Pump side scattering in ultrapowerful backward Raman amplifiers

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Extremely large laser power might be obtained by compressing laser pulses through backward Raman amplification (BRA) in plasmas. Premature Raman backscattering of a laser pump by plasma noise might be suppressed by an appropriate detuning of the Raman resonance, even as the desired amplification of the seed persists with a high efficiency. In this paper we analyze side scattering of laser pumps by plasma noise in backward Raman amplifiers. Though its growth rate is smaller than that of backscattering, the side scattering can nevertheless be dangerous, because of a longer path of side-scattered pulses in plasmas and because of an angular dependence of the Raman resonance detuning. We show that side scattering of laser pumps by plasma noise in BRA might be suppressed to a tolerable level at all angles by an appropriate combination of two detuning mechanisms associated with plasma density gradient and pump chirp.

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I. INTRODUCTION

The scheme of backward Raman amplification (BRA) of laser pulses in plasmas, proposed in [1], envisions a long laser pump beam nearly completely consumed by a short counter-propagating laser seed pulse downshifted by the plasma frequency. The theory predicts that the pumped pulse reaches huge nonfocused intensities of GW/λ^2 (where λ is the pump wavelength), before filamentation instabilities develop.

The very same high rate of BRA might, however, cause premature pump backscattering by plasma noise as the pump traverses the plasma toward the seed pulse. The danger is serious because the linear Raman backscattering instability of the pump, responsible for unwanted noise amplification, has a substantially larger growth rate than its counterpart responsible for the advanced nonlinear stage of the intentionally seeded amplification.

A remedy proposed in [2] is an appropriate detuning of the Raman backscattering resonance. The detuning can suppress the parasitic Raman backscattering of the pump by noise, while not suppressing the desirable seed pulse amplification. This filtering effect is possible due to the broadening of the pumped pulse frequency bandwidth in the nonlinear BRA regime. Specifically, the pumped pulse duration decreases in the nonlinear regime inversely proportional to the pulse amplitude (because the pumped pulse, getting stronger, depletes the pump faster). The growing frequency bandwidth makes the pumped pulse robust to the growing detuning that can suppress the relatively narrower bandwidth linear instability.

It was furthermore noted in [2] that, combining two different detuning mechanisms, associated with the plasma density gradient and the pump frequency chirp, one could suppress a broader class of deleterious instabilities, including, in particular, Raman near-forward scattering of the pumped pulse. The major idea is that two large detunings can be selected to nearly cancel each other for the useful resonance, while producing a large total detuning for parasitic resonances. The quantitative theory for suppression of the pumped pulse Raman near-forward scattering was developed in [3].

In the present paper we develop a quantitative theory for suppression of the premature pump side scattering by noise in BRA. A longitudinal plasma density gradient alone is, in general, insufficient for suppressing the parasitic pump side scattering. This is because the Raman scattered waves propagating in directions perpendicular to the pump are then not detuned at all. Rather they are resonantly amplified as long as the pump and the plasma last. In ultrapowerful BRA of large enough transverse sizes of plasmas, this could result in considerable side scatter.

We show that the pump frequency chirp alone can suppress parasitic pump side scattering in all directions approximately to the same extent as in the longitudinal direction. Moreover, we show that jointly the pump frequency chirp and the longitudinal plasma frequency gradient can suppress the parasitic pump side scattering in BRA to an even greater extent. Also, we show that these two detuning mechanisms jointly can strongly suppress parasitic pump scattering by noise in the more general case of side injection in BRA, when the angle between propagation directions of pump and seed laser pulses differs from π. By an appropriate selection of the detuning gradients, the smallest suppression can be arranged precisely in the desired direction of the useful amplification for any angle of side injection.

II. BASIC EQUATIONS

We are looking for BRA regimes wherein the pump is not depleted through Raman scattering arising from noise. In such regimes, a parasitic signal evolution can be described within the linear theory for three-wave interaction. In canonical variables, the basic equations for wave envelopes have the form

$$[\partial_t + (\mathbf{v}_1 \cdot \boldsymbol{\nabla})]a_1 = V a_0 f^*,$$

$$(\partial_t - i\delta\omega) f^* = V^* a_0^* a_1. \qquad (1)$$

Here ∂_t is the time derivative, $\mathbf{v_1}$ is the group velocity of a scattered electromagnetic wave (Stokes component), the Langmuir wave group velocity is neglected, $\delta\omega$ is the frequency detuning of Raman resonance: $\delta\omega=\omega_1+\omega_L-\omega_0$, where ω_0, ω_1, and ω_L are frequencies of pump, Stokes, and Langmuir waves, respectively, $\omega_j^2=\omega_p^2+k_j^2c^2$ $(j=0;1)$, c is the speed of light in vacuum, $\omega_L=\omega_p=\sqrt{4\pi n_0 e^2/m_e}$, n_0, e, and m_e are electron concentration, charge, and mass, respectively. The wave vectors of pump $\mathbf{k_0}$, Stokes $\mathbf{k_1}$, and Langmuir $\mathbf{k_L}$ waves satisfy the resonance condition $\mathbf{k_0}=\mathbf{k_1}+\mathbf{k_L}$. We normalize the wave amplitudes in such a way that the average square quiver velocity of an electron in the electromagnetic wave j ($j=0$ for the pump and $j=1$ for Stokes waves) be $c^2a_j^2$. We also restrict our consideration by the case of strongly undercritical plasma, where the plasma frequency ω_p is much smaller than the pump laser frequency ω, i.e., $\omega_p \ll \omega$. In such a plasma, $v_1\approx c$.

The Raman coupling constant V is the largest for the case of backscattering, when it is equal to

$$V_0=\sqrt{\omega_0\omega_p/2}.$$

To get the coupling constant for arbitrary angles, first note that the Raman coupling occurs through the resonant excitation of the Langmuir wave by the beating ponderomotive force of electromagnetic waves. The ponderomotive potential is proportional to the scalar product of the electromagnetic wave polarizations, $(\mathbf{e_1^*}\cdot\mathbf{e_0})$. The ponderomotive force, which is the gradient of the ponderomotive potential, is also proportional to the Langmuir wave number $k_L \approx 2k_0\sin(\theta/2)$, where θ is the angle between directions of the pump and Stokes waves propagation. It follows that the Raman coupling constant has the form

$$V=V_0\sin(\theta/2)(\mathbf{e_1^*}\cdot\mathbf{e_0}). \tag{2}$$

The growth rate for the monochromatic wave instability is then $\gamma=|a_0V|$. This formula was derived in [4] for linear wave polarizations.

So far, polarizations were considered to be arbitrary. However, the polarization of the growing Stokes wave is no longer arbitrary under conditions when the Stokes wave grows from noise and the dispersion law is degenerate, i.e., the Stokes frequency does not depend on polarization (nevertheless electromagnetic waves can still be described by polarization vectors, rather than polarization tensors). The polarization is fixed because the Stokes wave of polarization $\mathbf{e}_\perp=\mathbf{e_0^*}\times\mathbf{k_1}/|\mathbf{e_0^*}\times\mathbf{k_1}|$ Hermite orthogonal to $\mathbf{e_0}$ is not coupled to the pump, so that the only Stokes wave that grows is that of complimentary polarization (Hermite orthogonal to $\mathbf{e}_\perp$), namely,

$$\mathbf{e_1}=\frac{\mathbf{k_1}\times\mathbf{e}_\perp^*}{|\mathbf{k_1}\times\mathbf{e}_\perp^*|}=\frac{\mathbf{e_0}-(\mathbf{e_0}\cdot\mathbf{k_1})\mathbf{k_1}/\mathrm{k}_1^2}{\sqrt{\mathbf{1}-|(\mathbf{e_0}\cdot\mathbf{k_1})/\mathrm{k}_1|^{\mathbf{2}}}}. \tag{3}$$

It follows,

$$V=V_0\sin(\theta/2)\sqrt{1-|(\mathbf{e_0}\cdot\mathbf{k_1})/\mathrm{k}_1|^{\mathbf{2}}}. \tag{4}$$

For a pump of elliptic polarization

$$\mathbf{e_0}=(\mathbf{e_{01}}+\imath\mu\mathbf{k_0}\times\mathbf{e_{01}}/\mathrm{k}_0)/\sqrt{1+\mu^2},$$

with $\mathbf{e_{01}}=\mathbf{e_{01}^*}\perp\mathbf{k_0}$, $\mathbf{e_{01}}\cdot\mathbf{e_{01}}=1$ and $-1\leqslant\mu\leqslant 1$,

$$V=V_0\sin(\theta/2)\sqrt{1-\frac{\sin^2\theta(\cos^2\phi+\mu^2\sin^2\phi)}{1+\mu^2}}, \tag{5}$$

where ϕ is the azimuthal angle of Stokes wave vector $\mathbf{k_1}$ in the coordinate frame built on the main axis of the pump polarization, i.e., the angle between vectors $\mathbf{e_{01}}$ and $\mathbf{k_1}-\mathbf{k_0}(\mathbf{k_0}\cdot\mathbf{k_1})/\mathrm{k}_0^2$.

III. EFFECTIVE DETUNING

Note that a simple phase transformation $a_1\to a_1\times\exp[-\imath\Phi(\mathbf{r}-\mathbf{v}_1t)]$, $f^*\to f^*\exp[-\imath\Phi(\mathbf{r}-\mathbf{v}_1t)]$, with an arbitrary function $\Phi(\mathbf{r}-\mathbf{v_1}t)$ preserves the form of Eq. (1), transforming the detuning as follows:

$$\delta\omega\to\delta\omega-(\mathbf{v_1}\cdot\boldsymbol{\nabla})\Phi. \tag{6}$$

This transformation can be used to simplify the expression for detuning. In particular, a detuning smoothly varying in space and time, that can be linearized near the spatial-temporal location of exact resonance, can be reduced by the transformation (6) to a linear function of a single variable $z=(\mathbf{v_1}\cdot\mathbf{r})/v_1$.

Thus, the detuning produced by a plasma density gradient and a pump frequency chirp, linearized near the exact resonance $\mathbf{r}=0$, $t=0$, namely,

$$\delta\omega=(\mathbf{r}\cdot\boldsymbol{\nabla})\omega_p-\omega_{0t}(t-\mathbf{k_0}\cdot\mathbf{r}/\omega_0), \tag{7}$$

(ω_{0t} is the time derivative of pump frequency) can be reduced to the effective detuning

$$\delta\omega=[(\mathbf{v_1}\cdot\boldsymbol{\nabla})\omega_p-\omega_{0t}(1-\cos\theta)]z/v_1\equiv Qz. \tag{8}$$

($v_1\approx c$) by the transformation (6) with

$$\Phi=(z/c-t)\{\mathbf{r}_\perp\cdot\boldsymbol{\nabla}\,\omega_p+\omega_{0t}[(z/c-t)/2+\mathbf{r}_\perp\cdot\mathbf{k}_{0\perp}/\omega_0]\},$$

$[\mathbf{k}_{0\perp}=\mathbf{k_0}-\mathbf{k_1}(\mathbf{k_0}\cdot\mathbf{k_1})/k_1^2$, $\mathbf{r}_\perp=\mathbf{r}-\mathbf{k_1}(\mathbf{r}\cdot\mathbf{k_1})/k_1^2]$.

Near an exact resonance location, the basic equations (1) (with real V and a_0) then take the form

$$\begin{aligned}(\partial_t+c\partial_z)a_1&=Va_0f^*,\\ (\partial_t-iQz)f^*&=Va_0a_1.\end{aligned} \tag{9}$$

In new variables,

$$\zeta=Va_0(t-z/c),\tau=Va_0z/c, \tag{10}$$

Eqs. (9) reduce to

$$\partial_\tau a_1=f^*,\ (\partial_\zeta+iq\tau)f^*=a_1 \tag{11}$$

with

$$q = \frac{2q_2 - \mathbf{s}\cdot\mathbf{q_1}/\sin^2(\theta/2)}{1-|(\mathbf{e_0}\cdot\mathbf{s})|^2},$$

$$\mathbf{s} \equiv \frac{\mathbf{k_1}}{k_1}, \quad \mathbf{q_1} \equiv \frac{2c\,\nabla\,\omega_p}{\omega_p\omega_0 a_0^2}, \quad q_2 \equiv \frac{2\omega_{0t}}{\omega_p\omega_0 a_0^2}. \tag{12}$$

Equations (9) are the same as for the case of detuned Raman backscattering analyzed in [2,5]. The solution found therein shows that the Stokes wave amplitude can make $\pi/|q|$ exponentiations before the instability is suppressed by the detuning. Formula (12) gives the effective detuning gradient q, and thus the value of parasitic Raman scattering, for arbitrary directions of the Stokes wave propagation **s** and plasma density gradient $\mathbf{q_1}$. It is the major result of this paper. What follows describes some implications of this theory for BRA.

IV. SUPPRESSION OF PARASITIC RAMAN SIDE SCATTERING

Formula (12) gives a simple criterion for suppression of parasitic Raman scattering in all directions simultaneously,

$$\min_{\mathbf{s}}|q| > q_m \equiv 2\pi/N_M, \tag{13}$$

where N_M is largest tolerable number of exponentiations in Stokes wave intensity. This cannot be accomplished by the plasma density gradient alone ($q_2=0$), since $\mathbf{s}\cdot\mathbf{q_1}$ turns into zero for some directions of the scattering.

For the pump frequency chirp alone ($q_1=0$), it follows from Eq. (12) that $\min_{\mathbf{s}}|q|=2|q_2|$, so that condition (13) is satisfied for $2|q_2|>q_m$. The minimum of $|q|$ is reached at $\sin\theta=0$, i.e., $\theta=\pi$ and $\theta=0$, for $\mu\neq 0$, and also at $\cos\phi=0$, i.e., the entire plane $\phi=\pi/2$, for the linear polarization, $\mu=0$. Even for $\mu\neq 0$, the minimum of $|q|$ is broad enough, which means the side-scattered light makes approximately the same number of exponentiations as the backscattered light. The larger phase volume of the side-scattered light implies that a somewhat larger detuning gradient is needed to prevent premature pump energy depletion via the side scattering.

We will show now that, combining the pump frequency chirp with a plasma density gradient, it is possible to achieve a much more profound suppression of side scattering, effectively making the BRA problem one dimensional. The idea is to select two large detunings nearly canceling each other in the direction of desirable amplification, so that $\min_{\mathbf{s}}|q|\ll 1$. It means that the detuning-dependent multiplier in the expression for $q/2q_2$, namely,

$$F \equiv 1 - \frac{\mathbf{s}\cdot\mathbf{q_1}}{2q_2\sin^2(\theta/2)} \tag{14}$$

should be positive and should have minimum close to zero (i.e., to the threshold of absolute Raman instability) in the desirable direction of **s**.

To analyze this expression, it is convenient to allow for the value q_1 of plasma density gradient to be negative (as well as positive). Then one can reduce to $\pi/2\leqslant\theta_1\leqslant\pi$ the domain of variation of angle θ_1 between directions of plasma density gradient $\mathbf{q_1}$ and pump propagation $\mathbf{k_0}$. To keep F positive at $\theta\to 0$, one needs $\mathbf{s}\cdot\mathbf{q_1}/q_2\to q_1\cos\theta_1/q_2$ be negative, which is so for $q_1/q_2>0$.

The function (14) is minimized over the azimuthal angle ϕ of the Stokes wave at $\phi=\phi_1$, where ϕ_1 is the azimuthal angle of plasma density gradient $\mathbf{q_1}$. At $\phi=\phi_1$, (14) takes the form

$$\begin{aligned} F &= 1 - \frac{q_1\cos(\theta-\theta_1)}{2q_2\sin^2(\theta/2)} \\ &= 1 + \frac{q_1}{2q_2\cos\theta_1}\left[1-\left(\cos\theta_1\cot\frac{\theta}{2}+\sin\theta_1\right)^2\right]. \end{aligned} \tag{15}$$

This is minimized over the polar angle θ of the Stokes wave at

$$\cot(\theta/2) = -\tan\theta_1 \quad \Rightarrow \quad \theta = 2\theta_1 - \pi, \tag{16}$$

and the minimal value is

$$\min_{\mathbf{s}} F = 1 + \frac{q_1}{2q_2\cos\theta_1}. \tag{17}$$

The function F (and hence q/q_2) is positive everywhere, so that an absolute Raman instability is absent in the parameter domain

$$0 \leqslant q_1/q_2 < -2\cos\theta_1. \tag{18}$$

This domain can also be presented in the form

$$0 \leqslant q_1/q_2 < 2, \tag{19}$$

$$\arccos(-q_1/2q_2) < \theta_1 \leqslant \pi. \tag{20}$$

It means, in physical terms, that the absolute Raman instability is absent when the projection of pump frequency gradient on the direction opposite to the plasma frequency gradient $\nabla\omega_p$ is larger than half of the latter value, $|\nabla\omega_p|/2$.

At the stability boundary, $q_1/q_2+2\cos\theta_1\to -0$, the effective detuning tends to zero for the Stokes wave propagating in the plane of the pump and plasma frequency gradients at the angle θ given by formula (16). This can also be rewritten in the form $\theta_1-\theta=\pi-\theta_1$ showing that the Stokes direction is mirror symmetric to the counter-propagating one with respect to the plasma frequency gradient $\nabla\omega_p$. It looks like the pump reflection from the line perpendicular to $\nabla\omega_p$ within the incident plane and with the reflection angle equal to the incident angle.

Near the stability boundary, within the stability domain, the Stokes wave intensity amplification factor $\exp(2\pi/|q|)$ has a large narrow maximum near the reflected direction defined by the polar angle $\theta=2\theta_1-\pi$ and azimuthal angle $\phi=\phi_1$. Outside this narrow cone the scattering is strongly suppressed. Such conditions are favorable for a symmetric side injection of the seed and pump under angles $\pi-\theta_1$ and θ_1 to the plasma density gradient in the same plane.

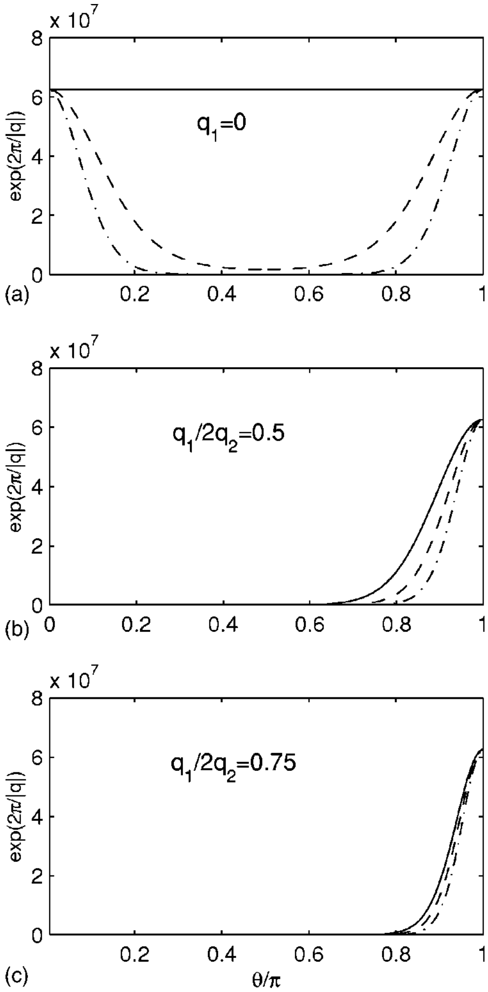

FIG. 1. The intensity amplification factor for Stokes wave scattered at the angle θ to the direction of pump propagation in the plasma with longitudinal plasma density gradient, $\theta_1 = \pi$ (which is optimal for counter-propagating BRA geometry) for $|q(\theta=\pi)| = |2q_2 - q_1| = 0.35$, and three different values of the ratio $q_1/2q_2$, namely, (a) $q_1/2q_2 = 0$, (b) $q_1/2q_2 = 0.5$, (c) $q_1/2q_2 = 0.75$; solid lines correspond to the linear polarization ($\phi = \pi/2$), dashed lines correspond to the elliptic polarization ($\mu = 0.5$, $\phi = \pi/2$), and dash-dotted lines correspond to the circular polarization.

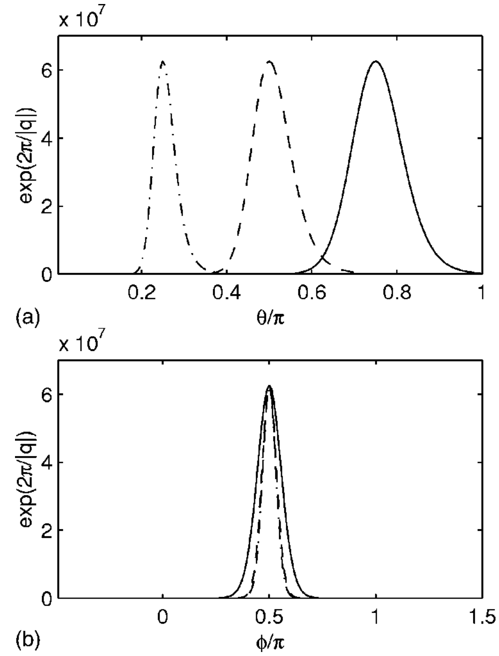

FIG. 2. Side scattering for a pump of linear polarization perpendicular to the plasma density gradient. (a) Stokes wave intensity amplification factor vs θ for $\phi = \phi_1 = \pi/2$ and (b) vs ϕ for $\theta = 2\theta_1 - \pi$; solid lines correspond to $\theta_1 = 7\pi/8$, dashed lines correspond to $\theta_1 = 3\pi/4$, and dash-dotted lines correspond to $\theta_1 = 5\pi/8$; in all cases $\min|q| = 0.35$, $q_2 = 0.7$, $q_1/q_2 = -3\cos(\theta_1)/2$.

Less close to the stability boundary, the maximum intensity amplification factor $\exp(2\pi/|q|)$ is less profound and the scattering cone around it is broader. The maximum location, generally speaking, somewhat deviates from $\theta = 2\theta_1 - \pi$, $\phi = \phi_1$, because of the denominator angular dependence in formula (12) for q. Some examples of this behavior are given in the appendix.

Here, we present just few simple formulas for the counter-propagation BRA geometry. In this case, the optimal plasma density gradient is directed opposite to the pump frequency gradient ($\theta_1 = \pi$) and satisfies condition (19) for suppression of absolute Raman instability.

The ratio q/q_2 is minimized over ϕ at $\phi = \pi/2$ (for $-1 < \mu < 1$, while for $\mu = \pm 1$, q does not depend on ϕ at all), and in the minimum

$$\frac{q}{2q_2} = \left[1 + \frac{q_1 \cos\theta}{q_2(1-\cos\theta)}\right] \frac{1+\mu^2}{1+\mu^2\cos^2\theta}. \tag{21}$$

This function is minimized over θ at $\cos\theta = -1$, i.e., $\theta = \pi$, and the minimum is

$$\frac{q}{2q_2} = 1 - \frac{q_1}{2q_2} \quad \Rightarrow \quad q = 2q_2 - q_1. \tag{22}$$

Thus, for a longitudinal plasma density gradient, satisfying condition (19), the effective detuning is the smallest for backscattering, while the parasitic side scattering is even more suppressed. This is favorable for useful backward amplification. For $0 < 1 - q_1/2q_2 \ll 1$ the suppression of side scattering is much stronger than that of backscattering for all angles outside of a very narrow cone around the backward direction. Figure 1 presents examples of the Stokes amplification factor $\exp(2\pi/|q|)$ as a function of θ calculated according to formula (21) for the minimum value of $|q|$, $|q| = 0.35$, reached at $\theta = \pi$, and different values of the ratio $q_1/2q_2$. As this ratio approaches to 1, the side-scattering suppression strongly increases.

V. SUMMARY

We have shown that the pump frequency chirp alone can suppress parasitic pump side scattering in all directions approximately to the same extent as in the longitudinal direc-

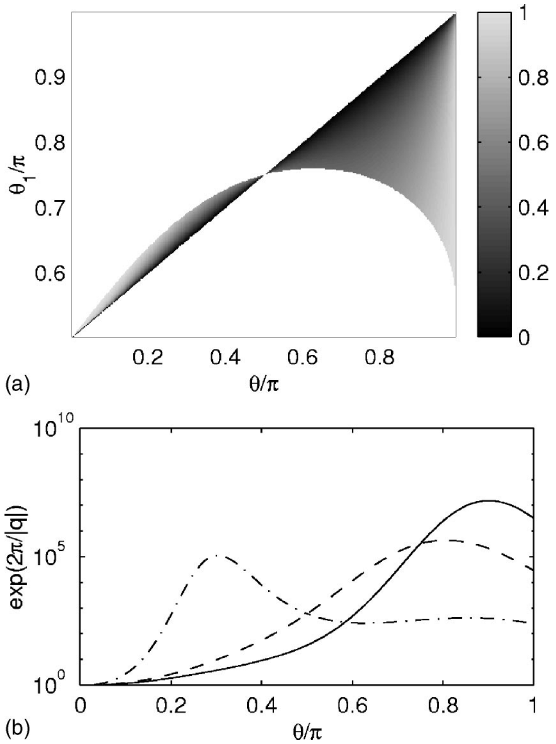

FIG. 3. The plot (a) shows the effective detuning gradient $|q|$ for a pump of circular polarization. The shaded area in the plane (θ,θ_1), shows at which θ's the value of $|q|$ is minimized for the plasma density gradient directed at given angle θ_1 and all possible values of q_1/q_2 within the stability domain (18). The darkness bar gives the minimal value of $q/2q_2$. The plot (b) shows the θ dependence of the Stokes wave intensity amplification factor at $\phi=\phi_1$: the solid line corresponds to $\theta_1=11\pi/12$, the dashed line corresponds to $\theta_1=5\pi/6$, and the dash-dotted line corresponds to $\theta_1=2\pi/3$; for all lines, $q_2=0.7$ and $q_1/q_2=-3\cos(\theta_1)/2$.

tion. However, much more profound suppression of parasitic pump side scattering can be accomplished when the pump frequency chirp is accompanied by an appropriate plasma frequency gradient. Jointly, these two detuning mechanisms can strongly suppress parasitic pump scattering by noise everywhere outside a narrow cone around any desired direction.

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APPENDIX: MORE NUMERICAL EXAMPLES FOR PUMPS OF LINEAR AND CIRCULAR POLARIZATIONS

For a pump of linear polarization perpendicular to the plasma density gradient, the effective detuning gradient $|q|$ is minimal at $\phi=\phi_1=\pi/2$ and $\theta=2\theta_1-\pi$ [see Eq. (16)]. The angular dependence of Stokes wave intensity amplification factor $\exp(2\pi/|q|)$ is shown in Fig. 2.

For a pump of circular polarization, the effective detuning gradient $|q|$ is minimized over ϕ at $\phi=\phi_1$ and the result does not depend on ϕ_1. The minimization over θ is done numerically. Figure 3(a) shows interval of values θ at which $|q|$ is minimized for a given θ_1 and all possible values of q_1/q_2 within the stability domain (18). The darkness of shaded area gives the minimal value of $q/2q_2$; in particular, black corresponds to $q=0$ reached at the boundary of the stability domain, namely, at $\theta=2\theta_1-\pi$ and $q_1/q_2=-2\cos\theta_1$. Note that for $3\pi/4<\theta_1<\pi$ detuning is minimal at $\theta\geqslant 2\theta_1-\pi$, while for $\pi/2<\theta_1<3\pi/4$ at $\theta\leqslant 2\theta_1-\pi$. Figure 3(b) shows the θ dependence of the Stokes wave intensity amplification factor at $\phi=\phi_1$.

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