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Recent papers claim that a one dimensional (1D) diode with a time-varying voltage drop can transmit current densities that exceed the Child-Langmuir (CL) limit on average, apparently contradicting a previous conjecture that there is a hard limit on the average current density across any 1D diode, as t $\rightarrow \infty$, that is equal to the CL limit. However, these claims rest on a different definition of the CL limit, namely, a comparison between the time-averaged diode current and the adiabatic average of the expression for the stationary CL limit. If the current were considered as a function of the maximum applied voltage, rather than the average applied voltage, then the original conjecture would not have been refuted.
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This short research note points out that a conjecture regarding the Child-Langmuir (CL) limiting current density, while yet to be demonstrated, has not been refuted. The Child-Langmuir limit [Eq. (1)] gives the maximum current density of charged particles that can be forced across an un-neutralized gap by a certain voltage. It applies to a wide range of devices and natural phenomena such as ion beam sources, vacuum diodes, and plasma sheaths. There has been recent interest in generalizing this limit to situations where the current emission is a function of time. For example, the photoemission electron microscopy requires a series of electron pulses whose time interval can be distorted if the parameters violate a certain space charge limit.[1] In other cases, time-dependent current emission can arise spontaneously in low-voltage diodes even though the voltage drop is constant, with the cathode emitting intermittent bursts of single electrons[2,3] or sheets of electrons.[4]

In a previous paper, we investigated whether a current source with time-dependent emissivity could drive more current across a fixed-voltage gap, on average, than steady-state emission would allow. We were not able to beat this limit through empirical investigations, leading us to conjecture that there is a hard limit on the average current density across any one dimensional (1D) planar diode (as t $\rightarrow \infty$) that is equal to the CL limit[5]

$$J_{CL} = \frac{4}{9}\epsilon_0\sqrt{\frac{2q}{m}}\,\frac{V^{3/2}}{d^2}, \tag{1}$$

where q and m are the charge and mass of the current-carrying particles, V is the voltage across the diode, d is the width of the gap between electrodes, and ϵ_0 is the free space permittivity.

Since then, there have been several attempts to refute the conjecture, leading to refinements of the original conjecture. For example, in the regime where space charge effects limit the current across the diode to only a few electrons at a time, the average of the spontaneously time-varying current across a constant-voltage diode was shown to exceed J_{CL} by 13%.[2] Here, the increase in average current was not caused by the time-dependence of the current, but rather by special boundary conditions that pertain when the space charge electric field from a single electron is not negligible compared to the vacuum electric field in the diode.[3] The refined conjecture then ruled out discreteness effects, by excluding the case when the electric field at the cathode turns negative.

More recently, it has been claimed that even current in a traditional macroscopic diode could exceed J_{CL} if the voltage drop across the diode varied in time.[6,7] The point hinges on the question of how to define "exceeding the CL limit" for a diode whose voltage varies in time, because J_{CL} is only defined for diodes with constant voltage. In References 6 and 7, a new figure of merit, J_1, is defined, which is equal to the adiabatic average of the expression for J_{CL} [Eq. (2)]. A time-varying diode is then considered to exceed the CL limit when its average current exceeds J_1, namely,

$$J_1 = \frac{4}{9}\epsilon_0\sqrt{\frac{2q}{m}}\,\frac{1}{d^2}\cdot\frac{1}{T}\int (V_T(t))^{\frac{3}{2}}dt, \tag{2}$$

where V_T is the time-varying voltage across the diode, and the adiabatic average is taken over the time interval T. It is then shown[6] that current in time-varying diodes can exceed J_1 by as much as 50%. Note that exceeding this current does not refute the original conjecture, since what is called the CL limit is redefined.

However, the new definition represented by Eq. (2) raises the question of whether this redefined current limit might represent a meaningful current to exceed. First, note that the re-definition does not really pertain to the CL limit because the CL limit is not defined as a function of time. Second, note that a more physically meaningful measure, and the intent of our original conjecture, would be to compare the performance of a time-varying diode, with voltage V_T, to some stationary diode that has a constant voltage V_S. But what is the proper choice for V_S? In our previous paper,[5] only the current emission varied in time and V_T was constant, so it was straightforward to set $V_S = V_T$. The choice is less clear when V_T is a function of time.

The definition of J_1 is equivalent to comparing the average time-varying current to the CL limit of a stationary diode with constant voltage $V_{S,1}$ defined as follows:

$$V_{S,1} = \left[\frac{1}{T}\int \left(V_T(t) \right)^{\frac{3}{2}} dt\right]^{2/3}. \tag{3}$$

The physical significance of this comparison is not clear. Practical considerations generally constrain the maximum voltage that drives charged particle current, not the time-average of $V^{3/2}$. Accordingly, the test for exceeding the Child-Langmuir law should be a comparison to a stationary diode with voltage $V_{S,2}$ equal to the maximum voltage reached by the time-varying diode

$$\begin{aligned} V_{S,2} &= Maximum[V_T(t)], \\ J_2 &= \frac{4}{9}\epsilon_0\sqrt{\frac{2q}{m}}\frac{1}{d^2}\left\{ Maximum[V_T(t)] \right\}^{3/2}. \end{aligned} \tag{4}$$

Stated another way, the time-varying diode "exceeds the CL limit" when its average current is larger than J_2. Note that J_2 is always larger than J_1, so it is harder to exceed the CL limit in this formulation and neither Reference 6 or 7 exceeded the CL limit using this stricter criteria. Thus, the original conjecture, with the more standard definition of the CL limiting current, captures succinctly the physical case and remains to be either refuted or proved.

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