Nonresonant Diffusion in Alpha Channeling

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The gradient of fusion-born alpha particles that arises in a fusion reactor can be exploited to amplify waves, which cool the alpha particles while diffusively extracting them from the reactor. The corresponding extraction of the resonant alpha particle charge has been suggested as a mechanism to drive rotation. By deriving a coupled linear-quasilinear theory of alpha channeling, we show that, for a time-growing wave with a purely poloidal wave vector, a current in the nonresonant ions cancels the resonant alpha particle current, preventing the rotation drive but fueling the fusion reaction.

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Introduction.—A particle gyrating in a magnetic field with a velocity $v_\perp$ greater than the phase velocity $v_p \equiv \omega_r/k_\perp$ of an electrostatic wave will become Landau resonant at some point in its orbit, allowing for efficient wave-particle energy exchange. Each time the particle energy changes (Fig. 1), its gyrocenter position also changes, leading to diffusion on a 1D path in the 2D energy-gyrocenter coordinate space [1,2]. If, along the path, there are more particles at higher than lower energy, the diffusion on average cools particles, and the wave amplifies. This effect is known as alpha channeling, so named because it cools and extracts alpha ash from the hot core of a fusion reactor, and channels their energy into wave power useful for current drive [1,2] or ion heating [3–5].

For alpha channeling in a slab geometry, the diffusion path slope in this energy-gyrocenter space has the simple form $\partial\mathbf{X}/\partial K = \mathbf{k}\times\hat{b}/m_\alpha\omega\Omega_\alpha$, where K is the perpendicular kinetic energy, $\mathbf{X}$ is the gyrocenter position, m_α and Ω_α are the alpha particle mass and gyrofrequency, and ω and $\mathbf{k}$ are the wave frequency and wave number. Thus, the condition for wave amplification from channeling is

$$\left(\frac{\partial}{\partial K}+\frac{\mathbf{k}\times\hat{b}}{m_\alpha\omega\Omega_\alpha}\cdot\frac{\partial}{\partial\mathbf{X}}\right)F_{\alpha 0}>0, \qquad (1)$$

where $F_{\alpha 0}$ is the zeroth-order distribution function in energy-gyrocenter space.

What remains unknown is whether or not the alpha particles carry net charge out of the plasma as a result of the wave-induced diffusion. If charge is in fact carried out, then alpha channeling can be used to drive $\mathbf{E}\times\mathbf{B}$ rotation in the plasma, providing an advantageous mechanism for shear rotation drive and centrifugal confinement in mirror fusion reactors [6]. Understanding whether such schemes are possible at all requires evaluation of the effect of the wave on the nonresonant particles, which has never been examined for alpha channeling. Such reactions in the nonresonant particles are extremely important in enforcing momentum and energy conservation [7,8], making theories that ignore them liable to error.

The reason the nonresonant response has proved elusive is that there is no existing linear theory of alpha channeling. Typically, a coupled linear wave and quasilinear particle system is necessary to calculate the nonresonant particle response. The elusivity of the linear theory is related to the fact that Landau damping cannot be derived from the magnetized dispersion relation, a conundrum sometimes termed the Bernstein-Landau paradox [9,10]. Derivation of the diffusion thus requires a nonlinear calculation, which allows for stochastic diffusion of the particle throughout phase space above a certain wave amplitude (see Supplemental Material [11]) at which Landau-resonant particles dephase from the wave [15–19]. This dephasing effectively destroys the gyrophase-dependent structure of the resonant particle distribution.

In this Letter, we show that a linear-quasilinear system can be derived by *assuming* this wave-particle dephasing, which we do by transforming the familiar *unmagnetized* kinetic theory to gyrocenter coordinates, and then forcing the resonant particle distribution to be independent of gyroangle. To show that the system describes alpha channeling, we show that it recovers both the amplification condition [Eq. (1)] and the nonlinear diffusion coefficient [16] for channeling by lower hybrid (LH) waves.

Treating the channeling problem in this way positions us to answer the question of whether alpha channeling extracts charge from the fusion reactor. We find that for the initial value problem, where an electrostatic wave with purely poloidal wave number grows in time, the charge flux from the resonant alphas is canceled by an equal and opposite charge flux in the nonresonant particles, so that no reactor charging occurs. We also determine which particles carry this nonresonant return current, in both single- and multi-ion-species plasmas. For LH waves, the nonresonant return current is carried exclusively by fuel ions, so that alpha

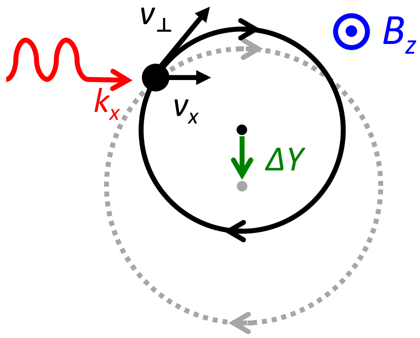

FIG. 1. Schematic of the alpha channeling process. A hot particle with $v_\perp > v_p$ resonates with the wave at some point in the orbit, leading to a change in both energy and gyrocenter. Thus, the particles diffuse along a specified path in gyrocenter-energy space. The amplification condition Eq. (1) depends on the derivative of the distribution function along this path.

channeling has the added benefit of fueling the fusion reaction while extracting alphas.

Linear theory.—For any electrostatic wave, the dispersion relation obtained by linearizing and Fourier transforming Poisson's equation can be expressed as

$$0 = 1 + \sum_s D_s; \qquad D_s \equiv -\frac{4\pi q_s}{k^2}\frac{\tilde{n}_s}{\tilde{\phi}}, \tag{2}$$

where q_s and n_s are the charge and density of species s, ϕ is the potential, and tildes denote Fourier transforms. In the standard limit $|\omega_i/\omega_r| \ll 1$, this becomes

$$0 = 1 + \sum_s D_{r,s}, \tag{3}$$

$$0 = \sum_s \left(i\omega_i \frac{\partial D_{r,s}}{\partial \omega_r} + iD_{i,s} \right), \tag{4}$$

where ω_r and ω_i are the real and imaginary components of the wave frequency, and $D_{r,s}$ and $D_{i,s}$ are the real and imaginary parts of the dispersion function D_s in Eq. (2) evaluated at real ω, k.

To treat the alpha channeling initial value problem, we take the wave number $\mathbf{k}\|\hat{x}$, the magnetic field $\mathbf{B}\|\hat{z}$, and the gradient of the gyrocenter distribution function to be along $\hat{y}$. Thus, y corresponds to the "radial" direction, and x to the "poloidal" direction. For simplicity, we specialize to a LH wave, assuming cold fluid populations of magnetized electrons e and unmagnetized ions i, and a hot, unmagnetized population of alpha particles α. Our dispersion components are thus given by $D_{r,e} = \omega_{pe}^2/\Omega_e^2$, $D_{r,i} = -\omega_{pi}^2/\omega^2$, $D_{i,e} = D_{i,i} = 0$, and

$$D_\alpha = -\frac{\omega_{pi}^2}{k_x^2}\int dv_y dv_x \frac{\partial f_{\alpha 0}/\partial v_x}{v_x - \omega/k_x - i\nu/k_x}, \tag{5}$$

where ω_{ps} is the plasma frequency of species s, and $\nu \to 0^+$ determines the pole convention. We further take $D_{r,\alpha} = 0$, which is a good approximation when the alpha particles are hot and sparse compared to the ions.

Plugging these dispersion components into the real dispersion Eq. (3), we find the familiar LH wave:

$$\omega_r = \pm\omega_{\mathrm{LH}} \equiv \pm(\omega_{pi}^{-2} + |\Omega_e\Omega_i|^{-1})^{-1/2}. \tag{6}$$

Taking $\omega_r > 0$, the imaginary dispersion yields

$$\omega_i = \frac{\pi}{2} S_{k_x} \frac{\omega_{p\alpha}^2}{\omega_{pi}^2}\frac{\omega_{\mathrm{LH}}^3}{k_x^2}\int dv_y \frac{\partial f_{\alpha 0}}{\partial v_x}\bigg|_{y=y_0, v_x=v_p}, \tag{7}$$

where $S_{k_x} = \mathrm{sgn}(k_x)$ determines the direction of the phase velocity $v_p \equiv \omega_r/k_x$.

To recover alpha channeling, we need to transform the distribution function and derivatives from phase space coordinates $x^i \equiv (x, y, v_x, v_y)$ to gyrocenter-energy coordinates $X^i \equiv (X, Y, K, \theta)$:

$$X = x + \frac{v_y}{\Omega_\alpha} \qquad Y = y - \frac{v_x}{\Omega_\alpha} \tag{8}$$

$$K = \frac{1}{2}m(v_x^2 + v_y^2) \qquad \theta = \arctan(-v_y, v_x). \tag{9}$$

In this coordinate system, the phase space density function transforms as

$$F_{\alpha 0} = \sqrt{|g|} f_{\alpha 0} \tag{10}$$

$$|g| = |g_{ij}| = \left| \frac{\partial x^m}{\partial X^i}\frac{\partial x^n}{\partial X^j}\delta_{mn} \right| = m_\alpha^{-2}. \tag{11}$$

Thus, we can rewrite our derivatives in terms of X^i as

$$\frac{\partial f_{\alpha 0}}{\partial v_x}\bigg|_{y, \frac{\omega_r}{k_x}} = \frac{\partial X^i}{\partial v_x}\frac{\partial}{\partial X^i}(m_\alpha F_{\alpha 0})_{y_0, \frac{\omega_r}{k_x}} \tag{12}$$

$$= m_\alpha^2 \frac{\omega_r}{k_x}\left(\frac{\partial F_{\alpha 0}}{\partial K} - \frac{k_x}{m_\alpha \omega_r \Omega_\alpha}\frac{\partial F_{\alpha 0}}{\partial Y} \right)_{Y^*, K^*}, \tag{13}$$

where $Y^* \equiv y - v_p/\Omega_a$, $K^* \equiv m(v_p^2 + v_y^2)/2$, and we have taken $F_{\alpha 0}$ independent of θ to capture the gyrophase structure loss induced by the stochasticity.

The quantity in parentheses in Eq. (13) can be recognized as derivative along the diffusion path in Eq. (1), and thus describes wave amplification from alpha channeling. Interestingly, this means that the condition that there be (on average) a population inversion along the channeling diffusion path in gyrocenter-energy space is identical to the condition that there be a bump-on-tail instability in the local hot alpha particle distribution.

Under this new formalism, in contrast to the nonlinear formalism, it is possible to straightforwardly calculate the wave amplification rate. For instance, for a Maxwellian with a gradient in Y, $F_{\alpha 0} = e^{-K/T_\alpha} e^{-Y/L}/2\pi T_\alpha$, with thermal velocity $v_{\text{th}\alpha} = \sqrt{T_\alpha/m_\alpha}$, we have at $y = 0$:

$$\omega_i = -|\omega_r| \sqrt{\frac{\pi}{8}} \left| \frac{v_{px}}{v_{\text{th}\alpha}} \right|^3 e^{-1/2(\frac{v_{px}}{v_{\text{th}\alpha}})^2} \frac{\omega_{p\alpha}^2}{\omega_{pi}^2} \times \left[\left(1 - \frac{v_{\text{th}\alpha}}{v_{px}} \frac{\rho_{\text{th}\alpha}}{L} \right) e^{-(y-\rho_{p\alpha})/L} \right], \tag{14}$$

where $\rho_{th\alpha} = v_{th\alpha}/\Omega_\alpha$ and $\rho_{p\alpha} = v_p/\Omega_\alpha$. The first line is the familiar Landau damping on a minority species [8], and the second line contains the channeling effect.

Resonant diffusion.—Identifying alpha channeling with the unmagnetized bump-on-tail instability allows us to compactly derive the diffusion tensor, by performing the same coordinate transformation. From unmagnetized quasilinear theory, we have for a single wave mode [8,20]:

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial x_i} \left(D_{\mathbf{x}}^{ij} \frac{\partial f_{\alpha 0}}{\partial v_j} \right), \tag{15}$$

$$D_{\mathbf{x}}^{v_x v_x} = \frac{\omega_{p\alpha}^2}{m_\alpha n_\alpha} \frac{W(y)}{k_x i} \left(\frac{1}{v_x - \omega_r/k_x - i\omega_i/k_x - i\epsilon} - \text{c.c.} \right), \tag{16}$$

where $W(y) = E_0(y)^2/16\pi$ and $E_0(y)$ are the wave electrostatic energy density and amplitude at y, respectively. The diffusion equation (15) transforms as

$$\frac{\partial F_{\alpha 0}}{\partial t} = \frac{\partial}{\partial X^i} \left[\sqrt{|g|} D_{\mathbf{X}}^{ij} \frac{\partial}{\partial X^j} \left(\frac{F_{\alpha 0}}{\sqrt{|g|}} \right) \right], \tag{17}$$

where $D_{\mathbf{X}}^{ij}$ is determined from $D_{\mathbf{x}}^{ij}$ by the same tensor transformation law as for the metric in Eq. (11).

Performing the coordinate transformation, taking $F_{\alpha 0}$ independent of θ, and averaging Eq. (17) over θ, we find

$$\left\langle \frac{\partial F_{\alpha 0}}{\partial t} \right\rangle_t = \frac{\partial}{\partial \bar{X}^i} \left(\langle D_{\bar{\mathbf{X}}}^{ij} \rangle_\theta \frac{\partial F_{\alpha 0}}{\partial \bar{X}^j} \right), \tag{18}$$

where the gyroaveraged coordinates $\bar{X}^i \equiv (K, Y)$, and

$$\langle D_{\bar{\mathbf{X}}}^{ij} \rangle_\theta = \frac{1}{2\pi} \frac{\omega_{p\alpha}^2}{m_\alpha n_\alpha} \frac{W(y)}{k v_\perp i} \begin{pmatrix} \Omega_s^{-2} I_d^0 & -\frac{m_\alpha v_\perp}{\Omega_\alpha} I_d^1 \\ -\frac{m_\alpha v_\perp}{\Omega_\alpha} I_d^1 & m_\alpha^2 v_\perp^2 I_d^2 \end{pmatrix} \tag{19}$$

with $v_\perp = \sqrt{2K/m_s}$ and $I_d^a = I_-^a - I_+^a$, where

$$I_\pm^a \approx \left(1 \mp i\omega_i \frac{\partial}{\partial \omega_r} \right) \int_0^{2\pi} d\theta \frac{\sin^a \theta}{\sin\theta - \omega_r/k v_\perp \pm i\epsilon}. \tag{20}$$

This integral can be evaluated with the u substitution $u = \sin\theta$. As discussed in Ref. [20], I_d^a then consists of two terms: one from the "1" and the pole, corresponding to resonant particles, and one from the $i\omega_i \partial/\partial\omega_r$ and the principle value, corresponding to nonresonant particles. We will focus on the resonant diffusion, which gives at $Y^* = y - v_p/\Omega_\alpha$:

$$\left\langle \frac{\partial F_{\alpha 0}}{\partial t} \right\rangle_t = \frac{d}{dK}\bigg|_{\text{path}} \left[D^{KK} \frac{d}{dK}\bigg|_{\text{path}} F_{\alpha 0} \right] \tag{21}$$

$$D^{KK} \equiv \frac{m_\alpha^2}{2} \left(\frac{q_\alpha E_0(y)}{m_\alpha} \right)^2 \frac{v_p^2}{\sqrt{k^2 v_\perp^2 - \omega_r^2}} H(v_\perp - v_p) \tag{22}$$

$$\frac{d}{dK}\bigg|_{\text{path}} \equiv \left(\frac{\partial}{\partial K} - \frac{k}{m_s \omega_r \Omega_\alpha} \frac{\partial}{\partial Y} \right). \tag{23}$$

Equation (22) is the same as Karney's [16] diffusion coefficient in $v_\perp$ in the limit of large $k_x \rho$ as used in [2] (see Supplemental Material [11]). Furthermore, the diffusion is seen to occur along the diffusion path in Eq. (1), confirming that this approach recovers alpha channeling.

The diffusion coefficient corrects the energy-space diffusion coefficient in Refs. [1,21]. This discrepancy is discussed in the Supplemental Material [11]. This error did not affect the study of alpha channeling in toroidal geometry due to ion-Bernstein waves (IBWs) [4,22], which relied on a different diffusion coefficient from orbit-averaging the cyclotron-resonant response [23,24].

Nonresonant reaction.—Having established that the linear-quasilinear system recovers alpha channeling, we are now in a position to examine the nonresonant response. In contrast to the resonant particles, which remain on largely unperturbed gyro-orbits except at the resonance points [Fig. 2(a)], and which have their θ-dependent structure destroyed by nonlinear effects, the nonresonant particles experience sloshing motion along v_x only, and thus have a nongyrotropic distribution function at $\mathcal{O}(E^2)$ [Fig. 2(b)].

Thus, instead of transforming the nonresonant diffusion coefficient to the coordinates X^i, we find the nonresonant response by first calculating the total force density on species s from the field-particle correlation:

$$F_{sx} = q_s \langle E_{1x} n_1 \rangle_x. \tag{24}$$

This approach is equivalent to finding the force from the full (nongyroaveraged) quasilinear theory.

Linearizing and Fourier transforming the above gives

$$F_{sx} = \lim_{L\to\infty} \frac{q_s}{L} \int_{-L/2}^{L/2} dx \int \frac{dp_x'}{2\pi} \frac{dk_x'}{2\pi} \tilde{E}_{p_x'} \tilde{n}_{s,k_x'} \times e^{i(p_x'+k_x')x - i[\omega(p_x')+\omega(k_x')]t}, \tag{25}$$

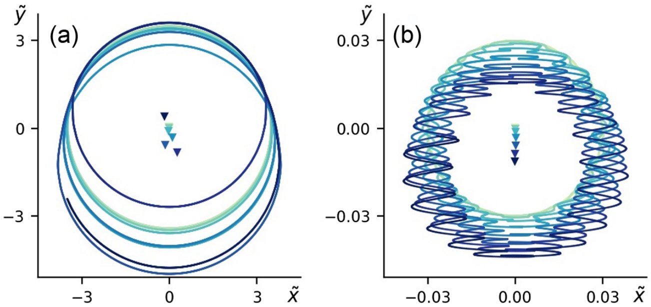

FIG. 2. Simulated single-particle trajectories in the x-y plane of (a) hot particles ($v_0 = 3.5v_p$) and (b) cold particles ($v_0 = 0.03v_p$) in a growing electrostatic wave. Lines are particle positions, and triangles are orbit-averaged gyrocenter positions. Axes are normalized to $\rho_{ps} \equiv \omega/k\Omega_s$, i.e., $\tilde{x} = x/\rho_{ps}$ and $\tilde{y} = y/\rho_{ps}$. The color indicates time, light to dark. The hot particles diffuse stochastically. The cold particles have a clearly nongyrotropic velocity distribution due to the oscillations, and exhibit a clear shift in gyrocenter downward.

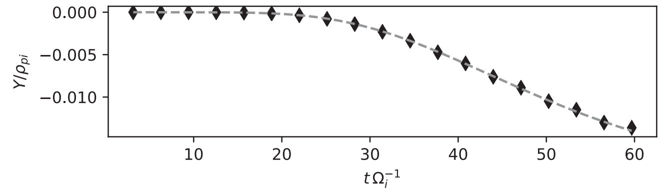

FIG. 3. Change in gyrocenter position Y for the particle in Fig. 2(b) due to the slow ramp-up of the electrostatic wave. The gyro-period-averaged position in the simulations (solid black diamonds) agrees well with the predicted shift (gray dashed line) from Eq. (31) due to the nonresonant reaction.

which can be expressed entirely in terms of $\tilde{\phi}$ by using $\tilde{E}_{p'_x} = -ip'_x\tilde{\phi}_{p'_x}$ and $\tilde{n}_{s,k'_x}$ from Eq. (2). Then, in this Fourier convention, the wave $\phi = \phi_0 \cos(k_x x - \omega t)e^{\omega_i t}$ corresponds to

$$\tilde{\phi}_{k'_x} = \pi\phi_0[\delta(k'_x - k_x) + \delta(k'_x + k_x)]. \tag{26}$$

Plugging this in to Eq. (25) and making use of the symmetry property $D(-k_x) = D^*(k_x)$ allows us to calculate the total force on species s as

$$F_{sx} = \frac{\phi_0^2 e^{2\omega_i t}}{8\pi}\mathrm{Im}[k_x k^2 D_s] \tag{27}$$

$$\approx 2Wk_x\left[D_{is} + \omega_i\frac{\partial D_{rs}}{\partial\omega_r}\right]. \tag{28}$$

Here, the first term on the right-hand side is the force density on the resonant particles due to the wave-induced diffusion, and the second term is the force density on the nonresonant particles. This derivation generalizes the result for an unmagnetized plasma in Ref. [20] to any electrostatic wave. Summing over all species, we recover the imaginary component of the dispersion function, Eq. (4). Thus the total force applied to the plasma sums to 0, as demanded by momentum conservation for the electrostatic wave.

The fact that the forces cancel in turn means that the total cross-field currents from resonant and nonresonant particles cancel, as can be seen by calculating the total gyrocenter current from the resulting $\mathbf{F}\times\mathbf{B}$ drifts:

$$\sum_s j_{sy} = \sum_s -q_s n_s\frac{(F_{sx}/n_s)B_z}{q_s B_z^2} = -\frac{1}{B_z}\sum_s F_{sx} = 0. \tag{29}$$

Thus, for a purely poloidal wave mode growing in time, alpha channeling does not charge the plasma.

In addition to revealing the conservation of total charge, Eq. (28) also tells us which species carries the canceling nonresonant current. Specializing to an LH wave, with $\partial D_{r,e}/\partial\omega_r = 0$, we see that the nonresonant reaction is exclusively in the ions, which experience a force density:

$$F_{ix} = 4Wk_x\omega_i\frac{\omega_{pi}^2}{\omega_r^3} = n_i m_i\left(\frac{q_i E_0}{m_i}\right)^2\frac{k_x\omega_i}{\omega_r^3}. \tag{30}$$

Thus, for every alpha particle brought out of the plasma by the LH alpha channeling instability, two fuel ions are brought in, fueling the fusion reaction.

The total shift in the nonresonant ion gyrocenter due to the ponderomotive force for the LH wave can be expressed nicely by integrating the $\mathbf{F}\times\mathbf{B}$ drift over the growth of the wave, using $dE_0^2/dt = 2\omega_i E_0^2$. This gives

$$\Delta Y = -\frac{1}{2}\frac{q_i}{m_i}\left(\frac{k_x}{\omega_r}\right)^3\frac{\Delta\phi_0^2}{B_z}, \tag{31}$$

where $\Delta\phi_0^2$ is the change of the wave potential squared. In a multi-ion-species plasma, Eq. (31) reveals to what extent each ion species moves inward as a result of LH alpha channeling. For instance, in a p-B11 fusion plasma, Boron ions would flow inward $Z_B/\mu_B = 5/11$ as much as the protons. For other electrostatic waves such as the IBW, the general force density from Eq. (28) can be used to determine each species' response. Equation (31) can also be easily checked against single-particle Lorentz force simulations in which a wave is ramped up from 0 initial amplitude, which are found to agree well (Fig. 3). Details for these simulations, which use the Boris algorithm [25,26], are given in the Supplemental Material [11].

It is important to note that the cancellation of the resonant and nonresonant gyrocenter currents is not locally exact. Because the nonresonant ions are cold, the electric field that enters this equation is evaluated locally at $y \approx Y$, in contrast to the case for the hot resonant particles, where it is evaluated at $y = Y + v_p/\Omega_\alpha$. Thus, if there is variation in the electric field in y on some scale length L, the slight offset of the resonant and nonresonant currents will produce a net current ordered down from the resonant current by $\mathcal{O}(\rho_{p\alpha}/L) \ll 1$. The resulting charge accumulation could in principle drive shear flow in the plasma, albeit at a much reduced rate than that suggested by the resonant current alone.

Discussion.—The force in Eq. (30) is the same time-dependent force that arises from the unmagnetized ponderomotive potential in the form

$$\Phi = \frac{e^2 E_0^2}{4m(\omega - \mathbf{k}\cdot\mathbf{v})^2}, \tag{32}$$

from whence the force is derived via

$$(m\delta_{ij} - \Phi_{v_i v_j})\frac{dv_j}{dt} = \Phi_{v_i x_j} v_j + \Phi_{v_i t} - \Phi_{x_i}, \tag{33}$$

where the subscripts represent derivatives. The force in Eq. (30) appears as the second term on the right-hand side.

Equation (32) can be derived from *unmagnetized* plasma susceptibility using the K-χ theorem [27], which relates the linear susceptibility to the ponderomotive potential. Interestingly, application of the K-χ theorem to the *magnetized* hot plasma dispersion relation [13] does *not* yield the nonresonant force we observe here, as we show in the Supplemental Material [11]. Nevertheless, single-particle simulations confirm the effect. The failure of the hot plasma dispersion to capture the effect is likely related to the gyroaverage in the hot plasma dispersion, which has previously been found to obscure the derivation of perpendicular resonant quasilinear forces [28–30].

The approach used here, of taking the lowest-order alpha particle motion to be a straight trajectory, and then averaging over a gyroperiod, is similar to how neoclassical wave-particle interactions are treated. In those interactions, one does not generally use the full constant-of-motion space dispersion relation [23] to calculate the quasilinear diffusion, but rather averages the effect of the diffusion derived from the magnetized dispersion relation over the neoclassical orbit [5,24]. This destroys resonances associated with the neoclassical orbit period, which are assumed to be destroyed by nonlinearities anyway. In each case, the long-term orbit is ignored in the calculation of the dispersion, allowing in the neoclassical case cyclotron damping for banana orbits, and in the LH alpha channeling case Landau damping at resonance points on the gyro-orbit.

Note that, while the charge transport cancellation result is general for any purely poloidal electrostatic wave, the channeling path in Eq. (1) and diffusion coefficient in Eq. (22) apply only to the case of gyroaveraged Landau resonance [1,2,21,31], and not to channeling via cyclotron resonances, as for the IBW [3,4,22,32–34].

Conclusion.—The alpha channeling interaction, which releases the free energy of particles through diffusion in coupled energy-space coordinates, can rigorously be transformed to the classic bump-on-tail instability in velocity space only. Applying the traditional mathematical apparatus then shows that, in an initial value problem, where resonant ions are ejected as the electrostatic wave grows at the expense of the ion energy, those same waves must pull in a return current of nonresonant ions so as to draw no current. This unexpected result is related to the cancellation of resonant and nonresonant currents in the bump-on-tail instability [8,20], except that these newly found currents are perpendicular to the magnetic field, rather than parallel. We also calculated for the first time the contribution to the imaginary component of the dispersion relation due to alpha channeling, a useful quantity for ray tracing calculations.

We not only prove rigorously the current cancellation, but we also determine the extent to which each species contributes to this cancellation. For LH waves, the nonresonant ions are pulled into the plasma core; thus, while no rotation is driven, the fusion reaction is beneficially fueled as ash is expelled. In a p-B11 reactor, we showed that protons are drawn in at twice the rate of boron.

While the nonresonant particles have been ignored in alpha channeling theory up to this point, our analysis shows that they can have important zeroth-order effects on the plasma dynamics. However, the specific problem we considered here is only part of the story; in the most useful scenarios, channeling is driven by a stationary wave propagating radially inward from an antenna at the boundary, requiring a fundamentally 2D analysis. While the 2D problem is outside the scope of this Letter, the 1D self-consistent theory of alpha channeling laid out here provides a sound basis for examining the nonresonant response in more general scenarios.

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