed fields, for simplicity, with $B_1=-B_2=B_0$. The $\vec{E}\times\vec{B}$-drift velocity is then always directed toward the boundary, so all interior particles drift to the magnetic boundary.

In the presence of friction, the particle motion might then be described by

$$\ddot{\vec{r}}=\omega_0(\dot{\vec{r}}\times\vec{z}^0)\,\mathrm{sgn}\,x+q\vec{E}/m+\vec{R}, \quad \omega_0=qB_0/mc,$$

where $\vec{R}=-\nu\dot{\vec{r}}$ is a friction force. In both the collisionless ($\nu=0$) and collisional ($\nu\neq 0$) cases, the only possible equilibrium trajectory is the trajectory $x(t)\equiv 0$. Particles with nonzero initial x will eventually be attracted to the boundary. Transverse oscillations decay in both the collisional and collisionless cases. For long time [$t\gg\tau$, where τ is the period of transversal oscillations, and $t\gg mV_y(0)/qE$ in case $\nu=0$, or after the amplitude of oscillation becomes much less than $qE\omega_0/m\nu^3$ in case $\nu\neq 0$, and $\nu\tau(t)\ll 1$], the rate of decay can be shown to obey

$$x_M\sim t^{-1/3}, \quad \dot{x}_M\sim t^{1/3}, \quad \tau\sim t^{-2/3} \quad (\nu=0),$$

$$x_M\sim \exp(-2\nu t/3), \quad \dot{x}_M\sim\exp(-\nu t/3),$$

$$\tau\sim\exp(-\nu t/3) \quad (\nu\neq 0),$$

where x_M and $\dot{x}_M$ are the amplitudes of transversal coordinate and velocity oscillations, respectively. Thus, the bounding surface can be easily made an attractor point for the boundary particles either in dissipative or nondissipative systems.

In summary, we have identified a new class of effects associated with charged particles undergoing constrained motion near abrupt magnetic boundaries. Magnetic discontinuities abrupt compared to a particle gyroradius are easily produced in the laboratory, and may occur naturally, for example, in fields undergoing magnetic reconnection. The classical adiabatic invariant for motion in slowly varying fields is generalized to account for the abrupt discontinuities. The complexity of the motion of boundary particles is constrained by an unusual ''wetting effect,'' which is a profound property of the boundary particles. Apart from academic interest in the wetting effect, there may be practical consequences in the manipulation of particles with such constrained motion, including the directed transportation of boundary particles, as well as the merging or separation of magnetic boundary plasma flows.

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