AS551 General Plasma Physics I
October 6, 1997

Problem Set #4 (due October 13)

1. Consider a toroidal device with major radius $R_0 = 5\,\mathrm{m}$, minor radius $r = 1\,\mathrm{m}$, toroidal field $B(R = R_0) = 5\,\mathrm{T}$. Determine approximately how long it takes electrons (or ions) with energy $10\,\mathrm{keV}$ to drift out of the device due to the inhomogeneities in $\mathbf{B}$. Suppose that a $10\,\mathrm{keV}$ plasma with density $10^{20}\,\mathrm{m}^{-3}$ is introduced into the device. The ∇B drifts will set up a time-varying electric field in the plasma, causing both polarization and $\mathbf{E}\times\mathbf{B}$ drifts. Verify that the resulting $\mathbf{E}\times\mathbf{B}$ motion is outwards and estimate the time it takes for the plasma to be ejected.

2. Suppose a plasma of electrons and ions is immersed in an infinite homogeneous magnetic field $\mathbf{B} = B(t)\hat{z}$. The function of time $B(t)$ is something that we can control.

The plasma is now taken repeatedly through the following four steps.

(i) The field strength is increased from $B(t = 0) = B_0$ to αB_0 on a time scale slow compared to an ion cyclotron period, but fast compared to a collision time, i.e., $1/\nu \gg B/(dB/dt) \gg 1/\Omega_i$.

(ii) The plasma is allowed to relax to equilibrium by waiting several collision times.

(iii) The field strength is decreased to B_0 as fast as in (i).

(iv) The plasma is again allowed to relax to equilibrium.

(a) Derive an expression for the plasma energy gain per cycle.

(b) Does any heating take place in the event that the collision frequency is much greater than the rate of change of the field in steps (i) and (iii), i.e., $B/(dB/dt) \gg 1/\nu \gg 1/\Omega_i$? Explain.

(c) Assume that electrons are isotropized in energy faster than are ions, but that energy transfer between electrons and ions is slower than the ion isotropization time. How might you modify steps (i)–(iv) in order to:

(1) pump energy mainly into electrons, but not into ions?

(2) pump energy mainly into ions, but not into electrons?

3. Consider an axisymmetric mirror configuration with a vector potential given by $A_\theta(r,z)\hat{\theta}$ (using cylindrical coordinates $[r,\theta,z]$).

(a) Write down the components of the magnetic field. Show that the magnetic field lines lie in "flux surfaces" given by $rA_\theta = \mathrm{const}$. Show that the magnetic flux enclosed by a flux surface is proportional to rA_θ.

(b) Because the configuration is axisymmetric, a charged particle moves in such a way that the canonical angular momentum $r(mv_\theta + qA_\theta)$ is constant. Show that this means that the particle is effectively confined to a flux surface. Make this statement precise.

(c) The magnetic field is now varied in time (i.e., the vector potential is given by $A_\theta(r,z,t)\hat{\theta}$). Show that the third adiabatic invariant, the flux enclosed by the orbit as it encircles the axis of the device, is conserved.