Parallel rf Force Driven by the Inhomogeneity of Power Absorption in Magnetized Plasma

Zhe Gao,[1,2,*] Jiale Chen,[2] and Nathaniel J. Fisch[3]

[1]*Department of Engineering Physics, Tsinghua University, Beijing 100084, China*
[2]*Institute of Plasma Physics and Center for Magnetic Fusion Theory, Chinese Academy of Sciences, Hefei 230031, China*
[3]*Princeton Plasma Physics Laboratory, Princeton University, Princeton, New Jersey 08543, USA*
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A nonlinear parallel force can be exerted through the inhomogeneity of rf resonant absorption in a magnetized plasma. While providing no integrated force over a plasma volume, this force can redistribute momentum parallel to the magnetic field. Because flows and currents parallel to the magnetic field encounter different resistances, this redistribution can play a large role, in addition to the role played by the direct absorption of parallel momentum. For nearly perpendicular propagating waves in a tokamak plasma, this additional force is expected to affect significantly the toroidal rf-driven current and the toroidal flow drive.

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Toroidal current and flow play key roles in confining and stabilizing magnetized plasma in tokamaks. The use of rf power to drive plasma current and flow is, therefore, of great interest. To optimize the steady-state operation of high-performance plasma, it remains vital to increase the drive efficiency and to improve the precision of rf-driven current and flow profiles.

When the injected rf power is absorbed in the plasma, the amplitudes of rf fields decay in space. The inhomogeneous field exerts the well-known ponderomotive force [1] in the direction of the inhomogeneity. However, in the symmetric directions, for example, inside the flux surface of torus, the nonresonant forces vanish upon flux-surface averaging [2,3]. In particular, the parallel force, along the background magnetic field, is resonant (i.e., dissipative) locally [4]. The parallel current and flow is, therefore, unlikely to be driven with high efficiency through nonresonant mechanisms.

There do remain possibilities for high efficiencies within the resonant mechanism. One is the rearranging of phase space. If a gradient in the ponderomotive potential is introduced in the direction of the current, the current drive arises from rearranging particle phase space through the Maxwell demonlike effects [5–9]. These ratchet-type effects can be quite efficient compared to the traditional current-drive mechanisms [10]. However, a sharply inhomogeneous, parallel magnetic field is required, which is difficult to produce in a torus. Another idea is the rearranging of the absorbed momentum in real space through the stress. The dissipative stress can introduce a poloidal resonant force even when the poloidal momentum of waves vanishes [11,12]. The poloidal force arises, because, due to the gyromotion, the poloidal perturbed motion is dominated by the radial wave field, and *vice versa* [13]. Then the quasilinear stress in the perpendicular surface is proportional to the perpendicular power absorption. Thus, the inhomogeneity of power absorption induces a poloidal force. This mechanism works even in cold plasmas; however, it obviously cannot be applied to the parallel force. A new mechanism will be required to enhance the parallel current or flow drive.

Here, we show that, because of the nonlinear stress in thermal plasmas, a parallel rf force can be driven through the inhomogeneity of resonant absorption. The poloidal rf momentum injection is necessary to generate this force; therefore, for nearly perpendicular propagating waves, the total parallel rf force is expected to be significantly enhanced over the direct drive force (DDF) which corresponds to the parallel momentum absorption rate. Fine control of parallel current and flow is important in tokamaks, since precise profiles of current and flow usually determine the plasma stability. Waves are often introduced in a narrow absorption layer, which maximizes the ponderomotive effect. The parallel rf force driven by the inhomogeneity of resonant absorption uncovered here, therefore, may be particularly important in shaping correctly these profiles.

The basic idea of driving the parallel rf force through the inhomogeneity of the resonant absorption is as follows. The inhomogeneous transport of parallel momentum generates a distributed parallel force without providing an integrated force over a plasma volume. Since the parallel momentum is orthogonal to the gyromotion, the Reynolds-type stress with two first-order fluctuating velocities cannot contribute to the transport of parallel momentum after gyro-averaging. However, a second-order slowly varying radial drift velocity (we may call it "the transporter") can transport the zero-order parallel momentum (we may call it "the passenger") as well. Considering a symmetric Maxwell distribution, the actual "passenger" is only the resonant absorbed parallel momentum, whose direction is decided by that of the wave. Meanwhile, to generate a "qualified transporter", i.e., an inhomogeneous radial drift, poloidal rf momentum absorption is necessary, which is due to the resonant mechanism as well. Thus, this kind of force depends on the resonant absorption of rf waves.

In the following, this parallel rf force driven by the inhomogeneity of resonant absorption will be derived. The rf force on a plasma fluid element is calculated based on the force on a single particle and averaging over the velocity distribution. This approach is helpful to clarify the physics of this additional force. The same general approach has been helpful in explaining the physics mechanism of collisionless damping [14] or in clearing up the dispute on nonresonant current drive [4].

First, we show that this parallel rf force due to inhomogeneity of resonant absorption does NOT appear in the force on a single particle. Assuming a constant magnetic field $\mathbf{B}_0 = B_0\hat{\mathbf{z}}$, wave field $\mathbf{E}_1 = \hat{\mathbf{E}}_1(x)\exp(-i\omega t + i\mathbf{k}\cdot\mathbf{r})$, and a spatially constant unperturbed velocity $\mathbf{v}_0$ (which implies finite temperature), then the perturbed velocity $\mathbf{v}_1$ and displacement $\mathbf{r}_1$ can be solved from the motion equation of a single particle. For simplicity, $q = 1$ and $m = 1$ are used and then the power absorption of a single particle is

$$\dot{w}_{\rm sp} = \mathbf{v}_1 \cdot \mathbf{E}_1 + \mathbf{v}_0 \cdot (\mathbf{r}_1 \cdot \nabla)\mathbf{E}_1. \tag{1}$$

Here the nonlinear product of two oscillating quantities AB indicates the abbreviation of $AB = \mathrm{Re}\langle AB^*/2\rangle$ unless otherwise specified, where $\langle\cdot\rangle$ means averaging over the period of wave field, and the eikonal approximation $\nabla\mathbf{E}_1 = i\mathbf{k}\mathbf{E}_1 + \nabla_E\mathbf{E}_1$ is used, where ∇_E means the gradient operator only on the amplitude of fields. Then the nonlinear rf force on a single particle can be written as

$$\mathbf{f}_{\rm sp} = \mathbf{k}\dot{w}_{\rm sp}/\omega + \mathbf{f}_{\rm pond}. \tag{2}$$

Here, the first term is the resonant momentum absorption rrate, which is along the direction of wave propagation; the second is the ponderomotive force, $\mathbf{f}_{\rm pond} = -\nabla_E \mathrm{Im}(\dot{w}_{\rm sp}/\omega)$, which is nonresonant, but along the direction of the gradient of wave fields. Clearly, unless the asymmetry along the field is specifically built-in [5,6], there is no parallel force due to the inhomogeneity of wave fields within the single particle picture.

Now let us analyze the force on a fluid element. Neglecting the second-order spatial derivatives of second-order quantities. i.e., the $O(\nabla^2)$ terms, the rf force density on a fluid element can be written as [3]

$$\begin{aligned}\mathbf{F}_f/n_0 = \mathbf{f}_{\rm sp} - \nabla\cdot[&\mathbf{r_1}(\mathbf{E}_1 + \mathbf{v}_0\times\mathbf{B}_1)\\ &+ \mathbf{v}_0\mathbf{v}_2 + \mathbf{v}_1\mathbf{v}_1 + \mathbf{v}_2\mathbf{v}_0],\end{aligned} \tag{3}$$

where $\mathbf{v}_2$ is a second-order quantity, which is averaged over the period of wave and therefore slowly varying in time and space. Besides the force on individual particles, the fluid element encounters force from surface stress, including the polarization stress $\mathbf{r_1}(\mathbf{E}_1 + \mathbf{v}_0\times\mathbf{B}_1)$, Reynolds stress $\mathbf{v}_1\mathbf{v}_1$, and the nonlinear stress $\mathbf{v}_0\mathbf{v}_2 + \mathbf{v}_2\mathbf{v}_0$ which describes the transport of second-order momentum by a zero-order flux and of zero-order momentum by a second-order slowly varying flux. Using the single particle motion equation, we have $\mathbf{r_1}(\mathbf{E}_1 + \mathbf{v}_0\times\mathbf{B}_1) + \mathbf{v}_1\mathbf{v}_1 = d_t(\mathbf{r}_1\mathbf{v}_1) - \mathbf{r}_1\mathbf{v}_1\times\mathbf{B}_0$. When $\mathbf{v}_0 = 0$, $\mathbf{r}_1\mathbf{v}_1$ is a pure oscillating term, and then $\mathbf{F}_f/n_0 = \mathbf{f}_{\rm sp} + \nabla\cdot[\mathbf{r}_1\mathbf{v}_1\times\mathbf{B}_0]$. The force reduces to that in the cold fluid limit [3], where the surface force exists only perpendicularly. Therefore, the enhanced parallel rf force, due to inhomogeneity of the resonant absorption, only appears in thermal plasma, i.e., with finite $\mathbf{v}_0$.

To proceed, we need integrate the force in Eq. (3) over unperturbed velocity $\mathbf{v}_0$ for Maxwell distribution with consideration of the behavior at steady status, so only the resonant terms with the form of $\sin(\omega t)/\omega$ remains. One can then get the averaged parallel force on a fluid element. To simplify the analysis, we assume $\mathbf{v}_0 = v_{0z}\hat{\mathbf{z}}$ so that finite Larmor radius effects are neglected. Since the inhomogeneity is only in the x direction, the parallel component of Eq. (3) simplifies to

$$F_{fz}/n_0 = k_z\dot{w}_{\rm sp}/\omega - \partial_x d_t(x_1 v_{1z}) - \partial_x(v_{2x}v_{0z}). \tag{4}$$

Here, the first term is the parallel momentum absorption rate by individual particles, which is proportional to the power absorption of single particle,

$$\begin{aligned}\dot{w}_{\rm sp} = &\sum_{\pm}\frac{\sin(\bar{\omega}_{\pm 1}t)}{\bar{\omega}_{\pm 1}}\Big\{\frac{\bar{\omega}_0}{4\omega}|E_\pm^2| + \frac{v_{0z}^2}{4\omega\bar{\omega}_0}(k_\perp^2 \mp k_y\partial_x)|E_{1z}^2|\\ &+ \mathrm{Re}\Big[\frac{v_{0z}}{2\omega}(k_x - i\partial_x \pm ik_y)E_{1z}E_\pm^*\Big]\Big\}\\ &- \frac{1}{2}\partial_{\bar{\omega}_0}\Big[\frac{\omega\sin(\bar{\omega}_0 t)}{\bar{\omega}_0}\Big]|E_{1z}^2| + \frac{k_y v_{0z}^2}{2\omega\Omega}\frac{\sin(\bar{\omega}_0 t)}{\bar{\omega}_0}\partial_x|E_{1z}^2|\\ \equiv &\ \dot{w}_c + \dot{w}_L + \dot{w}_a,\end{aligned} \tag{5}$$

where $\bar{\omega}_l = \omega - l\omega_c - k_z v_{0z}$, $k_\perp^2 = k_x^2 + k_y^2$, and $E_\pm = E_x \pm iE_y$. Here, $\dot{w}_c$ is the cyclotron damping with lowest order (due to the neglecting of perpendicular temperature), which includes X-mode damping, O-mode damping, and their mixture, corresponding to three terms in the braces, respectively; $\dot{w}_L$ is Landau damping, and $\dot{w}_a$ is the work of the electric field variation due to instantaneous vertical displacement, which appears at finite k_y and finite v_{0z} and therefore is not shown in previous similar derivations [14]. The second term in the right-hand side of Eq. (4) comes from the polarization stress and Reynolds stress, whose effective component is just $-2k_z\dot{w}_a/\omega$, and the third term comes from the nonlinear stress, where the nonlinear radial drift velocity is driven by the poloidal rf force, i.e., $v_{2x} = f_{{\rm sp}y}/\Omega = k_y\dot{w}_{\rm sp}/(\omega\Omega)$. Thus, Eq. (4) is rewritten as

$$\frac{F_{fz}}{n_0} = \frac{k_z}{\omega}(\dot{w}_c + \dot{w}_L - \dot{w}_a) - \frac{k_y v_{0z}}{\omega\Omega}\partial_x(\dot{w}_c + \dot{w}_L + \dot{w}_a). \tag{6}$$

It may seem that these two terms are somewhat inconsistent. This inconsistency arises from different definitions of electromagnetic forces (and, therefore, the energy-momentum absorption rates) in the pictures of single particle and fluid, but it can be eliminated after ensemble averaging. Multiplying Eq. (6) to the parallel equilibrium distribution function $f_z(v_{0z})$ and integrating the product, one can get the averaged parallel force on a fluid element, $\bar{F}_{fz}$. Before this, it is useful to introduce the power absorption density due to a different mechanism,

$$S_{c,L,a} \equiv \int_{-\infty}^{+\infty} \dot{w}_{c,L,a} f_z dv_{0z} = \frac{\pi}{k_z} \dot{w}_{c,L,a} f_z|_{v_{0z}=v_l^{\mathrm{res}}}, \quad (7)$$

where $v_l^{\mathrm{res}} = (\omega - l\Omega)/k_z$ is the resonant velocity for $\bar{\omega}_l = 0$. Noted that $\int_{-\infty}^{+\infty}[k_z\dot{w}_a/\omega + k_y v_{0z}\partial_x \dot{w}_L/(\omega\Omega)]f_z dv_z = k_y v_0^{\mathrm{res}} \partial_x S_L/(\omega\Omega)$ and, also, that $\partial_x \dot{w}_a$ is a term of $O(\nabla^2)$, which can be neglected. Then we get a perfect simplification that S_a vanishes. Defining a new power absorption density $S = S_c + S_L$, the parallel force $\bar{F}_{fz}$ is now rewritten in a more compact form,

$$\bar{F}_{fz} = \frac{k_z}{\omega} S - \frac{k_y v_l^{\mathrm{res}}}{\omega\Omega} \partial_x S, \quad (8)$$

where $l = 0$ and ± 1, corresponding to different terms in S, i. e., the Landau damping term or R or L wave cyclotron damping terms. In fact, S has a more clear physical meaning than the power absorption defined from the single particle, $\dot{w}_{\mathrm{sp}}$. It includes Landau damping and cyclotron damping, but excludes the contribution from the $\dot{w}_a$. It is just the local power absorption density by a mass of plasma. The following analysis from the kinetic theory indicates this point.

The derivation within the kinetic theory is rather direct. The time averaging of the second-order distribution function can be obtained as [11,12]

$$f_2 = -\lim_{\gamma\to 0} \int_0^t dt' e^{\gamma(t'-t)} (\mathbf{E}_1{}' + \mathbf{v}' \times \mathbf{B}_1{}') \cdot \partial_{\mathbf{v}'} f_1(\mathbf{r}', \mathbf{v}', t'). \quad (9)$$

Here, f_2 consists of two parts: $f_2^{(a)}$ is the divergence in velocity space and $f_2^{(b)}$ is the divergence in real space, i.e., $\int f_2 d\mathbf{v} = \int f_2^{(b)} d\mathbf{v} = n_2$. Excluding the power flux into or out of the volume element due to the density variation, the local power absorption density in the kinetic theory is defined as $S = \int (v^2/2)\partial_t f_2^{(a)} d\mathbf{v}$. After a complicated calculation we get

$$S = \iint dk_R dk_L e^{i(k_R - k_L)x} \sum_l W_l, \quad (10)$$

where $k_{R,L}$ are the radial wave numbers of two interacting wave fields and $W_l = e^{il(\theta_R - \theta_L)} \int d\mathbf{v}[f_0/(-i\bar{\omega}_l T)](\mathbf{H}_l \cdot \mathbf{E}_R)(\mathbf{H}_l \cdot \mathbf{E}_L)$ with $\mathbf{H} \cdot \mathbf{E} = E_z v_z J_l + \sum_{\pm} E_{\pm} v_{\perp} J_{l\mp 1} e^{\mp i\theta}/2$ and $\theta_{R,L} = \tan^{-1}(k_{R,L}/k_y)$. Once f_2 is solved, one can easily get the nonlinear rf force. Its parallel component, $\bar{F}_{kz} = \partial_t \int v_z f_2 d\mathbf{v}$, is

$$\bar{F}_{kz} = \frac{k_z}{\omega} S - \partial_x \left[\iint dk_R dk_L e^{i(k_R - k_L)x} \sum_l \frac{k_y v_l^{\mathrm{res}}}{\omega\Omega} W_l \right]. \quad (11)$$

Comparing Eq. (11) with Eq. (8), which is from the particle-fluid picture, they are almost the same except for high-order cyclotron damping retained in Eq. (11). It implies that the assumption of zero perpendicular temperature does not alter the physics that interests. Therefore, the mechanism can be clarified from the particle-fluid picture. Poloidal momentum absorption drives a radial nonlinear drift flux. This flux is radially inhomogeneous, and, therefore, transports different parallel momentum fluxes into and out of the surface of the fluid element. Since only resonant particles feel the momentum absorption and nonlinear drift, only resonant absorbed parallel momentum is transported. Then, besides the DDF due to direct parallel momentum absorption, an additional parallel force is generated, which is resonant, but is related to the radial inhomogeneity of poloidal momentum absorption. Since this force is proportional to the gradient of the second-order field amplitude products (although not along the direction of the gradient), we may name it the resonant ponderomotive force (RPF).

The RPF is a gradient force. If the rf waves are completely absorbed, the integral of the RPF over the whole plasma region is zero. Obviously, it has opposite signs on opposite sides of the strongest deposition layer. It enhances the parallel drive on one side, and has a weakening effect on the other side. However, since there are different collisionality and different background magnetic fields at different radial positions, the driven current or flow cannot be canceled altogether. At the same time, the profiles of current and flow are shaped by the RPF as well.

It is valuable to estimate the magnitude of the RPF. Assuming that the scale length of power deposition is L_{rf}, the ratio of the RPF to the DDF is $F_{\mathrm{RPF}}/F_{\mathrm{DDF}} = k_y \rho v_l^{\mathrm{res}}/(k_z L_{\mathrm{rf}} v_t)$. It implies that for nearly perpendicular propagating wave the effect of the RPF might be significant.

One example is the lower hybrid wave (LHW), where $k_\perp^2/k_z^2 \sim m_i/m_e$. As the LHW propagates into the plasma, $k_\perp$ gradually turns from the radial direction to tangential direction. A recent calculation [15] shows that a very high poloidal wave vector is developed, which is more than ten times larger than the toroidal wave vector. This poloidal wave vector increases the parallel refractive index, and generates a tangential momentum input as well. For the LHW, the resonant velocity v_l^{res} is several times of v_{te} (for example, $v_l^{\mathrm{res}}/v_{\mathrm{te}} = 6.5$ for $n_\parallel = 2$ and $T_e = 3$ keV) and the deposition layer is usually a fraction of minor radius (for example $L_{\mathrm{rf}} = a/3$). Then, one can estimate $k_y \rho_e v_l^{\mathrm{res}}/(k_z L_{\mathrm{rf}} v_{\mathrm{te}}) \sim 20\rho_i/a$. It indicates that the RPF may give a considerable correction to the DDF. Moreover, the signs of k_y/k_z and $\partial_x S$ decide the sign of the effect. When the parallel force is enhanced in the inner region, the LHW-driven current may increase due to fewer collisions in the inner. While, when the force in the outer is enhanced, where there are more trapped electrons and a lower magnetic field, the flow drive due to the effect of resonant trapped electron pinch [16] is enhanced.

It is noted that the RPF effect is influenced by the parallel wave number. Recalling that $v_l^{\rm res} \sim \omega/k_z$, then $k_y \rho v_l^{\rm res}/(k_z L_{\rm rf} v_t)$, is proportional to k_z^{-2} for fixed k_y and $L_{\rm rf}$. It means that the RPF effect is more significant for lower k_z, or, in other words, it enhances the efficiency of fast electron current drive. The increase of total driven current can be attributed to both the increase of the current drive efficiency and that of the absorption efficiency. The latter is usually explained by the so-called "spectral gap" or "spectral broadening" phenomena, which may be caused by the toroidicity effect [17], parametric decay [18], or other nonlinear effects. In practice, if one does not separate the increase of current drive efficiency from the increase of absorption efficiency, the RPF effect may appear to bridge the spectral gap as well. However, the evolution of parallel and tangential wave vector in toroidal geometry and its effect on power deposition should be considered in a more consistent study.

Another kind of nearly perpendicular propagating wave is Bernstein wave. The effect of the RPF on the electron Bernstein wave (EBW) current drive is expected to be similar to the LHCD case, i.e., to provide a correction on the order of several to tens of ρ_i/L. However, the effect might be significant for the ion Bernstein wave (IBW). The IBW has a large perpendicular wave vector $k_\perp \rho_i \sim 1$ and a rather low parallel wave vector $ck_z/\omega \sim 1$. In fact, in the case of the mode-converted IBW the parallel wave vector actually vanishes as it flips sign [19]. Its resonant velocity $v_l^{\rm res}$ is on the order of v_{ti}, and the thickness of the L-mode cyclotron resonance layer is about $\rho_i k_z R_0$, where R_0 is the major radius. Then, one has $k_y \rho_i v_l^{\rm res}/(k_z L_p v_{\rm ti}) \sim c^2/(\omega^2 \rho_i R_0)$, which is much larger than the unity in typical IBW experiments. Therefore, the force due to the inhomogeneity of rf resonant absorption, which is discovered in this letter, is expected to play a key role in the scheme of IBW flow drive. Moreover, since the IBW is proposed as the carrier of power from fusion alpha particles to fuel ions through the so-called "*alpha-channeling*" effect [20,21], we expect that, besides its role in power transfer, the IBW can simultaneously drive a significant toroidal flow to meet the requirement of steady state operation in fusion devices.

It is noted that here the inhomogeneity is assumed only in the x direction. In toroidal geometry, however, there is poloidal asymmetry. Thus, even with the classical nonresonant ponderomotive force, the nonlinear drift in the poloidal direction may induce significant effect on the parallel force by a mechanism similar to that derived here [22].

In summary, the parallel rf force is examined from both the particle-fluid picture and kinetic theory, with special attention on the effect of the inhomogeneity of rf resonant absorption. The major conclusions include (i) the local parallel force is completely resonant (i.e., dissipative), but it consists of two parts: one is the DDF due to parallel momentum absorption, the other is the RPF due to the radial inhomogeneity of poloidal momentum absorption, whose expression is shown in Eq. (8) or (11); (ii) the RPF is an actual nonlinear force in thermal plasmas, which originates from the transport of parallel momentum of resonant particles by the inhomogeneous nonlinear drift flux; (iii) the RPF enhances the total parallel driving force on one side of the layer of strongest deposition and weakens the total force on the other side. Although its integral over the plasma region vanishes, the RPF-driven current and flow cannot be altogether canceled due to different plasma responses in different positions and this redistribution can modify the profiles of current and flow as well; (iv) rf waves are usually employed to drive the current and flow in a narrow absorption layer, which maximizes the ponderomotive effect, therefore the RPF effects uncovered here may be particularly important in shaping the profiles of current and flow. Especially, for nearly perpendicular propagating waves, such as LHWs or Bernstein waves, the RPF effects are expected to be significant.

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*gaozhe@tsinghua.edu.cn

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