

## Alpha channeling in rotating plasma with stationary waves

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An extension of the alpha channeling effect to supersonically rotating mirrors shows that the rotation itself can be driven using alpha particle energy. Alpha channeling uses radiofrequency waves to remove alpha particles collisionlessly at low energy. It is shown that stationary magnetic fields with high  $n_\theta$  can be used for this purpose, and simulations indicate that a large fraction of the alpha energy can be converted to rotation energy. © 2010 American Institute of Physics.  
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### I. INTRODUCTION

Mirror machines are attractive fusion devices due to their relative simplicity, high  $\beta$ , and naturally steady state operation. Supersonic rotation in mirror machines can produce increased confinement and robustness to instability. The additional confinement is due to the centrifugal force, which confines ions axially.<sup>1-3</sup> Electrons are then confined axially by the ambipolar potential. When the centrifugal potential greatly exceeds the ion temperature, the ion distribution is isotropic and there are no loss cone type instabilities. Rotating plasmas may also be stable to magnetohydrodynamic (MHD) modes for appropriate density profiles, or with significant rotation shear.<sup>4-8</sup>

The rotation is produced by a radial electric field, which is usually maintained by electrodes that are in contact with the plasma. These electrodes introduce many complications into the design of centrifugal confinement devices.<sup>1,9</sup> The foremost problem has been the Alfvén critical ionization velocity (CIV).<sup>1,10,11</sup> This critical velocity limits the speed at which the plasma may flow past a stationary surface, and is several orders of magnitude too small to produce centrifugal confinement of fusion plasmas.<sup>12</sup> Additional limits are imposed on the plasma density by the need for conductivity between the plasma and electrodes.<sup>9</sup> For these reasons it was suggested that rotation could be maintained instead by alpha channeling within the bulk plasma.<sup>13</sup>

Alpha channeling was originally developed in tokamaks to take advantage of the free energy available from alpha particles, as well as to rapidly remove fusion ash from the system.<sup>14</sup> Alpha particles interact stochastically with radio frequency (rf) waves, causing average outward diffusion due to a population inversion.<sup>15,16</sup> Wave energy may then be used to drive electron currents or to heat fuel ions. By removing alpha particles quickly, the fusion reactivity might be doubled in a system with the same plasma pressure.<sup>17</sup>

The alpha channeling concept was recently extended to mirror machines.<sup>18-20</sup> In this case, the loss cone produces a convenient phase space boundary through which alpha particles can be removed. Many of the benefits of alpha channeling in tokamaks, such as increased fusion reactivity, also apply to mirrors. Wave energy in this case might be used for heating ions in the central cell, or electrons in the plug region of a tandem mirror.

An alternate use of alpha particle energy is possible in supersonically rotating mirrors.<sup>13,21,22</sup> By moving particles across the radial electric field, alpha particle kinetic energy can be transferred into potential energy directly. This not only allows efficient removal of alpha particles and direct use of their energy but also eliminates the electrodes that have been a major limitation in producing centrifugally confined plasmas. It was found that particle energy, wave energy, and potential energy could be interchanged using these waves, and a parameter describing the exchange of energy between these sources was called the branching ratio.

To convert a significant amount of the alpha particle energy into rotation energy, waves with large  $n_\theta$  are necessary. We propose that a simple way to produce these waves is to introduce a stationary perturbation on the outside of the plasma, which will appear in the rotating frame as a wave. Because the wave is stationary, the particle's total energy in the rest frame will be conserved. Therefore, only the kinetic and potential energies will be interchanged. The momentum in the system is balanced by forces holding the antenna in place. This is similar to the stationary magnetic resonance used in autoresonant ion cyclotron isotope separation.<sup>23</sup>

We show here that a stationary antenna can produce waves that penetrate the plasma layer and may be used for alpha channeling. By simulating a specific set of waves, we demonstrate that these waves efficiently convert alpha particle kinetic energy to potential energy. Finally, we consider a reactor based on the simulated parameters and find that rotation may be maintained in the absence of electrodes.

Our paper will be organized as follows. In Sec. II, we will describe how rf waves produce diffusion paths in phase space, and introduce the branching ratio. Then in Sec. III we will examine a rotating plasma's response to a stationary perturbation. Lastly, in Secs. IV and V, we will describe our simulation of alpha channeling and its results.

### II. DIFFUSION PATHS

The behavior of an alpha particle interacting with a wave may be considered by adopting a rotating frame with no electric field. In this frame, when a particle interacts with a wave, the changes in its perpendicular and parallel energies are correlated through the wave number and frequency of the wave.<sup>14</sup> The change in energy is also related to a change in

the gyrocenter position of the particle. Because of the radial electric field in the rest frame, this change translates to a change in the particle's potential energy.<sup>13,22</sup>

All of these changes occur simultaneously in the wave region. By relating these variables to the constants of motion— $E$ ,  $\mu$ , and  $P_\theta$ —we can determine the corresponding changes after the particle moves from the wave region to the mirror midplane.<sup>13,18</sup> As the particle interacts repeatedly with the wave, assuming the phase is random, the particle will undergo diffusion along a one-dimensional path in three-dimensional phase space  $\tilde{W}_{\perp 0}-\tilde{W}_{\parallel 0}-\Phi_0$ . Here we have defined  $\tilde{W}_{\perp 0}$  and  $\tilde{W}_{\parallel 0}$  to be the rotating frame perpendicular and parallel energies at the midplane, and  $\Phi_0$  to be the potential energy of the gyrocenter at the midplane.

The unique aspect of alpha channeling in rotating plasmas is the role played by the potential energy. In stationary plasmas there is no such energy sink so the alpha particle energy could only be transferred to the wave. In a rotating plasma, however, alpha particle energy may be transferred to wave energy as well as to potential energy. We defined the branching ratio  $f_E$  to be the fraction of the kinetic energy that is transferred to potential energy,  $f_E = -q\Delta\Phi/\Delta W$ .<sup>13,22</sup> A simple estimate in a slab geometry gives  $f_E = v_E/(v_E - v_{ph})$ , where  $v_E = E_y/B_z$  is the drift velocity and  $v_{ph} = \omega/k_x$  is the wave phase velocity. Including terms related to the cylindrical geometry, we find

$$f_E = \frac{-n_\theta \Omega_E \Omega_i}{\omega \tilde{\Omega}_i - n_\theta \Omega_E \Omega_i}, \quad (1)$$

where  $n_\theta$  is the azimuthal mode number,  $\Omega_E = -E/rB$ ,  $\Omega_i = eB/m$ , and  $\tilde{\Omega}_i = \Omega_i + 2\Omega$  is the cyclotron frequency in the ion frame rotating about the mirror axis at frequency  $\Omega$ .

The branching ratio can be used to characterize three regimes of wave interactions. For  $f_E \approx 0$ , alpha channeling in the rotating plasma will be the same as in the nonrotating mirror case—energy transfer will be primarily between the wave and the particle kinetic energy. Near  $f_E = 1$ , energy is primarily exchanged between kinetic and potential energy. Finally for  $|f_E| \rightarrow \infty$ , the kinetic energy is constant while energy is transferred between the wave and potential energy.

Several conditions are necessary for alpha channeling to be successful. First, there must be a diffusion path that connects the alpha particle source to a region of phase space in which the alpha particle will be lost. The alpha particles should have low energy when they are lost so that their energy can be used in the plasma. Also, there should be a limitation on the heating of alpha particles so that all particles eventually exit by the desired path.

Particles that interact with a wave satisfy the resonance condition  $\omega - n_\theta \Omega - k_{\parallel} v_{\parallel} = n \tilde{\Omega}_i$ . In rotating midplane coordinates, these resonant particles lie on a line in phase space with slope  $(R_{rf} - 1)^{-1}$ , where  $R_{rf} = B_{rf}/B_0$  is the mirror ratio of the rf region. By placing multiple wave regions at different positions in the mirror, we can thus access a large fraction of phase space using few waves, as in Fig. 1.<sup>18,19</sup>

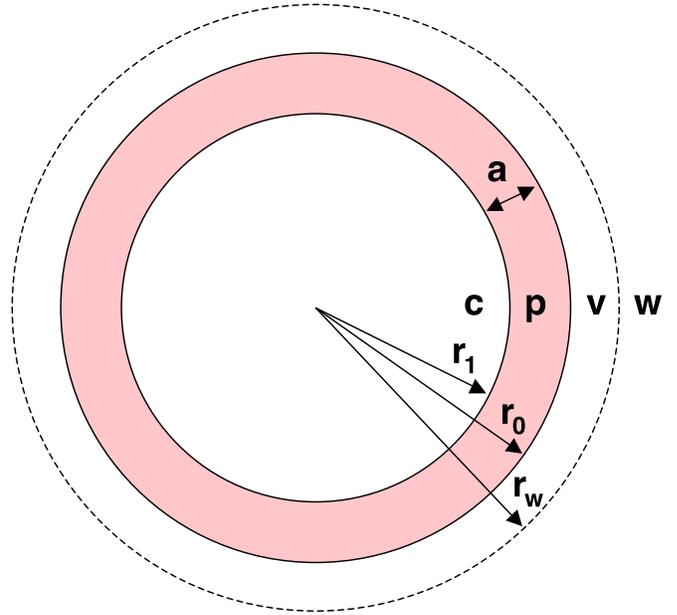


FIG. 1. (Color online) A cross section of the geometry used in determining the plasma response. The central (c), plasma (p), vacuum (v), and wall (w) regions are labeled.

The region will also be offset from the origin along  $W_{\parallel 0}$  by the parallel resonant energy in the wave region  $W_{\parallel \text{res}} = \frac{1}{2} m v_{\parallel}^2$ , plus the centrifugal confinement potential between the wave region and the midplane  $\frac{1}{2} m \Omega^2 r^2 (1 - R_{rf}^{-1})$ . If we use perpendicular diffusion paths, we can ensure that particles will leave at low energy by choosing a resonant parallel energy just above the centrifugal confinement potential. We will also guarantee that there is a population inversion along the diffusion path between the empty region of phase space at the loss cone and the source of alpha particles at high energy.

The final requirement for alpha channeling is that the energy gained by particles is limited. Two methods suggested for limiting heating in stationary mirror machines are by taking advantage of the radial limitation of the plasma and through a zero in the diffusion coefficient due to finite  $k_{\perp} \rho$  effects.<sup>19</sup> It is also possible to limit heating by using a wave that is not entirely perpendicular. In this case, as a particle gains energy from the wave, its parallel energy changes to move it out of resonance. If there is no overlapping wave on the high energy side of the diffusion path, the heating will be limited. This is the limitation that will be used in this paper. Disadvantages to this method are that multiple waves must be used and alpha particles may leave at higher energies.

Once the requirements for alpha channeling are met, we consider the most effective way to use the alpha particle energy. To convert a significant amount of the kinetic energy into rotation energy, the waves must have  $f_E \geq 1$ . The equality  $f_E = 1$  holds for stationary waves with  $\omega = 0$ . The resonance condition then dictates that these waves have  $n_\theta \approx -\tilde{\Omega}_i/\Omega \sim -20$  for practical values of  $\Omega$  and  $\tilde{\Omega}_i$ . In a vacuum such a mode would decay like  $r^{|n_\theta|}$ . Fortunately, the plasma in a centrifugal trap is localized near the outside of the cylinder (as in Fig. 2) so that even with this decay, there

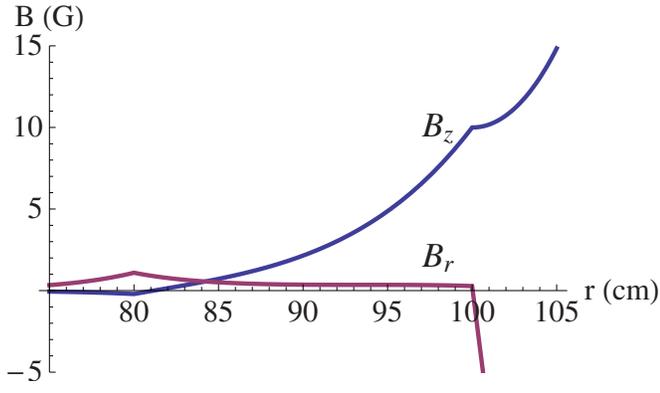


FIG. 2. (Color online) Axial and radial magnetic field vs radius, for  $r_1 = 80$  cm,  $r_0 = 100$  cm, and  $r_w = 110$  cm. The wave is stationary in the laboratory frame, with  $n_\theta = -19$  and  $k_{\parallel} = -0.0463$  cm, and plasma parameters described in the text. The two components are out of phase with each other, and the azimuthal field is not shown.

may be sufficient magnitude for alpha channeling throughout the plasma region. The rotating plasma response to a stationary magnetic perturbation will be calculated in Sec. III.

### III. PLASMA RESPONSE

We will estimate the plasma response to a magnetic perturbation by treating the local geometry as an infinite cylinder. We assume that the plasma occupies a region of width  $a$  from radius  $r_1$  to  $r_0$ , and that the plasma is surrounded by a wall at radius  $r_w$  (see Fig. 2). The wall carries an externally imposed current  $\mathbf{j}^*$  that produces the stationary magnetic wave. We will first solve for the normal mode solutions in each of the vacuum and plasma regions. We will then connect the solutions using appropriate boundary conditions, including the current  $\mathbf{j}^*$  on the wall boundary. This will allow us to determine the fields produced by the antenna everywhere.

For a given magnetic field strength and rotation speed, there is an optimum value for the plasma thickness  $a$ . The reason it is not advantageous to use a thicker plasma region is that the centrifugal force produces an outward pressure everywhere that must be balanced by the magnetic pressure. We can define the ratio of the plasma pressure to magnetic pressure as  $\beta_c = 8\pi(nT + nmar_0\Omega^2)/B_0^2$ .<sup>24</sup> Thus for fixed  $\beta_c$ , with supersonic rotation the achievable density  $n$  is inversely proportional to the plasma thickness. To maximize the fusion power output  $P_{\text{fus}} \propto n^2 a$ , we find  $a/r_0 \approx T/m\Omega^2 r_0^2 \approx 1/2R_m$ , where  $R_m$  is the mirror ratio.<sup>2</sup>

We can simplify calculation of the plasma response by neglecting the difference in rotation frequency between species due to the centrifugal drift. The difference in ion and electron drifts leads to an azimuthal current and a radial variation in the zero-order magnetic field. If  $k_{\parallel} \gg \Omega/v_A$ , the axial variation in the field will be much greater than the radial variation in first order terms, and so we may ignore the contribution of the zero-order azimuthal current. For different ion species rotating at different frequencies, the same wave will appear at different frequencies due to the Doppler

shift. Because this difference is a small fraction of the overall frequency, we may ignore this effect as well, assuming that the wave is far from any resonance.

Thus we are assuming in calculating the plasma response that every component of the plasma is rotating at the same frequency  $\Omega$ . In the frame rotating with  $\Omega$ , the problem is then simply the response of a stationary plasma to an oscillating field with  $\omega = -n_\theta\Omega$ . We can easily calculate the response of a two species cold plasma to such a field using the MHD equations.<sup>25,26</sup> Assuming  $\omega \ll ck_{\parallel}$  and  $\omega \ll \Omega_e$ , one finds

$$B_{pr} = \Lambda c_3 \frac{n_\theta}{r} B_z + \Lambda c_4 \frac{\partial B_z}{\partial r}, \quad (2)$$

$$B_{p\theta} = i\Lambda c_4 \frac{n_\theta}{r} B_z + i\Lambda c_3 \frac{\partial B_z}{\partial r}, \quad (3)$$

$$B_{pz} = BJ_{n_\theta}(k_r r) + CK_{n_\theta}(k_r r). \quad (4)$$

Here  $B$  and  $C$  are constants,  $J$  and  $K$  are Bessel functions, and we have omitted a factor of  $\exp i(k_{\parallel}z + n_\theta\theta - \omega t)$ . We define the constants  $x_i = n_i/n_e$ ,  $\bar{m} = x_i m_i + (1-x_i)m_j$ ,  $v_A^2 = B^2/4\pi n_e \bar{m}$ ,  $\Omega_s = eB/\bar{m}c$ ,  $\Omega_x = \Omega_i \Omega_j / \Omega_s$ ,  $\omega_H^2 = \Omega_x^2 \Omega_s / [x_i \Omega_i + (1-x_i)\Omega_j]$ ,

$$c_1 = 1 + \frac{(\Omega_x - \Omega_i)(\Omega_x - \Omega_j)}{\omega^2 - \Omega_x^2}, \quad (5)$$

$$c_2 = 1 + \frac{\omega^2/\omega_H^2 - 1}{\omega^2/\Omega_x^2 - 1}, \quad (6)$$

$$c_3 = ic_1 \frac{\omega}{\Omega_s} k_{\parallel} \frac{\omega^2}{v_A^2}, \quad (7)$$

$$c_4 = -ik_{\parallel} \left( c_2^2 k_{\parallel}^2 - \frac{\omega^2}{\Omega_s^2} c_1^2 k_{\parallel}^2 - c_2 \frac{\omega^2}{v_A^2} \right), \quad (8)$$

$$\Lambda^{-1} = \left( c_2 k_{\parallel}^2 - \frac{\omega^2}{v_A^2} \right)^2 - \frac{\omega^2}{\Omega_s^2} c_1^2 k_{\parallel}^4, \quad (9)$$

$$k_r^2 = -\frac{1}{\Lambda} \frac{1}{c_2^2 k_{\parallel}^2 - c_2 \omega^2/v_A^2 - c_1^2 \omega^2 k_{\parallel}^2/\Omega_s^2}. \quad (10)$$

Equation (10) may be seen to be a formulation of the two-ion cold plasma dispersion relation so that the wave is equivalent to the fast Alfvén wave.<sup>26</sup>

The solution in the vacuum regions satisfies  $\mathbf{B} = \nabla\psi$  and  $\nabla^2\psi = 0$ . The  $z$ -component of the solution in each region may be written, using only bounded solutions,

$$B_{cz} = AI_{n_\theta}(k_{\parallel}r), \quad (11)$$

$$B_{vz} = DI_{n_\theta}(k_{\parallel}r) + EK_{n_\theta}(k_{\parallel}r), \quad (12)$$

$$B_{wz} = FK_{n_\theta}(k_{\parallel}r), \quad (13)$$

where  $I$  and  $K$  are the modified Bessel functions, and we again omitted the factor of  $\exp i(k_{\parallel}z + n_\theta\theta - \omega t)$ . The other

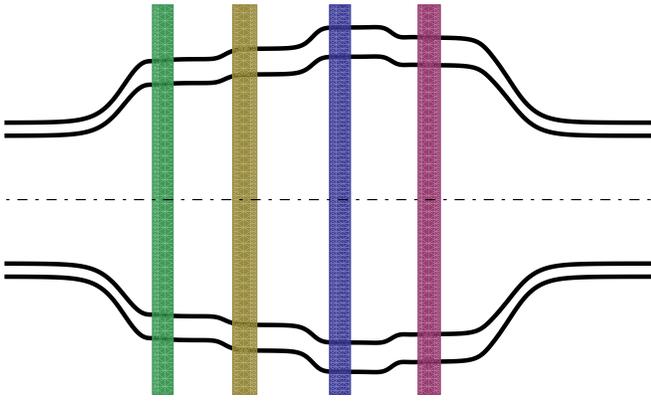


FIG. 3. (Color online) A cross section of the plasma with the wave regions indicated. The black lines denote the inner and outer plasma boundaries. The dashed-dotted line indicates the axis of symmetry.

components of the field are simple functions of the above equations.

We finally consider the boundary conditions. At all boundaries, the perpendicular field  $B_r$  must be continuous. At the free boundaries of the plasma, at  $r_1$  and  $r_0$ ,  $B_z$  must be constant across the boundary for magnetic pressure balance, while at  $r_w$  the current produces the jump  $B_{wz}=B_{vz}+(4\pi/c)j_\theta^*$ .

Solving the system equations (2), (4), and (11)–(13), with the above boundary conditions, we will have a solution to the magnetic field everywhere. To find the electric field, we can simply use Faraday's law,  $\nabla \times \mathbf{E} = -(1/c)\partial \mathbf{B}/\partial t$ . In this case  $E_z=0$  so we can calculate the rotating frame fields  $E_r = \omega/(ck_\parallel)B_\theta$  and  $E_\theta = -\omega/(ck_\parallel)B_r$ .

The results for a stationary wave in a plasma with  $n_e = 1.5 \times 10^{13} \text{ cm}^{-3}$ ,  $B = 2 \text{ T}$ ,  $\chi = 0.046$ ,  $a = 20 \text{ cm}$ ,  $r_0 = 100 \text{ cm}$ , and an equal density of deuterium and tritium ions are shown in Fig. 3. One can see that while the axial field dies off rapidly, the radial field is nearly constant over the plasma region because the derivative of the axial field remains large. Since  $E_\theta = -\omega/(ck_\parallel)B_r$ , this will lead to little radial variation in the phase space diffusion coefficient, which is proportional to  $|E|^2$ .<sup>19</sup>

Because we have assumed the plasma to be of uniform density and rotation frequency, we may be excluding important resonances influencing the accessibility of the wave.<sup>27,28</sup> Since the wave frequency in the rotating frame is proportional to the rotation frequency, a single wave will appear at a broad range of frequencies across a plasma with strong rotation shear. Should there be a resonance, the wave may be absorbed at that layer or may undergo mode conversion. Details of the density, temperature, and rotation profile will be critical in determining whether the wave will reach the inner plasma boundary.

#### IV. SIMULATION

To demonstrate the effectiveness of alpha channeling using these waves, we simulated alpha particles by calculating their full equations of motion in a mirror machine with four separate wave regions (see Fig. 4). We have chosen specific parameters for the simulation, but these are not optimized

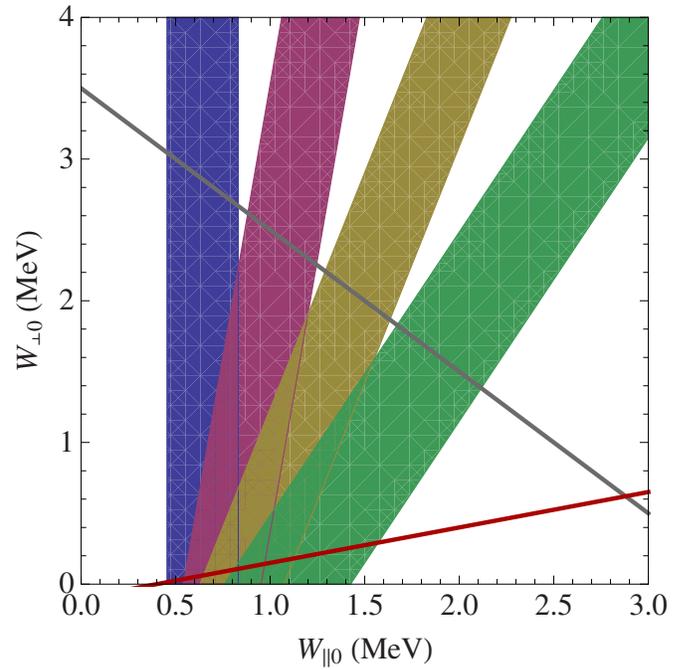


FIG. 4. (Color online) The resonance regions for the waves in rotating, midplane phase space. The diagonal line at 3.5 MeV indicates the alpha particle birth energy. The upward line by the  $x$ -axis indicates the boundary of the alpha particle loss cone.

and only provide an example for the present discussion. The midplane magnetic field is  $B_0 = 2 \text{ T}$ , the ratio of rotation to deuterium cyclotron frequency is  $\chi = 0.046$ , the plasma thickness is 20 cm, the outer radius is 100 cm, and the mirror ratio is 5.  $B_0$ ,  $\chi$ , and  $r_0$  require a radial electric field  $E_0 = 88 \text{ kV/cm}$  and lead to a rotation energy  $W_{E0D} = 200 \text{ keV}$ , which with  $R_m = 5$  gives the ion temperature  $T_i = 40 \text{ keV}$  and electron temperature  $T_e = 12 \text{ keV}$ .<sup>2</sup> With a midplane density of  $n_0 = 1.5 \times 10^{13} \text{ cm}^{-3}$ , the centrifugal beta is  $\beta_c = 0.20$ .

The properties of the waves are shown in Table I. The field everywhere is determined by solving the system of equations described in Sec. III, with the radii  $r_1$ ,  $r_0$ , and  $r_w$  varying like  $R_{\text{rf}}^{-1/2}$  to remain on the same field lines. The currents in the wall layer are calculated based on a specific value of  $B_{pz}(r_0)$  and the given wave parameters. Waves are localized axially using a Gaussian envelope with scale length  $L$ .

Due to centrifugal confinement, we expect the plasma density to be reduced in regions with higher magnetic fields. Both reduced density and increased field lead to an increase in the Alfvén velocity,  $v_A^2 \propto R_{\text{rf}}^2 \exp[W_{E0}(1 - R_{\text{rf}}^{-1})/T_i]$ . With higher fields we also find that larger azi-

TABLE I. Properties of the simulated waves.

$R_{\text{rf}}$	$n_\theta$	$k_\parallel$ ( $\text{cm}^{-1}$ )	$L$ (cm)	$B_{pz}(r_0)$ (G)
1.0	-19	-0.0463	125	10
1.13	-22	-0.0435	100	10
1.3	-26	-0.0401	100	5
1.5	-30	-0.0390	75	5

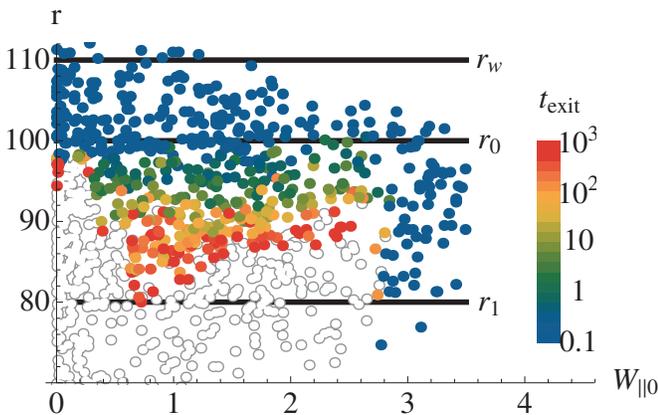


FIG. 5. (Color) The amount of time to exit (in milliseconds) vs initial midplane gyrocenter radius (centimeter) and parallel energy (MeV). Dots indicate individual particles in the simulation and white dots have not left the plasma after 1 s.

muthal mode numbers are required to satisfy the resonance condition  $-n_\theta \Omega \approx \Omega_i$ . If we consider the low frequency limit of the dispersion relation  $k_r^2 = \omega^2/v_A^2 - k_\parallel^2 - k_\theta^2$ , we see that since  $k_\theta^2 = n_\theta^2/r^2 \propto R_m^3$ , the wave will decay more rapidly in regions with higher magnetic fields. For this reason we have located waves in regions with relatively low mirror ratios.

The results of the simulations are shown in Figs. 5 and 6. Since particles were removed at the wall  $r_w = 110$  cm, alpha particles born at the very edge of the plasma have a sufficient gyroradius to exit immediately. This is why the exit time is smaller for these particles and the exit energy higher. Some energy is recovered from these particles, however, since the outer wall is the positive electrode.

For the remaining particles, one can see that alpha channeling efficiency is much better at small initial  $W_\parallel$ . Although the waves are mostly perpendicular, they do have  $\Delta W_\parallel / \Delta W_\perp < 0$ , leading to a slope outward toward larger  $W_\parallel$  for decreasing  $W_\perp$ . Also, particles with large  $W_\parallel$  are nearer to the loss cone and more poorly confined by the magnetic mirror force.

## V. DISCUSSION

It is clear from the simulations that a significant fraction of the alpha particle energy can be transferred to the potential. To discuss the efficiency of alpha channeling, we note that alpha particles in this system are born with significant rotation energy (0.4 MeV) and thermal energy (3.5 MeV). Without alpha channeling or radial losses, alpha particles would have to overcome the centrifugal potential and exit at small radius, giving 0.64 MeV to the potential.

In our simulations, 26% of the particles exit the system immediately, either radially or through the loss cone. 32% of the particles were removed by alpha channeling after 1 s, and the remaining 42% remained in the system. The particles subject to alpha channeling converted 64% of their energy (2.5 MeV) to potential energy on average. If we assume that the remaining particles eventually leave the plasma axially, the average energy that alpha particles add to the potential is 1.38 MeV. Since the energy used to create an alpha particle

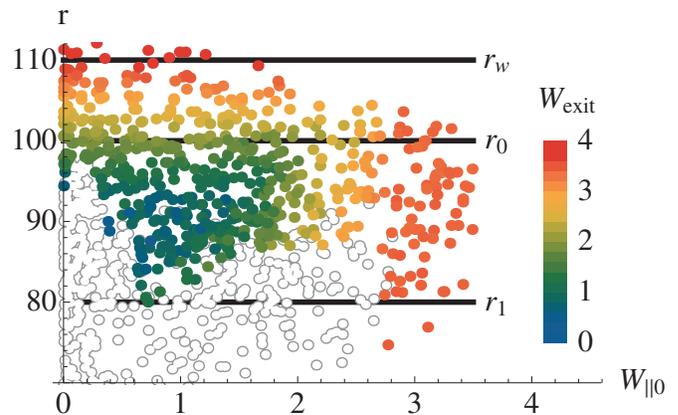


FIG. 6. (Color) The same as Fig. 5, for the alpha particle energy at exit (in MeV).

is 0.4 MeV for the deuteron and 0.6 MeV for the triton, 0.38 MeV remains to compensate other energy loss from the plasma.

Energy loss in centrifugal traps is primarily due to longitudinal ion loss.<sup>2</sup> Deuterons scatter into the loss cone after a time  $\tau_{\parallel D} \approx \tau_{ii} \exp[W_{ED}(1 - R_m^{-1})/T_i]$ , where  $\tau_{ii}$  is the ion-ion collision time and  $R_m$  is the mirror ratio, and tritons have a similar dependence.<sup>29</sup> In the plasma described above,  $\tau_{ii} = 1$  s so that  $\tau_{\parallel D} = 55$  s and  $\tau_{\parallel T} = 400$  s. The fusion timescale is  $\tau_{fus} = 300$  s. Combining these times with the simulated parameters and a plasma length of 40 m, we expect the power lost to be approximately 110 kW. If each fusion event produces 1.38 MeV of potential energy and requires 1 MeV, the net power from alpha particles left in the potential will be 150 kW, exceeding the power loss. The total fusion power produced will be 9 MW. Note that this reactor is not optimized for power production. The power could be increased by increasing the magnetic field or allowing for higher plasma beta.

Before alpha channeling, the efficiency  $Q$  would have been about 30. The above discussion demonstrates that alpha channeling can allow global energy balance without recirculating power, implying  $Q \rightarrow \infty$ . While this efficiency improvement is substantial, more critical is that power does not need to be supplied to the plasma through end electrodes. The electrodes have typically reduced the rotation velocity to the Alfvén CIV limit, or else required extensive conditioning to avoid this limit.<sup>1,12,30</sup> In addition, the electrodes could be a source of instability and require sufficient density for conductivity.<sup>9</sup>

Because eliminating the electrodes entirely is a critical issue, we note that this principle can still be used to produce rotation even if power loss is much larger than expected. One way to compensate for a higher loss rate is to decrease the rotation energy of the plasma, reducing the temperature by the same factor. This would reduce the energy used to create the plasma so that the alpha particle power produced is a larger fraction of the overall power consumed. Alternatively, recirculating power could be used to drive rotation directly with waves.<sup>21,22</sup>

Two effects have been considered in other works that contribute to the power loss, charge exchange and perpen-

dicular heat loss.<sup>2,12</sup> While the cross section for charge exchange is much smaller than the ionization cross section, the energy that is lost in each charge exchange event is larger since neutrals return no energy to the potential on exiting the trap. Regarding the perpendicular heat loss, it will be important to determine the plasma density between the hot plasma region and wall. Because the parallel loss time is much faster than the perpendicular loss time, this density may be negligible. It is still possible that energy will be lost due to heat transfer from a region of fast rotation to slow rotation, where the confinement potential is smaller.

Another effect that should be included in overall power balance is energy recovered at shaped endplates. It was assumed in the simulation that particles are removed at their gyrocenter after exiting the trap axially. However, using shaped endplates the particles may be removed at the high potential side of their gyro-orbit.<sup>2</sup> In past proposals this contribution has been significant but it has been necessary to use many closely spaced electrodes to achieve this effect. Because end electrodes are not necessary in the present scenario, a continuous surface may be used to absorb the outgoing plasma flux, improving the efficiency of energy recovery.

## VI. CONCLUSION

We have shown that stationary waves can sustain rotation in a supersonically rotating mirror. The waves interact with alpha particles, converting alpha particle kinetic energy into potential energy and ejecting the alpha particles. Because the waves are stationary in the laboratory frame, they may be produced by static magnetic coils and do not require significant energy input.

The most significant consequence of alpha channeling is that electrodes are not necessary to maintain rotation in the plasma. Electrodes have been a source of numerous issues with rotating plasmas, most prominently limiting rotation speeds to the Alfvén CIV. It is expected that with the added flexibility in design, systems will be able to exceed the CIV without the need for extensive surface conditioning.

Many steps remain before this system could be considered as a practical fusion device. It remains to be shown that a system driven in this way can achieve a MHD stable equilibrium in its radial density, temperature, and rotation profile. Parameters such as rotation speed, temperature, magnetic field, and radius also need to be optimized for a reactor scenario. Finally, the method of driving rotation and existence

of specific waves require experimental validation, especially in exceeding the CIV limit.

Nevertheless, results so far are promising, indicating that self-sustaining rotation can be produced in centrifugal mirrors with fusion. Because ions are born with the rotation energy, the hot ion mode is automatically produced and heating requires no recirculating power. Thus the system would make an attractive fusion reactor, with simplicity in design and high efficiency.

## ACKNOWLEDGMENTS

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