Laser duration and intensity limits in plasma backward Raman amplifiers

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The shortest duration and the largest non-focused intensity of laser pulses produced by means of backward Raman amplification (BRA) in plasmas are calculated. These limits occur in moderately undercritical plasmas and are imposed by combined effects of moderately small group velocity dispersion and relativistic electron nonlinearity of the amplified pulses. The efficient BRA range covered by this theory is broader than one known previously. This can be useful for BRA of x-ray pulses in regular or compressed solids and ultra-powerful optical pulses in the lowest density solids. © *2012 American Institute of Physics*. [doi:10.1063/1.3683558]

I. INTRODUCTION

The resonant backward Raman amplification (BRA) of laser pulses in plasmas[1] makes feasible producing laser intensities exceeding the currently available intensities by several orders of value at frequencies ranging from optical to x-ray band.[2–4] In strongly undercritical plasmas, where plasma frequencies are much smaller than laser frequencies, the largest achievable non-focused output intensity is, roughly speaking, proportional to the plasma frequency. This tendency could change, however, in moderately undercritical plasmas, where the plasma frequency is closer to the amplified laser pulse frequency. The larger dispersion of the group velocity is then more capable of interfering with the pulse compression.

The extension of the BRA theory to moderately undercritical plasmas has significant practical implications in both the optical and x-ray laser frequency bands. In fact, for sufficiently short-wavelength x-rays, the entire parameter range for efficient BRA is located in moderately undercritical plasmas.[4] In the optical band, the parameter range for efficient BRA is broader and, in principle, includes strongly undercritical plasmas. Using gas jet technology to make the plasma, experimental demonstrations of the backward Raman amplification output have already been carried out in the optical range[5–10] achieving the output laser intensities hundreds times higher than the input pump intensities and the efficiencies at several percent level[7,8] (although not necessarily reaching the theoretically described pump depletion regimes[11,12]). While the gas jet technology worked well at low power, it might be technologically challenging to extrapolate the present gas jet techniques to produce such plasmas in the volumes needed for compression at the highest power levels, which might be in the range of multi-exawatt to zetawatt powers.[2,13,14] An attractive alternative method to producing the plasma would be to ionize the lowest density solids, like foams or aerogels, but then the resulting plasma would be relatively dense for optical pulses, unless the plasma were allowed to expand, which likely would make most of the density profile non-resonant. The resonant optical BRA could be accomplished prior to the plasma expansion, which would be exactly in the moderately undercritical regime considered here.

The simplest model describing effects of interest here includes Raman backscattering, and moderately small relativistic electron nonlinearity and dispersion of group velocity. The dispersion and self-nonlinearity need to be taken into account only for the amplified pulse which is shorter and grows to intensities greater than that of pump. The parasitic forward Raman scattering (FRS) of the amplified pulse need not to be taken into account here, even though it is potentially capable of limiting the allowed BRA length.[1] This is because the FRS can be suppressed, for example by detuning techniques.[11] In addition, in the moderately undercritical plasmas considered here, the amplified pulse FRS is absolutely impossible at plasma frequencies larger than half of the amplified pulse frequency. The BRA length in this one-dimensional model is limited by the onset of the longitudinal self-phase modulation instability of the amplified pulse. This instability is associated with the relativistic electron nonlinearity. The transverse mode of this filamentation instability need not to be taken into account here, because it is grows from small transverse noise and therefore takes logarithmically larger time to develop.[1] The damping of the resonant Langmuir wave can also be neglected in a significant part of the efficient BRA range.[4] The inverse bremsstrahlung of the laser pulses is negligible for sufficiently high temperature of plasma electrons, except of the laser wavelengths close to the theoretical BRA short-wavelength limit.[4] However, at plasma densities currently accessible in practical laboratory settings, this limit is not reached, so the inverse bremsstrahlung of the laser pulses can be neglected here.

Thus, the effects that must be taken into account here, in addition to 3-wave Raman coupling, are the cubic relativistic electron nonlinearity and group velocity dispersion of the amplified pulse. These effects can be taken into account as independent small corrections, and crossover effects can be neglected as having a higher order of smallness. Although smaller than the Raman coupling, both the dispersion and cubic nonlinearity affect the BRA by producing phase shifts that accumulate over the amplification distance.

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II. BASIC EQUATIONS

The equations describing BRA, taking into account moderately small dispersion of the group velocity and relativistic electron nonlinearity can be presented in the form,[15]

$$a_t + c_a a_z = V_3 f b, \quad f_t = -V_3 a b^*, \tag{1}$$

$$b_t - c_b b_z = -V_3 a f^* - \imath c_b' b_{tt}/2c_b + \imath R|b|^2 b. \tag{2}$$

Here a, b, and f are envelopes of the pump pulse, counterpropagating shorter pumped pulse and resonant Langmuir wave, respectively; subscripts t and z signify time and space derivatives; c_a and c_b are group velocities of the pump and pumped pulses; c_b' is the derivative of the pumped pulse group velocity over the frequency; V_3 is the 3-wave coupling constant (real for appropriately defined wave envelopes), R is the coefficient of nonlinear frequency shift due to the relativistic electron nonlinearity.

The group velocities c_a and c_b are expressed in the terms of the respective laser frequencies ω_a and ω_b as follows:

$$c_a = c\sqrt{1 - \omega_e^2/\omega_a^2}, \quad c_b = c\sqrt{1 - \omega_e^2/\omega_b^2}, \tag{3}$$

where c is the speed of light in vacuum,

$$\omega_e = \sqrt{4\pi n_e e^2/m_e} \tag{4}$$

is the electron plasma frequency, n_e is the electron plasma concentration, m_e is the electron rest mass, and e is the electron charge. Note that

$$\frac{c_b'}{c_b} = \frac{\omega_e^2}{\omega_b(\omega_b^2 - \omega_e^2)} = \frac{\omega_e^2 c^2}{\omega_b^3 c_b^2}. \tag{5}$$

The pump pulse envelope, a, will further be normalized such that the average square of the electron quiver velocity in the pump laser field, measured in units c^2, will be $|a|^2$,

$$\overline{v_{ea}^2} = c^2|a|^2. \tag{6}$$

Then, the average square of the electron quiver velocity in the seed laser field and in the Langmuir wave field will be given by the formulas

$$\overline{v_{eb}^2} = c^2|b|^2\frac{\omega_a}{\omega_b}, \quad \overline{v_{ef}^2} = c^2|f|^2\frac{\omega_a}{\omega_f}. \tag{7}$$

The respective value of the 3-wave coupling constant is[16]

$$V_3 = \frac{k_f c}{2}\sqrt{\frac{\omega_e}{2\omega_b}}, \tag{8}$$

where k_f is the wave number of the resonant Langmuir wave,

$$k_f = k_a + k_b, \quad k_a c = \sqrt{\omega_a^2 - \omega_e^2}, \quad k_b c = \sqrt{\omega_b^2 - \omega_e^2}. \tag{9}$$

The frequency resonance condition is

$$\omega_b + \omega_f = \omega_a, \tag{10}$$

where $\omega_f \approx \omega_e$ is the Langmuir wave frequency in a cold plasma.

The nonlinear frequency shift coefficient R, corresponding to the above normalization of wave envelopes, is[17–19]

$$R = \omega_e^2\omega_a/4\omega_b^2. \tag{11}$$

Under condition of interest here, the dispersion and cubic nonlinear terms in Eq. (2) are small during the first stages of the amplification. As long as these terms are neglected, the solution of basic equations (1) and (2) is well known.[1,20–26] For sufficiently short initial seed pulses, the amplified pulse tends, during an advanced nonlinear stage, to the so-called π-pulse which completely depletes the pump. The π-pulse grows and contracts, so that the amplified pulse duration becomes much smaller than the time of amplification. This allows one to simplify the basic equations by splitting two different time scales, the "fast time" $\tilde{t}$, counted from the seed pulse arrival to a given location, and the "slow time" $\bar{t}$ which is the time of seed arrival to a given location,

$$\bar{t} = \frac{L - z}{c_b}, \quad \tilde{t} = t - \bar{t}. \tag{12}$$

Here L is the width of the amplifying plasma layer. Neglecting the "slow time" derivative compared to the "fast time" derivative in the Eq. (1) for the pump amplitude a, one can present the basic equations (1) and (2) for an advanced nonlinear stage of the amplification in the form,

$$(1 + c_a/c_b)a_{\tilde{t}} = V_3 f b, \quad f_{\tilde{t}} = -V_3 a b^*, \tag{13}$$

$$b_{\bar{t}} = -V_3 a f^* - \imath c_b' b_{\tilde{t}\tilde{t}}/2c_b + \imath R|b|^2 b. \tag{14}$$

As long as the dispersion and cubic nonlinear terms in Eq. (14) are neglected, Eqs. (13) and (14) have a purely real solution of the form,

$$a = a_0\cos(u/2), \quad f = a_0\sqrt{1 + c_a/c_b}\sin(u/2), \tag{15}$$

$$b = -\sqrt{1 + c_a/c_b}\,u_{\tilde{t}}/2V_3, \tag{16}$$

where u satisfies the equation,

$$u_{\tilde{t}\bar{t}} = V_3^2 a_0^2 \sin u. \tag{17}$$

This equation has a particular self-similar solution,

$$u(\bar{t}, \tilde{t}) = U(\xi), \quad \xi = 2V_3 a_0\sqrt{\bar{t}\,\tilde{t}}, \tag{18}$$

where U satisfies the ordinary differential equation,

$$U_{\xi\xi} + U_\xi/\xi = \sin U. \tag{19}$$

The solution of this equation depends on the parameter $U_0 = U(\xi \to +0)$. This parameter has the physical meaning of the integrated amplitude of the initial seed pulse (supposed to be short enough),

$$U_0 = -\frac{2V_3}{\sqrt{1+c_a/c_b}} \int d\tilde{t} b \Big|_{\bar{t}=0}. \tag{20}$$

The amplitude of the amplified pulse growing from such a short seed pulse is expressed in the terms of the function $U(\xi)$ as follows:

$$b = -\sqrt{1+c_a/c_b} V_3 a_0^2 \bar{t} U_\xi / \xi. \tag{21}$$

At $\xi \to \infty$, the function $U(\xi)$ tends to π, oscillating around this value. The respective amplified wavetrain b is called therefore "π-pulse."

For a small initial seed, $U_0 \ll 1$, the leading (and the largest) spike of the "π-pulse" wavetrain can be approximated, around its maximum, by the formula,

$$U \approx 4 \arctan\left(\frac{U_0 \exp \xi}{4\sqrt{2\pi\xi}}\right). \tag{22}$$

It is close to the stationary soliton of Eq. (17), called "2π-pulse." Namely, replacing Eq. (22) by,

$$U \approx 4 \arctan[\exp(\xi - \xi_M)], \quad \frac{U_0 \exp \xi_M}{4\sqrt{2\pi\xi_M}} = 1, \tag{23}$$

leads to the soliton

$$U_\xi = \frac{2}{\cosh \tilde{\xi}}, \quad \tilde{\xi} = \xi - \xi_M. \tag{24}$$

Using formulas (24) and (21) for the shape of amplified pulse, one can calculate the fluence acquired by the leading spike within the amplification time $\bar{t}$ as follows:

$$w_b = \frac{\omega_a k_b}{\pi \xi_M}\left(1 + \frac{c_a}{c_b}\right) a_0^2 \bar{t} G, \tag{25}$$

$$G = \frac{m_e^2 c^4}{e^2} \approx 0.3 \frac{J}{cm}. \tag{26}$$

The duration of the leading amplified spike, defined as the ratio of the fluence to the peak intensity, is given by the formula,

$$\Delta\tilde{t} = \frac{\xi_M}{V_3^2 a_0^2 \bar{t}}, \tag{27}$$

which immediately follows from Eqs. (18), (24), and (21). The peak intensity is then given by the formula,

$$\max_{\tilde{t}} I = \frac{w_b}{\Delta \tilde{t}}. \tag{28}$$

III. LIMITS FOR BRA OUTPUT PARAMETERS

When the dispersion and cubic nonlinear terms are taken into account in Eq. (14), the solution cannot longer be real, but rather has to be searched in general complex form,

$$b = |b| \exp(\imath\phi). \tag{29}$$

For small enough dispersion and nonlinear cubic terms, the phase ϕ is small. Substituting the π-pulse solution into the cubic nonlinear and dispersion terms of Eq. (14), one can calculate the first order approximation to the phase ϕ,

$$\phi = \phi_{nl} + \phi_{dsp}, \tag{30}$$

where

$$\phi_{nl} = R(1 + c_a/c_b) V_3^2 a_0^4 \bar{t}^3 U_\xi^2 / 3\xi^2 \tag{31}$$

is the phase associated with the cubic nonlinearity, and

$$\phi_{dsp} = -\frac{2c_b' V_3^4 a_0^4 \bar{t}^3}{3c_b U_\xi} \frac{d}{d\xi} \frac{d}{\xi d\xi} \frac{U_\xi}{\xi}. \tag{32}$$

is the phase associated with the dispersion.

For small seed pulses, $U_0 \ll 1$, these formulas can be presented in the form,

$$\phi_{nl} = \frac{Q_{nl}}{\cosh^2 \tilde{\xi}}, \quad Q_{nl} = R\left(1 + \frac{c_a}{c_b}\right) \frac{4V_3^2 a_0^4 \bar{t}^3}{3\xi_M^2}, \tag{33}$$

$$\phi_{dsp} = Q_{dsp}\left(1 - 2\tanh^2 \tilde{\xi}\right), \quad Q_{dsp} = \frac{2c_b' V_3^4 a_0^4 \bar{t}^3}{3c_b \xi_M^2}. \tag{34}$$

Interestingly, the ratio of the phase-shift coefficients associated with the dispersion and the cubic nonlinearity,

$$Q = \frac{Q_{dsp}}{Q_{nl}} = \frac{k_f^2 c^4 \omega_e}{4c_b(c_a + c_b)\omega_a \omega_b^2}, \tag{35}$$

depends neither on the pump pulse amplitude a_0, nor on the amplification time $\bar{t}$.

As seen from formulas (33) and (34), the phase shift is maximal at the top of the leading amplified spike, where $\tilde{\xi} = \xi - \xi_M = 0$,

$$\phi(\xi_M, \bar{t}) = Q_{nl} + Q_{dsp}. \tag{36}$$

Note that the phase shift exceeding $\pi/2$ would reverse the energy flow from the amplified pulse back to the pump pulse. In fact, the π-pulse compression/amplification regime becomes significantly affected even earlier, by the phase shift $\phi(\xi_M, \bar{t}) \sim 1$. Therefore, the largest allowed amplification time $\bar{t} = \bar{t}_M$ can be approximated by,

$$\begin{aligned} \delta &= Q_{nl} + Q_{dsp} = Q_{nl}(1 + Q) \\ &= R(1 + Q)\left(1 + \frac{c_a}{c_b}\right) \frac{4V_3^2 a_0^4 \bar{t}_M^3}{3\xi_M^2}, \end{aligned} \tag{37}$$

where $\delta \ll 1$, and δ may be thought of as a "safety factor."

As seen from here, for constant δ, $\bar{t}_M \propto a_0^{-4/3}$, so that, according to Eq. (25), $w_b \propto a_0^{2/3}$. Thus, the larger output fluence of the leading amplified spike can be obtained at larger pump amplitudes. However, the pump amplitude cannot exceed much the wavebraking threshold,[1,16]

$$a_{br} = \frac{\omega_a}{2k_f c}\left(\frac{\omega_e}{\omega_a}\right)^{3/2}. \tag{38}$$

This leads to the following formula for the maximal achievable fluence of the leading amplified spike,

$$w_{bM} = \frac{G\omega_b k_b}{\pi k_f c}\left[\frac{6\omega_a(1+c_a/c_b)^2\delta}{(1+Q)\xi_M k_f c}\right]^{1/3}. \tag{39}$$

The duration $\Delta\tilde{t} = \Delta\tilde{t}_m$, which is the shortest achievable duration of the leading amplified spike, is given by the formula,

$$\Delta\tilde{t}_m = \frac{8}{\omega_e}\left[\frac{\omega_a^2(1+c_a/c_b)(1+Q)\xi_M}{6k_f^2c^2\delta}\right]^{1/3}. \tag{40}$$

These fluence and duration are not very sensitive to specific values of parameters δ, Q, and ξ_M.

The largest achievable peak intensity of the leading amplified spike is given by,

$$I_M = \frac{w_{bM}}{\Delta\tilde{t}_m}. \tag{41}$$

In the limit of highly undercritical plasma, $\omega_e \ll \omega_b$, where $Q \approx \omega_e/2\omega_b \ll 1$ (i.e., the dispersion is much weaker than the cubic nonlinearity), formulas (39) and (40) reduce to,

$$w_{bM} = \frac{G\omega_b}{\pi c}\left(\frac{3\delta}{2\xi_M}\right)^{1/3}, \tag{42}$$

$$\Delta\tilde{t}_m = \frac{4}{\omega_e}\left(\frac{2\xi_M}{3\delta}\right)^{1/3}. \tag{43}$$

In this case, extending the amplification time beyond $\bar{t}_M$ would lead to the splitting of the leading amplified pulse, making no more energy available to the leading pulse.

In the opposite limit of nearly critical plasma, $\omega_b - \omega_e \ll \omega_e$, where $Q \approx \sqrt{3}c/4c_b \gg 1$ (i.e., the dispersion is much stronger than the cubic nonlinearity), formulas (39) and (40) reduce to,

$$w_{bM} = \frac{G\omega_e}{\pi c}\left(\frac{\sqrt{3}\delta}{4Q^2\xi_M}\right)^{1/3}, \tag{44}$$

$$\Delta\tilde{t}_m = \frac{8}{\omega_e}\left(\frac{4Q^2\xi_M}{9\delta}\right)^{1/3}. \tag{45}$$

In the general case, the fluence, duration and peak intensity of leading spike achievable within the amplification time $\bar{t}_M$ can be presented in the form,

$$w_{bM} = \frac{G\omega_b}{\pi c}\left(\frac{\delta}{\xi_M}\right)^{1/3}\hat{w}(r), \quad r = \frac{\omega_e}{\omega_b}, \tag{46}$$

$$\Delta\tilde{t}_m = \frac{8}{\omega_b}\left(\frac{\xi_M}{\delta}\right)^{1/3}\hat{t}(r), \tag{47}$$

$$I_M = \frac{w_{bM}}{\Delta\tilde{t}_m} = \frac{G\omega_b^2}{8\pi c}\left(\frac{\delta}{\xi_M}\right)^{2/3}\hat{I}(r), \tag{48}$$

$$\hat{w}(r) = \frac{\hat{k}_b}{\hat{k}_f}\left[\frac{6\hat{\omega}_a(1+\hat{c}_a/\hat{c}_b)^2}{\hat{k}_f(1+Q)}\right]^{1/3}, \tag{49}$$

$$\hat{t}(r) = \frac{1}{r}\left[\frac{\hat{\omega}_a^2(1+\hat{c}_a/\hat{c}_b)(1+Q)}{6\hat{k}_f^2}\right]^{1/3}, \tag{50}$$

$$\hat{I}(r) = \frac{\hat{w}(r)}{\hat{t}(r)}, \quad Q = \frac{\hat{k}_f^2 r}{4(\hat{c}_a+\hat{c}_b)\hat{c}_b\hat{\omega}_a}, \tag{51}$$

$$\hat{\omega}_a = \frac{\omega_a}{\omega_b} = 1 + r, \quad \hat{k}_a = \frac{k_a c}{\omega_b} = \sqrt{1+2r}, \tag{52}$$

$$\hat{k}_b = \frac{k_b c}{\omega_b} = \sqrt{1-r^2}, \quad \hat{k}_f = \hat{k}_a + \hat{k}_b, \tag{53}$$

$$\hat{c}_b = \frac{c_b}{c} = \hat{k}_b, \quad \hat{c}_a = \frac{c_a}{c} = \frac{\hat{k}_a}{1+r}. \tag{54}$$

The maximal averaged square of the electron quiver velocity in the amplified spike field is given by the formula,

$$\overline{v_{bM}^2} = c^2\left(\frac{\delta}{\xi_M}\right)^{2/3}\hat{v}_b^2, \quad \hat{v}_b^2 = \frac{\hat{I}(r)}{2\hat{k}_b}. \tag{55}$$

Due to the small factor $\delta/\xi_M \ll 1$, the electron motion stays non-relativistic $\overline{v_{bM}^2} \ll c^2$.

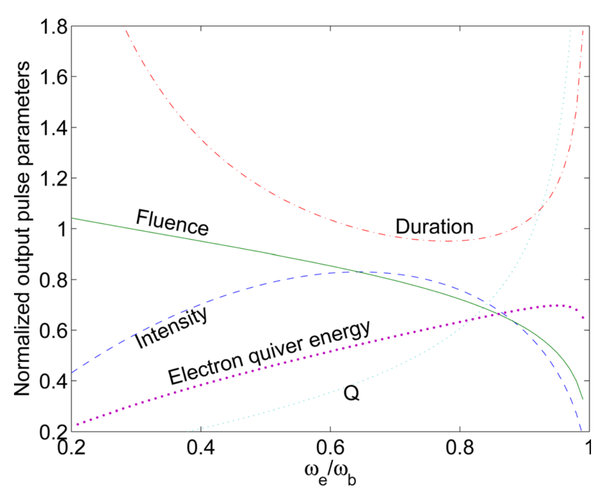

FIG. 1. (Color online) The normalized fluence $\hat{w}(r)$ (solid line), duration $\hat{t}(r)$ (dash-dotted line), peak intensity $\hat{I}(r)$ (dashed line), and electron quiver energy $\hat{v}_b^2$ (bold-dotted line) of the leading amplified spike as functions of the ratio ω_e/ω_b plasma to seed laser frequency. The ratio Q of the phase shifts associated with the dispersion and cubic nonlinearity is shown by the thin-dotted line.

Plots of the normalized fluence $\hat{w}(r)$, duration $\hat{t}(r)$, peak intensity $\hat{I}(r)$, and electron quiver energy $\hat{v}_b^2$ of the leading amplified spike are shown in Fig. 1, along with the ratio Q of the phase shifts associated with the dispersion and cubic nonlinearity. As seen, the shortest duration $\hat{t} \approx 1$ and the largest intensity $\hat{I} \approx 0.8$ of the BRA output laser pulses can be obtained specifically in moderately undercritical plasmas with $r = 0.5 \leftrightarrow 0.85$ ratios of the plasma to seed laser frequency. In this range, the output fluence $\hat{w} \sim 0.8$ is also reasonably close to the maximum value $\hat{w} = (3/2)^{1/3} \approx 1.14$ achievable in strongly undercritical plasmas. The coefficient Q grows with plasma density in this range from $Q \approx 0.3$ to $Q \approx 0.7$. Since $Q < 1$, the dispersion effect is somewhat smaller than the effect of cubic nonlinearity. However, such a dispersion may already be capable of delaying the development of the self-phase modulation instability associated with the cubic nonlinearity. This could enable obtaining even larger output fluences, as explained in Ref. 15, in situations when the highest fluence is more desirable than the shortest duration of the output pulse. In such situations, the "safety factor" δ could be allowed to grow up to values of the order of 1.

IV. DISCUSSION

The above theoretical results are applicable for all laser wavelengths not too close to the BRA short-wavelength limit. The most important applications of these results are associated with backward Raman amplification and compression in the x-ray and optical bands. The x-ray BRA appears possible, so long as it is restricted, in fact, primarily to moderately undercritical plasmas.[4] In the optical band, the final step to reach ultra-powerful BRA should be accomplished in moderately undercritical plasmas.[14] Such plasmas could be produced by ionizing foams or aerogels. For example, the wavelength of NIF laser pulses, $\lambda = 0.351\mu$m, corresponds to the critical plasma electron concentration $n_{cr} = 9 \times 10^{21}$ cm^{-3}. Such plasma could be produced by direct ionization of a foam of density $\rho = 30$ mg/cm^3. To enable the Raman backscattering of pump laser pulses, the plasma concentration should be smaller than the critical concentration for seed laser pulses, which is approximately 4 times smaller than for pump pulses, so that respective foam density should not exceed $\rho = 7.5$ mg/cm^3. In fact, it must be even smaller, because of the thermal addition to the resonant Langmuir wave frequency.

Note that the results obtained here are in a very different regime than those considered in recent particle-in-cell (PIC) simulations,[27,28] which suggested operation in strongly undercritical plasmas, noticeably above the Langmuir wavebreaking threshold. Those studies were apparently driven towards the strongly undercritical regime in order to avoid FRS. However, as noted here, FRS is completely absent for plasma frequencies larger than half of the amplified pulse frequency, and there are methods for suppressing FRS (through detuning) at smaller plasma frequencies. Thus, the moderately undercritical plasma considered here was not taken into consideration in the recent simulations.[27,28] Note as well that in the moderately undercritical plasma regimes considered here, the wavebreaking threshold is not exceeded and relatively less powerful seed pulses can be employed compared to the regimes described in these recent simulations.

V. SUMMARY

To summarize: the BRA theory is extended to moderately undercritical plasmas taking into account the dispersion of group velocity and relativistic electron nonlinearity of amplified pulses. Approximate scalings are found analytically, through which the BRA output pulses in multi-dimensional parameter space are expressed in terms of universal functions of the single parameter—the ratio of the plasma frequency to the seed laser frequency. The shortest duration and the largest non-focused intensity of the BRA output pulses are calculated explicitly and shown to occur specifically in moderately undercritical plasmas. This provides a much needed guide for further experimental and numerical studies, which otherwise would be hardly capable of capturing the most important BRA regimes in multi-dimensional parameter space.

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