Collective Deceleration of Relativistic Electrons Precisely in the Core of an Inertial-Fusion Target

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The energy deposition of a relativistic electron beam in a plasma can be managed through turning on or off fast beam-plasma instabilities in desirable regions. This management may enable new ways of realizing the fast-igniter scenario of inertial fusion. Collisional effects alone can decelerate electrons of at most a few MeV within the core of an inertial-fusion target. Beam-excited Langmuir turbulence, however, can decelerate even ultrarelativistic electrons in the core.

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Beam-plasma instabilities can produce electromagnetic fields that exceed by orders of magnitude fields associated with thermal fluctuations. These strong fields can decelerate relativistic electron beams (REB) within distances much shorter than Coulomb collisional deceleration lengths. The conditions for these instabilities to develop depend on parameters such as the beam and plasma concentrations, n_b and n_e, plasma temperature T_e, and plasma density gradients. By appropriately managing the system parameters, one might be able, in principle, to produce instabilities and REB energy deposition in desirable regions, while suppressing instabilities in undesirable regions.

In this Letter, we show how collective effects can produce deposition of REB energy in the core of an inertial-fusion target, thereby accomplishing fast ignition [1]. In the conventional paradigm, which we challenge, the electrons are assumed to be decelerated primarily by collisions. This gives an upper limit of just a few MeV on the energy of an electron that could deposit a significant fraction of its energy in the core.

The fast-igniter scenario requires energy $\gtrsim 10$ kJ invested within time $\lesssim 10$ psec to a deuterium-tritium fuel core compressed to a high density $\sim$300 g/cm^3. The core, being completely ionized at fusion temperature $\gtrsim 5$ keV, will have an electron concentration $n_e \sim 10^{26}$ cm^{-3}. This is about 4 orders higher than the critical concentration which modern powerful lasers might directly penetrate.

The energy necessary for the fast ignition has been imagined to be delivered to the core by an energetic electron beam produced by an intense laser-plasma interaction near the critical surface. The beam transport from the critical layer to the core might be inhibited, however, by beam-plasma instabilities, in particular, the Weibel instability (see, for instance, [2], and references therein) and the instability to Langmuir waves, as described below. We propose to create conditions whereby Langmuir wave turbulence is excited to high levels in the flat hot core, while the turbulence is suppressed by electron-ion collisions or by convection due to plasma density gradients in dense enough plasma layers surrounding the core. Then the REB might be decelerated, depositing its energy precisely in the core.

Assuming that the transport could be efficiently accomplished, consider, first, the problem of how to decelerate fast electrons within the core in order to deposit their energy there. It has been thought that the collective modes of beam-plasma interactions are suppressed in the dense core, so that beam electrons could be stopped there primarily by Coulomb collisions with the plasma particles [3]. To be significantly decelerated through collisions within a 50 μm radius core with density $n_e \sim 10^{26}$ cm^{-3}, a fast electron should not have energy much higher than a MeV. However, if electrons of, say, 30 MeV could be significantly decelerated in the core, it would open up possibilities for more energy transport at less current. It would also permit the use of higher intensity lasers, since the energy of a fast electron, produced in the laser-plasma interaction near the critical surface, increases with the laser intensity I (like $\sqrt{I}$, according to [4]).

In contexts other than fast ignition, it is well documented both theoretically and experimentally that a REB can deposit a significant fraction of its energy collisionlessly through the excitation of Langmuir wave turbulence [5–12]. For instance, 65%–70% and 40% collective decelerations of electron beams were reported in [5,6], respectively. This will not happen in a dense core automatically, however, as the Langmuir waves are strongly damped because of very frequent electron-ion collisions. For a core density $n_e \sim 10^{26}$ cm^{-3} with 5 keV electron temperature, the damping rate is $\nu = \nu_{\mathrm{core}} \sim 5 \times 10^{14}$ sec^{-1}. Note that the Langmuir wave frequency, approximately equal to the electron plasma frequency, $\omega_e \approx 5.64 \times 10^4 \sqrt{n_e \times \mathrm{cm}^3}\,\mathrm{sec}^{-1}$, is about 1000 times larger, in the core, than the collisional damping rate, so that Langmuir waves still can be treated as long-lived collective excitations. Note also that the relativistic beam electrons are far less collisional than the bulk plasma electrons, so that weak collisional stopping of relativistic

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electrons coexists with the strong collisional damping of Langmuir waves.

In order to overcome the damping and to excite Langmuir waves by the beam in the core, the collisionless growth rate of the beam-plasma instability to Langmuir waves maximized over wave vectors $\vec{k}$ should exceed the damping rate, $\Gamma \equiv \max_{\vec{k}} \Gamma(\vec{k}) > \nu$. In this condition, Γ must be calculated in the kinetic regime of the beam-plasma instability, since the instability is kinetic near the threshold. For a small beam-to-plasma electron concentration ratio, $n_b/n_e \ll 1$, the collisionless kinetic instability growth rate is calculated by an expansion in n_b/n_e, so that in the leading order $\Gamma \propto \omega_e n_b/n_e \propto 1/\sqrt{n_e}$. Since the collisional damping rate scales (up to a logarithm) as $\nu \propto n_e T_e^{-3/2}$, the growth-to-damping rate ratio scales as $\Gamma/\nu \propto (T_e/n_e)^{3/2}$, indicating that Langmuir waves are more easily excited in a beam-plasma system at larger T_e/n_e ratios.

If the ratio T_e/n_e is maximized in the core, there would be a parameter range for which Langmuir waves are excited in the core, where the energy should be deposited, while not excited during the beam propagation through the surrounding core plasma layers where the energy deposition is not desirable. Incidentally, the favorable parameter range might be extended, in principle, by a dip in plasma concentration somewhere on the path of the beam propagation through the core. This would resemble conceptually the so-called "two-step scheme" of plasma heating by a relativistic electron beam in mirror traps (see [7] and references therein).

In the complementary case when the ratio T_e/n_e is maximized outside the core, the excitation condition for Langmuir waves in the core implies automatically that the collisional threshold of the instability is exceeded outside the core as well. Then, to suppress the beam-decelerating turbulence outside the core, one may try to employ convective stabilization effects. These effects have been considered theoretically [8]. Several experiments indicate that, for sufficiently steep plasma density gradients, electron beams can, in fact, be efficiently transported through inhomogeneous plasmas. These beams then deposit most of their energy in a reasonably short flat density region [5].

To evaluate this mechanism of stabilization by plasma density gradients, consider that, by the eikonal equation for ray optics, the longitudinal wave number of a resonant Langmuir wave evolves as

$$\frac{dk_\parallel}{dt} = -\frac{\omega_e}{2}\frac{\partial \ln n_e}{\partial z} \equiv -\frac{\omega_e}{2L_n}. \tag{1}$$

While crossing the resonant interval $\Delta k_\parallel$ (located near $k_\parallel \approx \omega_e/c$), the wave amplitude undergoes

$$\Lambda \approx 2L_n \int \Gamma(\vec{k}) dk_\parallel/\omega_e \tag{2}$$

exponentiations. If Λ_0 wave amplitude exponentiations can be tolerated without premature beam relaxation occurring, then, to suppress the instability, the inverse logarithm slope of the plasma density L_n should satisfy the condition

$$L_n < \Lambda_0 \omega_e/2 \int \Gamma(\vec{k}) dk_\parallel. \tag{3}$$

For the kinetic instability of an ultrarelativistic beam with angular spread $\Delta\theta$, the integrated growth rate $\int \Gamma(\vec{k}) dk_\parallel$ is largest at $k_\perp \sim \omega_e/c$, where $\max_{k_\parallel} \Gamma(\vec{k})$ is approximately twice smaller than the absolute maximum of $\Gamma(\vec{k})$ (reached at $k_\perp \sim \Delta\theta\omega_e/c$), while the width of the resonant domain in $k_\parallel$ can be estimated at $k_\perp \sim \omega_e/c$ as $\Delta k_\parallel \sim \Delta\theta\omega_e/c$ [9]. Then, the above formula for L_n takes the form

$$L_n < L_{nc} \sim \Lambda_0 c/\Gamma\Delta\theta. \tag{4}$$

For $\Gamma = 10^{15}\ \mathrm{sec}^{-1}$, $\Delta\theta = 0.1$, and $\Lambda_0 = 7$, it gives for the critical scale of the plasma density variation $L_{nc} \sim 20\ \mu$m.

To assure an instability in the core, Γ is chosen twice larger than the collisional damping of Langmuir waves. The scale of the plasma density variation, $L_n < 20\ \mu$m, sufficient for convective stabilization of Langmuir waves growing with the rate Γ, applies to the plasma layer immediately surrounding the core. For more peripheral plasma layers, $\Gamma \propto 1/\sqrt{n_e}$ is larger (assuming fixed parameters of the beam). The respective critical scale of the plasma density variation $L_{nc} \propto 1/\Gamma \propto \sqrt{n_e}$ must then be smaller yet. For instance, at a plasma density 100 times smaller than in the core, the critical stabilization scale would be $L_{nc} \sim 2\ \mu$m. At smaller densities, the convective stabilization condition tends to be more challenging. Stabilization could occur collisionally, however, if the outer regions have lower temperature, as noted above. Moreover, the relevant density regions may be not so small, since the igniting pulse is going to be injected into the target through a bored hole, at the end of which plasma density rises steeply up to the local density in the plasma past the bore. For a hole bored to the plasma layers of concentration, say, $10^{25}\ \mathrm{cm}^{-3}$, the local scale of the plasma density variation $L_n < L_{nc} \sim 6\ \mu$m would be sufficient for the convective stabilization of Langmuir waves.

Furthermore, it should be taken into account that the dependence of L_{nc} on the plasma density reverses when Γ exceeds the width of the resonant domain in frequency and the instability becomes hydrodynamic. In the hydrodynamic regime, the growth rate, maximized over the longitudinal wave number $k_\parallel \approx \omega_e/c$, is [9]

$$\begin{gathered}\max_{k_\parallel} \Gamma(\vec{k}) \sim \omega_e (n_b/n_e \gamma_b \gamma_{b\parallel}^2)^{1/3}, \\ \gamma_{b\parallel}^{-2} = 1 - (\vec{k}\cdot\vec{v}_b)^2/k^2c^2 \approx \gamma_b^{-2} + k_\perp^2/k^2,\end{gathered} \tag{5}$$

where γ_b is the relativistic factor of electrons in the beam. In contrast to the kinetic regime, the hydrodynamic growth rate gets smaller at smaller plasma densities, since in this regime $\Gamma \propto n_e^{1/6}$.

The hydrodynamic regime formula for the growth rate is valid when the resulting Γ is larger than the spread of the wave frequency Doppler shifts experienced by the beam electrons, $\Gamma > k\Delta v_k \sim kc(\sin\theta\,\Delta\theta + \Delta\theta^2 + \Delta\mathcal{E}/\gamma_b^2)$, where θ is the angle between directions of the wave and beam propagation, and $\Delta\mathcal{E}$ is the relative energy spread of the beam electrons.

Under the opposite condition, the instability at a given $\vec{k}$ is kinetic. For simplicity, we assume further that the condition $\gamma_b\Delta\theta > 1$ is satisfied, since we are interested primarily in ultrarelativistic beams. The following crude estimate then applies for the growth rate maximized over the longitudinal wave number $k_{||} \approx \omega_e/c$:

$$\max_{k_{||}} \Gamma(\vec{k}) \sim \frac{\omega_e}{\gamma_b \Delta\theta^2} \frac{n_b}{n_e} \frac{k_{||}^2}{k^2}. \tag{6}$$

This estimate does not describe a spike at small $\theta \lesssim \Delta\theta$, where the maximized growth rate is about twice higher [9]. Including this factor (although in our crude calculation the uncertainties are at least that large), we get

$$\Gamma \sim \frac{2\omega_e}{\gamma_b \Delta\theta^2} \frac{n_b}{n_e}. \tag{7}$$

For example, for $n_b = 5 \times 10^{22}$ cm^{-3}, $\Delta\theta = 0.1$ and $\gamma_b = 50$ gives $\Gamma \sim 10^{15}$ sec^{-1} in the core, which satisfies both the conditions of instability ($\Gamma > \nu$) and the kinetic formula applicability ($\Gamma < \omega_e\Delta\theta^2 \sim 6 \times 10^{15}$ sec^{-1}).

The excitation of Langmuir waves in the core does not necessarily imply an efficient energy deposition there. To describe the beam relaxation and, in particular, to evaluate the energy deposited in the core, one needs to know some major spectral properties of Langmuir waves excited by the beam. These properties depend substantially on specific mechanisms of nonlinear stabilization of Langmuir waves. There are several such mechanisms, and the leading one may change even in the process of the same beam relaxation in the plasma as the system parameters evolve. Precise spectra of the turbulence and laws of the beam relaxation have been calculated [10] for the weakly turbulent regimes that might be expected when the growth rate of the instability, $\Gamma_{\rm eff} = \Gamma - \nu$, is sufficiently small. For strong Langmuir turbulence, only qualitative theories have been formulated, which include several different regimes (see, for instance, [11]).

To evaluate the possibility of strong Langmuir turbulence excitation in the target core, note that such turbulence is primarily associated with the instability of uniform wave intensity distributions to spatial modulations. This instability leads to formation of cavities in the plasma density which trap Langmuir waves and then deepen, because of a pressure of the trapped waves, with the simultaneous blowup in the trapped wave intensity — the phenomena known as Langmuir wave collapse. The modulational instability develops at turbulent energy densities exceeding a threshold value $W_{\rm th}$, which depends on the wave energy distribution in wave-vector $\vec{k}$ space. A crude threshold estimate for spectra with the typical $k \sim \omega_e/c$ is $W_{\rm th} \sim n_e T_e^2/m_e c^2$. For a turbulent energy density W moderately exceeding the threshold $W_{\rm th}$, the growth rate of the modulational instability can be crudely evaluated as

$$\Gamma_{\rm md} \sim \omega_e \left(\frac{W}{n_e T_e} \frac{m_e}{m_i} \right)^{1/2}, \tag{8}$$

where m_i is the plasma ion mass. For $\omega_e \sim 6 \times 10^{17}$ sec^{-1}, $T_e \sim 5$ keV, and $m_i \sim 4000 m_e$, we get

$$\Gamma_{\rm md} \sim 10^{15} (W/W_{\rm th})^{1/2} \text{ sec}^{-1}. \tag{9}$$

For $W \sim W_{\rm th}$, $\Gamma_{\rm md}$ is comparable with the Langmuir wave collisional damping ν. Since the weakly turbulent processes are usually slower than the modulational instability, the latter is likely needed to stabilize Langmuir waves for not too small $\Gamma_{\rm eff} = \Gamma - \nu \gtrsim \nu$. Thus, strong Langmuir turbulence is likely excited.

To evaluate the length of beam relaxation caused by Langmuir turbulence, one needs to know the energy density W_r of Langmuir waves resonantly interacting with the beam. Then, the rate of beam energy loss could be evaluated as $\Gamma_{\rm eff} W_r$ and the local relaxation length could be evaluated as

$$L_r \sim \gamma_b n_b m_e c^3 / \Gamma_{\rm eff} W_r. \tag{10}$$

For $\Gamma_{\rm eff} \sim \Gamma$, it follows

$$L_r \sim (c/2\omega_e)(m_e c^2/T_e)^2 (\gamma_b \Delta\theta)^2 (W_{\rm th}/W_r). \tag{11}$$

For $\omega_e \sim 6 \times 10^{17}$ sec^{-1} and $T_e \sim 5$ keV, this simplifies to

$$L_r \sim 2.5 (\gamma_b \Delta\theta)^2 (W_{\rm th}/W_r)\ \mu\text{m}. \tag{12}$$

If $W_r > W_{\rm th}$, this length would not exceed 50 μm for

$$\gamma_b \Delta\theta \lesssim 5. \tag{13}$$

The beam relaxation also depends on where specifically the Langmuir energy is located in the resonant $\vec{k}$-space domain. If the resonant wave energy is located primarily at small angles, $\theta \sim k_\perp/k \lesssim \Delta\theta$, the beam primarily loses energy without much angular scattering. However, for the resonant wave energy located primarily at large angles, say, $\theta \sim 1$, the beam would undergo substantial angular scattering with only a relatively small, of order $\Delta\theta$, portion of the energy lost. The growth rate $\Gamma \propto \Delta\theta^{-2}$ quickly decreases as the beam angular spread $\Delta\theta$ increases. It might become smaller than the Langmuir wave collisional damping ν, so that the stabilization might occur before the beam is collectively decelerated. Such regimes should be avoided.

Note that the ideas described here must be applied with caution to contemporary experiments that do not appear to observe collisionless slowing down. For example, in recent experiments on stopping of fast electrons [13], relativistic electrons were produced with a large angular spread $\Delta\theta \sim 1$ and presumably with a Maxwellian-like initial distribution function. During propagation of such electrons through targets, beamlike distributions are likely to form that might lead to excitation of Langmuir waves and reduction in the electron penetration depth. However, for the large Z target materials (from Cu to Al) used in these experiments and the relatively low plasma temperatures achieved, the collisional damping of the Langmuir wave was likely strong enough to suppress the instability. Interestingly, for lighter target materials, such as CH, the electrons were clearly less able to penetrate the target, which might indicate a larger role of collective stopping effects.

To be specific, for $Z = 15$, $T_e = 300$ eV, and $n_e = 10^{23}$, the collisional damping would be nearly the same as in our case, namely, $\nu \sim 5 \times 10^{14}$ sec^{-1}. This is just 30 times smaller than the plasma frequency for $n_e = 10^{23}$, so that a broad, moderately relativistic beam is stable for $n_b/n_e < 1/30$. In the presence of a cold plasma component, which apparently was the case in [13], the beam stability condition can be much softer. Thus, the penetration of the beam through the target, as observed in [13], is not surprising.

In summary, we challenged the prevailing paradigm that ultrarelativistic electrons cannot be significantly decelerated in cores of inertial-fusion targets. We showed that Langmuir waves can be excited precisely in the core of an inertial-fusion target by an ultrarelativistic electron beam in a reasonably broad parameter range of practical interest. Although more detailed calculations of the spectra are needed, these Langmuir waves are potentially capable of significantly decelerating the beam within the core. The relevant spectral properties could be determined by extending REB experiments, such as [12], to larger relativistic factors.

While there are possibilities for convective or collisional suppression of the beam instability to Langmuir waves on the beam way to the core, this instability, in addition to the Weibel instability, might inhibit electron beam transport to the core. Note that the possibilities for suppression of Langmuir instabilities by plasma density gradients and collisions might also be useful in suppressing the Weibel instability, also modified in inhomogeneous plasmas [14].

Our findings indicate possible suitability of stiffer ultrarelativistic electron beams, with smaller currents, for a fast-igniter inertial fusion. Such beams correspond to much higher igniting laser intensities. Larger energies might be deposited in smaller target regions within shorter times, which is especially important for advanced fuels. While these methods of managing REB energy deposition are clearly important for fast-igniter targets, the methods are certainly much more widely applicable to a broad class of plasma targets.

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