Inverse problem for bremsstrahlung radiation

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Information about the momentum distribution of electrons in a plasma is often of great importance, for example, when superthermal electrons are created by noninductive means. For such predominantly one-dimensional distribution functions, an analytic inversion has been found that yields the velocity distribution of superthermal electrons given their bremsstrahlung radiation.

In general, it is not possible to write an analytic expression for the distribution function of electrons simply on the basis of observing their bremsstrahlung radiation in a plasma. There is a set of circumstances, however, in which just such a data inversion is possible: Consider a homogeneous plasma of electrons and ions of charge Z_i. The bremsstrahlung emission at energy $\hbar\omega$ is given by

$$R(\hbar\omega,\theta)\equiv\frac{dP}{d(\hbar\omega)d(\theta)}$$

$$=\int_{v_{\min}}^{\infty} d^3\mathbf{v}\, f(\mathbf{v}) n_e n_i v\hbar\omega \frac{d\sigma}{d(\hbar\omega)d(\theta)}, \qquad (1)$$

where $f(\mathbf{v})$ is the electron distribution function, and $d\sigma/d(\hbar\omega)d(\theta)$ is the bremsstrahlung radiation cross section. If the electrons are nonrelativistic, yet are of large enough energy for the Born approximation to be valid ($v/c \gg 2\pi Z_i/137$), then the cross section is given by Koch and Motz[1] as

$$\frac{d\sigma}{d(\hbar\omega)}=\frac{16Z_i^2 r_0^2}{137}\frac{1}{\hbar\omega}\frac{c^2}{v^2}\ln\left(\frac{\left(\sqrt{\frac{1}{2}mv^2}+\sqrt{\frac{1}{2}mv^2-\hbar\omega}\right)^2}{\hbar\omega}\right). \qquad (2)$$

This cross section has been integrated over photon emission angle, hence the resulting total bremsstrahlung emission must also be averaged over emission angle.

In certain cases, the distribution of the most energetic electrons is primarily one dimensional, i.e., $f(\mathbf{v})=f(v_\parallel)$, where $v_\parallel$ might be the dimension along a strong imposed magnetic field. This is the case in tokamak plasmas for a slide-away discharge[2] or for discharges sustained by rf-driven currents.[3] Using Eq. (2) and $f(\mathbf{v})=f(v_\parallel)$ in Eq. (1), we have

$$R(s)=R_0\int_s^{\infty} dx\, f(x)\frac{1}{x}\ln\left(\frac{x}{s}+\sqrt{\frac{x^2}{s^2}-1}\right)$$

$$=R_0\int_s^{\infty} dx\, f(x)\frac{1}{x}\cosh^{-1}\left(\frac{x}{s}\right), \qquad (3)$$

where energies are normalized such that $x^2\equiv\frac{1}{2}mv_\parallel^2$, $s^2\equiv\hbar\omega$. Note that the lower bound takes into account the cutoff of electrons below the photon energy.

If we now take the derivative of Eq. (3), we obtain

$$\frac{dR}{ds}=R_0\int_s^{\infty} dx\, f(x)\left(-\frac{1}{s(x^2-s^2)^{1/2}}\right)$$

$$=-\frac{R_0}{s}\int_s^{\infty} dx\left(\frac{f(x)}{2x}\right)\frac{2x}{(x^2-s^2)^{1/2}}, \qquad (4)$$

which is the familiar form of an Abel invertible equation,[4] so we can invert to yield

$$f(x)=\frac{2x}{R_0\pi}\int_x^{\infty}\frac{d}{ds}\left(s\frac{dR}{ds}\right)\frac{ds}{(s^2-x^2)^{1/2}}. \qquad (5)$$

A similar inversion can be made for a distribution that is primarily in the perpendicular direction, e.g., a distribution of highly energetic electrons created by electron cyclotron resonant heating. In this case (where we label the energy as $y^2\equiv\frac{1}{2}mv_\perp^2$), the resulting inversion is

$$f(y)=\frac{2}{R_0\pi}\int_y^{\infty}\frac{d}{ds}\left(s\frac{dR}{ds}\right)\frac{ds}{(s^2-y^2)^{1/2}}. \qquad (6)$$

Application of this inversion technique to an experimental situation requires radiation measurements along several sightlines, at different viewing angles. For nonrelativistic electrons, this allows for a representative sample of angles from which the angle-averaged emission can be calculated. For electrons that are mildly relativistic, the emission has a more pronounced angular dependence, and therefore requires a larger number of viewing angles to properly determine the angle-averaged emission. The more complicated data inversion of a spatially inhomogeneous plasma is not addressed here, as it would require many more sightlines. If, however, the superthermal population produced is highly localized, as it often is, the inversion outlined here can be applied.

The restrictions on this successful inversion of bremsstrahlung data are that of high energy (Born approximation), yet nonrelativistic electrons, and a primarily one-dimensional distribution. All three of these conditions can be met by the superthermal electrons in an extended tail distribution of a typical tokamak discharge (1–10 keV), where the excited tail extends to about five thermal speeds. Thus it should be possible to apply experimentally this inversion technique to determine, for example, the very important features of rf-driven tokamak discharges.

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