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The double well mass filter

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Various mass filter concepts based on rotating plasmas have been suggested with the specific purpose of nuclear waste remediation. We report on a new rotating mass filter combining radial separation with axial extraction. The radial separation of the masses is the result of a "double-well" in effective radial potential in rotating plasma with a sheared rotation profile. © *2014 AIP Publishing LLC*. [http://dx.doi.org/10.1063/1.4864325]

Plasmas have long been used for separating elements, with an emphasis on isotope separation.[1,2] Among plasma techniques, rotating configurations are of particular interest, since centrifugal forces offer a direct ion separation scheme based on mass.[3] This led to the development of plasma centrifuges,[4,5] in which collisional diffusion produces radial separation of ions within a plasma column. Plasmas have the advantage of high rotation velocities compared to gas centrifuges.

Within the last decade, the interest for a high throughput plasma filter in the context of nuclear waste processing[6] motivated the development of alternative concepts. Waste remediation differs, in particular, from isotope separation in that the mass discrimination requirement is less severe. In the DC Ohkawa mass filter,[7] known as the Archimedes Plasma Filter,[6] ions are separated as a result of a charge to mass ratio threshold for radial confinement. Heavy ions are unconfined radially and collected at the outer wall, while light ions are radially confined and collected axially along the magnetic field lines. This collisionless separation scheme relies on a plasma solid body rotation, obtained by means of biased electrodes at each end of the device. More recently, an alternative filter concept featuring a rotating axisymmetric plasma has been proposed.[8,9] In this concept, called the Magnetic Centrifugal Mass Filter (MCMF), a particular magnetic field topology is used to provide asymmetric confinement properties at each end of the device. Ions are collected along the field lines, with light and heavy streams preferentially collected at different axial end of the device.

In this letter, we propose a new rotating plasma filter combining both radial separation and axial extraction, while only requiring a simple linear magnetic field topology. It however differs from plasma centrifuges in that ion populations of different masses will have their density peak at different positions. The filter relies on the existence, through the production of a sheared rotation profile, of two distinct effective potential wells along the radial direction for two ions of different mass and identical charge, making it what we call a double well filter. In contrast with the Ohkawa filter where only one ionic species is radially confined, the sheared rotation profile allows both ionic species to be radially confined in this filter.

Let us consider in the laboratory frame a uniform axial magnetic field $\boldsymbol{B} = B_0\hat{z}$ and an arbitrary radial electric field $\boldsymbol{E} = E_r\hat{\boldsymbol{r}} = -\nabla\Phi$. In a frame rotating at constant angular velocity $\boldsymbol{\Omega} = \Omega\hat{z}$, the fields read

$$\tilde{\boldsymbol{E}} = \boldsymbol{E} + (\boldsymbol{\Omega}\times\tilde{\boldsymbol{r}})\times\boldsymbol{B}, \tag{1a}$$

$$\tilde{\boldsymbol{B}} = \boldsymbol{B}, \tag{1b}$$

where $\tilde{x}$ is the laboratory frame variable x written in the rotating frame. Assuming the fields in the rotating frame are time independent, the Newton-Lorentz equation can be rewritten (see, for example, Refs. 10, p. 328 or Ref. 11) as

$$m\frac{\partial\tilde{\boldsymbol{v}}}{\partial t} = q(\boldsymbol{E}_\star + \tilde{\boldsymbol{v}}\times\boldsymbol{B}_\star), \tag{2}$$

with

$$\boldsymbol{E}_\star = \tilde{\boldsymbol{E}} + \nabla\left(\frac{m\Omega^2\tilde{r}^2}{2q}\right), \tag{3a}$$

$$\boldsymbol{B}_\star = \tilde{\boldsymbol{B}} + \frac{2m}{q}\boldsymbol{\Omega}. \tag{3b}$$

Introducing

$$\Phi_\star(\tilde{r}) = \Phi(r) + \left(\frac{qB_0^2}{4m} - \frac{m\Omega^2}{2q}\right)\tilde{r}^2, \tag{4}$$

we may write $\boldsymbol{E}_\star = -\nabla\Phi_\star$. Equation (3b) shows that there exists a particular rotation frequency value, $\Omega = -qB_0/(2m) = -\omega_c/2$, for which the magnetic field in the rotating frame cancels, in which case

$$\Phi_\star(\tilde{r}) = \Phi(r) + \frac{qB_0^2}{8m}\tilde{r}^2. \tag{5}$$

Note that since the gyro-frequency ω_c depends on the mass, the frame in which the motion of a particle appears purely electrostatic will be different for two particles having the same charge but different masses. Looking at Eq. (5), one sees that, by specifying a parabolic radial profile Φ in the lab frame, the potential $\Phi_\star$ in the rotating frame can be made concave up or concave down. More specifically, an ion will be radially confined if

$$\frac{d^2}{dr^2}[\Phi(r)] > -\frac{qB_0^2}{4m}, \tag{6}$$

and unconfined otherwise. Introducing Φ_d the potential difference between the axis and the outer wall at $r = a$, one recovers the charge to mass ratio threshold

$$\frac{m}{q} = \frac{a^2B^2}{8\Phi_d}, \tag{7}$$

identified by Ohkawa and Miller[7] for ion confinement in a DC parabolic electric potential profile. In other words, under the assumption of solid body rotation at the angular rotation frequency $\Omega = E_r/(B_0 r)$, the electric potential in the rotating frame $\Phi_\star$ has a constant curvature sign.

Considering now a sheared rotation profile, as obtained, for example, by means of an applied potential Φ with higher polynomial dependency in radius, one may produce a radial separation while confining both ionic species. Let us consider the simple case of the following biquadratic laboratory frame potential:

$$\Phi(r) = \frac{k_bT}{e}\left[\frac{r^4}{\lambda^2a^2} - \frac{r^2}{\lambda^2}\right], \tag{8}$$

with

$$\lambda^2 = 8\rho^2 = 8\frac{k_bT}{A^\diamond m_p {\omega_c^\diamond}^2} = 8\frac{k_bTA^\diamond m_p}{e^2B^2}, \tag{9}$$

where k_b is the Boltzmann constant, T is the reference ion temperature, m_p is the proton mass, ρ is the thermal ion gyro-radius, $A^\diamond$ is the reference atomic mass number, and $\omega_c^\diamond$ is the corresponding gyro-frequency. Combining Eqs. (8) and (5) in the rotating frame of a singly charged ion of atomic mass number A, one gets

$$\Phi_\star(\tilde{r}) = \frac{k_bT}{e}\left[\frac{\tilde{r}^4}{\lambda^2a^2} + \left(\frac{A^\diamond}{A} - 1\right)\frac{\tilde{r}^2}{\lambda^2}\right]. \tag{10}$$

An ion such that $A < A^\diamond$ will therefore see a monotonically increasing potential $\Phi_\star(\tilde{r})$. On the other hand, an ion such that $A > A^\diamond$ will see a potential $\Phi_\star$ decreasing close to the axis, but increasing further off axis, that is to say a potential well off axis. The evolution from a second-order to a fourth-order polynomial dependence for the electric potential profile thus changes the nature of the separation scheme in an essential way. Heavy ions are no longer radially unconfined, but are now collected axially along the magnetic field lines.

In contrast to the Archimedes filter,[6] which relies on a collisionless plasma, the double well filter requires a collisional plasma to produce the radial ion separation. Collisional diffusion is indeed necessary in order to separate a mixture of ions introduced in a finite radius plasma column on-axis. Assuming now that the length l of the device is chosen long enough for an ion population to reach equilibrium as it diffuses axially, the spatial distribution will be of the Boltzmann type, with $n = n_0 \exp[-e(\Phi_\star - \Phi_\star^{min})/(k_bT_i)]$, where n_0 is the ion species density, T_i is the ion population temperature, and $\Phi_\star^{min}$ designates the minimum of $\Phi_\star$ over $[0, a]$. Ions such that $A < A^\diamond$ will be localized in an on-axis cylindrical column, while ions such that $A > A^\diamond$ will be localized in an annular ring, as illustrated in Fig. 1. Although the aspect ratio l/a required to reach such a distribution will depend on the ratios $A/A^\diamond$ considered, an estimate can be obtained as follows. Denoting $\chi = a/\rho$ and approximating the perpendicular diffusion coefficient as $D_\perp \sim \rho^2\nu$, where ν is the collision frequency, one gets that the ion residence time τ has to satisfy $\tau \geq \chi^2/\nu$ for an ion to diffuse on a distance R. Introducing the ion collision cross section σ and n the ion number density, it yields $l \geq \chi^2/(\sigma n)$. The minimum length is hence a function of how strongly is an ion magnetized (through χ), of the element considered (through σ), and of the density. It is worth noting here that, in this case, a collisionless regime, which is of course not what is contemplated here, would lead to a mixture of heavy and light elements in the plasma core, surrounded by an annular region made of heavy elements.

Although the Φ form chosen in Eq. (9) conveniently illustrates the existence of an inflection point as the atomic mass number of an ion gets larger than a reference value, the lack of independent control over the second and forth degree monomial is over-constraining. By a proper choice of the laboratory potential Φ monomial coefficients, one can make the heavy ions potential well deep enough to remove the heavy ions from the axis, while maximizing the radial separation of heavy from light elements. Such a separation is illustrated in Fig. 2 for two 5 eV singly charged ion populations of 40 and 80 amu. The ratio of light to heavy elements, as plotted in Fig. 2(c), shows high separation factors close to the axis, and even higher ones further away from the axis, with less than one light ion for an hundred heavy ions for $r > 0.6$ m. In between these regions ($0.2 \leq r \geq 0.6$ in Fig. 2), there exists a region where separation is much weaker and varies significantly, but the ion density is lower (Fig. 2(b)). Elements collected over this radial position range would have in principle to be processed at another time.

One way to avoid losing the part of the stream where the separation is worst is simply to provide axial confinement within that radial region. This could be accomplished by

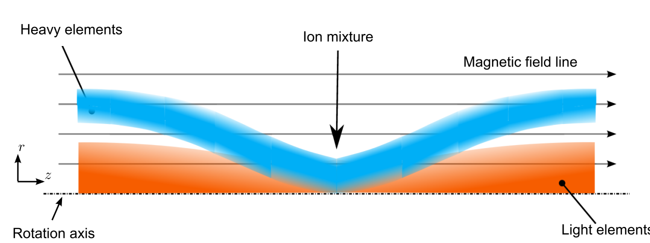
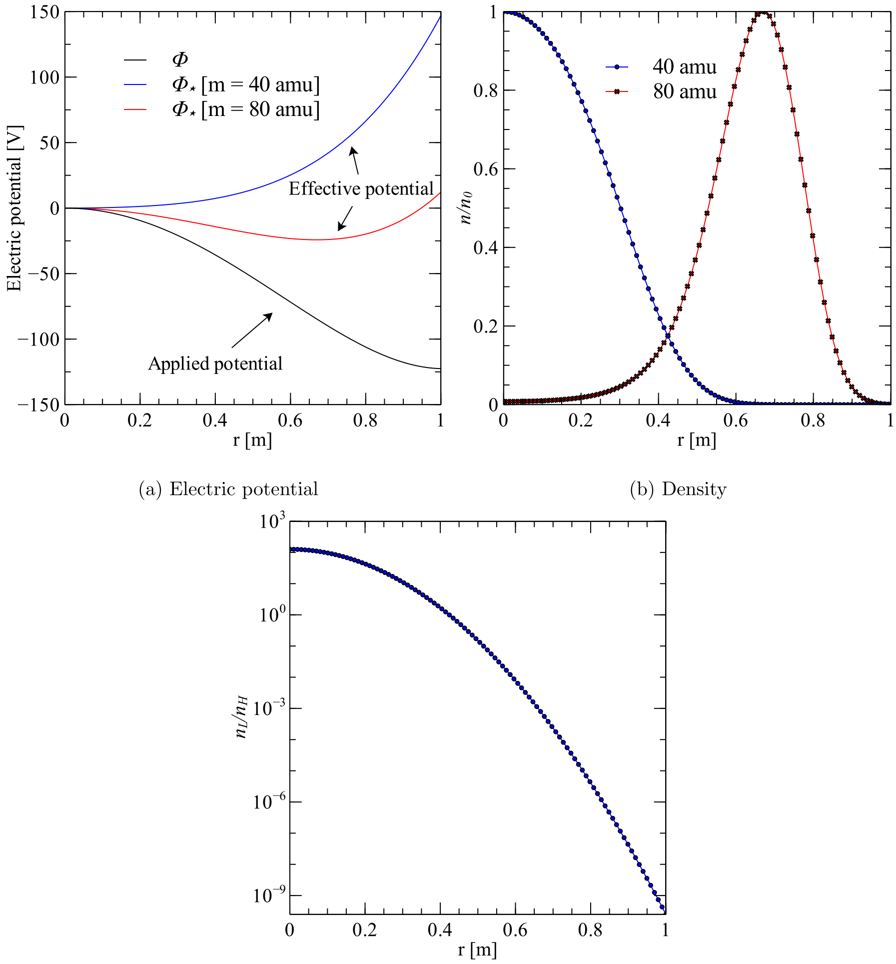


FIG. 1. Schematic of the double well filter (cut-view in the r–z plane). Light elements are eventually confined in a plasma column on-axis while heavy elements are confined in an annular ring. Both species are radially confined and extracted axially.



(a) Electric potential

(b) Density

(c) Light to heavy element ratio

FIG. 2. Electric potential radial profile in both the laboratory (Φ) and the rotating ($\Phi_\star$) frame (a), calculated Boltzmann radial distributions for two singly charged ion populations, and light (40 amu) to heavy (80 amu) elements ratio radial profile (c). Ion temperature of both population is 5 eV, $B_0 = 0.03$ T.

pinching the magnetic field lines within this radial zone, so as to produce a mirror force reflecting the particles towards the device mid-plane. However, since the plasma is collisional, this solution would only offer partial reflection due to the diffusion of the particles in the mirror loss cone. In addition, such a modification would be made at the expense of the magnetic field topology simplicity. Alternatively, higher degree polynomial form for the applied electric potential would allow optimizing the radial distance between the two population peaks, as well as the width of these peaks, for a given pair of ion masses and ion temperature. More complex applied potential profiles Φ are however expected to present an experimental challenge, since they will require larger number of biasing electrodes to properly represent the spatial variations.

In summary, the double well filter is a new concept combining the radial separation feature offered by the Ohkawa filter with the axial extraction property exhibited by the MCMF. This is made possible through the production of a sheared rotation profile, which allows in turn setting up confining potential wells at different radial locations depending only on the ion mass for a given ion charge. Although the large heavy to light separation factors predicted at large radial positions make this concept particularly interesting for waste remediation, for which the main objective consists in removing radio-active heavy elements from a contaminated stream, the double well filter appears to be a promising filter for other challenging separation processes too, notably such as those encountered in nuclear spent fuel reprocessing.

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