AS551 General Plasma Physics 1
December 2, 1996

In-class Exam II.

1. **Fluid derivations (10 minutes).**

In the derivation of fluid equations from the Vlasov Eq., closure approximations have to be made. Briefly describe why this is so. Briefly describe 2 different closure approximations.

2. **MHD overview (10 minutes).**

Write down a complete set of ideal MHD equations.

Briefly describe 2 important assumptions (other than fluid closure approximations) in the derivation of the ideal MHD equations. Briefly mention what some of the consequences of these assumptions are. [For example, what phenomena are lost or ordered out of the equations by these assumptions.]

3. **Choice of a wave or equilibrium derivation (25 minutes).**

Do *either* problem 3A or problem 3B:

(3A) Shear-Alfvén wave. Starting from the ideal MHD equations, derive the dispersion relation for the shear-Alfvén wave, propagating in a uniform plasma with a uniform equilibrium magnetic field $\vec{B}_0 = B_0\hat{z}$.

(3B) Bennett pinch equilibrium.

Consider a standard z-pinch equilibrium configuration (current in the z-direction, magnetic field only in the θ direction), with cylindrical symmetry ($\partial/\partial\theta = \partial/\partial z = 0$). Calculate the volume integrated plasma energy (per length in z):

$$W = \int_0^a dr 2\pi r \frac{3}{2} p$$

Assuming that ideal MHD holds, and that the plasma is confined to $r < a$ (so $p = 0$ at $r = a$), show that W can be expressed in terms of the total current carried by the plasma.

4. Quickie (5 minutes). Briefly describe what is meant by a flux surface. Show that in ideal MHD equilibria, surfaces of constant pressure are flux surfaces.