Relativistic electron acceleration in focused laser fields after above-threshold ionization

I. Y. Dodin and N. J. Fisch
Princeton Plasma Physics Laboratory, Princeton, New Jersey 08543, USA
(Received 4 April 2003; revised manuscript received 19 August 2003; published 18 November 2003)

Electrons produced as a result of above-threshold ionization of high-Z atoms can be accelerated by currently producible laser pulses up to GeV energies, as shown recently by Hu and Starace [Phys. Rev. Lett. **88**, 245003 (2002)]. To describe electron acceleration by general focused laser fields, we employ an analytical model based on a Hamiltonian, fully relativistic, ponderomotive approach. Though the above-threshold ionization represents an abrupt process compared to laser oscillations, the ponderomotive approach can still adequately predict the resulting energy gain if the proper initial conditions are introduced for the particle drift following the ionization event. Analytical expressions for electron energy gain are derived and the applicability conditions of the ponderomotive formulation are studied both analytically and numerically. The theoretical predictions are supported by numerical computations.

DOI: 10.1103/PhysRevE.68.056402 PACS number(s): 52.38.Kd, 52.35.Mw, 52.20.Dq, 52.27.Ny

I. INTRODUCTION

Recent advances in high-power laser technology have resulted in the development of petawatt laser systems [1–3]. Those can deliver superstrong laser pulses with intensities of focused output radiation as high as 10^{21} W/cm^2, which makes them especially attractive from the standpoint of studying the interaction between electromagnetic radiation and matter. One of the aspects of this interaction is the problem of obtaining ultrarelativistic beams of charged particles. Numerical computations [4–11] and experiments [12–15] show that laser waves can accelerate charged particles to ultrarelativistic energies. For electrons, the energies achievable with currently available laser systems can be as large as several GeV [4–7,9].

The electron energy gain can be characterized by the dimensionless parameter $a_0 = eE_0/m\omega c$, where E_0 is the amplitude of the laser electric field, e and m are the electron charge and the mass, respectively, ω is the frequency of the laser field, and c stands for the speed of light. The value of a_0 can be understood as the ratio of the momentum imparted by the wave field in a single oscillation to mc, meaning that relativistic effects become important at $a_0 \gtrsim 1$. (For the wavelength of the laser radiation equal to 1 μm, the intensity corresponding to $a_0 \sim 1$ is about 10^{18} W/cm^2.) Thus, under the influence of currently available laser intensities ($a_0 \lesssim 10^2$), the ultrarelativistic electron oscillatory motion itself has gamma factor ($\gamma = \mathcal{E}/mc^2$) of the order of a_0, which though large is still small compared to the experimentally and numerically observed values $\gamma \sim a_0^2$ [4–15].

Under the plane-wave approximation, the particle motion is exactly integrable (Sec. II) and the scaling $\gamma \sim a_0^2$ follows naturally. However, for focused laser beams used in experiments, the plane-wave model does not capture the important dynamics, because particles can escape from the interaction region in the direction perpendicular to the wave vector. In this case, the same scaling can be more adequately explained in terms of the ponderomotive formulation (Sec. III, see also Refs. [10,11,16]), which can be applied to experimentally realizable conditions. However, for electrons produced as a result of above-threshold ionization, the applicability of such formulation is not obvious: the ionization event itself cannot be described directly in terms of the ponderomotive approach, because the electron leaves the ion on a time scale small compared to the oscillation period [9].

What we show here is that the ponderomotive formulation can still give good results if the ionization is accounted for properly, and the laser field, in which the acceleration takes place changes, is smooth enough (for the nonrelativistic problem, see Ref. [17]). We show how ionization can be included in a model based on the fully relativistic ponderomotive approach. For certain cases of interest, we calculate the electron energy gain precisely. Also, we address the applicability conditions of the model, both analytically and numerically. These conditions turn out to be significantly more relaxed than those given in Ref. [11].

The paper is organized as follows. In Sec. II, we formulate the actual model studied in the paper and introduce the basic notation through revising the known problem of particle acceleration by a plane wave. In Sec. III, we give the ponderomotive treatment of nonadiabatic particle acceleration in spatially nonuniform laser fields. In Sec. IV, we compare our analytical results with those obtained from numerical computations of particle motion. Sec. V summarizes our main ideas.

II. IONIZATION EVENT AND ADIABATICITY VIOLATION

In a smooth laser field, with a characteristic spatial scale of the wave envelope L, large compared to the wavelength λ, a charged particle experiences oscillatory "figure-eight" motion (in a linearly polarized wave) or circular motion (in a wave with circular polarization) [18,19]. Its guiding center drifts as a quasiparticle with an effective mass $m_{\text{eff}} = m\sqrt{1+\overline{a^2}}$, where the bar above $a(\eta) = eE(\eta)/m\omega c$ stands for averaging over the phase $\eta = \omega t - \boldsymbol{k}\cdot\boldsymbol{r}$ [20–25]. Slow variations of the laser intensity "seen" by the particle result in variations of its effective mass and produce a ponderomotive force, proportional to the intensity gradient. Because of the conservative nature of this force, as the pulse passes over the particle, the particle is decelerated almost precisely down to its initial energy. In this case, the net acceleration appears to be exponentially small with respect to the parameter λ/L. In order to prevent adiabatic deceleration and extract the energy from the laser field *irreversibly*, one needs to violate the adiabatic approximation for particle drift. (See also the discussion on the so-called Lawson-Woodward theorem in Ref. [26], and references cited therein.)

Laser fields with sufficiently large transverse gradients of average intensity, which can lead to the loss of adiabaticity, can be obtained by focusing laser radiation onto a small spot, whose size is only limited by diffraction spreading. The focal waist $w=N\lambda$ (where the factor N is often of the order of several units) is usually small compared to the longitudinal scale of the focal region $b=kw^2$. Thus, nonadiabatic acceleration in a focal spot can occur primarily due to particle escape in the transverse direction at $N\sim 1$ [4–7]. (For more precise condition, see Sec. III.) However, in this case, charged particles must be delivered to the interaction region *after* the laser field has been turned on. Otherwise, as the laser intensity seen by a charged particle grows slowly, the particle is swept away by the very front of the pulse even before the intensity reaches its maximal value, which results in inefficient acceleration [9].

It is advantageous to keep charged particles in the focal region until the field reaches its maximal amplitude. In Ref. [9] (see also Refs. [10,13,27]), it was proposed to keep the electron trapped by the Coulomb field of a high-Z ion for the time needed for the laser field to reach its maximal intensity. During this time, the bounded electron remains practically at rest but, after the laser field intensity becomes large enough, it is swept off by the wave. After the electron has left the ion, the Coulomb field does not influence the electron dynamics significantly. Thus, the electron can be considered as accelerated in free space, assuming for the initial conditions that, effectively, it starts seeing the intense laser field instantaneously, immediately after ionization. The actual value of the ionization potential does not influence the acceleration but rather determines the amplitude of the laser field, at which ionization occurs (see also Sec. III).

Once the adiabaticity is violated by an abrupt ionization event, the ponderomotive force generally does not bring the particle to rest after the interaction is over, even if the laser intensity seen by the particle decreases adiabatically to zero at $t\to\infty$. Therefore, in this case, the particle can retain a significant part of its energy even if leaving adiabatically the region of interaction with the laser field. In this paper, we will consider precisely this case below. Namely, we will assume that after ionization, which we model as an instantaneous jump of the laser intensity seen by the electron, the particle moves in smooth laser field (see Secs. III, IV) and leaves the interaction region adiabatically. We will develop a ponderomotive formulation to study electron acceleration after ionization and show that, though the above-threshold ionization represents an abrupt process compared to laser oscillations, the ponderomotive approach can still adequately predict the resulting energy gain if the proper initial conditions are introduced for the particle drift following the ionization event.

Under the plane-wave approximation, the problem of particle motion yields an exact solution for arbitrary polarization and longitudinal profile of the pulse [8,18,28]: Consider a particle moving under the action of a plane laser wave propagating in vacuum with vector potential given by

$$\boldsymbol{A}(\boldsymbol{r},t)=(mc^2/e)\boldsymbol{\mathcal{A}}(\eta), \tag{1}$$

where $\eta=\omega t-kz$ stands for the phase of the wave and $k=\omega/c$ represents the wave vector. Several exact integrals of particle motion under the action of such a wave can be figured out readily. First, the canonical momentum $\boldsymbol{P}_\perp=\boldsymbol{p}_\perp+(e/c)\boldsymbol{A}$ is conserved because of the symmetry of the laser field with respect to the transverse displacement ($\boldsymbol{\nabla}_\perp A_{x,y}=0$). Assuming zero field at $t\to-\infty$ [$\mathcal{A}(-\infty)=0$], one gets that $\boldsymbol{P}_\perp$ equals the initial kinetic momentum of the particle before the interaction $\boldsymbol{p}_{\perp,0}$. This yields a solution for the kinetic momentum

$$\boldsymbol{p}(\eta)=\boldsymbol{p}_0-(e/c)\boldsymbol{A}(\eta). \tag{2}$$

Also, because of the fact that the wave field depends on the phase (that is, on the combination $z-ct$, rather than z and t separately), there exists an additional integral of motion given by

$$u=\gamma-p_z/mc=\gamma_0-p_{z,0}/mc=\mathrm{const}, \tag{3}$$

where $\gamma_0=\sqrt{1+(p_0/mc)^2}$ is the initial normalized energy of the particle. (Note that the obtained formula is valid only in the case when the laser pulse is propagating in vacuum. If the refractive index of the medium differs from unity, the u conservation law needs to be modified, and further analysis becomes more complicated [24].) The equations for the particle energy γ and the phase η,

$$\frac{d\gamma}{dt}=\frac{e}{mc^2}(\boldsymbol{v}\cdot\boldsymbol{E}),\quad \frac{d\eta}{dt}=\omega-kv_z, \tag{4}$$

where $\boldsymbol{E}=-(1/c)(\partial\boldsymbol{A}/\partial t)$, with Eqs. (2) and (3) taken into account, can be put in the dimensionless form

$$\frac{d\gamma}{d\tau}=\frac{1}{2\gamma}\frac{d\hat{\mathcal{A}}^2}{d\eta},\quad \frac{d\eta}{d\tau}=\frac{u}{\gamma}, \tag{5}$$

where we introduce the dimensionless time $\tau=\omega t$ and the quantity $\hat{\boldsymbol{\mathcal{A}}}==\boldsymbol{\mathcal{A}}-\boldsymbol{p}_{\perp,0}/mc$. Solving those, one readily gets the function $\gamma(\eta)$ for arbitrary initial conditions and arbitrary shape of the laser pulse:

$$\gamma(\eta)=\gamma_0+\frac{[\boldsymbol{\mathcal{A}}(\eta)-\boldsymbol{p}_{\perp,0}/mc]^2-(\boldsymbol{p}_{\perp,0}/mc)^2}{2(\gamma_0-p_{z,0}/mc)}. \tag{6}$$

Let us now precisely point out the connection between $\boldsymbol{A}$ that enters the above formula and the actual laser electric field $\boldsymbol{E}$. From Eq. (1), it follows that

$$\boldsymbol{\mathcal{A}}(\eta)=-\frac{e}{m\omega c}\int_{-\infty}^{\eta}\boldsymbol{E}(\eta')\,d\eta'. \tag{7}$$

In the case when the field seen by the charged particle is smooth enough ($\lambda/L \ll 1$), one has

$$\boldsymbol{\mathcal{A}} \approx \boldsymbol{a} = \mathrm{Re}\left(-i\frac{e\boldsymbol{E}_c}{m\omega c}\right), \quad \boldsymbol{E}_c = \boldsymbol{E}_0(\eta)e^{-i\eta}, \tag{8}$$

with $\boldsymbol{E}_0(\eta)$ being a slow function of η; $\boldsymbol{E}=\mathrm{Re}\boldsymbol{E}_c$. For such a pulse, $\boldsymbol{\mathcal{A}}(+\infty)$ is exponentially small (with respect to the parameter λ/L), since it represents the average of a rapidly oscillating function. Thus, after such a smooth laser pulse has passed over the particle, the particle energy γ reverts to its initial value γ_0. This regime corresponds to the adiabatic motion, when no substantial acceleration takes place. As discussed above, in order to provide significant acceleration, one needs to make the laser field amplitude $\boldsymbol{E}_0(\eta)$ an abrupt function.

Consider the case when the particle initially rests, which will be the case of our further primary interest. Equation (6) now simplifies to

$$\gamma(\eta) = 1 + \mathcal{A}^2(\eta)/2. \tag{9}$$

Suppose that the laser field seen by the particle grows instantaneously at some phase η_0 from zero value up to a finite amplitude. If, later, the pulse amplitude decreases to zero slowly, the value of integral (7) at $\eta \to \infty$ is determined by the intensity only at $\eta = \eta_0$. Precisely, for field (8), Eq. (9) can be written as

$$\gamma(\infty) = 1 + a^2(\eta_0)/2 = 1 + a_0^2\sin^2(\eta_0)/2. \tag{10}$$

The model of instantaneous increase of the field seen by the particle can adequately describe the acceleration following an ionization event [9,13] and, generally, represents a good qualitative model describing any nonadiabatic effects during acceleration. As mentioned above, the scaling $\gamma \sim a_0^2$ predicted by Eq. (10) matches well with the values of γ actually measured in experiments [12–15]. However, for focused laser beams used in experiments, the plane-wave model does not capture the important transverse dynamics of particles, which can be more adequately approached in terms of the ponderomotive formulation discussed in the following section.

III. PONDEROMOTIVE TREATMENT

Treating the acceleration process under the approximation of a plane laser pulse gives a simple estimate for the energy gained by a particle during the interaction with a laser wave. In reality, however, to get the most efficient acceleration for a given fluence, the radiation is usually focused into a tiny spot, where the field gradients become essential for particle dynamics. As discussed in Sec. II, the particle energy gain depends on the transverse structure of the field because particles primarily escape from the focal region in the direction transverse to the wave propagation. In addition, in focused fields, the longitudinal component of the laser electric field appears, which, though small, can influence the acceleration process substantially [11,29]. Therefore, a more accurate model (rather than the one assuming the plane-wave approximation for a laser pulse) is needed to describe electron acceleration in real experiments.

In this section, we employ a ponderomotive formulation of acceleration of an electron produced as a result of above-threshold ionization in a smooth (though arbitrarily nonuniform) laser field. Again, below we will assume that the field seen by the particle is switched on instantaneously at $t=t_0$ (or $\eta=\eta_0$), when the particle remains at rest. Note that, if the spatial scale of the field is large enough (the exact conditions are discussed below), the particle dynamics is adiabatic at any time $t>t_0$. The nonadiabatic "jump" of the laser field intensity at $t=t_0$ can be attributed to proper initial conditions for the *drift* motion, which is superimposed on the *known* laser-driven oscillations and is considered only at $t \geqslant t_0$. The requirement on these proper initial conditions is that they must provide the particle *total* velocity to be zero at $t=t_0$ (similar approach for the case of nonrelativistic particle energies is discussed, e.g, in Ref. [17]).

At $t>t_0$, the particle average motion is determined by the effective ponderomotive force [11,20,21,23–25] caused by inhomogeneity of a smooth laser field with the vector potential satisfying Eq. (8). As shown in Refs. [20,24], the particle drift is described by the Hamiltonian function

$$\mathcal{H}(\boldsymbol{r}_\mathrm{d},\boldsymbol{p}_\mathrm{d};t) = \sqrt{m_\mathrm{eff}^2c^2 + p_\mathrm{d}^2c^2}, \tag{11}$$

where $\boldsymbol{r}_\mathrm{d} = \langle \boldsymbol{r}\rangle$ stands for the guiding center location, $\boldsymbol{p}_\mathrm{d} = m_\mathrm{eff}\gamma_\mathrm{d}\boldsymbol{v}_\mathrm{d}$ equals the particle *phase-averaged* momentum $\overline{\boldsymbol{p}}$, $m_\mathrm{eff}(\boldsymbol{r}_\mathrm{d},t) = m\sqrt{1+\overline{a^2}}$ is the slowly variable effective mass, and $\gamma_\mathrm{d} = 1/\sqrt{1-v_\mathrm{d}^2/c^2}$ is the gamma-factor associated with the drift velocity $\boldsymbol{v}_\mathrm{d}$, which coincides with the *time-averaged* particle velocity $\langle \boldsymbol{v}\rangle$. (The bar denotes averaging over the phase η, and the angular brackets stand for the time averaging [20].) As follows from the definition, the transverse drift momentum $\boldsymbol{p}_{\mathrm{d},\perp}$ at $t=t_0$ equals the canonical momentum $\boldsymbol{P}_\perp = (e/c)\boldsymbol{A}(\eta_0)$. The value of $p_{\mathrm{d},z}(t=t_0)$ can be readily obtained from Refs. [20,24],

$$\mathcal{H} = m_\mathrm{eff}\gamma_\mathrm{d}c^2 = m\overline{\gamma}c^2, \quad \overline{\gamma} = \gamma_\mathrm{d}\sqrt{1+\overline{a^2}}, \tag{12}$$

considered together with the expressions $\overline{\gamma} = u + \overline{p}_{z,0}$ and $u(t=t_0)=1$. (Note that, in nonuniform fields, which are not only phase dependent but may also vary in time and space independently, the quantities u and $\boldsymbol{P}_\perp$ are subject to slow but substantial variations as the particle travels across the laser field.) Finally, one gets the expressions for the drift momentum and the normalized energy:

$$\begin{gathered} \boldsymbol{p}_{\mathrm{d},\perp}(t=t_0) = mc\,\boldsymbol{a}, \\ p_{\mathrm{d},z}(t=t_0) = mc\,(a^2+\overline{a^2})/2, \\ \overline{\gamma}(t=t_0) = 1 + (a^2+\overline{a^2})/2, \end{gathered} \tag{13}$$

where the right-hand side is assumed to be evaluated at $t=t_0$ (or $\eta=\eta_0$), and the quantity $\boldsymbol{a}$ is defined according to Eq. (8).

Equations (13) represent the initial conditions for the particle drift motion, which, at $t>t_0$, can be solved for in the framework of the ponderomotive canonical description with the Hamiltonian function given by Eq. (11). Using those, one can finally formulate the limitations of the ponderomotive treatment. (For the problem of free electron scattering off a laser pulse, those were addressed in Ref. [11].) In addition to the restriction that the amplitude of oscillations must remain small compared to the characteristic scale of the laser field, the particle needs to undergo at least several oscillations before it leaves the interaction region. The number of oscillations can be calculated as the ratio of the time $T_{\rm int}$ spent by the particle within the interaction region to the Doppler-modified period of oscillations $T\sim 2\pi a_0^2/\omega$. Denoting the transverse and the longitudinal scales of the laser field with $L_\perp$ and $L_{||}$ respectively, one gets $T_{\rm int}=\min\{L_\perp/v_{\rm d,\perp}\,;\, L_{||}/v_{\rm d,||}\}$. For $a_0\gg 1$, from Eqs. (13), it follows that $v_{\rm d,\perp}\sim c/a_0$ and $v_{\rm d,||}\approx c$. Thus, finally, the condition for the ponderomotive approach to be valid can be written as

$$L_\perp \gg \lambda a_0, \quad L_{||}\gg \lambda\, a_0^2 \tag{14}$$

(see also Sec. IV). The condition of small amplitude of particle oscillations (compared to $L_\perp$ and $L_{||}$) can also be shown to coincide with Eqs. (14).

It is important to emphasize that, for the ponderomotive approach to stay valid, the laser pulse may not satisfy the plane-wave approximation on the global scale. Note that it is the potential of the field that enters the drift motion equations rather than the field itself. Thus, the possible inhomogeneity of the laser intensity is automatically taken into account in the proposed treatment, and so is the particle acceleration by a small longitudinal field component, which is hard to take into consideration in the conventional plane-wave model [29].

In the framework of the ponderomotive approach utilizing the concept of a quasiparticle (that is, a guiding center drifting with the effective mass $m_{\rm eff}$), the scaling for the retained energy $\gamma\sim a_0^2$ of an electron produced as a result of ionization becomes apparent. Indeed, consider the initial zero total velocity as a superposition of a drift velocity $\boldsymbol{v}_{\rm d}$ in the laboratory frame of reference K and the quiver velocity $\boldsymbol{v}'$ in the frame K' moving relatively to K at the speed $\boldsymbol{v}_{\rm d}$. From Lorentz addition of velocities, one finds for $t=t_0$ that $\boldsymbol{v}_{\rm d}=-\boldsymbol{v}'$. Since the magnitude of a is relativistically invariant [20,21], it follows that $\gamma_{\rm d}=\gamma'\sim a_0$, where $\gamma'=1/\sqrt{1-v'^2/c^2}$ stands for the relativistic factor of quiver motion. Recall that it is the guiding center with an effective mass $m_{\rm eff}\sim m a_0$ (rather than the true particle with mass m) that is accelerated up to the velocity v'. Thus, immediately after the laser field is switched on, the guiding center energy $m_{\rm eff}\gamma_{\rm d}c^2$ increases up to the value of the order of $mc^2a_0^2$. On the other hand, as follows from Eq. (12), the guiding center energy coincides with the average energy of the true particle, which yields the predicted scaling $\gamma\sim a_0^2$.

The results obtained in the preceding section for charged particles acceleration by a plane wave with $\boldsymbol{a}=\boldsymbol{a}(\eta)$ can be readily derived from the proposed ponderomotive formulation. Indeed, consider a particle instantaneously injected into a plane laser wave, whose amplitude decreases adiabatically at $\eta\to\infty$ $[\boldsymbol{a}(\infty)=0]$. Using $d\mathcal{H}/dt=\partial\mathcal{H}/\partial t$ and $\partial(\overline{a^2})/\partial t=\omega\{d(\overline{a^2})/d\eta\}$, one gets equations similar to Eqs. (5):

$$\frac{d\bar{\gamma}}{d\tau}=\frac{1}{2\bar{\gamma}}\frac{d(\overline{a^2})}{d\langle\eta\rangle},\quad \frac{d\langle\eta\rangle}{d\tau}=\frac{1}{\bar{\gamma}}. \tag{15}$$

Here $\langle\eta\rangle=\omega t-kz_{\rm d}$ is the time-averaged phase, which we can be approximately used as the argument of a slow function $\overline{a^2}(\eta)$, that stands for the normalized average intensity. From Eqs. (15), the previously obtained formula for the particle net energy gain (10) follows immediately: As follows from Eqs. (15), one has $\bar{\gamma}-\overline{a^2}/2=\text{const}$, which yields

$$\bar{\gamma}(\infty)=\bar{\gamma}(\eta_0)+\frac{\overline{a^2}(\infty)-\overline{a^2}(\eta_0)}{2}=1+\frac{a^2(\eta_0)}{2}, \tag{16}$$

where we substituted Eq. (13) for $\bar{\gamma}(\eta_0)$ and used that $\overline{a^2}(\infty)=0$.

On the other hand, from the ponderomotive treatment, more results of interest can be derived. For example, suppose the nonuniform intensity profile remains static during a time larger than that required for the accelerated particle to leave the region of interaction (e.g., in a focal region). In this case, the phase-average particle energy $\bar{\gamma}$ is conserved, since $\partial\mathcal{H}/\partial t=0$. Thus, assuming $\overline{a^2}(\infty)=0$, one gets $\gamma(\infty)=\bar{\gamma}(\infty)=\bar{\gamma}(0)$, meaning that

$$\gamma(\infty)=1+\tfrac{1}{2}(a^2+\overline{a^2})_0, \tag{17}$$

where the subscript 0 denotes the evaluation of the right-hand side of the formula at $t=t_0$ $(\eta=\eta_0)$. The additional term $\overline{a^2}/2$ [compared to Eq. (10)] results from the particle ponderomotive acceleration out of the interaction region, which, for static intensity profile, replaces a similar deceleration by the tail of a plane laser pulse. The detailed structure of the intensity profile appears to be unimportant for the net energy gain because of the conservative property of the ponderomotive force. Also, it is worth noting that, by above-threshold ionization, electrons are primarily produced at the maximum of electric field [9], when the instantaneous magnitude of the vector potential is zero $[a(\eta_0)=0]$, if the field intensity seen by electrons varies only slightly from one laser period to another. (If the field strength has not been large enough to unbind an electron on some period, the ionization event, if any, can occur on the next period only near the peak field: any other field strength has been previously experienced by the electron and has not caused an ionization event.) Therefore, electrons produced in the same region of a static laser field at different moments of time eventually must retain approximately the same energy $\gamma(\infty)=1+\overline{a_0^2}/2$.

One needs to keep in mind though that ionization in a laser field is a stochastic process [9]: Since the frequency of a bounded electron oscillations inside an atom is much larger than the laser frequency of interest, electron performs multiple rotations around the ion in the laser field before ionization. Therefore, in principle, there can be few electrons escaping at different phases. One might also imagine other

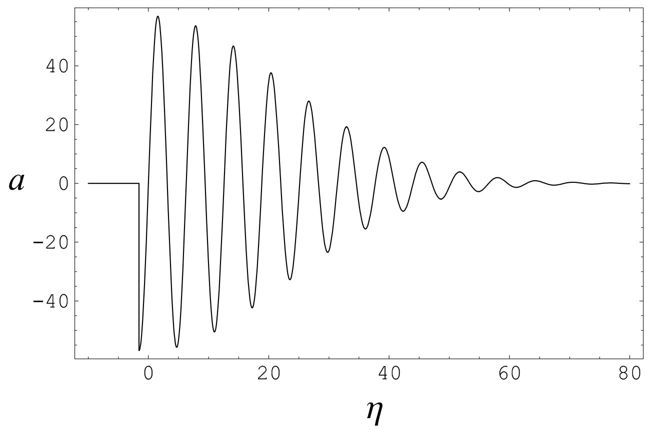

FIG. 1. Normalized vector potential a (''seen'' by electron) of a linearly polarized laser pulse vs phase η. The field is switched on at $\eta_0=-\pi/2$. For the peak intensity $I=8\times10^{21}$ W/cm^2 and the wavelength $\lambda=1.054$ μm, the dimensionless field strength parameter is $a_0\approx57$.

situations when the phase η_0 differs from the one corresponding to the peak field. However, working out the actual phase of particle escape remains out of the scope of the present study, which is primarily focused on particle dynamics after ionization.

IV. DISCUSSION

In this section, we discuss the possibility of ponderomotive acceleration by ultraintense electromagnetic pulses, available with existing laser systems, and present the results of our numerical computations. We consider particle acceleration both by a plane laser wave and by a focused wave with a static average intensity profile. In these two cases, the energy gain can be obtained analytically, which allows us to compare the numerical results with our theoretical predictions.

To start, let us calculate the actual value of the parameter a_0, which determines the characteristic energy of accelerated particles. In terms of the wavelength λ (measured in microns) and the laser intensity I (measured in W/cm^2), a_0 is given by

$$a_0=\lambda_{\mu\mathrm{m}}\sqrt{I_{\mathrm{W/cm}^2}/(2.74\times10^{18})}. \tag{18}$$

For the sake of definiteness, consider the parameters of laser radiation accepted in Ref. [9], namely, $\lambda=1.054$ μm and $I=8\times10^{21}$ W/cm^2 for the peak field. For those, Eq. (18) yields $a_0\approx57$. In the peak field, the maximal possible value of the oscillating electron energy $\gamma(\eta)=1+a_0^2(\sin\eta-\sin\eta_0)^2/2$ is therefore $\gamma_{\max}=1+2a_0^2\approx6500$ (or $\mathcal{E}\approx3.3$ GeV).

As discussed above, a significant fraction of the electron energy $\gamma\sim\gamma_{\max}$ can be retained by the particle after the interaction, only if the field amplitude seen by the particle changes abruptly in time. As in previous sections, we will assume a free electron to be born (for instance, by means of ionization) in a peak-intensity laser field rapidly in comparison with the wave period, so that instantaneous increase of the seen field intensity represents an adequate approximation.

First, consider the interaction with a linearly polarized plane laser pulse switched on at $\eta=\eta_0$, with intensity decreasing slowly at $\eta\to\infty$ (see Fig. 1). From Eqs. (10) and (16), it follows that the maximal retained energy is given by $\gamma_{\mathrm{ret}}=1+a_0^2/2\approx1630$ (or $\mathcal{E}=0.83$ GeV). This value matches precisely with the one obtained numerically. In Fig. 2, the normalized electron energy γ (solid line) is shown versus the normalized time ωt. It can be seen that $\gamma(\infty)=\gamma_{\mathrm{ret}}$, where the asymptotic value is represented by a dotted line. The electron drift energy $\overline{\gamma}=\mathcal{H}/mc^2$ (dashed line) is found to follow Eqs. (15), as predicted above.

Lower acceleration is found for initial phases, other than $\eta_0=\pm\pi/2$. For the same laser field switched on at $\eta_0=0$, the electron energy after acceleration is negligible (compared to $\gamma_{\max}$), exactly as predicted by Eq. (10) (Fig. 3). If the amplitude of the laser pulse decreases adiabatically at $\eta\to\infty$, substantial acceleration for $\eta_0=0$ can only be achieved in spatially nonuniform field (rather than the field

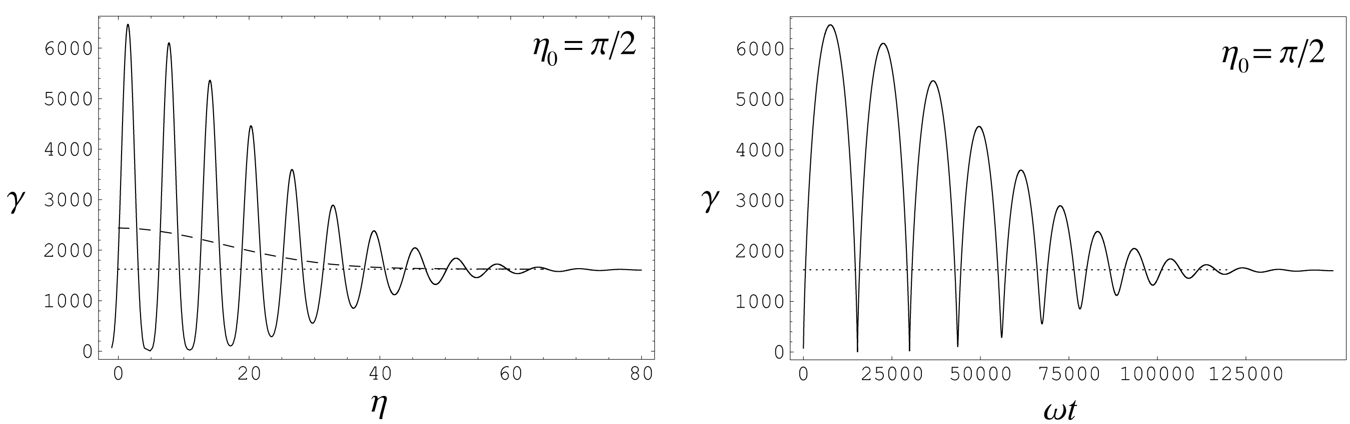

FIG. 2. Electron normalized relativistic energy $\gamma=\mathcal{E}/mc^2$ (solid line), normalized drift energy $\overline{\gamma}=\mathcal{H}/mc^2$ (dashed line), and predicted normalized retained energy $\gamma_{\mathrm{ret}}=1+a_0^2/2\approx1630$ (dotted line) vs phase η (upper plot) and time $\tau=\omega t$ (lower plot) for electron acceleration by the laser field shown in Fig. 1 ($a_0=57$).

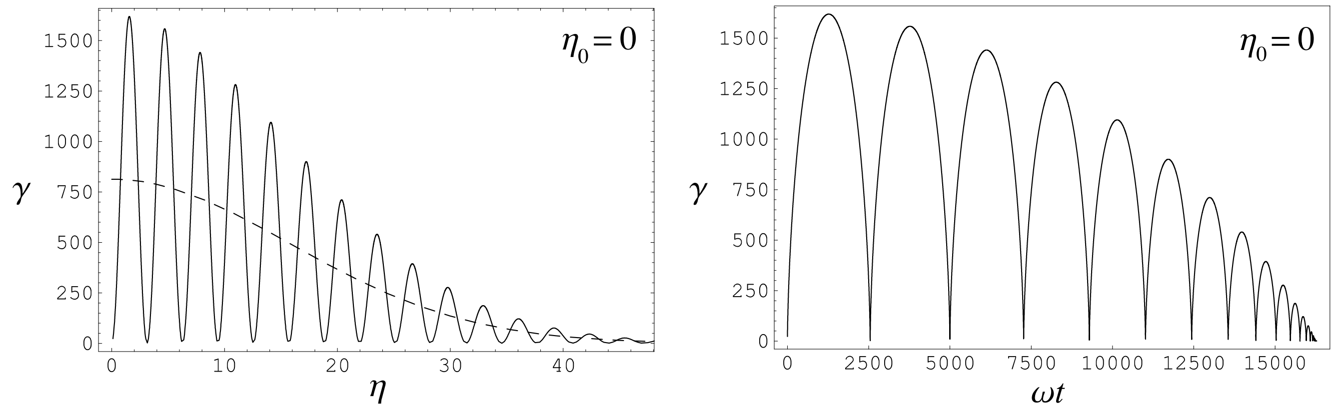

FIG. 3. Electron normalized relativistic energy $\gamma=\mathcal{E}/mc^2$ (solid line) and normalized drift energy $\overline{\gamma}=\mathcal{H}/mc^2$ (dashed line) vs phase η (upper plot) and time $\tau=\omega t$ (lower plot) for electron acceleration by the laser field shown in Fig. 1 ($a_0=57$) switched on at $\eta_0=0$. The theoretically predicted retained energy is $\gamma_{\rm ret}=1$.

which depends on the phase only), when the acceleration is provided by the ponderomotive force pushing the particle out of the region of strong field.

Simulations of electron acceleration of this type were performed for a linearly polarized focused field with $\boldsymbol{a}=\mathrm{Re}\boldsymbol{\alpha}$, where

$$\alpha_x=a_0\frac{ib}{b+2iz}\exp\left(-\frac{k(x^2+y^2)}{b+2iz}+i\eta\right), \tag{19}$$

$\alpha_z=(i/k)(\partial\alpha_x/\partial x)$, and the y component is neglected as being one of a higher order with respect to b/λ. Here $b=kw^2$ sets the longitudinal scale of the focal area, and $w=N\lambda$ is the focal waist size. For the factor N, three different values were chosen to simulate different regimes determined by the conditions Eqs. (14), which, for the focal area, can be put in a simple form

$$N\gg a_0. \tag{20}$$

Note that this condition is significantly more relaxed than the one given in Ref. [11], namely, $1-v_z/c\gg 1/kw$, which, for ultrarelativistic particles, yields $N\gg a_0^2$. As we show below, for the considered acceleration of electrons produced as a result of above-threshold ionization, condition (20) is consistent with results of our numerical computations.

In Fig. 4, the particle energy γ is shown versus time $\tau=\omega t$ for $N=140$ and initial particle location $\boldsymbol{r}(t=0)=\boldsymbol{0}$, also assumed below. As predicted from the ponderomotive approximation, the phase-average energy is conserved throughout the acceleration process. Thus, the retained energy is $\gamma_{\rm ret}=1+(a^2+\overline{a^2})/2$, where a is evaluated at the moment when the field was switched on. For the given field and $\eta_0=0$, one has $\gamma_{\rm ret}\approx 810$. Note that despite the fact that the energy plots in Fig. 4 seem to demonstrate nonadiabatic behavior, in fact, the dynamics remains adiabatic: Each period of oscillations contains two peaks of $\gamma(\eta)$, and each of those changes slightly from one period to another. Because of the interchange of the two types of peaks, the function $\gamma(\eta)$ contains multiple Fourier harmonics, but the amplitudes of those evolve slowly. As the interaction region is decreased and condition (20) is violated, the acceleration becomes nonadiabatic, as, for example, shown in Fig. 5 for $N=10$. On the other hand, even for $N=50\sim a_0$, the ponderomotive treatment predicts the energy gain fairly well, as seen in Fig. 6.

In the end, we would like to point out that, as we mentioned in Sec. II, all of the above analysis corresponds to laser-particle interaction *in vacuum*, where the phase and the group velocities of laser pulses coincide. It is often the case though that electron acceleration by laser pulses in a plasma appears to be of interest. In rare plasmas, with small background electron densities ($n\ll m\omega^2/4\pi e^2$), the refraction index remains close to unity, and the results obtained by applying the ponderomotive treatment represent a good approximation.

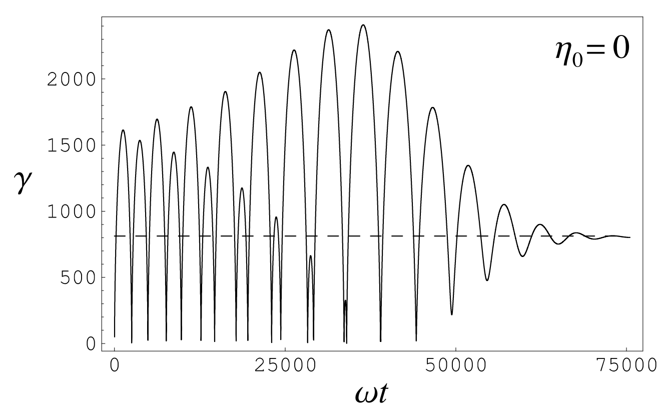

FIG. 4. Electron normalized relativistic energy $\gamma=\mathcal{E}/mc^2$ (solid line) vs time $\tau=\omega t$ for electron acceleration in a focused laser field with the focal waist $w=140\lambda$ and $a_0=57$, instantaneously switched on at $\eta_0=0$. Initial particle location is in the center of the focal region. For $\eta>\eta_0$, the average intensity profile is assumed static. Condition (20) is satisfied, so that the ponderomotive description is valid. The normalized drift energy $\overline{\gamma}=\mathcal{H}/mc^2$ (dashed line) is conserved equal to the theoretically predicted retained energy $\gamma_{\rm ret}\approx 810$.

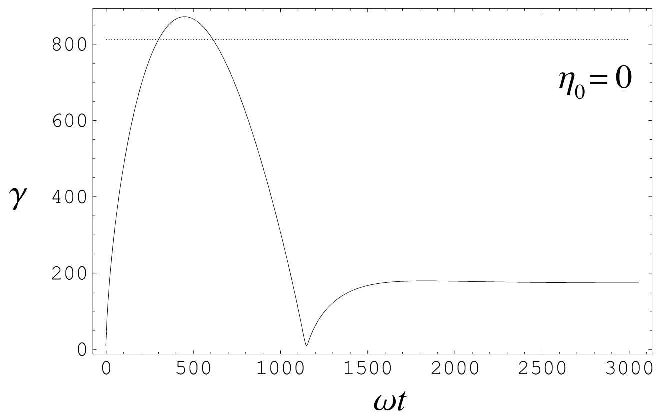

FIG. 5. Electron normalized relativistic energy $\gamma = \mathcal{E}/mc^2$ (solid line) vs time $\tau = \omega t$ for electron acceleration in a focused laser field with the focal waist $w = 10\lambda$ and $a_0 = 57$, instantaneously switched on at $\eta_0 = 0$. Initial particle location is in the center of the focal region. For $\eta > \eta_0$, the average intensity profile is assumed static. Condition (20) is not satisfied, and the electron motion in nonadiabatic. The theoretically predicted retained energy is $\gamma_{\rm ret} \approx 810$ (dotted line).

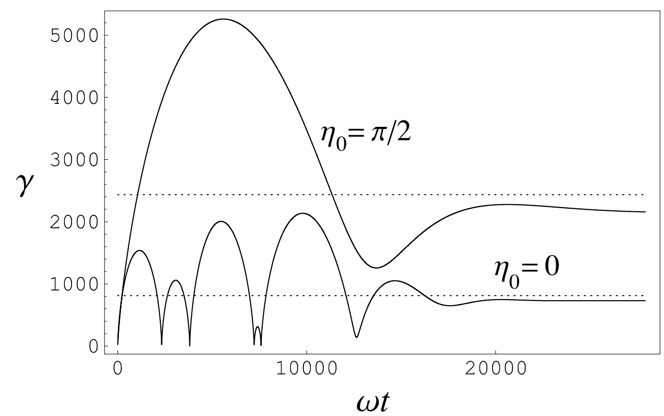

FIG. 6. Electron normalized relativistic energy $\gamma = \mathcal{E}/mc^2$ (solid line) vs time $\tau = \omega t$ for electron acceleration in a focused laser field with the focal waist $w = 50\lambda$ and $a_0 = 57$, instantaneously switched on at phase η_0 ($\eta_0 = 0$, $\pi/2$). Initial particle location is in the center of the focal region. For $\eta > \eta_0$, the average intensity profile is assumed static. Since $N \sim a_0$, condition (20) is satisfied only marginally, though the ponderomotive description predicts the energy gain fairly well. The theoretically predicted retained energy (dotted lines) is $\gamma_{\rm ret} = 1 + (a^2 + \overline{a^2})/2$, which gives $\gamma_{\rm ret} \approx 810$ for $\eta_0 = 0$ and $\gamma_{\rm ret} \approx 2440$ for $\eta_0 = \pi/2$.

V. SUMMARY

In summary, we employed an analytical model to describe acceleration of electrons produced as a result of above-threshold ionization up to ultrarelativistic energies. The model is based on a fully relativistic ponderomotive treatment. We showed that, though the above-threshold ionization represents an abrupt process compared to laser oscillations, the ponderomotive approach can still adequately predict the resulting energy gain of electrons if the proper initial conditions are introduced for the particle drift following the ionization event. The major result of the present work consists of obtaining the explicit expressions for those initial conditions [Eqs. (13)] and determining the applicability conditions of the ponderomotive model [Eqs. (14) and (20)], studied both analytically and numerically and shown to be significantly more relaxed than those given in Ref. [11]. The Hamiltonian formulation for the electron average motion allows us to simplify the problem of calculating the particle drift trajectory and the particle energy gain. Not only does it allow one to reproduce the well-known results of electron acceleration by plane laser pulses, but it also gives the precise energy gain for acceleration in smooth transversely nonuniform fields, for which the conventional plane-wave approximation does not hold.

To our knowledge, no precise measurements on electron acceleration after the above-threshold ionization have been carried out at ultrarelativistic laser fields. While experimental data remains unavailable for direct comparison with the obtained results, our analytical predictions show a good agreement with the results of our numerical computations.

ACKNOWLEDGMENT

The work was supported by the U.S. DOE, under Contract No. DE-AC02-76 CHO3073.

[1] M.D. Perry *et al.*, Opt. Lett. **24**, 160 (1999).
[2] S.P. Hatchett *et al.*, Phys. Plasmas **7**, 2076 (2000).
[3] M.H. Key *et al.*, Phys. Plasmas **5**, 1966 (1998).
[4] J.X. Wang, Y.K. Ho, Q. Kong, L.J. Zhu, L. Feng, S. Scheid, and H. Hora, Phys. Rev. E **58**, 6575 (1998).
[5] P.X. Wang, Y.K. Ho, X.Q. Yuan, Q. Kong, N. Cao, A.M. Sessler, E. Esarey, and Y. Nishida, Appl. Phys. Lett. **78**, 2253 (2001).
[6] Q. Kong, Y.K. Ho, J.X. Wang, P.X. Wang, L. Feng, and Z.S. Yuan, Phys. Rev. E **61**, 1981 (2000).
[7] Y.I. Salamin and C.H. Keitel, Phys. Rev. Lett. **88**, 095005 (2002).
[8] Y. Cheng and Z. Xu, Appl. Phys. Lett. **74**, 2116 (1999).
[9] S.X. Hu and A.F. Starace, Phys. Rev. Lett. **88**, 245003 (2002).
[10] A. Maltsev and T. Ditmire, Phys. Rev. Lett. **90**, 053002 (2003).
[11] B. Quesnel and P. Mora, Phys. Rev. E **58**, 3719 (1998).
[12] P.H. Bucksbaum, M. Bashkansky, and T.J. McIlrath, Phys. Rev. Lett. **58**, 349 (1987).
[13] C.I. Moore, J.P. Knauer, and D.D. Meyerhofer, Phys. Rev. Lett. **74**, 2439 (1995).
[14] G. Malka and J.L. Miquel, Phys. Rev. Lett. **77**, 75 (1996).
[15] G. Malka, E. Lefebvre, and J.L. Miquel, Phys. Rev. Lett. **78**, 3314 (1997).
[16] P.X. Wang, J.F. Hua, Y.Z. Lin, and Y.K. Ho, Phys. Lett. A **300**, 76 (2002).
[17] N.H. Burnett and P.B. Corkum, J. Opt. Soc. Am. B **6**, 1195 (1989).
[18] L. D. Landau and E. M. Lifshitz, *The Classical Theory of*

Fields (Pergamon Press, New York, 1971).
[19] E.S. Sarachik and G.T. Schappert, Phys. Rev. D **1**, 2738 (1970).
[20] I. Y. Dodin, N. J. Fisch, and G. M. Fraiman, JETP Lett. **78**, 202 (2003).
[21] J.E. Moore and N.J. Fisch, Phys. Plasmas **1**, 1105 (1994).
[22] T.W.B. Kibble, Phys. Rev. **150**, 1060 (1966).
[23] P. Mora and T.A. Antonsen, Jr., Phys. Plasmas **4**, 217 (1997).
[24] M.D. Tokman, Plasma Phys. Rep. **25**, 140 (1999).
[25] D. Bauer, P. Mulser, and W.H. Steeb, Phys. Rev. Lett. **75**, 4622 (1995).
[26] E. Esarey, P. Sprangle, and J. Krall, Phys. Rev. E **52**, 5443 (1995).
[27] C.I. Moore, A. Ting, T. Jones, E. Briscoe, B. Hafizi, R.F. Hubbard, and P. Sprangle, Phys. Plasmas **8**, 2481 (2001).
[28] F.V. Hartemann, S.N. Fochs, G.P. Lesage, L.C. Luhmann, Jr., J.G. Woodworth, M.D. Perry, Y.J. Chen, and A.K. Kerman, Phys. Rev. E **51**, 4833 (1995).
[29] K.T. McDonald, Phys. Rev. Lett. **80**, 1350 (1998).