Controlling Hot Electrons by Wave Amplification and Decay in Compressing Plasma

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Through particle-in-cell simulations, it is demonstrated that a part of the mechanical energy of compressing plasma can be controllably transferred to hot electrons by preseeding the plasma with a Langmuir wave that is compressed together with the medium. Initially, a wave is undamped, so it is amplified under compression due to plasmon conservation. Later, as the phase velocity also changes under compression, Landau damping can be induced at a predetermined instant of time. Then the wave energy is transferred to hot electrons, shaping the particle distribution over a controllable velocity interval, which is wider than that in stationary plasma. For multiple excited modes, the transition between the adiabatic amplification and the damping occurs at different moments; thus, individual modes can deposit their energy independently, each at its own prescribed time.

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Introduction.—Compressing and expanding plasmas are found both in nature and in the laboratory. In particular, target compression in facilities for inertial confinement fusion has been pursued recently through a variety of approaches using laser implosion, magnetized liner implosion, and Z pinches [1]. What is not generally addressed, though, is what happens to waves embedded in such plasmas. To the extent that the oscillations are undamped, it may be expected that they will persist and even grow as the medium is compressed. Then, very interesting phenomena become available that allow using wave compression as a means for plasma manipulation.

The purpose of this Letter is to suggest and demonstrate numerically one of the possible techniques. Specifically, it is shown here that waves can concentrate energy and deposit it inside plasma at a predetermined time much like a *switch*; the wave is first amplified adiabatically, and then its energy is suddenly redirected to heat a selected species of the plasma. Among modes that could exercise such heating, we choose the Langmuir wave as a prototype, for it can be described within the simplest, one-dimensional (1D) electrostatic approximation. Within a hydrodynamic model, Langmuir wave compression was addressed in Ref. [2], and references therein. The focus of the present Letter, though, is on kinetic effects such as the energy transfer to particles. Those are studied here through particle-in-cell (PIC) simulations.

Specifically, the following results are obtained. For the first time, it is shown explicitly in PIC simulations that the wave action (i.e., the number of the wave quanta, or plasmons) is conserved and the wave is amplified adiabatically during slow compression of bulk plasma as long as $\kappa \equiv k\lambda_D \ll 1$, with k being the wave number, and λ_D being the electron Debye length. It is also demonstrated that, as κ increases under 1D longitudinal compression, Landau damping is induced at a specific stage that can be prescribed. Hence, the wave energy is transferred to hot electrons, shaping the particle distribution over a controllable velocity interval, which can be wider than that in stationary plasma. For multiple excited modes, the transition between the adiabatic amplification and the damping occurs at different moments; thus, individual modes can deposit their energy independently, each at a predetermined moment.

Adiabatic amplification.—Suppose that plasma is collisionless, and $\kappa_0 \ll 1$, the index 0 henceforth standing for initial conditions. In this case, a Langmuir wave will not (at first) experience dissipation, so the wave action I is conserved, assuming that compression is slow enough [2]. Suppose also, for simplicity, that the bulk plasma is homogeneous; then, $I = W/\omega$, where W is the wave total energy, ω is the instantaneous frequency, $\omega = \omega_p(1 + 3\kappa^2/2)$, and $\omega_p \propto \mathcal{V}^{-1/2}$ is the electron plasma frequency, $\mathcal{V}$ being the plasma volume [2]. For 1D compression, one has $\mathcal{V} = \epsilon \times \text{const}$, where $\epsilon \equiv L/L_0$ is the normalized length of the plasma. Hence, the action conservation yields

$$W = W_0 g^{-1} \epsilon^{-1/2}, \tag{1}$$

where $g \equiv \omega_p/\omega$ is a factor close to 1 that is found as follows. First, notice that the velocity v of each individual electron changes adiabatically as

$$vL = \text{inv}. \tag{2}$$

Hence, the electron bulk distribution evolves self-similarly, thereby remaining Maxwellian with the thermal velocity $v_T \propto \epsilon^{-1}$. Therefore, $\lambda_D = v_T/\omega_p \propto \epsilon^{-1/2}$ and, since $k \propto \epsilon^{-1}$, one obtains $\kappa = \kappa_0 \epsilon^{-3/2}$, so

$$g(\epsilon) = \left(1 + \frac{3}{2}\kappa_0^2\right) \Big/ \left(1 + \frac{3}{2}\kappa_0^2 \epsilon^{-3}\right). \tag{3}$$

To demonstrate these scalings *ab initio*, an electrostatic PIC code was developed to describe plasma compression in a 1D box with a constant velocity of the right wall,

$\dot{L} = -V < 0$, and the left wall kept fixed. Hard-wall boundary conditions are assumed and total charge neutrality is maintained. Only standing waves with integer $m \equiv kL/(2\pi)$ can be excited; in the following those are termed the mth modes. In our simulations, electrons were initialized randomly as Maxwellian, and, to produce the figures below, ions were modeled as a charge-neutralizing homogeneous background. (For the specific parameters, see Table I.) As a check, PIC simulations of the ion motion were also performed for some cases (see Discussion).

Figure 1 shows that the wave action is indeed conserved in the PIC simulations, confirming the general theory. (Notice that the initial stage corresponds to the right of the figure, where $\epsilon = 1$, and the final stage corresponds to the left, where $\epsilon < 1$.) The same figure also shows that the electrostatic energy $\mathcal{E} = \frac{1}{8\pi}\int \tilde{E}^2 d\mathcal{V}$, where $\tilde{E}$ is the electric field, agrees with the scaling

$$\mathcal{E} = \mathcal{E}_0 g \epsilon^{-1/2}. \tag{4}$$

Since $\mathcal{E} = g^2 W/2$ [2], this supports Eq. (1), as expected.

Induced Landau damping.—What is remarkable is that after these modes are adiabatically amplified, they can be used to heat the electrons at a predetermined moment; this occurs because v_{ph} decreases under 1D compression, whereas v_T increases. To see this in detail, let us consider how the wave collisionless dissipation evolves throughout compression, again assuming that it is negligible initially. Since the plasma remains Maxwellian, the local Landau damping rate is given by [3]

$$\gamma \approx \frac{\omega_p}{\kappa^3}\sqrt{\frac{\pi}{8}}\exp\left(-\frac{1}{2\kappa^2}-\frac{3}{2}\right), \tag{5}$$

so γ increases with compression as κ grows. Hence, strong Landau damping is induced, and all the wave energy eventually dissipates.

Specifically, the damping causes exponential decay of the wave action, $\mathcal{I} = \mathcal{I}_0 \exp(-2\Gamma)$, where $\Gamma = \int_0^t \gamma(t')dt'$. Then, Eq. (4) is generalized as

$$\mathcal{E} = \mathcal{E}_0 g \epsilon^{-1/2}\exp(-2\Gamma). \tag{6}$$

For constant $V = V_0$ (henceforth assumed), one gets

$$\Gamma = \frac{1}{3}\sqrt{\frac{\pi}{2e^3}}\left(\frac{\omega_p}{\kappa\eta}\right)_0 [\epsilon^2 e^{-\epsilon^3/(2\kappa_0^2)} - e^{-1/(2\kappa_0^2)}], \tag{7}$$

where we used an asymptotic expansion at small κ and introduced the compression rate $\eta \equiv V/L$. This prediction is supported by numerical results, as seen in Fig. 2, which illustrates how adiabatic amplification is replaced by sudden decay when ϵ becomes small enough. The difference between the numerical and the analytical curves is due to the fact that the wave that we model is actually in the trapping regime [4]. Specifically, during the sudden decay, we have $\eta \ll \gamma \ll \Omega_b$, where $\Omega_b = \sqrt{qEk/m_e}$ is the characteristic bounce frequency of resonant electrons trapped in the wave, q/m_e is the electron charge-to-mass

TABLE I. The initial parameters used in our PIC simulations. Here $\kappa = k\lambda_D$, $\delta v = \sqrt{qE/(m_e k)}$, $\eta = V/L$; the index 0 denotes the initial conditions. The parameters were chosen arbitrarily to facilitate our proof-of-principle analysis and speed up simulations rather than model a specific experiment. The large value of $\delta v/v_T$ is reduced as $\epsilon^{9/8}$ during adiabatic compression and much faster at dissipation.

Figure	κ_0	$(\delta v/v_T)_0$	$(\eta/\omega_p)_0$	m
Figure 1	0.012	3.04	2×10^{-4}	1
Figure 2	0.12	0.96	3×10^{-4}	1
Figure 3	0.18	2.48	5×10^{-5}	3
Figure 4	0.06	1.92	1×10^{-4}	1
	0.18	1.11	1×10^{-4}	3
	0.30	0.86	1×10^{-4}	5

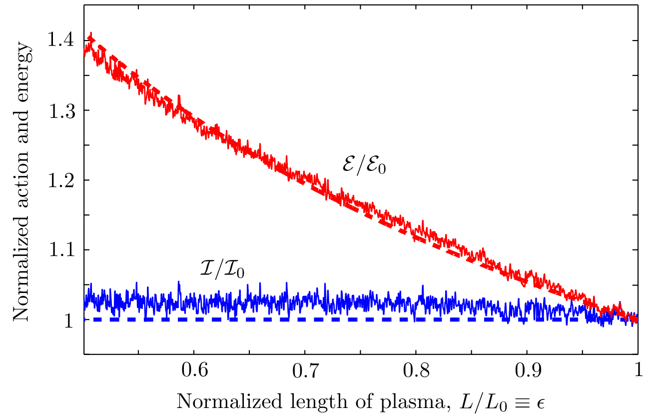

FIG. 1 (color online). The normalized total action, $\mathcal{I}/\mathcal{I}_0$, and the normalized electrostatic energy $\mathcal{E}/\mathcal{E}_0$ (see Table I). Solid: numerical, smoothed over oscillations; dashed: analytical, i.e., $\mathcal{I} = \mathcal{I}_0$ for the action and Eq. (4) for the energy.

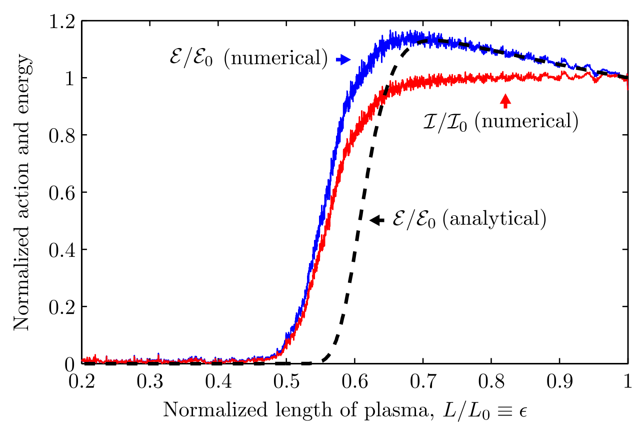

FIG. 2 (color online). Same as in Fig. 1, but showing smaller ϵ and for different parameters (Table I). The figure shows how adiabatic amplification is replaced with decay at small ϵ. The analytical plot corresponds to Eq. (6).

ratio, and E is the electric field amplitude. In this case, γ is smaller than that predicted by Eq. (5), which explains why the damping is delayed during compression as compared to the linear theory. In the example of Fig. 2, note how sudden, or *switchlike* damping sets in once the plasma extent reaches a critical value.

Hot electron distribution.—The induced collisionless dissipation is due to the phase velocity v_{ph} decreasing down to several v_T, after which resonant suprathermal electrons start to absorb the wave energy efficiently. Still, the ratio v_{ph}/v_T continues to decrease gradually after the onset of damping. Thus, within the resonant interval Δv that increases with V, particles are, on average, propelled to higher energies. Unlike in stationary plasma, where a wave would affect only electrons within the interval about $\delta v = \Omega_b/k$ around fixed v_{ph} [5], the original distribution is now essentially shifted by δv over the wider range Δv. For waves in this trapping regime, the distribution also tends to flatten [4,5], and this effect is further augmented when multiple modes are present (see below). After the wave has dissipated, the hot tail undergoes further heating self-similarly through adiabatic compression [Eq. (2)]. The suprathermal tail therefore has a distinct signature that persists as a recognizable feature of a distribution which has once experienced switchlike heating.

To calculate the precise shape of the suprathermal electron distribution by plasma compression is a matter of a separate study, but numerically this distinct effect is evident in our simulations. Figure 3 illustrates the tail formation and flattening for a standing wave, in which case both positive and negative v_{ph} are present, yielding a final distribution even in v. This confirms that part of the mechanical energy of compressing plasma can be controllably transferred to suprathermal electrons by preseeding a Langmuir wave that compresses together with the medium. Notice also that, for a traveling wave, an *asymmetric* suprathermal tail could form. (However, a different, e.g., toroidal, geometry is required to avoid velocity isotropization by the walls.) Hence, a current-drive effect could be produced at a predetermined time in a switchlike manner, greater than that in stationary plasma [6], since it utilizes all the wave energy.

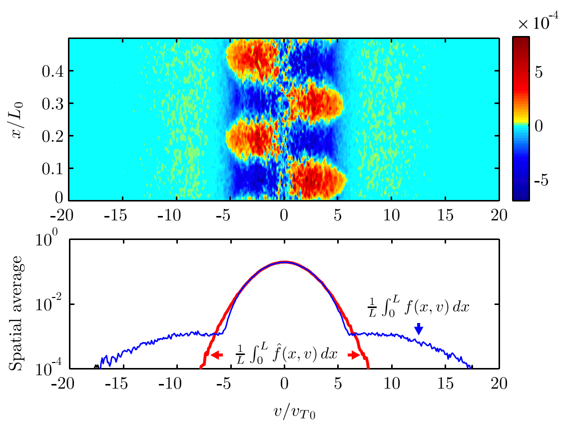

FIG. 3 (color online). The electron distribution function $f(x, v)$ compressed together with a wave (Table I; arbitrary units). Top: $[f(x, v) - \hat{f}(x, v)]$, where $\hat{f}(x, v)$ is the same distribution compressed without the wave; bottom: space-averaged f and space-averaged $\hat{f}$, close to Maxwellian. The figures demonstrate the production of suprathermal electrons. The upper figure also shows remnants of the parasitic second mode, not yet damped unlike the third mode that was most intense initially (cf. Fig. 4).

Multiple modes.—Now consider evolution of multiple linear waves during compression of bulk plasma. In general, only the total number of plasmons is conserved in the absence of dissipation [7], and no invariants exist if *any* of the modes involved are damped. However, if a mode is separated from the rest by a frequency gap $\Delta\omega$ large compared to η, it will, in fact, decouple from the rest. Since higher-k waves have smaller $v_{ph} \approx \omega_p/k$, they will interact sooner with resonant particles and thereby damp more rapidly, as can be seen from Eq. (5). This means that higher-k waves will dissipate earlier during compression, while those with lower k may still be well inside the adiabatic domain. Thus, the energy deposition into hot particles can be timed through choosing which k_0 to preseed.

These predictions are confirmed in our simulations. Figure 4 shows how two standing modes with different spatial numbers m evolve after being seeded in plasma together. For the two cases that are depicted featuring the pairs $m = 1$, 3 and $m = 1$, 5, higher-m modes decay at earlier stages (i.e., larger ϵ) than lower-m ones. Thus, multiple linear modes indeed can affect the electron distribution additively, shaping it (as in Fig. 3) one after another. Particularly, a flatter tail is produced then.

Correlated damping.—Although the modes are largely independent, Figs. 4(a) and 4(b) do reveal a small correlation between the wave damping rates. Notice that the first

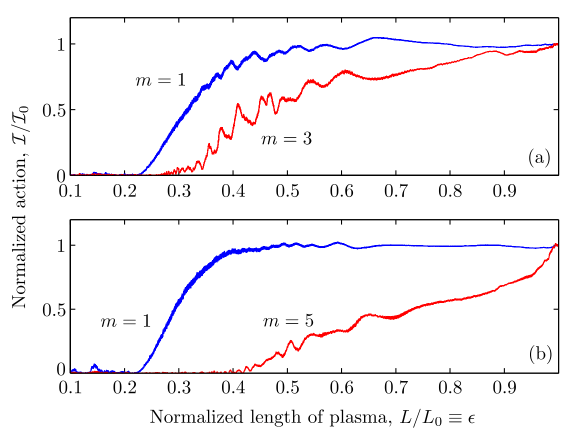

FIG. 4 (color online). The normalized total actions, I/I_0, of paired excited modes with different m (Table I). The figures illustrate independent onsets of collisionless damping. Specifically, modes with larger k dissipate earlier, i.e., at larger ϵ.

mode decays somewhat faster in the presence of the third mode than in the presence of the fifth mode. This is due to nonlinear coupling of the paired waves, which can be understood as follows. Since the field amplitudes are large enough, suprathermal electrons are strongly affected by both modes simultaneously and thereby move stochastically [8], allowing the waves to exchange plasmons. Whereas alone the lowest-order mode would have decayed more slowly, it can now decay faster by transferring action to a higher-m mode, the latter acting like a plasmon sink due to its stronger Landau damping. Since the underlying stochastic effects are more pronounced at smaller differences between the paired v_{ph} [8], the damping of the first mode is more affected by the third mode than the fifth mode.

Discussion.—Insofar as 1D compression of Langmuir waves serves as a realizable model, it remains to check whether the main effects here persist when ion dynamics is taken into consideration, introducing the possibility of nonlinear instabilities and inhomogeneities of the bulk plasma under compression. Among the former, potentially deleterious in our case are the modulational and decay instabilities [9]. However, modulational modes are stabilized at small enough $\delta v/v_T$, which need only be satisfied initially, since $\delta v/v_T \propto \epsilon^{9/8}$ will further decrease during compression. The decay instability, which sets in when κ^2 reaches about m_e/m_i (here m_i is the ion mass), has the rate $\Gamma_d \propto \epsilon^{-1/2}$. Since $\eta \propto \epsilon^{-1}$, this means that having $\eta > \Gamma_d$ at this threshold ensures that the plasma will be compressed faster than the instability develops [10].

Now let us consider the influence of compression-driven gradients. Assume a low-m linear wave, such that $\Delta\omega \gg \eta$, so it is isolated from other modes; then, it can couple to others only through scattering at plasma inhomogeneities. However, the scattering is suppressed with exponential accuracy with respect to L/ℓ, where ℓ is the inhomogeneity spatial scale. This ratio is small at $V/v_{Ti} \ll 1$, where v_{Ti} is the ion thermal velocity. Hence, the latter condition is sufficient for k to be conserved (unlike in the geometrical optics limit [2]), so the inhomogeneity will have little effect on the mode dynamics [10]. Also notice that compression-driven gradients can be avoided completely with ballistic compression [10], such as that of space-charge waves in velocity-chirped beams [11].

In summary, it is predicted here that a part of the mechanical energy of compressing plasma can be controllably transferred to selected species in a switchlike manner by preseeding the plasma with a wave which is compressed together with the medium. Langmuir oscillations are considered as a paradigmatic example. Initially, a Langmuir wave is undamped, so it is amplified under compression due to plasmon conservation. Later, as the phase velocity also changes under compression, Landau damping can be induced at a predetermined instant of time. Then the wave energy is transferred to hot electrons, shaping the particle distribution over a controllable velocity interval, which is wider than that in stationary plasma. Other types of waves in plasma could also exhibit similar effects, and different types of oscillations may be better suited for different applications.

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