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Maximizing ion current by space-charge neutralization using negative ions and dust particles

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Ion current extracted from an ion source (ion thruster) can be increased above the Child–Langmuir limit if the ion space charge is neutralized. Similarly, the limiting kinetic energy density of the plasma flow in a Hall thruster might be exceeded if additional mechanisms of space-charge neutralization are introduced. Space-charge neutralization with high-mass negative ions or negatively charged dust particles seems, in principle, promising for the development of a high current or high energy density source of positive light ions. Several space-charge neutralization schemes that employ heavy negatively charged particles are considered. It is shown that the proposed neutralization schemes can lead, at best, only to a moderate but nonetheless possibly important increase of the ion current in the ion thruster and the thrust density in the Hall thruster. © *2005 American Institute of Physics*. [DOI: 10.1063/1.1897715]

I. INTRODUCTION

The maximum current density of an ion beam that can be extracted from a plasma source with a conventional extraction system is limited by the beam space charge. This fact is well known as the Child–Langmuir law,[1] which in the one-dimensional case reads

$$J_{i\,\max} = J_i^{\mathrm{CL}} = \frac{\sqrt{2}U^{3/2}}{9\pi d^2\sqrt{qM_i}}. \tag{1}$$

Here, d is the distance between the extracting grids, U is the applied voltage, q and M_i are the ion charge and mass, respectively. As was first pointed out by Langmuir in 1929,[2] the extracted current can be raised above the natural limit (1) if the positive ion space charge is neutralized by negatively charged particles. In the past, several experiments demonstrated successful space-charge neutralized ion beam extraction. Partial neutralization of the ion space charge was achieved by supplying electrons from an external source, such as filament cathode,[3] dual plasmatron,[4] or ferroelectric electron gun.[5] Similar techniques are routinely used for beam charge compensation in numerous technical fields.[6] For example, in ion thrusters for spacecraft propulsion,[7] the ejected ion beams are charge and current neutralized with electrons supplied by hollow plasma cathodes. We note, however, that the beam extraction stage of an ion thruster is not neutralized, and, therefore, the beam current is space-charge limited.

Positive ion space charge can be neutralized also by negative ions. Such neutralization naturally occurs, for example, in negative ion sources with surface conversion,[8,9] in the negative ion rich plasma used in plasma processing for notch-free etching.[10] In the present paper, we investigate the feasibility of a source of positive light ions with space-charge neutralization by heavy negatively charged particles. Suppose that what is wanted is an ion (plasma) injector of the highest possible current density of light ions. It seems promising to use high-mass negative ions or negatively charged dust particles for the space-charge neutralization of the ion beam. Suppose that we bleed into the ion thruster at the cathode end high-mass negative ions, such that their mass exceeds the mass (per charge) of the positive ions. The advantage of high-mass negative ions is that they neutralize the space charge in the thruster, but, by virtue of moving slowly, they do not extract much power. If negative ions of mass M_- were more massive than the positive ions by a factor $R=M_-/M_i$, and if the space charge were completely neutralized by the negative ions, then the rate of momentum input into the two species would be the same, but the rate of energy input into the negative ions would be down by a factor of $R^{1/2}$ in order that both the space charge be neutralized and the momentum be balanced. Thus, for negligible power, one achieves space-charge neutralization. If one finds a way to perfectly neutralize the space charge, the current density of positive ions can be significantly increased, and, in principle, possibly without limit.

An apparatus in which heavy ions come out at one end, and light ions come out at the other end, with momentum balanced, is not a thruster, but it could be a light ion injector of very high current density and very high energy density. In principle, such an injector, with heavy negative ions injected at the cathode, could beat the space-charge limit for non-neutral flow. However, a much more stable device is likely to ensue from leaking the negative ions into a Hall thruster.[11–13] The advantage of the Hall thruster is that the electrons can both smooth out space-charge differences between the two ion species as well as charge or maintain the charge on the high-mass negative ions, which could be dust particles that are susceptible to charging by electron impact.

In a Hall thruster, the ions flow from a neutral plasma across which there is an axial potential drop. The electrons neutralize the positive space charge, so that the ion flow is not space-charge limited. The electrons, however, do not flow easily in the axial direction, since the radial magnetic

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field of the Hall thruster (HT) inhibits axial electron motion. The electrons instead, to the extent that they do not suffer collisions, execute azimuthal motion. Under such conditions, the achievable ion flow kinetic energy density is limited by the peak magnetic field energy density:[11,13]

$$M_i J_i V_{if} = \frac{B_{\max}^2}{8\pi}. \tag{2}$$

Here, V_{if} is the final ion velocity and $B_{\max}$ is the maximum magnetic field strength. By providing additional means for neutralizing the positive ion space charge, the ion current J_i can be increased and kinetic energy density limit (2) might, in principle, be exceeded. It is worth mentioning here that negative ion injection can naturally occur under typical conditions of laboratory HT experiments. For instance, the appearance of a nonconductive coating[14] on the HT channel walls was proposed to be due to the backstreaming of the negative ions born upon the ion bombardment of the vacuum vessel walls.[15]

The goal of the present study is to investigate, both analytically and numerically, several schemes of space-charge neutralization with heavy negative ions and dust in the ion and Hall thrusters. Specifically, we first consider the ion thruster (ion diode) with negative ion injection from the cathode, from the anode, and from an intermediate location between the electrodes. Then, we analyze the possibility of space-charge compensation with dust particles in the ion thruster. Finally, we consider negative ion injection at the cathode end in the Hall thruster. We show that although the ion current density can, in principle, be increased without limit by neutralizing the positive space charge, the straightforward neutralization schemes proposed here either lead to a moderate increase of the current density or appear to be impractical.

II. NEGATIVE ION INJECTION IN THE ION THRUSTER

In this section we, following and extending the work of Langmuir,[2] consider an ion thruster (ion diode) with negative ion injection. Throughout this paper, we use terms "ion thruster" and "ion diode" as synonyms, keeping in mind the fact that the total thrust produced by the ion thruster with negative ion injection can be, in fact, very small (see the introduction). The schematic one-dimensional (1D) model of the ion thruster is shown in Fig. 1. We first analyze negative ion injection from the cathode and from an arbitrary location between the electrodes. In the latter case, one may imagine that there is an auxiliary intermediate electrode that emits negative ions in the $-z$ direction. We assume that both positive and negative ions are emitted with zero initial velocity.

A. Negative ion injection from the cathode

In the presence of the negative ion current injected from the cathode, Poisson's equation in the one-dimensional case reads

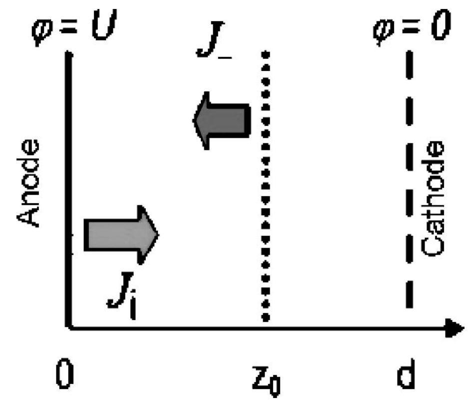

FIG. 1. Schematic of an ion thruster (ion diode) with negative ion injection from an intermediate electrode.

$$\frac{d^2\varphi}{dz^2} = -4\pi\left(\frac{qJ_i}{(2q/M_i)^{1/2}[U-\varphi(z)]^{1/2}} - \frac{QJ_-}{(2Q/M_-)^{1/2}[\varphi(z)]^{1/2}}\right), \tag{3}$$

where J_i and J_- are the positive and negative ion flux densities, which are assumed to be constant, q and Q are the positive and negative ion charges, respectively. Throughout this paper, quantity $J=NV$ will be called the "ion current" for brevity. For further analysis, it is convenient to introduce dimensionless variables $x=z/d$, $\phi=\varphi/U$, $j_i=J_i/J_i^{\mathrm{CL}}$, $j_- = J_-/J_-^{\mathrm{CL}}$. Here, J_i^{CL} and J_-^{CL} are the Child–Langmuir currents of the positive and negative ions, respectively [see Eq. (1)]. Integrating Eq. (3) once, we obtain

$$\frac{1}{2}\left(\frac{d\phi}{dx}\right)^2 = \frac{8}{9}(j_i\sqrt{1-\phi} + j_-\sqrt{\phi}) + C, \tag{4}$$

where C is a constant of integration. When $j_-=0$, Eq. (4) can be integrated analytically. The maximum ion current $J_{i\max}$ that can be drawn from the anode is limited by the space-charge saturation, i.e., $d\phi/dx(0)=0$. Using this boundary condition together with $\phi(0)=1$, $\phi(1)=0$, we obtain the Child–Langmuir law (1) and find the potential distribution

$$\phi_{\mathrm{CL}}(x) = 1 - x^{4/3}. \tag{5}$$

To illustrate how the ion current $J_{i\max}$ can be increased, we consider Eq. (4) when small negative ion current $j_- = \delta j_-(\delta j_- \ll 1)$ is injected from the cathode. The contribution of the negative ions in the right-hand side of Eq. (4) is treated as a perturbation. We seek a solution in the form

$$\phi(x) = \phi_{\mathrm{CL}}(x) + \psi(x), \quad |\psi(x)| \ll \phi_{\mathrm{CL}}(x), \quad j_i = 1 + \delta j_i.$$

The boundary conditions for $\psi(x)$ are $\psi(0)=\psi(1)=0$. The maximum ion current is achieved when the electric field at the anode vanishes, therefore, $d\psi/dx(0)=0$ as well. Solving Eq. (4) to first order in δj_-, one finds

$$J_i = J_i^{\mathrm{CL}} + \delta J_i, \quad \delta J_i = 3\kappa\sqrt{\frac{QM_-}{qM_i}}\delta J_-,$$

$$\kappa = 1 - \int_0^1 (1-s^4)^{1/2}ds \approx 0.126, \tag{6}$$

$$\psi(x) = 2(\delta J_-/J_-^{\mathrm{CL}})x^{1/3}\left(x^{1/3} - \int_0^{x^{1/3}} (1-s^4)^{1/2}ds - \kappa x\right). \tag{7}$$

Thus, due to the presence of the negative ions, the space-charge saturation of the ion current occurs at the value of J_i larger than that determined by the Child–Langmuir law. For heavy negative ions the factor $(QM_-/qM_i)^{1/2}$ can be large, therefore, as follows from Eq. (6), it is possible to get a significant increase in J_i while supplying a moderate negative ion current. For example, for $Q=q=1$ and $M_-/M_i=100$, $\delta J_i/\delta J_- \approx 3.8$.

Now let us consider the situation when there is an infinite supply of negative ions at the cathode. Quite obviously, the maximum current of the negative ions that can be injected in the diode is limited itself by the uncompensated negative space charge in the immediate vicinity of the cathode. Thus, the maximum positive ion current is obtained when the diode is space-charge saturated at both ends. Solving Eq. (4) in this case, one finds that $j_i=j_-=K$, where $K\approx 1.865$.[2] Thus, with an unlimited supply of negative ions at the cathode, the positive ion current approaches the limiting value that is $\approx$1.865 times larger than the Child–Langmuir current (1) without space-charge neutralization. The negative ion current is $(qM_i/QM_-)^{1/2}$ times the positive ion current, both currents being limited by space charge. It is worth noting here that if electrons were used for neutralization of the positive ion space charge, their required current would be much larger than that of the ions. The ultimate reason of the fact that the ion current is increased by factor of about 2 only is that it is impossible to match everywhere the density distributions of the two beams counterpropagating in a diode.

B. Negative ion injection from an intermediate electrode

If the negative ions are injected at some intermediate location x_0 between the anode and cathode (see Fig. 1), the electric potential spatial distribution is determined from the following two equations:

$$\frac{1}{2}(\phi')^2 = \frac{8}{9}(j_i\sqrt{1-\phi} + j_-\sqrt{\phi-\phi_0}) + C_1 \quad \text{if } 0 \leqslant x \leqslant x_0, \tag{8a}$$

$$\frac{1}{2}(\phi')^2 = \frac{8}{9}j_i\sqrt{1-\phi} + C_2 \quad \text{if } x_0 \leqslant x \leqslant 1. \tag{8b}$$

Here, ϕ_0 is a yet unknown value of the potential at the location of the negative ion injection. Physically, this situation is equivalent to two ion diodes connected in series (a triode), with the left diode being space-charge compensated. We consider solutions without reflections, i.e., only monotonic distributions of the electric potential. We assume also that we have an infinite supply of positive and negative ions. In order to find the increase in the positive ion current due to space-charge neutralization, the set of Eq. (8) is supplemented with the boundary conditions:

$$\phi(0) = 1, \quad \phi(1) = 0, \quad \phi(x_0) = \phi_0,$$

$$\phi'(0) = \phi'(x_0 - 0) = 0, \quad [\phi]_{x_0} = [\phi']_{x_0} = 0.$$

Solving Eq. (8) with these boundary conditions, we find that the amount of positive ion current that can be passed through the diode depends on the location of the negative ion injection x_0,

$$j_i = K/x_0^2y^3, \quad y = \sqrt{1 + \frac{\phi_0}{U-\phi_0}}, \tag{9}$$

where y is related to x_0 through $x_0^{-1} = 1 + (2+y)\sqrt{(y-1)/K}$, and $K\approx 1.865$. Maximizing j_i with respect to x_0, we find that

$$J_{i\,\max} \approx 4.59 J_i^{\mathrm{CL}} \tag{10}$$

is achieved when the negative ions are injected at $z_0 \approx 0.44d$.[2] The increase in the ion current above the Child–Langmuir value is due partly to the space-charge compensation, and partly to the fact that the effective lengths of the two diodes connected in series are smaller than d.

Now, in view of result (10), the following scheme of space-charge neutralization may, in principle, seem promising. Suppose we inject negative ions with a certain velocity distribution $f(V)$ from the anode, together with the positive ions. The negative ions will undergo deceleration in the applied electric field, thus providing space-charge neutralization. A group of negative ions with initial velocity in the interval $(V^*, V^*+\delta V)$ will neutralize the positive ion space charge best of all near the reflection point z^* determined by $M_-V^{*2}/2 = Q[U-\varphi(z^*)]$. By choosing the appropriate shape of $f(V)$ it might seem possible then to perfectly neutralize the positive ion space-charge. We consider the feasibility of this neutralization scheme next.

C. Negative ion injection from the anode

Suppose we found shape $f(V)$ that provides the complete space-charge neutralization in the ion diode. The entire population of the negative ions consists, as shown in Fig. 2, of reflected particles that cannot overcome the potential barrier and escaping particles that reach the cathode. It is clear that regardless of the details of the distribution, its characteristic width has to be on the order of $(QU/M_-)^{1/2}$. If the positive ion density near the anode is N_{i0}, then, by quasineutrality assumption

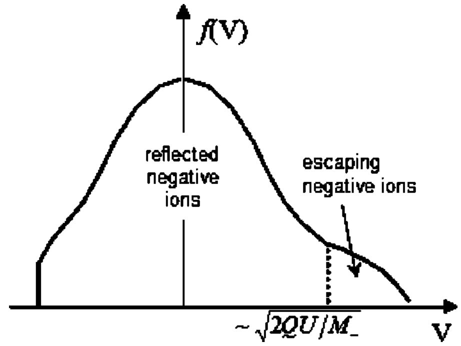

FIG. 2. Approximate shape of the distribution of the negative ions injected from the anode.

$$N_- \sim \frac{qN_{i0}}{Q} \sim \frac{qJ_i}{Q\sqrt{\varepsilon_{i0}/M_i}},$$

where ε_{i0} is the initial energy of the positive ions at the anode, $\varepsilon_{i0} \ll qU$. The negative ion current that has to be injected is approximately equal to

$$J_- \sim \frac{qN_{i0}}{Q}\sqrt{\frac{QU}{M_-}}. \tag{11}$$

Note that J_- is much larger than the current of escaping ions, because the reflected negative ions, even though they return back to the anode, have to be generated in the negative ion source. Since the ion source that generates negative ions must itself operate below the Child–Langmuir limit, we impose the constraint

$$J_- < \frac{\sqrt{2}U_-^{3/2}}{9\pi d_-^2\sqrt{QM_-}}, \tag{12}$$

where U_- and d_- are the parameters of the negative ion source extraction system. From Eqs. (11) and (12) we derive that

$$J_i < \frac{\sqrt{2}U_-^{3/2}}{9\pi d_-^2\sqrt{QM_-}}\sqrt{\frac{\varepsilon_{i0}}{qU}}.$$

Thus, for the proposed scheme of space-charge neutralization to be useful, the right-hand side of the last inequality should be significantly larger than J_i^{CL}. We therefore obtain the condition

$$U_- > U\left(\frac{d_-}{d}\right)^{4/3}\left(\frac{qU}{\varepsilon_{i0}}\right)^{1/3} \quad \text{where } qU/\varepsilon_{i0} \gg 1.$$

As we see, the considered scheme of space-charge neutralization is likely to be impractical because it requires the negative ion source to be operated in the Child–Langmuir limit with a very high extraction voltage. The same kind of limitation would arise if, instead of the negative ions, dust particles charged negatively by surface charging[16] were coinjected together with the positive ions.

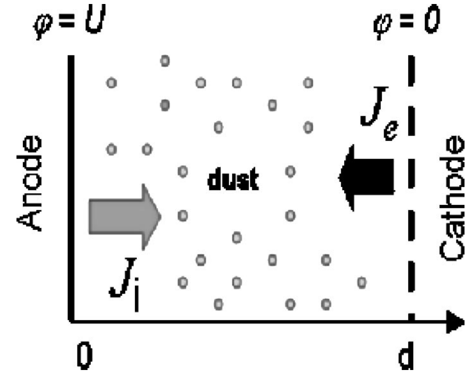

FIG. 3. Scheme of space-charge neutralization with the dust particles and electron beam.

III. SPACE-CHARGE NEUTRALIZATION WITH DUST PARTICLES

The positive ion space charge in the ion thruster can be neutralized by high-mass dust grains that are charged negatively either by contact charging[16] or, in the presence of a plasma or an external electron beam, by electron impact.[17] The dust can be introduced into the thruster from a side with a certain axial distribution of dust density, so that the ion beam moving through the cloud of dust is neutralized at each axial location. The typical charge-to-mass ratios of dust particles are so small that for the purposes of our analysis it is reasonable to assume the dust grains to be stationary.

A dust particle immersed into a plasma collects electrons and ions. Since electrons are more mobile than ions, a dust particle in a plasma acquires a negative charge and a negative floating potential with respect to the unperturbed surrounding plasma. The floating potential is adjusted so that the particle collects no net current at steady state. Dust particles of micron and submicron size in a laboratory plasma can carry negative charge of 10^4e–10^6e.[17] It seems that such particles, if introduced to the near-anode region of the ion diode, could neutralize the positive space charge very effectively. Therefore, it is natural to explore the space-charge neutralization scheme shown in Fig. 3.

We consider the electron and ion beams propagating in the opposite directions through a cloud of dust suspended in the diode. One may think of the interelectrode region shown in Fig. 3 as of the acceleration region in the ion thruster. In this case, the active discharge region where ionization takes place is located to the left from the anode in Fig. 3. We assume that there is no neutral gas in the acceleration region and that the ions can be only in charge state +1. Negatively charged dust particles together with free electrons neutralize the positive ion space charge, thus potentially increasing the maximum allowed ion current above the Child–Langmuir limit. On the other hand, dust grains also collect and scatter ions, thereby decreasing the ion current. To determine the net effect, we proceed with the detailed analysis of dust charging and space-charge neutralization.

Let us assume that the dust particles are spherical and made of electrically conducting material. In the presence of ion and electron flows, the sheath around a dust particle may be deformed and nonsymmetric. This effect, which is rather difficult to take into account since it requires a specific treatment of Poisson–Vlasov system, will be neglected in the following. We assume the sheath around a dust grain to be symmetric and characterized by an appropriate shielding parameter λ_D. To determine the dust grain floating potential, we use orbital motion limited (OML) approximation,[18,19] which, basically, postulates that for every charged particle energy there exists an impact parameter that makes the charged particle hit the dust grain with a grazing incidence. In these conditions, from the angular momentum and energy conservation it can be easily found that the collection cross section of a dust particle with radius a is[18,19]

$$\sigma_{\mathrm{coll}}^{e,i} = \pi a^2\left(1 \pm \frac{eV_d}{\varepsilon_{e,i}}\right), \tag{13}$$

where V_d is the dust particle potential, $\varepsilon_{e,i}$ is the incident electron (ion) energy, and the upper (lower) sign corresponds to electrons (ions). To find V_d, we equate the electron and ion fluxes to the dust grain:

$$\pi a^2 N_i V_i\left(1 - \frac{eV_d}{\varepsilon_i}\right) = \pi a^2 N_e V_e\left(1 + \frac{eV_d}{\varepsilon_e}\right), \quad V_d < 0.$$

Defining dimensionless dust potential $V_{\mathrm{De}}=e|V_d|/\varepsilon_e$, we obtain

$$V_{\mathrm{De}}(z) = \left(1 - \frac{J_i}{J_e}\right) \Big/ \left(1 + \frac{J_i\varepsilon_e}{J_e\varepsilon_i}\right). \tag{14}$$

Here, $J_{i,e}$, $\varepsilon_{e,i}$ vary along z. Both J_i, $J_e>0$ (J_e is the electron current in the $-z$ direction). The steady state negative charge of a dust grain is proportional to $|V_d|$,

$$Z_d = C_d|V_d|/e,$$

where C_d is the dust particle capacitance, which in general depends on the distribution of the shielding charge around the grain. For simplicity, we exploit the approximation $C_d = a$, which is valid for $a \ll \lambda_D$.[17]

Analyzing Eq. (14), we note several important limiting cases. First, near the anode, where $\varepsilon_i \to 0$, the dust particle charge goes to zero:

$$Z_d = \frac{a|V_d|}{e} \approx \frac{a\varepsilon_i}{\varepsilon_e}\left(\frac{J_e}{J_i} - 1\right) \propto \varepsilon_i \to 0. \tag{15}$$

Indeed, if a dust grain in the immediate vicinity of the anode had a substantial negative charge, it would be able to collect a huge amount of ions since the ions with evanescent energy would all fall in the potential well at the dust particle. Another way to think about this is as follows. We obtain from Eq. (14) that $V_{Di}=e|V_d|/\varepsilon_i \approx J_e/J_i - 1$ near the anode. For a diode that is space-charge saturated at both the anode and the cathode, this means that $V_{Di} \approx J_e/J_i \gg 1$. Thus, the ion collection cross section $\sigma_{\mathrm{coll}}^i \approx \pi a^2 J_e/J_i \gg \pi a^2$, and dust particles collect ions from the ion beam very effectively. Thus, the space-charge neutralization by dust particles is likely to be ineffective near the anode, which is exactly the place where the charge non-neutrality limits the ion current.

It is interesting to note also that for the diode which is space-charge saturated at both the anode and the cathode $J_i(z)/J_e(z) \ll 1$ and $V_{\mathrm{De}}(z) \approx 1$ everywhere, except for the near-anode region where $V_{\mathrm{De}}(z)$ abruptly goes to zero.

Now let us consider the Coulomb interaction between the charged dust grains and particles of the beams. In the quasineutral plasma, the Coulomb cross section can be written as[17]

$$\sigma_{\mathrm{Coul}} = 2\pi b_{\pi/2}^2 \ln\left(\frac{\lambda_D^2 + b_{\pi/2}^2}{b^2 + b_{\pi/2}^2}\right), \tag{16}$$

where

$$b = a\left(1 - \frac{qV_d}{\varepsilon}\right)^{1/2}, \quad b_{\pi/2} = \frac{Z_d q^2}{\varepsilon}.$$

These formulas are not quite applicable in the case of two counterpropagating beams, because it is not clear how, in fact, the dust grain shielding occurs (see the comment on shielding above). In our numerical simulations, for the sake of simplicity, we treat the Coulomb cross sections as given quantities and assume that $\sigma_{\mathrm{Coul}}=\alpha\sigma_{\mathrm{coll}}$ where coefficient α is the same for both ions and electrons. Nonetheless, it is instructive to compare the cross section given by Eq. (16) with $\sigma_{\mathrm{coll}}=\pi b^2$ to determine what mechanism—either ion slow down due to the Coulomb collisions or the ion absorption by the dust particles—leads to a stronger decrease of the limiting ion current.

We are mostly interested in seeing what kind of ion interaction with the dust grains dominates near the anode. From Eq. (16) we obtain for large V_{Di} that

$$\left(\frac{\sigma_{\mathrm{Coul}}}{\sigma_{\mathrm{coll}}}\right)^i \approx \frac{V_{Di}}{2}\ln\left(1 + \left(\frac{2\lambda_D}{aV_{Di}}\right)^2\right). \tag{17}$$

Now, $V_{Di} \approx J_e/J_i \sim (M_i/m_e)^{1/2}$ ranges from 42.7 for hydrogen to about 487 for xenon. Equation (17) was obtained in the approximation $a \ll \lambda_D$, which is reasonable for micron-size dust particles. Indeed, for $\varepsilon_e \sim 1$ keV (electrons near the anode) and $N_e \leq 10^{14}$ cm^{-3}, $\lambda_D \geq 25$ μm. Therefore, for $a \sim 1$ μm $(\sigma_{\mathrm{Coul}}/\sigma_{\mathrm{coll}})^i \geq 1$ even for xenon ions.

For electrons $V_{\mathrm{De}}(z) \approx 1$, except near the anode, where $V_{\mathrm{De}}(z)$ decreases to zero. We find accordingly that $(\sigma_{\mathrm{Coul}}/\sigma_{\mathrm{coll}})^e \sim V_{\mathrm{De}}^2/(1-V_{\mathrm{De}}) \gg 1$ when $V_{\mathrm{De}} \sim 1$, and $(\sigma_{\mathrm{Coul}}/\sigma_{\mathrm{coll}})^e \ll 1$ when $V_{\mathrm{De}}(z) \to 0$.

Propagation of the electron and ion beams through the cloud of dust can be described by the following 1D model:

$$\frac{dJ_i}{dz} = -\sigma_{\mathrm{coll}}^i N_d N_i V_i, \tag{18a}$$

$$\frac{dJ_e}{dz} = \sigma_{\mathrm{coll}}^e N_d N_e V_e, \tag{18b}$$

$$\frac{d}{dz}\left(\frac{M_i V_i^2}{2} + e\varphi\right) = -\sigma_{\mathrm{Coul}}^i M_i V_i^2 N_d, \tag{18c}$$

$$\frac{d}{dz}\left(\frac{m_e V_e^2}{2} - e\varphi\right) = -\sigma_{\mathrm{Coul}}^e m_e V_e^2 N_d, \tag{18d}$$

$$\frac{d^2\varphi}{dz^2} = -4\pi e\left(\frac{J_i}{V_i} - \frac{J_e}{V_e} - \frac{aN_d|V_d|}{e}\right), \tag{18e}$$

$$|V_d| = \frac{\varepsilon_e\varepsilon_i(J_e - J_i)}{e(J_e\varepsilon_i + J_i\varepsilon_e)}. \tag{18f}$$

Collection cross sections $\sigma^{e,i}_{\rm coll}$ are given by Eq. (16). $\sigma^{e,i}_{\rm Coul} = \alpha\sigma^{e,i}_{\rm coll}$, where parameter α was set, quite arbitrarily, to be equal to 2. The objective of the performed numerical simulations was to illustrate the ineffectiveness of the proposed scheme for space-charge neutralization. The conclusions that we make are quite insensitive to the value of parameter α.

To find the maximum electron and ion currents that can be passed through the dust cloud, we solve Eqs. (18) numerically with the boundary conditions:

$$\varphi(0) = U, \quad \varphi(d) = 0, \quad \varphi'(0) = \varphi'(d) = 0,$$

$$\varepsilon_i(0) = \varepsilon_e(d) = 0. \tag{19}$$

The set of Eq. (18), when written in the dimensionless form, has one free parameter, namely, $N_D = \pi a^2 d N_d$ which is the dimensionless density of the dust cloud. The results of numerical simulation are shown in Fig. 4 for $N_D = 0.1$. In Fig. 4, J_e and J_i are normalized by $J_e^{\rm CL}$ and $J_i^{\rm CL}$, respectively, ϕ, V_d are normalized by the applied voltage U, the electron and ion energies ε_e and ε_i are normalized by U/e, and the charge densities ρ_e, ρ_i, ρ_d are normalized by U/d^2.

As can be seen from Fig. 4, even a small amount of dust introduced in the diode ($\rho_d < \rho_e, \rho_i$ everywhere) attenuates the ion beam significantly due to absorption of ions on the dust particles. The electron beam attenuation is much weaker, and, in fact, cannot be resolved in Fig. 4(a), because $\sigma^e_{\rm coll} \propto (1 - e|V_d|/\varepsilon_e) \approx 0$ for $x \geqslant 0.1$, as follows from Fig. 4(b). The ion current that reaches the cathode, $J_i(z=d) \approx 1.14 J_i^{\rm CL}$, is smaller than the current in the "no dust" case, $J_i \approx 1.865 J_i^{\rm CL}$. With increasing dust cloud density N_D, $J_i(z=0)$ grows, while $J_i(z=d)$ decreases. The dust cloud appears to be a poor space-charge neutralizer and a good ion beam attenuator, thus making the proposed scheme of space-charge neutralization ineffective.

One potentially important effect that was left outside of the scope of the present study is secondary electron emission from the dust grains. Accelerated electrons can cause secondary electron emission (SEE) from the dust particles in the near-anode region of the diode. Note that for most materials the SEE yield is larger than one when the incident electron energy is larger than a few hundred electronvolts. (One exclusion is, for example, graphite, for which the SEE yield starts to decrease at the energy of primary electrons about 300 eV and is equal to ~0.7 at 1 keV.[20]) If the accelerating voltage is large enough, the dust grains in the near-anode region are likely to be charged positively due to SEE.[21,22] This interesting situation requires a thorough analysis, which can be a subject of a separate paper. However, positive charging of dust grains near the anode is likely to strengthen the conclusion of this section about the ineffectiveness of the proposed scheme of space-charge neutralization.

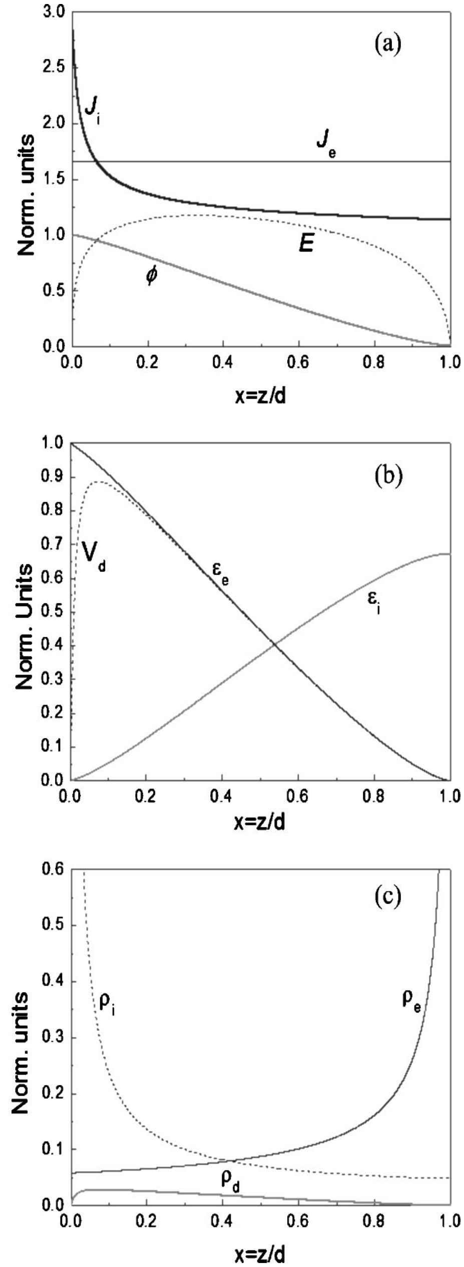

FIG. 4. Normalized axial distributions of (a) the electron and ion current densities, the plasma potential, and the electric field; (b) the electron and ion energies and the dust particle potential; (c) the electron, ion, and dust charge densities.

IV. NEGATIVE ION INJECTION IN THE HALL THRUSTER

In a Hall thruster the positive ion space charge is completely neutralized by magnetized electrons, which are locked in the azimuthal $\mathbf{E}\times\mathbf{B}$ drift. Since no space-charge limitation of the ion current occurs, HTs operate at much higher thrust densities than ion thrusters. However, the maximum thrust density (ion flow kinetic energy density) attainable in an HT is limited by the energy density of the applied magnetic field.[11,13] Let us investigate whether the additional space-charge neutralization with negative ions can help to exceed the HT thrust density limit (2). In our analysis we

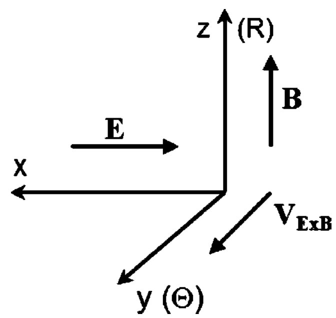

FIG. 5. The coordinate system for a Hall thruster in the slab geometry.

generalize the derivation performed originally by Zharinov and Popov in 1967.[13]

We consider HT in the slab geometry (Fig. 5), assuming that $\partial/\partial y = \partial/\partial z = 0$, $\partial/\partial x \neq 0$, where x is the axial coordinate. The applied magnetic and electric fields are $\mathbf{B} = \mathbf{z}_0 B(x)$, $\mathbf{E} = -\mathbf{x}_0 d\varphi/dx$. We assume the ions to be unmagnetized and the electrons to be strongly magnetized: $\omega_{ce}\tau \gg 1$, where τ is the characteristic electron collision time and ω_{ce} is the electron gyrofrequency. Under such conditions the electron fluid momentum equation simplifies to

$$eN_e \frac{d\varphi}{dx} = \frac{dp_e}{dx} + \frac{eBJ_y}{c}, \tag{20}$$

where N_e and p_e are the electron density and pressure and the azimuthal electron current J_y, from the Maxwell equations, is

$$J_y = \frac{c}{4\pi e}\frac{dB}{dx}. \tag{21}$$

For positive ions in the acceleration region we have

$$M_i V_i N_i \frac{dV_i}{dx} = -qN_i \frac{d\varphi}{dx}. \tag{22}$$

Now, suppose that we bleed negative ions at the cathode end of the thruster (in the same way as proposed for the ion thruster in Sec. II A). Similarly to the positive ions, the negative ions are assumed to be unmagnetized:

$$M_- V_- N_- \frac{dV_-}{dx} = QN_- \frac{d\varphi}{dx}. \tag{23}$$

Integrating Poisson's equation together with Eqs. (20)–(23), and assuming that the ion currents $J_i = N_i V_i$ and $J_- = N_- V_-$ are constant along x, we obtain the conservation law:

$$M_i J_i V_i + M_- J_- V_- + \frac{B^2}{8\pi} - \frac{E^2}{8\pi} + p_e = \text{const}. \tag{24}$$

Plasma in a Hall thruster is low β, so the electron pressure p_e can be neglected with respect to the magnetic pressure $B^2/8\pi$. For typical conditions of HT operation $E \ll B$ and the electric field pressure term can be dropped out in Eq. (24) as well. Finally, we equate the values of the conserved quantity at the beginning of the acceleration region ($x = x_0, B = B_0, V_i = 0, V_- = V_{-f}$) and at the cathode end ($x = x_1, B = B_1, V_i = V_{if}, V_- = 0$):

$$M_i J_i V_{if} = M_- J_- V_{-f} + \frac{B_0^2 - B_1^2}{8\pi}. \tag{25}$$

Note that when there are no negative ions and $B_1 \ll B_0$, Eq. (25) reduces to Eq. (2). Thus, the kinetic energy density of the positive ion beam does increase when negative ions are bled at the cathode end. However, the increase is additive with $B^2/8\pi$ term. The physical meaning of this fact is the following. In the present model, the magnetic field variation in the thruster is due entirely to the azimuthal Hall electron current:

$$B_0 - B_1 = \frac{4\pi e}{c}\int_{x_1}^{x_0} J_y dx. \tag{26}$$

Therefore, the magnetic pressure term in the right-hand side of Eq. (25) is, basically, the Lorentz force proportional to the mean magnetic field $(B_1 + B_0)/2$ and the Hall electron current. If the injected negative ion current is small so that the positive space charge is neutralized mainly by the magnetized electrons, then the negative ions contribution in the right-hand side of Eq. (25) is also small. In the opposite limiting case, when the positive space charge is neutralized primarily by the negative ions, the amount of electrons in the discharge is small and the Hall current vanishes, thus making the magnetic field term in Eq. (25) negligible. In this case,

$$M_i J_i V_{if} = M_- J_- V_{-f}, \tag{27}$$

which is the dimensional form of the result $j_i = j_-$ obtained in Sec. II A for the ion thruster. To the extent that there are no magnetized electrons to neutralize the space charge, the Hall thruster and the ion thruster are essentially equivalent. Therefore, the value of the ion current in this case is $J_i \approx 1.865 J_i^{\mathrm{CL}}$, which is much less than what would be obtained in the Hall thruster without the negative ions.

Thus, we conclude that the negative ion injection in a Hall thruster may only lead to a small increase of the kinetic energy density of the positive ion flux. When a large negative ion current is injected, the space-charge-neutralizing electrons are replaced with the negative ions and the Hall thruster is essentially transformed into the ion thruster. In this case the maximum attainable kinetic energy density is much smaller than that in the conventional Hall thruster.

V. CONCLUSIONS

The current of an ion beam extracted from an ion thruster is limited by the space charge. The limiting value of the ion current is determined by the Child–Langmuir law. By neutralizing the positive space charge, the ion current can be increased significantly and, in principle, possibly without limit. A high current or high energy density ion thruster might find many physical and technological applications. If the efficiency of the ion thruster is an issue, it seems promising to neutralize the positive ion space charge with high-mass negative ions or negatively charged dust particles. In the present paper we investigated a variety of space-charge neutralization schemes with heavy negatively charged particles in the ion and Hall thrusters.

If a small negative ion current δJ_- is injected at the cathode end in the ion thruster, the positive ion current rises above the Child–Langmuir limit J_i^{CL} as $\delta J_i \sim 0.378(QM_-/qM_i)^{0.5}\delta J_-$. However, the positive ion current cannot be increased indefinitely: With an unlimited supply of negative ions (negatively charged dust) at the cathode, J_i is limited to $J_i \sim 1.865 J_i^{\mathrm{CL}}$. The positive ion current can be further increased if the negative ions are injected at some intermediate location between the anode and cathode. The optimal injection location is $z_0 \approx 0.44d$, and the maximum attainable positive ion current is $\sim 4.59 J_i^{\mathrm{CL}}$. Negative ion injection from the anode appears to be impractical because it requires the negative ion source to be operated in the Child–Langmuir limit with a very high extraction voltage. Yet another scheme of space-charge neutralization in the ion thruster, considered in the present paper, employs dust grains charged negatively in the presence of an electron beam. Due to the ion collection by dust particles, this neutralization scheme is likely to be ineffective near the anode, where the charge non-neutrality limits the ion current. The positive ion current extracted from the "dusty" thruster is smaller than that in the no dust case. Finally, negative ion (dust) injection in the Hall thruster is shown to cause only a moderate increase of the kinetic energy density of the flux of positive ions.

Thus, the straightforward neutralization schemes proposed in this work lead, at their best, only to a moderate but nonetheless possibly important increase of the ion current in the ion thruster and the thrust density in the Hall thruster. The possibility to significantly increase the ion current in the ion thruster through space-charge neutralization with heavy negatively charged particles cannot be dismissed in general. Besides attempting to explore new schemes of space-charge neutralization, the analysis here is relevant to the physical situations that occur in negative ion sources with surface conversion, in negative ion rich plasma used for plasma etching, and in laboratory Hall thruster experiments.

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