Effect of nonlinear Landau damping in plasma-based backward Raman amplifier

N. A. Yampolsky and N. J. Fisch
Department of Astrophysical Sciences, Princeton University, Princeton, New Jersey 08544, USA

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A plasma wave can mediate laser coupling in a plasma-based resonant backward Raman amplifier for high power amplification of short laser pulses. The resonant nature of amplification requires a long lifetime of the plasma wave. However, the plasma wave can be heavily Landau damped in warm plasma. On the other hand, Landau damping can be saturated in the presence of a strong plasma wave. We study backward Raman amplifier in the nonlinear regime of Landau damping using a simplified fluid model. We find the regime in which initially high linear Landau damping can be significantly saturated. Because of the saturation effect, higher temperatures can be tolerated in achieving efficient amplification. The plasma temperature can be as much as 50% larger compared to the case of unsaturated Landau damping.
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I. INTRODUCTION

The plasma-based resonant backward Raman amplifier (BRA) is a promising scheme for compression of powerful laser pulses to ultrashort durations.[1] In this scheme, two counter-propagating laser pulses are coupled through resonant Raman scattering. In this three-wave interaction process, the generated plasma wave mediates energy transfer from the long pump pulse to its Stokes component, which is seeded with a short laser pulse. The counter-propagating geometry of the laser pulses allows continuous amplification of the seed pulse by the pump. The amplitude of the seed pulse grows in time and eventually becomes much larger than the amplitude of the original seed and the pump pulses. The duration of the amplified pulse remains short and is determined by the time required to deplete the pump pulse. This time is roughly the inverse growth rate of the backward Raman instability. While the seed pulse is being amplified, the interaction becomes stronger and the pump depletes at the front of the amplified pulse. Therefore, the back part of the amplified pulse is shadowed by the front part and is not amplified, which leads to the effective shortening of the amplified pulse. Earlier research showed that the regime of amplification and compression is efficient until the amplified pulse reaches the duration of the original seed pulse.[2]

The currently used chirped pulse amplification (CPA) scheme[3] approaches a technological limit at high fluencies. The solid state gratings, used in CPA, cannot tolerate high power laser radiation. The damage of the gratings can be reduced only through reducing the power density handled, which then requires larger gratings. That leads to the huge increase in their cost. On the other hand, plasma in BRA cannot be destroyed by powerful laser radiation. The plasma is reproduced every shot, which avoids the damage of expensive elements. Modern experiments now show good progress on the implementation of the BRA scheme.[4–7]

There are many issues that can prevent amplification in the BRA, such as plasma wavebreaking,[8,9] the self-focusing and the self-modulation instabilities,[9–11] and the generation of precursors.[12] In this paper we consider the nonlinear Landau damping and the nonlinear frequency shift of the plasma wave. These effects are caused by the change of the electron distribution function at velocities close to the phase velocity of the plasma wave. The influence of these effects on Raman scattering was noted before.[13,14] However, the detailed study of these effects is complicated due to their kinetic nature. We apply a simple but numerically justified fluid model of nonlinear Landau damping[15] to study the nonlinear kinetic effects in BRA.

First, we study the influence of nonlinear Landau damping on seed amplification. The generated plasma wave can experience strong damping. It can be either collisional damping at low plasma temperature or Landau damping at high plasma temperature. Therefore, there is an optimal plasma temperature at which damping of the plasma wave is the smallest. This temperature is the most favorable for BRA. However, the inverse Bremsstrahlung losses of the pump pulse might result in a significant increase in the plasma temperature.[16] Eventually, the amplification could enter a regime of strong Landau damping. Strong Landau damping reduces the amplitude of the plasma wave, which makes amplification inefficient, since the plasma wave mediates coupling between the laser pulses. On the other hand, the plasma wave strongly interacts with electrons, which travel with velocities close to the phase velocity of the wave. As a result, the electron distribution function itself changes, which significantly reduces the Landau damping.

The plasma wave is generated behind the seed pulse, which travels almost with the speed of light. The generated plasma wave is amplified through Raman scattering of the pump and experiences Landau damping at the same time. The electron distribution function is modified, which leads to the saturation of Landau damping. Depending on the amplitude, the plasma wave either damps prior saturation of Landau damping or saturates Landau damping and then gets amplified with high efficiency. We will find conditions for the regime of saturated Landau damping. Thus, we extend the parameters region for efficient amplification even at high plasma temperatures.

Then we take into account the effect of the nonlinear frequency shift caused by the resonant particles. The plasma wave, generated behind the seed pulse, matches the exact resonant conditions. Later its frequency downshifts due to the nonlinear dynamics of the resonant electrons. Therefore, some phase shift between three interacting waves (the pump, the amplified seed, and the plasma wave) appears. That reduces the efficiency of the three-wave interaction and eventually can saturate energy transfer from the pump to the amplified pulse.

The paper is organized as follows: In Sec. II we present a model which describes the nonlinear Landau damping and the nonlinear frequency shift of the plasma wave in BRA. This model describes the kinetic phenomenon of Landau damping in terms of fluid equations, which can be easily incorporated into the three-wave model for BRA. In Sec. III, we study the effect of the nonlinear Landau damping in BRA, neglecting the nonlinear frequency shift of the plasma wave. We find the regime for saturation of Landau damping and describe the amplitude profile of the amplified pulse. In Sec. IV we take into account the nonlinear frequency shift of the plasma wave caused by the nonlinear change in the electron distribution function in the domain close to the phase velocity of the plasma wave. We find a condition for neglecting the nonlinear frequency shift in Raman amplifier. In Sec. V we determine whether the nonlinear Landau damping or the nonlinear frequency shift of the plasma wave limit the amplification the most. We show that in Maxwellian plasma the nonlinear frequency shift can be neglected for typical parameters of BRA. In Sec. VI we summarize the main results of the paper.

II. MAIN EQUATIONS

The BRA can be described by the coupled equations for the wave envelopes of the laser pulses and the plasma wave. We consider plasma to be warm, so that Landau damping of the plasma wave can be significant. At the same time, the electron distribution function can significantly change in the presence of a strong plasma wave. That results in the partial saturation of Landau damping and the nonlinear frequency shift of the plasma wave. The equations describing BRA under these conditions take the following form (see, for instance, Ref. 17):

$$\partial_t a + c\partial_z a = -\sqrt{\omega\omega_p} bf, \qquad (1)$$

$$\partial_t b - c\partial_z b = \sqrt{\omega\omega_p} af^*, \qquad (2)$$

$$\partial_t f + \nu_{\rm nl} f - i\delta\Omega_{\rm nl} f = \sqrt{\omega\omega_p} ab^*/2, \qquad (3)$$

where a and b are the electric field envelopes of the pump and the seed, respectively, in units of $mc\omega_{a,b}/e$; f is the electric field of the plasma wave in units of $mc\sqrt{\omega\omega_p}/e$, $\Omega_{\rm nl}$ and $\nu_{\rm nl}$ are the nonlinear frequency shift and the Landau damping rate of the plasma wave, respectively, ω and ω_p are the laser and plasma frequencies, respectively, t is time and z is space coordinate in the direction of the pump propagation, and c is the speed of light.

The problem as modeled by Eqs. (1)–(3) requires a kinetic description since both the nonlinear Landau damping rate and the nonlinear frequency shift of the plasma wave depend on the time-dependent electron distribution function. As a result, the problem becomes very complicated for analytical and even numerical study. Alternatively, we propose to approximately describe the nonlinear Landau damping of the plasma wave through the model presented in Ref. 15. The numerical study of this model showed good accuracy for the problem of driven plasma waves. The advantage of this model is that the inherently kinetic effect of Landau damping is described in terms of fluid equations. This simplified description allows, in some cases, even analytical progress. Moreover, this model describes all the stages of the nonlinear saturation of Landau damping: the linear damping, the transition stage, and the advanced nonlinear stage when the plateau in the distribution function is formed. The detailed discussion of this model can be found in Ref. 15. Other simplified models for nonlinear Landau damping in the three-wave interaction were recently proposed.[17,18] The model used here[15] was found to be somewhat more precise for the regimes of interest to us here. However, the accuracy of the other models is approximately the same and either could have been chosen as well for studying the BRA problem.

The use of the simplified fluid model for nonlinear Landau damping[15] allows one to describe this effect in BRA in terms of a set of partial differential equations (PDEs),

$$\partial_\zeta a - \partial_\tau a = -bf, \qquad (4)$$

$$\partial_\tau b = af^*, \qquad (5)$$

$$\partial_\zeta f + \nu f - i\delta\Omega f = ab^*, \qquad (6)$$

$$\partial_\zeta((\nu_0 - \nu)|f|^{3/2}) = \alpha\nu|f|^2, \qquad (7)$$

$$\delta\Omega = -\Omega_0|f|^{1/2}. \qquad (8)$$

Here $\nu = 2\nu_{\rm nl}/\sqrt{\omega\omega_p}$ is the normalized nonlinear Landau damping rate and $\nu_0 = -\pi/2(\omega_p/\omega)^{3/2}c^2F'(V_{\rm ph})/(\omega\partial_\Omega D)$ is the normalized linear Landau damping rate; $\delta\Omega = 2\delta\Omega_{\rm nl}/\sqrt{\omega\omega_p}$ is the normalized nonlinear frequency shift of the plasma wave due to resonant electrons and $\Omega_0 = 0.385(\omega_p/\omega)^{3/2}[c^3F''(V_{\rm ph})]/(\omega\partial_\Omega D)$. $\alpha = 9\sqrt{2}\pi^2/64 \times(\omega/\omega_p)^{1/4} \approx 2(\omega/\omega_p)^{1/4}$ is the nonlinear coupling coefficient between the plasma wave and the resonant particles. $\omega \equiv \omega_a$, ω_b, and $\Omega \equiv \omega_f$ are the frequencies of the pump pulse, the seed pulse, and the plasma wave, respectively. $D = D(\Omega, k_f)$ is the plasma dielectric function, which corresponds to the electron distribution function $F(V)$. $V_{\rm ph} = \Omega/k_f$ is the phase velocity of the plasma wave. $\tau = -z\sqrt{\omega\omega_p}/c$ is the normalized amplification length. $\zeta = (t + z/c)\sqrt{\omega\omega_p}/2$ is the longitudinal coordinate in the frame moving with the seed pulse.

The electric field of each waves is a product of its envelope a, b, f and fast oscillations $e^{i\phi_{a,b,f}} = e^{i\omega_{a,b,f}t - ik_{a,b,f}z}$. The frequencies $\omega_{a,b,f}$ and the wave numbers $k_{a,b,f}$ satisfy the dispersion relations and the exact resonant conditions for the three-wave interaction,

$$D(\Omega,k_f)=0, \quad k_{a,b}^2 - D(\omega_{a,b},k_{a,b})\omega_{a,b}^2/c^2=0, \tag{9}$$

$$\omega_a=\omega_b+\Omega, \quad k_a=k_b+k_f. \tag{10}$$

Here, six variables (the frequencies and the wave numbers of two laser pulses and the plasma wave) are constrained by five equations. Therefore, only one independent variable remains (for example, the pump frequency ω). The dispersion relations for the laser pulses are almost not affected by the presence of plasma, since we consider very underdense plasma $\omega_p \ll \omega$. In this case $\omega_b=\omega_a-\Omega$, $|k_{a,b}|\approx\omega_{a,b}/c$, $k_f\approx(2\omega_a-\Omega)/c$, and $D(\Omega,k_f)=0$.

The physical picture of the pulse amplification in BRA can be described as follows: A short seed pulse travels toward a long pump pulse. In our model the seed pulse is approximated with the δ-function waveform of a given integrated amplitude $\int b d\zeta=\epsilon/2\ll 1$. The beatwave of the seed and the pump pulses generates the plasma wave, which remains behind the seed pulse while it travels with the speed of light. The pump scatters back on the generated plasma wave and amplifies it. The backscattered light forms the amplified pulse, which travels behind the seed pulse. The amplified pulse grows, since the undepleted pump constantly enters the interaction region. Initially, the amplified pulse amplitude grows exponentially in time with the growth rate of the monochromatic wave instability $b_{\max}\propto e^{\gamma\tau}$, $\gamma=a_0/\sqrt{2}$. The full pump depletion occurs when the amplified pulse becomes powerful enough, so that all the energy of the pump is consumed by the amplified pulse. At that point, amplification enters the so-called nonlinear stage. The amplified pulse can be approximated with the "π-pulse" self-similar solution during this stage.[1] The form of the amplified pulse remains the same during amplification while the peak amplitude and duration of the pulse change. The amplitude of the π-pulse grows linearly with the amplification length and its duration contracts inversely with the amplification length. As a result, the energy of the amplified pulse linearly grows with the amplification length, which corresponds to absorption of all the power of the incident pump. This regime of amplification will be described quantitatively in Sec. III [Eqs. (31)–(33)].

The plasma wave is amplified due to Raman scattering and it is Landau damped at the same time. The saturation of the growth can be observed at large Landau damping rate $\nu_0>\gamma$. However, this restriction is only applicable at the linear stage of amplification, when the amplified pulse grows exponentially. For example, if Landau damping was not taken into account, then amplification eventually would enter the nonlinear regime, which is described by compressing in time the π-pulse solution. The amplified pulse then becomes short enough, $\delta\zeta<1/\nu_0$, so that Landau damping could not affect amplification, since the plasma wave cannot be damped in such a short time. Moreover, in a small region behind the seed pulse, $\zeta<1/\nu_0$, the plasma wave is not affected by Landau damping during the full amplification process. Therefore, the amplification can reach the nonlinear stage of the π-pulse regime even at strong nonsaturated Landau damping. A long amplification length, however, may be required to reach this stage. That leads to reduced efficiency of amplification, since the pump remains undepleted longer than if there were no damping of the plasma wave.

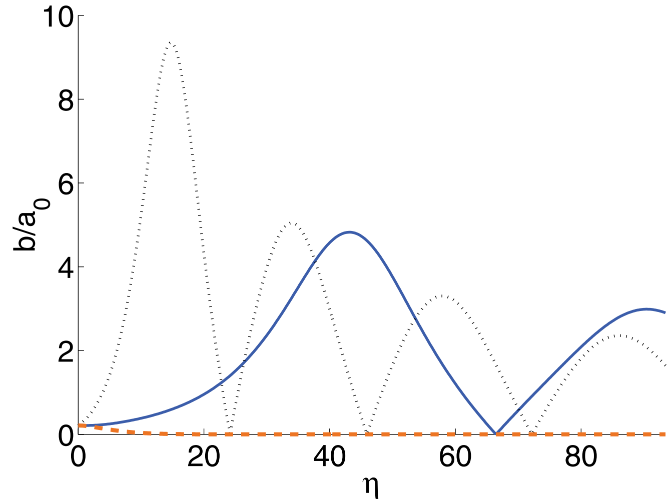

FIG. 1. (Color online) Amplified pulse in the model with quasilinear saturation of Landau damping (solid blue line) is compared to the amplified pulses in the models with linear Landau damping (dashed red line) and no Landau damping (dotted black line). $\eta=a_0^2\zeta\tau$ is the scaled longitudinal coordinate along the amplified pulse. The parameters of the simulations are the same as in Fig. 3.

The nonlinear saturation of Landau damping can reduce the duration of the linear stage of amplification (small pump depletion) compared to the case of the nonsaturated Landau damping. This idea is demonstrated in Fig. 1, where three different models are compared to each other: the model with no Landau damping, the model with linear Landau damping, and model (4)–(7) with the nonlinear saturation of Landau damping.

A plasma wave of large amplitude is required to suppress Landau damping. Earlier research showed that a given plasma wave of the amplitude $\omega_B\sim\nu_0$ can saturate Landau damping.[19,20] Here $\omega_B=\sqrt{2}(\omega^3\omega_p)^{1/4}\sqrt{|f|}$ is the bounce frequency of the plasma wave of amplitude f. The amplitude of the plasma wave generated by the seed pulse is typically small and cannot result in the saturated Landau damping. However, the plasma wave amplitude grows during amplification and eventually becomes large enough to saturate Landau damping. On the other hand, the plasma wave can be Landau damped to zero before reaching the level for saturation of Landau damping.

We are interested in the regime of saturated Landau damping. In this regime, the plasma wave eventually becomes strong enough that the seed amplification is not strongly affected by Landau damping. The seed is both amplified and compressed while it propagates, reaching the regime close to the π-pulse. The duration of the amplified pulse in this regime is much smaller than the Raman length, $b/\partial_\zeta b\ll\gamma^{-1}$. At the same time, the pulse grows much slower than during the linear stage, $b/\partial_\tau b\gg\gamma^{-1}$. Then the amplitude profiles of the waves are quasistationary in this regime, $\partial_\zeta\gg\partial_\tau$. Under this condition, the first integral of Eqs. (4)–(8) can be found,

$$\partial_\zeta\left(\frac{|a|^2}{2}+\frac{|f|^2}{2}+\frac{(\nu_0-\nu)|f|^{3/2}}{\alpha}\right)=0. \tag{11}$$

This expression is a combination of the Manley–Rowe relation and the energy conservation law. It states that each pump photon generates one plasmon. Some plasmons are Landau damped. The number of damped plasmons is related to the Landau damping rate through the energy conservation law.

The efficient amplification of the seed pulse is possible only if an insignificant part of the plasma wave energy is required to saturate Landau damping. This regime can be achieved at high enough pump amplitude.

$$\frac{\alpha}{2}\sqrt{a_0} \gg \nu_0. \tag{12}$$

This limit has the following interpretation: the amplitude of the pump should be high enough so that its full depletion results in a strong plasma wave, $\omega_B \gg 2\sqrt{2}\nu_0$. As mentioned above, the amplitude of the plasma wave on the same order of magnitude is required to saturate Landau damping. Constraint (12) is a necessary condition for efficient seed amplification. However, this condition does not ensure that the amplification reaches the nonlinear stage, since the plasma wave grows from a small amplitude and can be damped prior to saturation of Landau damping.

Equation (7) describes the nonlinear saturation of Landau damping in the presence of a strong plasma wave. For a given time dependence of the plasma wave amplitude $f(\zeta)$ this equation is a linear ordinary differential equation (ODE). We are interested in the regime when Landau damping can be significantly reduced and does not affect much the amplification. The amplitude of the plasma wave rapidly grows in this regime so that $\int_0^\zeta f d\zeta' \approx f^2/\partial_\zeta f$. Under this condition we find the asymptotic solution of Eq. (7):

$$\frac{\nu}{\nu_0} \approx 1, \quad \alpha\int_0^\zeta \sqrt{|f|}d\zeta' \ll 1, \tag{13}$$

$$\frac{\nu}{\nu_0} \approx \frac{3}{\alpha\int_0^\zeta\sqrt{|f|}d\zeta'}, \quad \alpha\int_0^\zeta \sqrt{|f|}d\zeta' \gg 1. \tag{14}$$

Using these asymptotic expressions we construct

$$\nu = \frac{\nu_0}{1+\alpha/3\int_0^\zeta\sqrt{|f|}d\zeta'}, \tag{15}$$

which meets both the limiting cases in Eqs. (13) and (14).

The Landau damping rate saturates much slower in this regime than if the amplitude of the plasma wave were constant: $\dot{\nu}/\nu = \dot{\omega}_B/\omega_B$ rather than $\dot{\nu}/\nu=\omega_B$. The physics of this effect is the following. The width of the resonant region in the distribution function is proportional to the bounce frequency of the plasma wave. A growing plasma wave results in the increase in the number of the resonant particles. The distribution of new resonant particles is the same as the original one (prior to the wave-particle interaction). As a result, extra cold particles enter the resonant region, since the original distribution function decreases with electron velocity. The excess in cold particles modifies the distribution function in the resonant region and increases its slope. That leads to the increase in the Landau damping rate, which tends to approach its linear value of ν_0. The saturation of Landau damping of an amplified plasma wave is a competition of two processes. Landau damping transfers energy from the plasma wave to the resonant particles tending to form the plateau in the distribution function. The growth of the plasma wave amplitude tends to restore the distribution function back to the original one. As a result, the saturation of Landau damping is slowed down. This effect plays an important role in BRA and requires detailed analysis.

III. SEED AMPLIFICATION IN THE REGIME OF NONLINEAR LANDAU DAMPING

The plasma wave starts to grow from a small amplitude $f(\zeta=0)=\epsilon/2 \ll 1$. The generated plasma wave is amplified by the pump and damps due to Landau damping at the same time. As discussed above, we are mostly interested in the regime of the efficient pulse amplification. Landau damping is expected to be nonlinearly saturated in this regime. We also expect that only a small fraction of the plasma wave energy is required to be lost in order to form the plateau in the electron distribution function [condition (12) is satisfied]. As a result, Landau damping is expected to be reduced in the domain where the pump is not depleted yet. Therefore, we analyze the saturation of Landau damping without taking into account the pump depletion. The pump depletion will be taken into account later in this section. Then the saturation of Landau damping in BRA can be described by the following set of equations:

$$\partial_\tau b = a_0 f, \tag{16}$$

$$\partial_\zeta f + \nu f = a_0 b, \tag{17}$$

$$\nu = \frac{\nu_0}{1+\alpha/3\int_0^\zeta\sqrt{|f|}d\zeta'}. \tag{18}$$

This set of equations should be solved together with the boundary conditions,

$$f(\zeta=0)=\epsilon/2, \quad b(\tau=0)=0. \tag{19}$$

The transformation $b=B\exp(-\int\nu d\zeta')$ together with $f=F\exp(-\int\nu d\zeta')$ reduces Eqs. (16) and (17) to the damping free case. This damping free system becomes linear and can be solved using Green's function.[21]

$$f = \epsilon a_0 e^{-\int_0^\zeta \nu d\zeta'} I_0(2a_0\sqrt{\tau\zeta})/2, \tag{20}$$

$$b = \epsilon e^{-\int_0^\zeta \nu d\zeta'} \partial_\zeta I_0(2a_0\sqrt{\tau\zeta})/2, \tag{21}$$

where I_0 is zero-order modified Bessel function (Green's function in the damping-free regime). Green's function has a self-similar form since it depends only on the self-similar variable $\eta=a_0^2\tau\zeta$.

If Landau damping is saturated, then the integral $\int_0^\infty \nu d\zeta'$ converges. Therefore, the asymptotic growth of the plasma wave is exactly the same as if no Landau damping were present in plasma. The only contribution of Landau damping to amplification is the effective reduction in the seed pulse amplitude. In the domain of saturated Landau damping the effective seed amplitude reaches the asymptotic value of

$$\epsilon_{\mathrm{eff}}(\zeta \to \infty) = \epsilon e^{-\int_0^\infty \nu d\zeta'}. \tag{22}$$

Equations (20) and (18) describe the self-consistent profile of the Landau damping rate and should be solved simultaneously. As a result, we derive the second order ODE describing the longitudinal profile of the plasma wave,

$$d^2_{\eta\eta}G(\eta) = -\frac{\alpha\sqrt{\epsilon a_0/2}}{3\nu_0}(d_\eta G)^2\sqrt{I_0(2\sqrt{\eta})}e^{-G/2}, \tag{23}$$

$$G(0) = 0, \quad d_\eta G(0) = \frac{\nu_0}{a_0^2\tau}, \tag{24}$$

$$G = -\ln\left[\frac{2f}{a_0\epsilon I_0(2\sqrt{\eta})}\right], \quad \eta = a_0^2\tau\zeta. \tag{25}$$

Here G is the attenuation factor, which describes the reduced plasma wave amplitude due to Landau damping. This attenuation factor is related to the effective seed pulse amplitude, $\epsilon_{\mathrm{eff}} = \epsilon e^{-G}$. The effective amplitude of the seed pulse is always smaller than the original seed pulse amplitude $\epsilon_{\mathrm{eff}} < \epsilon$, since the attenuation factor is positive. Note that the attenuation factor fully describes the solution of Eqs. (16)–(18).

The attenuation factor increases behind the seed pulse. It monotonically grows until it saturates, $G' \to 0$. When the attenuation factor saturates, the plasma wave grows until it depletes the pump. On the other hand, the attenuation factor can remain unsaturated. It can increase with a constant slope $G'(\eta \to \infty) = \mathrm{const}$. The amplitude of the plasma wave damps to zero in this regime, since $f \propto e^{-G}I_0(2\sqrt{\eta}) \to 0$. Either of these regimes can be observed at different amplification lengths. A small amplification length τ results in large initial slope of the attenuation factor $G'(0)$. The saturation of Landau damping does not occur in this regime. While the amplification length increases, $G'(0)$ decreases, and the saturation of the attenuation factor can be observed. Therefore, there exists a critical initial slope G'_* of the attenuation factor, which separates two regimes of Landau damping. It depends only on $\beta = \alpha\sqrt{0.5\epsilon a_0}/(3\nu_0)$, since it is the only parameter of Eq. (23). The critical initial slope G'_* results in the critical amplification length which separates two different regimes of amplification,

$$\tau_{\mathrm{Landau}} = \nu_0/(a_0^2 G'_*). \tag{26}$$

Landau damping cannot be suppressed at small amplification length $\tau < \tau_{\mathrm{Landau}}$. The plasma wave starts to grow from a small amplitude and damps before it becomes strong enough to modify the distribution function. At large amplification length $\tau > \tau_{\mathrm{Landau}}$, the plasma wave rapidly grows and becomes strong enough to change the distribution function. Landau damping is suppressed and further amplification is not affected by damping in this regime. The nonlinear regime of amplification, which is accompanied with the pump depletion, can be observed only if $\tau > \tau_{\mathrm{Landau}}$.

Equation (23) can be rewritten in the following form:

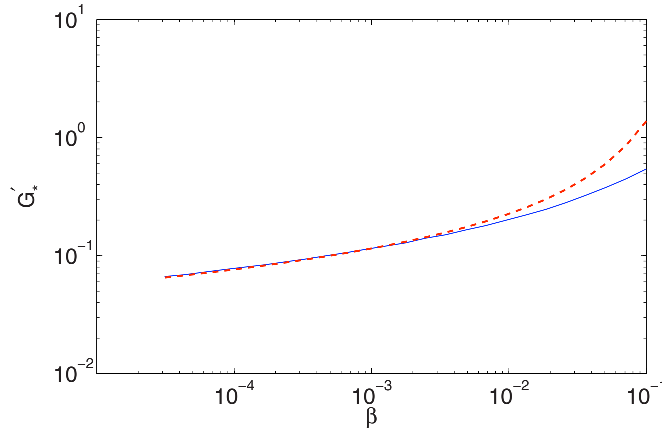

FIG. 2. (Color online) Numerically (solid blue line) calculated dependence of the parameter G'_* and its analytical (dashed red line) approximation with expression (28). Parameter G'_* characterizes the amplification length required for significant saturation of Landau damping as defined by Eq. (26); β is proportional to the ratio of the bounce frequency of the seeded plasma wave and the linear Landau damping rate as defined by Eq. (28).

$$G'(0) - G'(\eta) = \int_0^\eta -\beta(G')^2\sqrt{I_0(2\sqrt{\eta})}e^{-G/2}d\eta. \tag{27}$$

This integral can be evaluated using the saddle point approximation. In the zero-order approximation ($\beta \ll 1$) the attenuation factor G is close to its unperturbed value $G \approx \eta d_\eta G(0)$ for small values of η. The slope changes in a narrow domain, since the kernel of the integral has a well-defined narrow maximum. Thus, we can estimate the critical initial slope of the attenuation factor, which separates two types of solutions: saturated and not saturated.

$$G'_* = \frac{1}{2\ln\left[\frac{(G'_*)^{1/4}}{4(4\pi)^{1/4}\beta}\right]}, \quad \beta = \frac{\alpha\sqrt{\epsilon a_0/2}}{3\nu_0}, \tag{28}$$

where the numerical factor of 4 was introduced for a batter fit with numerical results. This expression approximates well the numerically found G'_* for $\beta < 0.01$ as demonstrated in Fig. 2. Note that G'_* logarithmically depends on α, the parameter describing saturation of Landau damping. Therefore, the critical amplification length τ_{Landau} is insensitive to the accuracy of a model describing nonlinear Landau damping. This fact justifies earlier approximation (15) for the nonlinear Landau damping rate.

Now we describe the longitudinal profile of the amplified pulse close to its maximum. In this domain, the pump depletion is significant at large enough amplification length, $\tau > \tau_{\mathrm{Landau}}$. The linear (no pump depletion) solutions (20) and (21) correspond to the linear part of the π-pulse, which corresponds to the effective seed amplitude ϵ_{eff}. One can expect the nonlinear profile of the amplified pulse to be close to the π-pulse.

The Landau damping rate is significantly reduced in the domain of the amplified pulse maximum. At the same time, the amplitude of the plasma wave does not change significantly in this domain. Thus, we introduce the effective undepleted pump amplitude $\tilde{a}_0$ as the maximum amplitude of the plasma wave, which can be excited. This amplitude is reduced in the presence of Landau damping, since some energy of the plasma wave is absorbed by the resonant particles. The effective undepleted pump amplitude $\tilde{a}_0$ can be determined from the Manley–Rowe relation (11),

$$\tilde{a}_0^2 + \frac{2\nu_0 \tilde{a}^{3/2}}{\alpha} = a_0^2. \tag{29}$$

The effective undepleted pump amplitude is well defined if it is close to the original pump amplitude, $a_0 - \tilde{a}_0 \ll a_0$ [condition (12)].

The nonlinear solution close to the amplified pulse maximum can be described with the following set of equations:

$$\partial_\zeta a = -bf, \quad \partial_\tau b = af, \quad \frac{a^2}{a_0^2} + \frac{f^2}{\tilde{a}_0^2} = 1. \tag{30}$$

This set of equations allows a self-similar solution similar to the damping-free case. We seek the nonlinear solution in the following form:

$$a = a_0 \cos\frac{U}{2}, \quad f = \tilde{a}_0 \sin\frac{U}{2}, \quad b = \frac{a_0}{2\tilde{a}_0}\partial_\zeta U. \tag{31}$$

This set of equations can be reduced to the Sine–Gordon equation. The solution tends to approach the self-similar π-pulse solution,

$$\tilde{\eta} U''(\tilde{\eta}) + U'(\tilde{\eta}) = \sin U, \tag{32}$$

$$U(0) = \epsilon_{\text{eff}}, \quad U'(0) = 0, \quad \tilde{\eta} = \tilde{a}_0^2 \zeta \tau. \tag{33}$$

The linear solution in the domain of the undepleted pumps (20) and (21) corresponds to the linear part of the π-pulse. The linear solution becomes inaccurate close to the amplified pulse maximum. The position of the pulse maximum can be determined from the linear solution found above,[2]

$$\epsilon I_0(2\sqrt{\eta_m}) e^{-G(\eta_m)} = 4. \tag{34}$$

Then the effective integrated seed amplitude ϵ_{eff} should be calculated based on the attenuation factor at the position of the amplified pulse maximum. As a result, the nonlinear solution close to the amplified pulse maximum,

$$a = a_0 \cos\frac{U}{2}, \quad f = \tilde{a}_0 \sin\frac{U}{2}, \quad b = \frac{a_0}{2\tilde{a}_0}\partial_\zeta U, \tag{35}$$

$$\tilde{\eta} U''(\tilde{\eta}) + U'(\tilde{\eta}) = \sin U, \tag{36}$$

$$U(0) = \epsilon_{\text{eff}}, \quad U'(0) = 0, \quad \tilde{\eta} = \tilde{a}_0^2 \zeta \tau. \tag{37}$$

$$d_{\eta\eta}^2 G(\eta) = -\frac{\alpha\sqrt{\epsilon a_0/2}}{3\nu_0}(d_\eta G)^2 \sqrt{I_0(2\sqrt{\eta})} e^{-G/2}, \tag{38}$$

$$G(0) = 0, \quad d_\eta G(0) = \frac{\nu_0}{a_0^2 \tau}, \quad \eta = a_0^2 \zeta \tau, \tag{39}$$

$$\epsilon I_0(2\sqrt{\eta_m}) e^{-G(\eta_m)} = 4, \quad \epsilon_{\text{eff}} = \epsilon e^{-G(\eta_m)}. \tag{40}$$

This solution should have a longitudinal shift, so that the position of the pulse maximum appears at η_m.

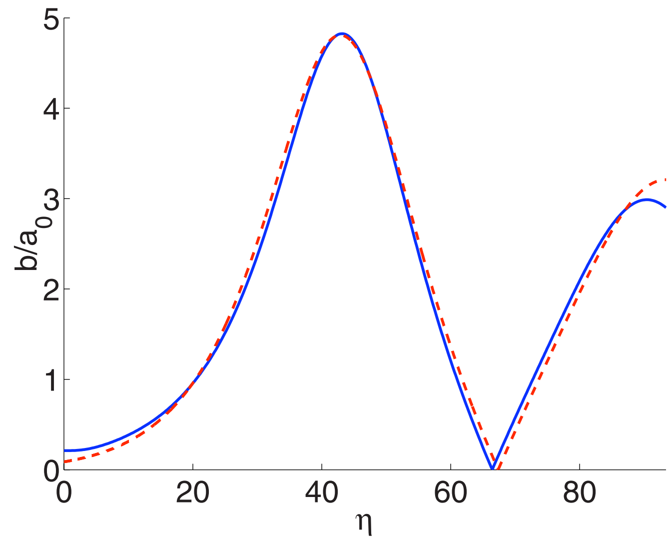

FIG. 3. (Color online) Longitudinal dependence of the amplified pulse amplitude. Solution of numerically solved set of Eqs. (4)–(7) (solid blue line) is compared with analytical approximation (35)–(40) (dashed red line). The parameters of simulation: $a_0 = 0.006$, $\gamma\tau = 30$, $\nu_0/\gamma = 15$, $\omega/\omega_p = 10$, and $\epsilon = 0.01$. Here γ is dimensionless growth rate of backward Raman instability $\gamma = a_0/\sqrt{2}$; $\eta = 2\gamma^2\zeta\tau$ is scaled longitudinal coordinate along the amplified pulse. For these parameters $\tau_{\text{Landau}} \approx \tau/2$.

This approximated solution is verified numerically through the numerical simulations of the original model (4)–(7) (here the nonlinear frequency shift is assumed to be zero). The results presented in Fig. 3 show good accuracy of the approximated solution.

We now verify that this regime of amplification falls into the parameter region appropriate for the simplified model of Landau damping.[15] The amplitude of the plasma wave should grow slowly compared to the bounce frequency timescale, $\partial_t \omega_B \ll \omega_B^2$. The growth rate of the plasma wave changes along the pulse profile and is time dependent, $\partial_t f \propto \partial_\zeta f \propto \tau/\tilde{\eta}$. It is possible that the growth rate of the plasma wave exceeds the bounce frequency close to the seed pulse. However, the Landau damping rate is close to the linear value in that domain, so violation of this restriction is not important. The simplified model should be valid in the domain of the nonlinearly modified Landau damping rate, $\alpha \int \sqrt{f} d\zeta \sim 1$. Using the π-pulse solution (35)–(37), we can show that $\partial_t \omega_B / \omega_B^2 \sim \sqrt{f} \ll 1$ in that domain. Therefore, the plasma wave grows slowly enough for the model to be accurate.

The simplified model describes the saturation of Landau damping of a growing plasma wave, $\partial_\zeta |f| > 0$. Therefore, it can be used to describe the plasma wave between the seed pulse and the amplified pulse maximum. The Landau damping rate is significantly reduced close to the maximum of the amplified pulse. As a result, the plasma wave is not affected by Landau damping in that domain and its evolution still can be described by the simplified model.

A further restriction that limits the amplitude of the pump is that the initial electron distribution function does not significantly change within the resonant domain $\delta v \sim \omega_B / k_f \ll v_T^2 / v_{\text{ph}}$. The condition is violated for the numerical simulations presented in Figs. 1 and 3. These simulations were performed to demonstrate that Eqs. (4)–(7) result in solution (35)–(40). Thus, this solution should be used with caution for analyzing amplification by a strong pump. However, the Landau damping rate is significantly reduced when the amplitude of the plasma wave becomes large so that model (35)–(40) is invalid. Further pulse amplification is not significantly affected by Landau damping and the simplified model can still be applied. Other effects of a large plasma wave amplitude, such as the wavebreaking, can be considered regardless to the effect of Landau damping.

IV. NONLINEAR FREQUENCY SHIFT

The change in the distribution function results in the change in the plasma susceptibility and, therefore, in the change of the frequency of the plasma wave mode. Both the real and the imaginary parts of the frequency change during the wave-particle interaction. The change in the imaginary part of the frequency corresponds to the saturation of Landau damping, which was considered in the previous section. The change in the real part of the frequency we consider in this section.

The nonlinear frequency shift prevents efficient amplification due to dephasing between the interacting waves,

$$a = |a| \exp^{i\phi_a(\zeta,\tau)}, \tag{41}$$

$$f = |f| \exp^{i\phi_f(\zeta,\tau)}, \tag{42}$$

$$b = |b| \exp^{i\phi_b(\zeta,\tau)}, \tag{43}$$

$$\Delta\phi = \phi_b + \phi_f - \phi_a. \tag{44}$$

The dephasing grows away from the seed pulse. The energy transfer from the pump to the amplified pulse changes its sign when the phase difference $\Delta\phi$ reaches the value of $\pi/2$. Therefore, a quantitative estimate of the dephasing is required.

The amplitude profiles of the waves are well approximated with the solution (35)–(37). The first spike of the π-pulse solution can be approximated well with the "2π-pulse" solution,[1]

$$U_{2\pi} = 4 \arctan(\epsilon_{\mathrm{eff}} I_0(2\sqrt{\tilde{\eta}})/4) = 4 \arctan(U_0). \tag{45}$$

We assume that the phase difference $\Delta\phi$ between the waves is much smaller than $\pi/2$, which is a favorable regime for amplification. Under these conditions, the equations for the wave phases can be deduced from Eqs. (4)–(6) and have the following form:

$$\partial_\zeta \phi_a = -\frac{4U_0 \partial_\zeta U_0}{1-U_0^4}(\phi_b + \phi_f - \phi_a), \tag{46}$$

$$\partial_\tau \phi_b = -\tilde{a}_0^2 \frac{1-U_0^2}{1+U_0^2} \frac{U_0}{\partial_\zeta U_0}(\phi_b + \phi_f - \phi_a), \tag{47}$$

$$\partial_\zeta \phi_f = -\frac{a_0^2}{\tilde{a}_0^2} \frac{1-U_0^2}{1+U_0^2} \frac{\partial_\zeta U_0}{U_0}(\phi_b + \phi_f - \phi_a) + \delta\Omega(\tilde{\eta}), \tag{48}$$

The expression for $\delta\Omega$ is described by Eq. (8) and has a self-similar form, since the amplitude of the plasma wave depends only on the self-similar coordinate $\tilde{\eta}$.

This set of equations allows a self-similar solution $\phi_{a,b,f} = \zeta \psi_{a,b,f}(\tilde{\eta})$. For small amplitudes of the seed pulse $\epsilon_{\mathrm{eff}} \ll 1$, the maximum of the amplified pulse is located far from the seed pulse $\tilde{\eta}_m = (\ln(8\sqrt{\pi}\tilde{\eta}_m^{1/4}/\epsilon_{\mathrm{eff}}))^2/4 \gg 1$ (Ref. 1) ($U_0 = 1$ at $\tilde{\eta} = \tilde{\eta}_m$). Therefore, the asymptotic expression for the modified Bessel function can be used.

$$\psi_a + \tilde{\eta}\psi_a' = -\frac{4U_0^2}{1-U_0^4}(\psi_b + \psi_f - \psi_a)\sqrt{\tilde{\eta}}, \tag{49}$$

$$\tilde{\eta}\psi_b' = -\frac{a_0^2}{\tilde{a}_0^2} \frac{1-U_0^2}{1+U_0^2}(\psi_b + \psi_f - \psi_a)\sqrt{\tilde{\eta}}, \tag{50}$$

$$\psi_f + \tilde{\eta}\psi_f' = -\frac{1-U_0^2}{1+U_0^2}(\psi_b + \psi_f - \psi_a)\sqrt{\tilde{\eta}} + \delta\Omega(\tilde{\eta}). \tag{51}$$

The phase of the pump ψ_a remains small in the domain of small pump depletion. It becomes significant only in a relatively small domain close to the amplified pulse maximum, $|\tilde{\eta} - \tilde{\eta}_m| \ll \tilde{\eta}_m$. In this domain, the phase shift between the waves decreases after reaching its maximum. In our analysis we neglect the nonzero phase of the pump and estimate the maximum phase difference between the waves caused only by the plasma wave and the amplified pulse. Consistent with the above assumption, we simplify Eqs. (50) and (51) using $U_0 \ll 1$ and $a_0 = \tilde{a}_0$. Then we find the solution far from the seed pulse, $\tilde{\eta} \gg 1$,

$$\phi_b = -\frac{4}{5} \frac{\delta\Omega(\tilde{\eta})}{\sqrt{\tilde{\eta}}}\zeta, \quad \phi_f = \frac{6}{5} \frac{\delta\Omega(\tilde{\eta})}{\sqrt{\tilde{\eta}}}\zeta. \tag{52}$$

The dephasing between the waves can be estimated at the position of the amplified pulse maximum,

$$\Delta\phi \approx \phi_b + \phi_f = -\frac{2}{5} \frac{\Omega_0 \sqrt{\tilde{\eta}_m}}{\tilde{a}_0^{3/2}\tau}. \tag{53}$$

The BRA should be operated in the regime $|\phi_b + \phi_f| \ll \pi/2$ for efficient amplification. This condition is of the order of magnitude correct but slightly overestimated, since we did not take into account the phase of the pump, which slightly compensates the dephasing between the waves.

The plasma wave is detuned, since the frequency of the plasma wave at the locations of the seed pulse and the amplified pulse maximum are separated by $\delta\Omega_{\max} = -\Omega_0\sqrt{\tilde{a}_0}$. The dephasing caused by the nonlinear frequency shift can be partially compensated by additional dephasing between the waves. One might think that increasing the plasma density by some constant value can partially compensate the nonlinear frequency shift (so that the exact resonant condition is observed at some intermediate amplitude of the plasma wave). However, the uniform change in the linear frequency of the plasma wave results in the same change of the amplified pulse frequency, which can be derived directly from the nonlinear Eqs. (4)–(6). Therefore, inhomogeneous detuning should be introduced in order to compensate the nonlinear frequency shift. Either the plasma density gradient or the pump chirp can result in this detuning. For efficient compensation of the dephasing, the detuning factor should be

$$q \sim \frac{\delta\Omega_{\max}}{\zeta_m} = \frac{\delta\Omega_{\max}}{\tilde{\eta}_m}\tilde{a}_0^2\tau, \tag{54}$$

where the detuning factor is described as $q=2(\Omega' -2\omega_a')c/(\Omega_p\omega_a\tilde{a}_0^2)$;[9] $\zeta_m \equiv \tilde{\eta}_m/\tilde{a}_0^2\tau$ is the spatial coordinate of the amplified pulse maximum. As a result, the detuning factor changes with the amplification length, which requires a parabolic plasma density profile.

The nonlinear frequency shift can also be compensated if the amplification is performed in partially ionized plasma. Ionization of the low ionization levels can be reached at high intensity of the amplified pulse due to tunnel or multiphoton ionization.[22] Therefore, the negative nonlinear frequency shift due to the resonant particles can be compensated by an increase in the plasma frequency. Both the ionization and the nonlinear frequency shift have the largest effect close to the amplified pulse maximum, which can result in their effective compensation during the entire amplification.

V. MAXIMUM ALLOWED TEMPERATURE FOR RAMAN AMPLIFIER

Kinetic effects, such as nonlinear Landau damping and the nonlinear frequency shift of the plasma wave, constrain the parameter region in which BRA can be operated. First, we determine which of the considered effects limit amplification the most. The amplification length should be large enough so that the saturation of Landau damping can be reached, $\tau > 2\tau_{\mathrm{Landau}}$. In this case at least half of the pump energy can be absorbed by the seed, which implies high efficiency of the amplifier. At this amplification length, the dephasing caused by the nonlinear frequency shift can be neglected if

$$\frac{2}{5\pi}\frac{\Omega_0\sqrt{\tilde{\eta}_m}}{\tilde{a}_0^{3/2}\tau_{\mathrm{Landau}}} \ll 1. \tag{55}$$

This condition can be simplified for Maxwellian plasma,

$$\frac{V_T^2}{c^2}\frac{\omega}{\omega_p}\sqrt{\tilde{\eta}_m}G_*'\sqrt{a_0} \ll 1, \tag{56}$$

where $V_T=\sqrt{T/m}$ is the plasma thermal velocity. This condition is satisfied automatically for typical BRA parameters: ($V_T^2/c^2 \sim 10^{-4}$ for $T\sim 100$ eV plasma, $\omega/\omega_p \sim 10$, $\sqrt{\tilde{\eta}_m}\sim 10$ for $\epsilon_{\mathrm{eff}} \sim 10^{-2}$–$10^{-4}$, $G_*' \sim 10^{-1}$, $a_0 \sim 10^{-2}$). Therefore, the amplification is limited by Landau damping rather than the nonlinear frequency shift of the plasma wave. However, the opposite case scenario is possible if the plasma is not Maxwellian, for example, if BRA is operated in the ionization front regime.[23] Thus, the constraints on the regime for backward Raman pulse compression differ from the restriction on the plasma wave growth due to Raman backscattering.[24] This difference arises because the backward Raman instability is seeded in the BRA. As a result, conventional Raman scattering seeded by the thermal noise develops over a longer time until it first reaches the nonlinear stage of saturated Landau damping and only then can it result in the pump depletion. The nonlinear frequency shift of the plasma wave then results in much larger dephasing between the waves. Technically, this effect can be estimated by considering really small seeding amplitude $\epsilon=\epsilon_{\mathrm{noise}}$, which results in significantly increased coordinate of the scattered pulse maximum $\tilde{\eta}_m$ in Eq. (56).

Let us consider the regime of amplification in which saturation of Landau damping does not take place. Strong Landau damping significantly reduces coupling, which can result in small pump depletion. The linear solution can be described by Eqs. (20) and (21) considering $\nu=\nu_0=$const. However, it is possible that the amplification enters the nonlinear regime accompanied by significant pump depletion even if Landau damping is strong. The linear solution fails when the amplitude of the plasma wave becomes on the order of the pump amplitude (34). Then the amplification enters the nonlinear stage similar to the π-pulse regime. The amplification length required for reaching the nonlinear stage in this regime is

$$\tau_{\nu_0} = 2\sqrt{\eta_m}\frac{\nu_0}{a_0^2}. \tag{57}$$

The duration of the linear stage (small pump depletion) can be reduced if the saturation of Landau damping takes place. In Sec. III it was shown that the amplified pulse approaches the π-pulse solution at the amplification length,

$$\tau_{\mathrm{Landau}} = \frac{\nu_0}{a_0^2 G_*'}. \tag{58}$$

Therefore, the nonlinear saturation of Landau damping allows one to tolerate $2G_*'\sqrt{\eta_m}$ times larger Landau damping rate compared to the case of nonsaturated Landau damping. This value becomes of the order of 10 for $\beta\sim 0.1$, $\epsilon \sim 10^{-2}$–10^{-4}. Note, that the asymptotic expression (28) is inaccurate for these parameters and the parameter G_*' should be calculated numerically. The larger tolerable linear Landau damping rate means larger tolerable plasma temperature. However, the increase in plasma temperature is much smaller since the linear Landau damping rate depends exponentially on the plasma temperature. As an example, we consider the BRA parameters described in Ref. 7. The parameters of the experiment are the laser wavelength $\lambda=0.8$ μm, the plasma density $n=1.45\times 10^{19}$ cm^{-3}, $\omega/\omega_p=11.5$, the pump intensity of $I_p=2.3\times 10^{14}$ W/cm^2, corresponding to the pump amplitude of $a_0=7.3\times 10^{-3}$, the plasma length of $l=3$ mm corresponding to the dimensionless amplification length of $\tau=7000$, and $\epsilon=0.01$ (empirical value). Then the saturation of Landau damping results in efficient amplification as high as $T=180$ eV plasma temperature rather than $T=120$ eV, which is the critical plasma temperature in the linear regime of Landau damping. At the same time, the ratio of the linear Landau damping rates at these temperatures is about a factor of 4.

VI. CONCLUSIONS

We studied kinetic effects in a BRA including the nonlinear Landau damping and the nonlinear frequency shift of the plasma wave. The analysis was performed using the fluid model developed in Ref. 15.

We found the regime of amplification in which Landau damping is nonlinearly saturated in the presence of a strong plasma wave. The amplitude profile of the amplified pulse can be well approximated with the "π-pulse," same as in damping-free plasma. The π-pulse solution in this regime corresponds to the reduced amplitude of the seed pulse. The effective seed amplitude can be determined by solving the second-order ODE (38)–(40).

The saturated Landau damping can be observed at large enough amplification lengths $\tau > \tau_{\text{Landau}}$, which is defined by Eq. (26). This critical length depends on the amplitude of the seed pulse. Therefore, the plasma wave, amplified from the thermal noise, saturates Landau damping much less efficiently than the plasma wave generated by the seed pulse, since much larger amplification length is required for saturation. This effect can be used to suppress premature pump depletion seeded by the thermal noise.

The nonlinear frequency shift of the plasma wave due to the resonant particles causes the dephasing between the interacting waves. This dephasing reduces the energy transfer from the pump to the amplified pulse. The derived quantitative estimate (53) defines the parameters for avoiding dephasing in BRA. It is shown that for typical BRA parameters the effect of the nonlinear frequency shift is much smaller than the effect of Landau damping in Maxwellian plasmas.

This study extends the parameter region in which BRA can be operated with high efficiency, since large linear Landau damping rate can be strongly reduced during amplification. As a result, higher plasma temperature can be tolerated compared to the regime of nonsaturated Landau damping. Raman compression regimes in which the Landau damping, if unsaturated, becomes problematic occur particularly for compression to very short times at relatively higher plasma density[25] and even more so for compressing very high frequency light (which necessitates even higher plasma densities).[26] It is important, therefore, that the maximum allowed linear Landau damping rate can be up to ten times larger compared to the situation in which nonlinear saturation of Landau damping was not taken into account. At the same time, the increase in plasma temperature, while significant, is smaller (about 50%), since the linear Landau damping rate depends exponentially on the plasma temperature. The increase in plasma temperature corresponds to about 25% increase in $k\lambda_D$.

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