Suppression of bremsstrahlung losses from relativistic plasma with energy cutoff

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We study the effects of redistributing superthermal electrons on bremsstrahlung radiation from hot relativistic plasma. We consider thermal and nonthermal distribution of electrons with an energy cutoff in the phase space and explore the impact of the energy cutoff on bremsstrahlung losses. We discover that the redistribution of the superthermal electrons into lower energies reduces radiative losses, which is in contrast to nonrelativistic plasma. Finally, we discuss the possible relevance of our results for open magnetic field line configurations and prospects of the aneutronic fusion based on proton-boron-11 (p-B11) fuel.

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I. INTRODUCTION

Bremsstrahlung emission emerge whenever a charged particle moves in the Coulomb field of other charged particles. bremsstrahlung emission is one of the primary mechanisms of radiative energy loss from plasma and is mainly characterized by its differential cross section. The relativistic differential cross section of bremsstrahlung emission has been derived in a seminal work of Bethe and Heitler [1,2], where quantum matrix elements in the Born approximation were calculated. Other notable papers and review works include Refs. [3–14].

Bremsstrahlung is used for diagnostics purposes [15–24] and as a source of x-ray production [25–28]; it is present in laser-plasma interactions [29,30] and determines dynamics of fast runaway electrons [31,32]. Bremsstrahlung emission is plentiful in astrophysics and has been extensively studied in this context [6,33–38]. It is responsible for x-ray production in galaxy clusters [39–41] and solar flares [24,42–44], plays a role in the physics of cosmic microwave background distortions [45,46], and can be an important emission process for plasma around compact objects [47,48].

An inverse process of bremsstrahlung absorption is one of the main mechanisms of laser energy transfer in inertial confinement fusion experiments [49–51]. Many features of inverse bremsstrahlung have been investigated [52–69]. For example, in Refs. [70,71] recoil effect in the electron-ion bremsstrahlung absorption was studied, while influence of strong laser fields on bremsstrahlung absorption was explored in Refs. [72–78]. Bremsstrahlung absorption is also critical for the opacity of astrophysical plasmas [79,80], such as high-temperature stellar plasma [81,82] and the intracluster plasma [83–85].

Both bremsstrahlung emission and absorption crucially depend on the distribution function of the charged particles. The distribution function can differ substantially from a thermal distribution either naturally or intentionally, through phase space engineering. In astrophysical settings, bremsstrahlung emission from nonthermal power-law distribution of electrons is present in supernova remnants [86–88], clusters of galaxies [40], and solar flares [24,43,44]. In laboratory settings, Langdon [89] showed that nonlinear effects in inverse bremsstrahlung absorption lead to a distortion of the electron distribution function towards a super-Gaussian, which decreases the effectiveness of the energy transfer from laser to plasma [90]. Intense radiation can even affect the electron distribution function leading to magnetogenesis effects [91–94]. The distribution function can also exhibit a significant degree of anisotropy, which in turn affects bremsstrahlung emission [95–99].

Fusion based on proton-boron-11 (p-B11) fuel has always been seen as a very attractive method for generating clean energy due to its aneutronic nature [100–102]. Because of the temperature dependence of the p-B11 reaction cross section, fusion with this fuel source requires plasma having a relativistic temperature on the order of hundreds keV. Such high-temperature plasmas of relativistic temperatures emit significant amounts of radiation with synchrotron and bremsstrahlung emission being the major loss mechanisms. These obstacles were deemed fatal for the feasibility of fusion devices utilizing p-B11 fuel [103–105]. However, recent research has shown that the p-B11 reaction cross sections could be larger than previously thought [106] and that the redistribution of fusion power from electrons to protons through alpha channeling [107–109] makes the economical p-B11 fusion energy potentially viable [110,111]. This inspired revival of interest in p-B11 fusion [110–112] including some recent experimental endeavors [113–115]. Besides a magnetically confined p-B11, there are also growing efforts with laser-based p-B11 fusion [116–124].

In regard to synchrotron radiation, it was recently shown in Ref. [125] that synchrotron radiation from relativistic plasma can be meaningfully reduced by redistribution of superthermal electrons into lower energies, introducing an effective energy cutoff. Such an effective cutoff in the energy distribution of electrons can emerge in open magnetic field line configurations, such as mirror machines and inertial electrostatic confinement devices. Relativistic bremsstrahlung has a

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certain important feature, that there is a long increasing tail in the probability of bremsstrahlung for large electron energies. This implies that the redistribution of high-energy electrons into lower energies should lead to a decrease in bremsstrahlung emission, similar to the effect seen for synchrotron radiation in Ref. [125].

In this paper, we show that it is indeed possible to suppress the production of bremsstrahlung radiation from relativistic plasma by redistributing superthermal electrons into lower energies. We evaluate the power density of bremsstrahlung radiation emitted from plasma with an energy cutoff for different temperatures and energy cutoff parameters, and determine the reduction in emission compared to the case of thermal plasma. Finally, we discuss the possible relevance of the present study for the p-B11-based fusion devices.

II. FORMULATION OF THE PROBLEM AND RESULTS

We consider relativistic plasma of electron density n_e with the electrons described by the Maxwell-Jüttner distribution with an energy cutoff:

$$f_e(\mathbf{p}) = \begin{cases} N_{\text{const}} \frac{e^{-\frac{\gamma}{\theta_{T_e}}}}{4\pi m_e^3 c^3 \theta_{T_e} K_2(1/\theta_{T_e})}, & \gamma \leqslant \gamma_{\max} \\ 0, & \gamma > \gamma_{\max} \end{cases}. \tag{1}$$

Here $\theta_{T_e} = T_e/(m_e c^2)$ is the electron temperature in the units of electron rest mass, $\gamma = \varepsilon/(m_e c^2) = \sqrt{1 + p^2/(m_e^2 c^2)}$ is the total electron energy in the units of electron rest mass or the Lorentz factor, $\gamma_{\max}$ is the energy cutoff parameter, and K_2 is the modified Bessel function of the second kind. The normalization constant N_{const} is determined through $\int f_e(\mathbf{p}) d\mathbf{p} = 1$, so that for a pure Maxwell-Jüttner distribution without a cutoff ($\gamma_{\max} = \infty$) the normalization constant is equal to unity. The total electron density n_e is kept fixed, i.e., we do not throw away the electrons but rather redistribute them.

Our goal is to determine the total power density of bremsstrahlung radiation emitted from such a plasma. Self-absorption of bremsstrahlung radiation is usually negligible for magnetic confinement plasma, and thus to calculate the radiative losses we will solely concentrate on spontaneous emission.

While for nonrelativistic plasma it is mainly electron-ion Coulomb collisions that contribute to emission, for relativistic plasma electron-electron bremsstrahlung becomes comparable or even exceeds electron-ion contribution and must be taken into account [126,127].

The effective expression for the bremsstrahlung power density emitted from thermal relativistic plasma with the Maxwell-Jüttner distribution was derived in Ref. [34]:

$$P_{\text{Br}} \approx 7.56 \times 10^{-11} n_e^2 \sqrt{\theta_{T_e}} \big[Z_{\text{eff}} \big(1 + 1.78\theta_{T_e}^{1.34}\big) + 2.12\theta_{T_e} \big(1 + 1.1\theta_{T_e} + \theta_{T_e}^2 - 1.25\theta_{T_e}^{2.5}\big)\big] \text{eV cm}^3/\text{s}. \tag{2}$$

Here Z_{eff} is the effective ion charge, and the formula is valid for relativistic, but not ultrarelativistic plasmas, up to $\theta_{T_e} \leqslant 1$. The first term in Eq. (2) proportional to Z_{eff} comes from electron-ion bremsstrahlung; it has a nonrelativistic leading order of $\sqrt{\theta_{T_e}}$, while the $1.78\theta_{T_e}^{1.34}$ term inside the first set of parentheses is a correction to it due to relativistic effects. The second term comes from electron-electron bremsstrahlung; it has a nonrelativistic leading order of $\theta_{T_e}^{1.5}$ with the $1.1\theta_{T_e} + \theta_{T_e}^2 - 1.25\theta_{T_e}^{2.5}$ term inside the parentheses being a relativistic correction.

Expression (2) was used in Refs. [110,111] (note that in Ref. [111] the $\theta_{T_e}^2$ term is missing in the second term due to electron-electron bremsstrahlung) to evaluate the energy budget of the p-B11-based fusion systems and can be considered as a benchmark.

In the next two subsections we calculate the emitted radiation from relativistic plasma described by the cutoff electron distribution (1) due to electron-ion (Sec. II A) and electron-electron (Sec. II B) bremsstrahlung and compare it with the thermal result given by Eq. (2). We will see that there is a reduction in bremsstrahlung losses as a result of introducing the energy cutoff and evaluate it.

A. Electron-ion bremsstrahlung

To calculate the radiative losses from relativistic plasma due to bremsstrahlung emission we need to know the corresponding differential cross section. The relevant cross section for relativistic electron-ion bremsstrahlung is the Bethe-Heitler differential cross section [1,2]. It was used to derive expression (2), and so Refs. [110,111] also implicitly use it to calculate the thermal bremsstrahlung losses from the p-B11 plasma. The Bethe-Heitler differential cross section for relativistic electron-ion bremsstrahlung, including the Elwert correction factor [129], is given by [83]

$$d\sigma_{\text{ei}}(\omega) = \alpha Z^2 r_e^2 \frac{p_f}{p} \frac{d\omega}{\omega} \frac{\eta_f}{\eta} \frac{1 - e^{-2\pi\eta}}{1 - e^{-2\pi\eta_f}} \left\{ \frac{4}{3} - 2\varepsilon\varepsilon_f \frac{p_f^2 + p^2}{p_f^2 p^2 c^2} + m_e^2 c^2 \left(\frac{l_f \varepsilon}{p_f^3 c} + \frac{l \varepsilon_f}{p^3 c} - \frac{l_f l}{p_f p} \right) + L \left[\frac{8}{3} \frac{\varepsilon\varepsilon_f}{p p_f c^2} + \frac{\hbar^2 \omega^2}{p^3 p_f^3 c^6} \left(\varepsilon^2 \varepsilon_f^2 + p^2 p_f^2 c^4 \right) + \frac{m_e^2 c^2 \hbar\omega}{2 p p_f} \left(\frac{\varepsilon\varepsilon_f + p^2 c^2}{p^3 c^3} l - \frac{\varepsilon\varepsilon_f + p_f^2 c^2}{p_f^3 c^3} l_f + \frac{2\hbar\omega\varepsilon\varepsilon_f}{p_f^2 p^2 c^4} \right) \right] \right\}. \tag{3}$$

Equation (3) is valid when the Born approximation is applicable, which requires $v/c = pc/\varepsilon \gg Z\alpha$. Here ε is the electron energy, ε_f is the electron energy after emission of a photon, p is the electron momentum, p_f is the electron momentum after emission of a photon, ω is the emitted photon angular frequency, Z is the ion charge, $\alpha = e^2/(\hbar c)$ is the fine-structure constant, and $r_e = e^2/(m_e c^2)$ is the classical electron radius, while

$$\varepsilon_f = \varepsilon - \hbar\omega, \tag{4}$$

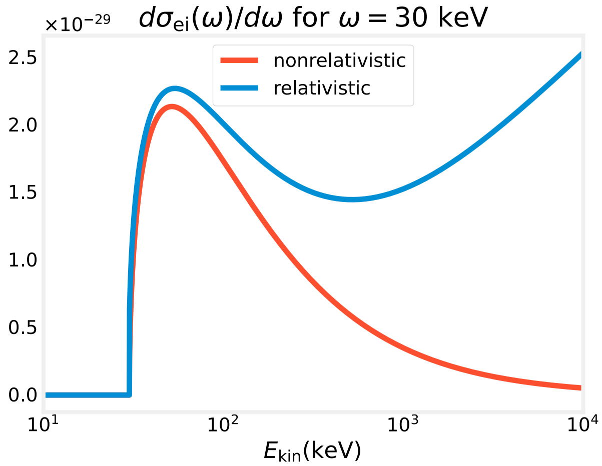

FIG. 1. The Elwert corrected relativistic Bethe-Heitler differential cross section $d\sigma_{ei}(\omega)/d\omega$ [blue line given by Eq. (3)] in arbitrary units for electron-ion bremsstrahlung emission of 30 keV photon as a function of electron kinetic energy ε_{kin}. The red line shows the nonrelativistic approximation of the Elwert corrected Bethe-Heitler differential cross section (see Ref. [128]).

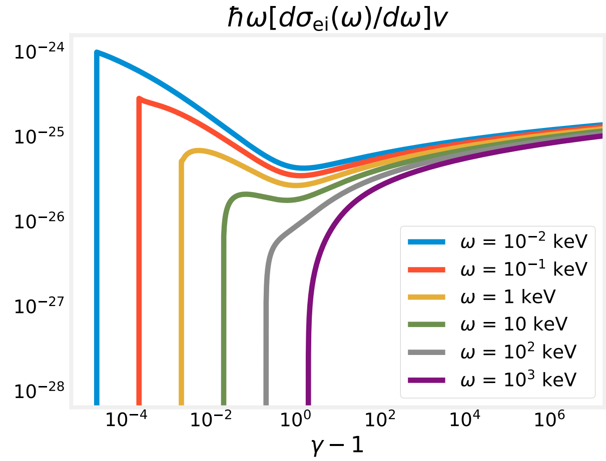

FIG. 2. The photon energy $\hbar\omega$ times the Elwert corrected Bethe-Heitler differential cross section $d\sigma_{ei}(\omega)/d\omega$ [Eq. (3)] times the electron speed $v = pc^2/\varepsilon$ in arbitrary units as a function of the dimensionless electron kinetic energy $\varepsilon_{kin}/(m_ec^2) = \gamma - 1$ for several values of the photon energy. This product of these three quantities enters formula (8) under the integral.

$$l_f = 2\ln\frac{\varepsilon_f + p_fc}{m_ec^2}, \quad l = 2\ln\frac{\varepsilon + pc}{m_ec^2}, \tag{5}$$

$$L = 2\ln\frac{\varepsilon_f\varepsilon + p_fpc^2 - m_e^2c^4}{m_ec^2\hbar\omega}, \tag{6}$$

$$\eta_f = \frac{\alpha Z\varepsilon_f}{p_fc}, \quad \eta = \frac{\alpha Z\varepsilon}{pc}. \tag{7}$$

Figure 1 shows the differential cross section as a function of electron kinetic energy ε_{kin}. We notice several important features from Fig. 1. First, below $\varepsilon_{kin} = \hbar\omega$ the cross section is zero, which is a manifestation of the fact that, due to energy conservation, the electron cannot emit a photon larger than its kinetic energy. Second, we notice that after reaching a local maximum the differential cross section does not decrease to zero for large values of the kinetic energy but instead increases. This second feature is of a relativistic nature and is not present in the nonrelativistic calculations (see the red line in Fig. 1, which denotes the nonrelativistic approximation given in Ref. [128]). It is this increased probability of bremsstrahlung for large values of the electron kinetic energy that is responsible for the additional radiative losses in the relativistic regime.

The differential cross section (3) together with the electron distribution function determine the total electromagnetic power density emitted from plasma due to electron-ion bremsstrahlung:

$$P_{ei} = n_en_i\iint \hbar\omega\frac{d\sigma_{ei}(\omega)}{d\omega}\frac{pc^2}{\varepsilon}f_e(\mathbf{p})d\omega\, d\mathbf{p}. \tag{8}$$

Figure 2 shows the product of the differential cross section, the energy of the emitted photon, and the electron speed that enters formula (8) under the integral versus the dimensionless electron kinetic energy $\varepsilon_{kin}/(m_ec^2) = \gamma - 1$ for a wide range of values of the emitted photon energy. We can see that for large electron energies, the value of $\omega d\sigma_{ei}(\omega)/d\omega$ increases for all values of the photon energy.

The basic intuition that we extract from Figs. 1 and 2 is that if we redistribute high-energy electrons into lower energies we could expect a reduction in the overall emission. To check whether it is indeed correct we perform a series of the numerical integrations for a range of electron temperatures T_e and the cutoff parameter γ_{max}.

The most important results of the paper are presented in Figs. 3 and 4. There we use Eqs. (3) and (8) to calculate the power density due to electron-ion bremsstrahlung emission for different values of the electron temperature and the energy cutoff. Figure 3 shows the power density of electron-

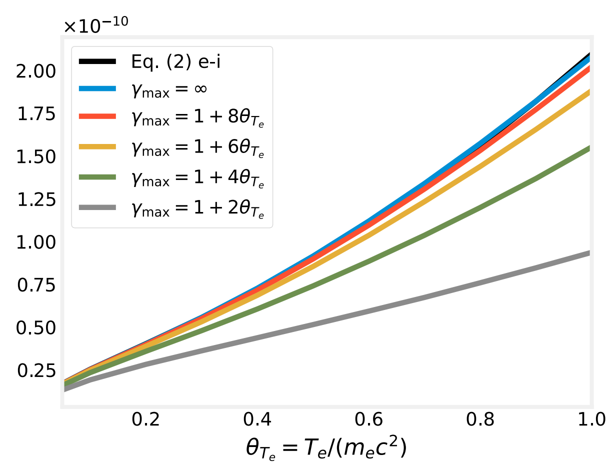

FIG. 3. Electron-ion bremsstrahlung emitted power density in arbitrary units as a function of the dimensionless electron temperature θ_{T_e} for several values of the energy cutoff γ_{max}.

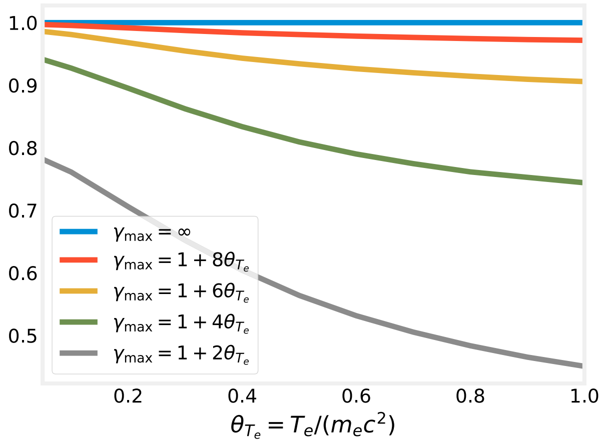

FIG. 4. Reduction in electron-ion bremsstrahlung emission relative to the thermal case as a function of the dimensionless electron temperature θ_{T_e} for several values of the energy cutoff $\gamma_{\max}$.

ion bremsstrahlung radiation generated by a plasma with the electron distribution function that has an energy cutoff as well as by a thermal plasma without a cutoff, both calculated numerically and using a fitting formula of Eq. (2); the graph shows the corresponding curves for several values of the cutoff parameter $\gamma_{\max}$ versus the dimensionless electron temperature $\theta_{T_e} = T_e/(m_e c^2)$. As one would expect, we can see that for thermal plasma without a cutoff ($\gamma_{\max} = \infty$) we reproduce the line given by Eq. (2). Figure 4 is similar to Fig. 3 but instead shows the power density of bremsstrahlung radiation from plasma with an energy cutoff relative to the emission power from the thermal plasma. We can clearly see the reduction in the emitted power for the distribution with an energy cutoff. The larger the cutoff depth, the greater the reduction, while for $\gamma_{\max} \gtrsim 1 + 8\theta_{T_e}$ the reduction becomes negligible. We also see that as plasma becomes more relativistic, i.e., θ_{T_e} approaches unity, the effect of the redistribution becomes more pronounced.

Thus, we demonstrated that by redistributing electrons into lower energies it is possible to mitigate electron-ion bremsstrahlung emission from relativistic plasma. Note that the opposite effect occurs, i.e., the bremsstrahlung losses increase, for nonrelativistic plasma. This is because in the nonrelativistic approximation, the differential cross section decreases to zero for large electron energies, so it is mainly thermal electrons that contribute to the emission. In the relativistic case, the superthermal electrons contribute disproportionally more to the emission, and thus moving them into more thermal part of the distribution reduces the overall radiative losses.

B. Electron-electron bremsstrahlung

We can expect that the redistribution of electrons into lower energies will have an even greater impact on electron-electron bremsstrahlung, which has a quadrupole nature as opposed to a dipole nature of electron-ion bremsstrahlung [130]. This is because the total electromagnetic power density emitted from plasma due to electron-electron bremsstrahlung is obtained by integrating twice over the electron distribution [8]:

$$P_{ee} = n_e^2 m_e c^3 \iint \frac{1}{2}\frac{\gamma_1+\gamma_2}{\gamma_1\gamma_2}\sqrt{\frac{1}{2}[(\mathbf{u}_1\cdot\mathbf{u}_2)-1]} \times \left(\int_0^{k_{\max}} k_{cm}\frac{d\sigma}{dk_{cm}}dk_{cm}\right) f_e(\mathbf{u}_1) f_e(\mathbf{u}_2) d\mathbf{u}_1\, d\mathbf{u}_2. \quad (9)$$

Here $\mathbf{u}_1$, $\mathbf{u}_2$ are the dimensionless momenta of two colliding electrons in the units of $m_e c$, i.e., $\mathbf{u} = \mathbf{p}/(m_e c)$, and $f_e(\mathbf{u})$ is the electron distribution as a function of $\mathbf{u}$ properly renormalized so that $\int f_e(\mathbf{u})d\mathbf{u} = 1$; k is the photon energy in the units of $m_e c^2$, $d\sigma/dk_{cm}$ is the differential cross section for electron-electron bremsstrahlung, the upper limit in the integral over dk_{cm} is $k_{\max} = u_{cm}^2/\gamma_{cm}$, and the subscript “cm” refers to the center of mass system of the colliding electrons; see Ref. [8] for the details.

The expression for the differential cross section of electron-electron bremsstrahlung is given in Refs. [7,131]. However, due to its complexity, a simplified treatment developed in Ref. [8] is commonly used. This simplified treatment was also employed in deriving Eq. (2) used in Refs. [34,110,132]. For consistency and computational benefits we will also use this approach.

In accordance with Ref. [8], we introduce the following function:

$$J(\gamma_1,\gamma_2) = \int_{\gamma_1\gamma_2 - u_1u_2}^{\gamma_1\gamma_2+u_1u_2} \sqrt{\frac{1}{2}(\mu-1)} \times \left(\int_0^{k_{\max}} k_{cm}\frac{d\sigma}{dk_{cm}}dk_{cm}\right) d\mu, \quad (10)$$

where $\mu = (\mathbf{u}_1\cdot\mathbf{u}_2)$ and $\int_0^{k_{\max}} k_{cm}(d\sigma/dk_{cm})dk_{cm}$ is a function of μ.

Then the emitted power density due to electron-electron Coulomb collisions can be written as [8]

$$P_{ee} = n_e^2 m_e c^3 \iint \frac{\gamma_1+\gamma_2}{4\gamma_1 u_1\gamma_2 u_2} J(\gamma_1,\gamma_2) f_e(\gamma_1) f_e(\gamma_2) d\gamma_1\, d\gamma_2, \quad (11)$$

where $f_e(\gamma)$ is the properly renormalized electron distribution as a function of the electron energy in the units of electron rest mass γ so that $\int f_e(\gamma)d\gamma = 1$.

It was demonstrated in Ref. [8] that the function $J(\gamma_1,\gamma_2)$ can be approximated as

$$J(\gamma_1,\gamma_2) = 4\alpha r_e^2 \Bigg\{ \left(\frac{\mu}{2}-2\right)\sqrt{\mu^2-1} - \frac{11}{12}\mu^2 + \frac{20}{3}\mu - \frac{8}{3}\ln(\mu+1) + \left[\frac{3}{2} + \left(\frac{\mu}{2} - \frac{8}{3}\frac{\mu+2}{\mu+1}\right)\sqrt{\mu^2-1}\right]\ln(\mu+\sqrt{\mu^2-1}) + \frac{7}{4}\ln^2(\mu+\sqrt{\mu^2-1})\Bigg\}\Bigg|_{\gamma_1\gamma_2-u_1u_2}^{\gamma_1\gamma_2+u_1u_2}. \quad (12)$$

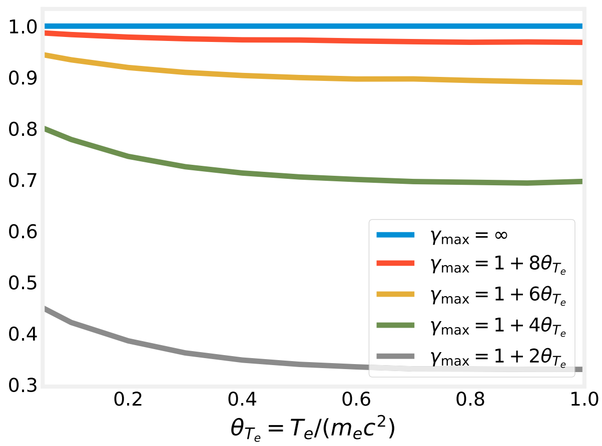

FIG. 5. Reduction in electron-electron bremsstrahlung emission relative to the thermal case as a function of the dimensionless electron temperature θ_{T_e} for several values of the energy cutoff γ_{max}.

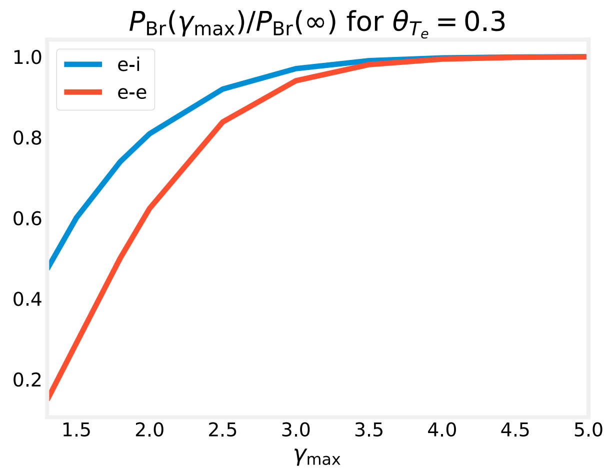

FIG. 6. Reduction in bremsstrahlung emission relative to the thermal case as a function the energy cutoff γ_{max} for $\theta_{T_e} \approx 0.3$ ($T_e \approx 150$ keV).

Using Eqs. (11) and (12) we can perform a numerical integration to obtain the emitted power density for various values of the electron temperature T_e and the cutoff parameter γ_{max}.

The reduction of the electron-electron bremsstrahlung losses as a function of the electron temperature for several values of the cutoff parameter γ_{max} relative to the power density of the thermal bremsstrahlung is shown in Fig. 5. We can see that similar to the case of electron-ion bremsstrahlung there is a reduction in the radiative losses; in fact, the reduction even exceeds that for the electron-ion case.

III. POSSIBLE RELEVANCE FOR ANEUTRONIC FUSION

In this section we speculate how the reduction in bremsstrahlung losses from relativistic plasma with an energy cutoff can potentially help with the energy balance of p-B11 fusion reactors.

What has been envisioned as, in principle, a possible p-B11 fusion reactor would operate at the typical electron temperature on the order of $T_e \approx 150$ keV [110–112], which corresponds to $\theta_{T_e} \approx 0.3$. We plot the decrease in bremsstrahlung emission relative to the thermal case as a function of the energy cutoff γ_{max} for such a temperature in Fig. 6.

Open magnetic field line plasma devices, which are thought to be used for the p-B11-based fusion, offer several methods for regulating the confinement of electrons and ions. These methods include magnetic [132,133], electrostatic [132,133], centrifugal [134,135], and ponderomotive [136] confinement. For example, slow electrons can be confined electrostatically, while fast electrons are magnetically deconfined. This can be realized continuously: as thermal electrons gain high energies through collisions and leave the device, fast electrons are simultaneously removed, allowing electrostatic forces, to which slow electrons are more sensitive, to replenish the electron population to ensure charge neutrality.

Therefore, one possible way that the energy cutoff can emerge is due to the electrostatic ambipolar potential. The typical electrostatic ambipolar potential that can be established in such devices is roughly estimated as $|e\varphi| \sim 7T_e$ [132,133], corresponding to the cutoff value $\gamma_{max} \approx 2$. We can see from Fig. 6 that for γ_{max} approximately 2, there is a 20% reduction in electron-ion and a 40% reduction in electron-electron bremsstrahlung emission. If we choose a more conservative value of $\gamma_{max} \approx 2.5$, we can observe a decrease of 10% in electron-ion bremsstrahlung emission and a decrease of 20% in electron-electron bremsstrahlung emission. Finally, as Fig. 6 indicates, when the cutoff parameter exceeds $\gamma_{max} \approx 3.5$, the reduction in emission becomes negligible. Thus, a reduction of at least 10% can be potentially achieved.

We caution, however, that removing the electron tail substitutes the power loss in radiation with a power loss in kinetic energy. Nonetheless, this effect can be advantageous in situations where it is much easier to capture this energy in the form of fast electrons than potentially damaging radiation coming from all over the place. Bremsstrahlung energy is very hard to capture in part because it is omnidirectional, requiring any energy capture apparatus to completely surround the fusion device. The power loss in electron kinetic energy is much more easily recovered for several reasons. First of all, in a magnetic field, the electrons are lost along the field lines, so the power loss is highly localized and therefore may be captured with localized devices. Second, it is in the form of charged particle flow, for which energy capture can be efficient. We also note that the distribution function in the mirror machines may exhibit a significant anisotropy, so that the effective energy cutoff for perpendicular and parallel energies can be noticeably different. In addition, the emission of the radiation itself (both synchrotron and bremsstrahlung) will deplete the high-energy tail and introduce a natural energy cutoff. However, the total self-consistent picture is outside of the scope of this work.

IV. CONCLUSION

We considered bremsstrahlung emission from a relativistic plasma of fixed electron density but varying phase space distributions. We investigated the impact of the energy cutoff on bremsstrahlung losses and found that for relativistic plasmas introducing energy cutoff through redistributing superthermal electrons into lower energies can significantly decrease radiative losses. This result is not entirely obvious even qualitatively, let alone quantitatively; the nonrelativistic approximation of the bremsstrahlung effect could actually show an increase in the bremsstrahlung losses.

We conducted calculations to determine the potential reduction in bremsstrahlung emission for a typical p-B11-based fusion device. We speculate that, if a number of assumptions are met, a meaningful reduction of 10% or even more can be potentially attained. Given the importance of bremsstrahlung losses for the operation of these systems, such a potential reduction can help to relax the constraints on the energy balance of the p-B11-based fusion. We caution, however, that such a strategy of trading radiation losses for kinetic losses must be accompanied with a means of effective capture of the deconfined electrons, which requires certain design choices.

Finally, we highlight the potential relevance of our findings in astrophysical scenarios. While it is power-law distributions that are typically observed in astrophysical settings, under certain conditions, such as formation of the effective magnetic mirror traps in the plasma surrounding neutron stars and black holes, the distribution with an energy cutoff can emerge, in which case the calculations presented here will pertain. For example, it might change the Eddington limit when it is influenced by bremsstrahlung radiation [137,138].

ACKNOWLEDGMENT

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