CURRENT DRIVE BY LOWER HYBRID WAVES IN THE PRESENCE OF ENERGETIC ALPHA PARTICLES

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ABSTRACT. Many experiments have proved the effectiveness of lower hybrid waves for driving toroidal current in tokamaks. However, the use of these waves to provide all the current in a reactor is thought to be uncertain because it may happen that the waves do not penetrate into the centre of the more energetic reactor plasma and, if they do, the wave power may be absorbed by alpha particles rather than by electrons. The paper addresses mathematically the interaction between lower hybrid waves and alpha particles.

1. INTRODUCTION

Lower hybrid waves have been found to be effective in driving toroidal current in tokamaks [1] and the central aspects of the theory have now been established by many experiments [2–5]. It would be of great benefit if these waves could be employed to provide the full toroidal tokamak current, thereby allowing tokamak reactors to operate in steady state. At present, >2 MA of current are routinely produced by lower hybrid waves in laboratory tokamaks, but it is still uncertain whether these waves can be used to provide all the current in a reactor. The problems connected with extrapolating the results relate to the physics of the very centre of a tokamak reactor: first, it may happen that the waves do not penetrate into the centre of the more energetic reactor plasma, and, second, if they do, as pointed out by Wong and Ono [6], the wave power may be absorbed by α-particles rather than by electrons.

The damping by α-particles can be avoided by using waves of high frequency. These higher frequency waves, however, are technically more difficult to inject into the plasma; the coupling to the plasma may be less efficient, and, if waveguides are employed, their structural dimensions will be smaller at high frequency and they will be more difficult to fabricate. Recent calculations by Bonoli and Porkolab [7], Barbato and Santini [8] and Spada et al. [9] explore the possibilities for avoiding the damping by α-particles. The calculations of Barbato and Santini include the quasi-linear effect of the waves on the α-particle distribution.

The question of α-particle damping would be moot, however, if the waves did not penetrate into the plasma centre. Penetration of the plasma centre is not likely to occur in the dense, hot, reactor plasma, because there are sufficient hot electrons at a distance from the plasma centre that absorb the lower hybrid power. In this case, the lower hybrid driven current would appear, but it would have a hollow profile. Hollow current profiles are not considered to be stable, so the injection of such a current would not be suitable for maintaining the complete plasma current. In a number of detailed reactor studies it was concluded that, in order to provide the full current in a reactor, lower hybrid current drive will need to be supplemented by other means of current drive. (Such a design is pursued in the present plans for ITER [10].) These conclusions are reached precisely because of the assumption that the current appears only on the magnetic surface in which the wave power is absorbed.

If the current could be provided through deposition of power away from the plasma centre, then there would be a number of favourable circumstances related to the interaction with α-particles: The energetic α-particles, produced primarily in the plasma centre, are fewer and less energetic away from the centre. The lower hybrid waves, on the other hand, have a decreasing phase velocity as the waves enter the higher density plasma centre, so the α-particle damping is much less away from the centre than it is at the plasma centre. Moreover, while an isotropic distribution of α-particles will always damp the lower hybrid waves (so long as the energetic

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particles can be considered unmagnetized in the wave fields), this is no longer the case if the distribution function is not isotropic. In fact, as the α-particles both slow down and diffuse away from where they are produced (near the plasma centre), their distribution could become anisotropic off the centre. Since the α-particle distribution is energy inverted, it is possible that, away from the centre, the α-particles might even amplify the lower hybrid waves.

The speculation that lower hybrid waves might provide the full current even in the presence of fusion generated α-particles can be analysed through a consideration of the details of the wave damping on α-particles, the evolution of the α-particle distribution function, including quasi-linear velocity diffusion and spatial diffusion, and the spatial diffusion of energetic current carrying electrons. In this paper, we examine the response of α-particles to intense radiofrequency (RF) heating power, in the light of the goal of generating the full current by RF waves.

The paper is organized as follows: In Section 2 we derive the one-dimensional α-particle distribution function, which is under the influence of intense RF diffusion, but no spatial diffusion. In Section 3 we derive the power dissipation based on the one-dimensional velocity distribution. A very interesting finding of Sections 2 and 3 is that an exact one-dimensional distribution function can be found analytically and that the exact power dissipation can be written in terms of this distribution function. We find a quasi-linear damping law for α-particle damping at high RF power that scales with the 4/5 power of the lower hybrid wave spectral intensity, while for the electron damping there is saturation at high power ('0 power' law scaling). Of practical interest is that, as other authors have found, the resonant interaction of lower hybrid waves with α-particles ought to be avoided altogether. In Section 4, we derive the one-dimensional α-particle distribution function in the presence of α-particle diffusion in configuration space, but in the absence of resonant wave-particle interactions. In Section 5 we present our conclusions and suggestions for experimentation to decide key issues.

2. FOKKER-PLANCK EQUATION

Energetic alpha particles collide primarily with electrons, so the slowing down of an α-particle with velocity $\vec{v}$ obeys

$$\frac{d\vec{v}}{dt} = -\nu\vec{v} \tag{1}$$

where the collision frequency $\nu = 16\sqrt{2\pi m_e}\, e^4 n_e \times \ln\Lambda/3T_e^{3/2} m_\alpha$. For a distribution of α-particles, we then have a kinetic equation

$$\frac{\partial}{\partial t} f(\vec{v}, t) = -\frac{\partial}{\partial \vec{v}} \cdot \vec{S} + \frac{\dot{N}}{4\pi v_\alpha^2}\, \delta(v - v_\alpha) \tag{2}$$

where the second term describes the birth of α-particles at 3.5 MeV at a rate $\dot{N}$ and the velocity space flux $\vec{S}$ is given by

$$\vec{S} = -\nu\vec{v} f - \overleftrightarrow{D}_{rf} \cdot \frac{\partial}{\partial \vec{v}} f \tag{3}$$

where the first term describes the slowing down of α-particles in collisions with electrons and $\overleftrightarrow{D}_{rf}$ is the quasi-linear diffusion coefficient.

For a Landau resonant interaction, with $k_\perp \gg k_\parallel$, the diffusion is essentially in the perpendicular direction, and the magnitude of the quasi-linear diffusion coefficient can be written, correctly for unmagnetized α-particles [11] as

$$\overleftrightarrow{D}_{rf} = D_{QL}(v_\perp)\hat{\iota}_\perp\hat{\iota}_\perp \tag{4}$$

with

$$D_{QL} = \begin{cases} 0 & \text{if } v_\perp < \omega/k_\perp \\ \frac{1}{2}(eZ_\alpha E/m_\alpha)^2(\omega/k_\perp v_\perp)^2 \\ \quad\times\ (k_\perp^2 v_\perp^2 - \omega^2)^{-1/2} & \text{if } v_\perp > \omega/k_\perp \end{cases} \tag{5}$$

where E is the electric field of the lower hybrid wave, ω is the wave frequency, $k_\perp$ is the perpendicular wave number and $Z_\alpha = 2$ is the α-particle charge state.

It is quite remarkable that the problem is then inherently one-dimensional. In contrast, other so-called 1-D problems, such as current drive by electron Landau damping, are really just approximations. The problem of α-particle damping is rigorously one-dimensional, because the collisions of α-particles with electrons result in a contraction of the α-particle distribution function in parallel velocity, and this contraction affects neither the rate of slowing down nor the wave absorption resonance. This can be seen as follows. Let us write

$$-\frac{\partial}{\partial \vec{v}} \cdot \nu\vec{v} f = -\left(\hat{\iota}_\perp \frac{1}{v_\perp}\frac{\partial}{\partial v_\perp} v_\perp + \hat{\iota}_\parallel \frac{\partial}{\partial v_\parallel}\right) \cdot \vec{v} f$$

$$= -\left(\frac{1}{v_\perp}\frac{\partial}{\partial v_\perp} v_\perp^2 f + \frac{\partial}{\partial v_\parallel} v_\parallel f\right) \tag{6}$$

We define

$$F(v_\perp, t) \equiv \int_{-\infty}^{+\infty} f(\vec{v}, t)\, dv_\parallel \tag{7}$$

Then, integrating Eq. (2) over $v_\parallel$, we obtain

$$\frac{\partial}{\partial t} F(v_\perp, t) = -\frac{1}{v_\perp}\frac{\partial}{\partial v_\perp} v_\perp(S_\perp(v_\perp, t) + Q_\alpha) \tag{8}$$

where the α-particle source flux Q_α is

$$Q_\alpha = H(v_\alpha - v_\perp)\frac{1}{v_\perp}\frac{\dot{N}}{2\pi v_\alpha}\sqrt{v_\alpha^2 - v_\perp^2} \tag{9a}$$

where H is a Heaviside function ($H(x) = 1$, $x > 0$, or else $H(x) = 0$), and the perpendicular flux is given by

$$S_\perp(v_\perp, t) = -\nu v_\perp F(v_\perp, t) - D_{QL}\frac{\partial}{\partial v_\perp} F(v_\perp, t) \tag{9b}$$

In general, supplementing Eq. (8) by an initial condition for $F(v_\perp, t = 0)$ and by boundary conditions, for example, at $v_\perp = v_0$, where v_0 is some finite velocity, and at $v_\perp \to \infty$ results in a well posed diffusion equation. Here, the physical boundary conditions of interest are for α-particles that slow down into a relatively cold thermal distribution, which we treat as a sink. In other words, we impose a perfectly reflecting (zero flux) boundary at $v_\perp \to \infty$ and a perfectly absorbing boundary at $v_\perp \to 0$. The first condition is realized if F or $S_\perp$ vanish at $v_\perp \to \infty$. The absorption condition as $v_\perp \to 0$ can be realized by matching the full flux to the collisional flux, $-\nu v_\perp F(v_\perp, t)$. In other words, particles escape the region of interest to lower $v_\perp$ at a rate dictated just by slowing down collisions.

For the interesting case of steady state, the flux $S_\perp$ must vanish in the sourceless, semi-infinite region $v_\perp > v_\alpha$. In the region $v_\perp < v_\alpha$, we can integrate Eq. (8) to obtain

$$\nu v_\perp F(v_\perp) + D_{QL}\frac{\partial}{\partial v_\perp} F(v_\perp) = \frac{1}{v_\perp}\frac{\dot{N}}{2\pi v_\alpha}\sqrt{v_\alpha^2 - v_\perp^2} \tag{10}$$

where the constant of integration is zero since the flux must vanish at $v_\perp = v_\alpha$. Note that as $v_\perp \to 0$, we have $2\pi v_\perp S_\perp \to \dot{N}$, since it is presumed that there is a sink at $v_\perp = 0$ to collect, in the steady state, the complete α-particle production.

Equation (10) is a first order ordinary differential equation and so it is readily solved exactly. Let us introduce the non-dimensional variables, $s = v_\perp/v_\alpha$, $\beta = \omega/k_\perp v_\alpha$ and $G = F/(\dot{N}/2\pi\nu v_\alpha^2)$. Using the quasilinear diffusion coefficient in Eq. (5), Eq. (10) may then be cast in the form

$$DH(s-\beta)\left(\frac{1}{s^2-\beta^2}\right)^{1/2}\frac{dG(s)}{ds} + s^3 G(s) = s\sqrt{1-s^2}\,H(1-s) \tag{11}$$

where D, a normalized measure of diffusion, is defined as

$$D \equiv \frac{1}{2}\left(\frac{eZ_\alpha E}{m}\right)^2\frac{\beta^3}{\nu\omega v_\alpha^2} = \frac{D_{QL}}{\nu v_\alpha^2} s^2\sqrt{s^2-\beta^2} \tag{12}$$

For $\omega/k_\perp > 0$, both the particle source and the diffusion vanish at the boundary with the collisional region, $v_\perp \to 0$, so that boundary conditions can be realized by matching the solution of Eq. (11) for $s > \beta$ to the solution valid for $s < \beta$, namely

$$G(s) = s^{-2}\sqrt{1-s^2} \qquad s < \beta \tag{13}$$

Let us define for $s > \beta$

$$\Psi(s) \equiv \frac{1}{D}\int_\beta^s x^3\sqrt{x^2-\beta^2}\,dx = \frac{1}{5D}(s^2-\beta^2)^{3/2}(s^2+2\beta^2/3) \tag{14}$$

Then the solution to Eq. (11) can be written for $s > \beta$ as

$$G(s) = e^{-\Psi(s)}\left[\frac{\sqrt{1-\beta^2}}{\beta^2} + \frac{1}{D}\int_\beta^s x e^{\Psi(x)}\sqrt{x^2-\beta^2}\sqrt{1-x^2}\,H(1-x)\,dx\right] \tag{15}$$

We note that the integrating factor can be expressed in terms of elementary functions, as done on the right-hand side of Eq. (14), but further analytic progress appears possible only in certain asymptotic limits, which may be of physical interest and are described in the following section.

3. POWER DISSIPATED

The power dissipated by the waves turns out to be available exactly in terms of the one-dimensional solution. Since the RF induced flux is perpendicular, we can write the wave power dissipated as

$$P_{rf} = \int d^3v\, m\vec{v}\cdot\vec{S}_{rf} = \int 2\pi v_\perp dv_\perp m v_\perp \int dv_\parallel \vec{S}_{rf} = \int 2\pi v_\perp dv_\perp m v_\perp D_{QL}\frac{\partial F}{\partial v_\perp} \tag{16}$$

where we carried out the integration over $v_\parallel$ trivially, since $\vec{S}_{rf}$ is independent of $v_\parallel$. This power can be contrasted to the α-particle heating power, which can be written as

$$P_\alpha = \dot{N} m v_\alpha^2/2 \tag{17}$$

Consider the normalized wave power (where a negative number indicates wave damping)

$$\begin{aligned} P_\omega &\equiv \frac{P_{rf}}{P_\alpha} = 2D \int_\beta^\infty (s^2 - \beta^2)^{-1/2} \frac{dG}{ds}\, ds \\ &= 2 \int_\beta^\infty ds \left[s\sqrt{1-s^2}\, H(1-s) - s^3 G \right] \end{aligned} \tag{18}$$

where in writing the first equality we used Eqs (10), (11) and (12), and in writing the second equality we used Eq. (11). The correct power dissipated may be found now by use of Eq. (15). Note that the calculation of this power dissipated is exact for the physical model at hand — namely that α-particles are subject to slowing down collisions only and that the RF power results in perpendicular diffusion only. No approximations are introduced through our mathematical use of the one-dimensional perpendicular distribution function. Accurately calculating the fully two-dimensional distribution function $f(v_\perp, v_\parallel)$ is more difficult, and might be approached approximately, for example, by an expansion in Legendre harmonics [8]. The fully two-dimensional distribution function $f(v_\perp, v_\parallel)$ is sufficient but not necessary to calculate the dissipated power, but an approximated two-dimensional distribution function will only give an approximated dissipated power; here, the one-dimensional distribution function $F(v_\perp)$, which is available exactly, not only suffices but is guaranteed to give the exact power dissipation.

Note, however, that this simplification would not be possible if the pitch angle scattering of the α-particles were important. This limits our consideration to wave phase velocities much higher than about 500 keV, which is where slowing down and pitch angle scattering are about equal. In practice, this limit is of little concern, since the waves must avoid entirely the less energetic α-particles.

The power dissipated can be estimated analytically in a number of interesting limits. If $\beta > 1$, then, of course, no power is dissipated to the α-particles. Of great interest, however, is how sensitive the dissipation is under high power to a relaxation of the condition that $\beta > 1$; in other words, whether some α-particles can be resonant with the wave, resulting in tolerable dissipation with $\beta \leq 1$. Consider the limit $\beta \to 1$; then, most of the wave power is dissipated through resonant interactions in the region $s > 1$, namely through acceleration of α-particles to velocities greater than their birth velocity. This can be seen as follows: The first term on the right-hand side of Eq. (18), representing interactions in the regime $s < 1$, can be calculated directly as

$$\begin{aligned} &\int_\beta^\infty s\sqrt{1-s^2}\, H(1-s)\, ds \\ &= \frac{1}{2} \int_{\beta^2}^1 \sqrt{1-x}\, dx = \frac{1}{3}(1-\beta^2)^{3/2} \end{aligned} \tag{19}$$

This integral can be compared with the second term on the right-hand side of Eq. (18), which, by using Eq. (15), can be written as the sum of two terms, namely

$$\begin{aligned} &-\int_\beta^\infty s^3 G\, ds \\ &= -\int_\beta^\infty s^3 e^{-\Psi(s)} \left[\frac{\sqrt{1-\beta^2}}{\beta^2} + \frac{1}{D} \cdots \right] \\ &\equiv A + B \end{aligned} \tag{20}$$

Term A can be put in the form

$$\begin{aligned} A &\equiv -\frac{\sqrt{1-\beta^2}}{\beta^2} \\ &\times \int_\beta^\infty s^3 \exp\left(-\frac{1}{5D}(s^2-\beta^2)^{3/2}(s^2+2\beta^2/3)\right) ds \\ &= -D^{4/5} \frac{\sqrt{1-\beta^2}}{2\beta^2}\, g(\beta^2/D^{2/5}) \end{aligned} \tag{21}$$

where

$$g(x) = \int_x^\infty y \exp\left(-\frac{1}{5}(y-x)^{3/2}(y+2x/3)\right) dy \tag{22}$$

Now, in the limit $\beta^2 \sim O(1)$, but $D \to \infty$, we have $x \to 0$, and we can approximate

$$\begin{aligned} \lim_{x\to 0} g(x) &= \int_0^\infty y \exp(-y^{-5/2}/5) \left(1 + \frac{xy^{3/2}}{6} + O(x^2)\right) \\ &= \frac{2\Gamma(4/5)}{5^{1/5}} + O(x) \end{aligned} \tag{23}$$

where we used the gamma function, $\Gamma(x) \equiv x\Gamma(x-1)$. Thus, we have for $D \to \infty$

$$A \equiv -\frac{\Gamma(4/5)}{5^{1/5}} \frac{\sqrt{1-\beta^2}}{\beta^2} D^{4/5} + O(D^{2/5}) \tag{24}$$

Note that the contribution of term A to the power dissipated is $D^{4/5}(1-\beta^2)^{-1}$ larger than that of the term considered in Eq. (19), which represents interactions in the regime $s < 1$.

Let us now show that term B is negligible compared to term A in the regime of interest. From Eq. (20), we have

$$B \equiv -\frac{1}{D}\int_{\beta}^{\infty} s^3 \exp\left(-\frac{1}{5D}(s^2-\beta^2)^{3/2}(s^2+2\beta^2/3)\right) I(s)\,ds \tag{25}$$

where, for most of the interval of interest, namely $s > 1$, but $\beta \to 1$,

$$\begin{aligned} I(s) &\equiv \int_{\beta}^{s} x\, e^{\Psi(x)} \sqrt{x^2-\beta^2}\sqrt{1-x^2}\, H(1-x)\,dx \\ &= \int_{\beta}^{1} x\, e^{\Psi(x)} \sqrt{x^2-\beta^2}\sqrt{1-x^2}\,dx \\ &\sim \int_{\beta}^{1} \sqrt{(x-\beta)(1-x)}\,dx \sim (1-\beta)^2 \end{aligned} \tag{26}$$

Thus, term B is smaller than term A for large D and $\beta \to 1$ by a factor of $(1-\beta)^{-3/2}/D$.

Since term A is the largest term both for large D and $\beta \to 1$, it follows that, asymptotically,

$$P_\omega \to 1.19 D^{4/5}\sqrt{1-\beta} \tag{27}$$

from which it follows that, in order for the RF dissipated power to be small compared to the α-particle heating power, the lower bound of phase velocities must obey $1-\beta \sim D^{-8/5}$, so that, for large D, β is essentially equal to one, from which we may infer that there is essentially no leeway in the requirement that none of the α-particles be resonant with the RF spectrum in the limit of high RF power.

While, in practice, the mathematical limit $D \to \infty$ is not physically realizable, the ITER scenarios considered by Barbato and Santini [8] do consider D (their parameter ξ) in the range $D \sim 3$, and $D \gg 1$ might be envisioned to occur in highly RF driven low density tokamak reactors.

The limit $D \to \infty$ is an interesting limit, and the ion response is fundamentally very different from the electron response [12] to the same waves in the limit of high power. Under low RF power and a Landau resonant interaction, the RF power dissipated both in the α-particles and in the electrons is proportional to the RF power. For the electrons, however, under intense RF excitation, there is a maximum absorbed power, and the absorbed power then scales as a constant with increasing RF power. In contrast, at high power, the α-particle absorption is proportional to the 4/5 power of the RF intensity. The physical reason for this difference is clear: For a Landau resonant interaction, the magnetized electrons exhibit a quasi-linear plateau, so their capability of absorbing power is limited. On the other hand, the α-particle absorption of power is highly non-linear in that, with more RF power, more α-particles are accelerated into the region $s > \beta$ ($v_\perp > \omega/k$) where they can then be subjected to further acceleration.

4. WAVE ABSORPTION IN THE PRESENCE OF SPATIAL DIFFUSION OF α-PARTICLES

Given the necessity of avoiding damping by α-particles, it is important to know, in detail, the α-particle distribution as a function of both perpendicular velocity and configuration space. Since non-linear effects at high power tend to exacerbate the damping, rather than, as for electrons, alleviate it, resonant interactions with the α-particles are best avoided altogether. In that case, the α-particle distribution function, $F(v_\perp, r)$, can be calculated in the absence of the RF interaction. The linear damping rate can then be calculated on the basis of that distribution function. On the basis of the result of Section 3, it would be anticipated that, if the α-particle source is isotropic in velocity space, quasi-linear effects do not decrease the damping, and so the wave phase velocity needs to be faster than the fastest α-particles to avoid damping of the wave.

The processes that determine the α-particle distribution function $f(\vec{v}, \vec{r}, t)$ are then the energy slowing down due to the collisions with electrons, the α-particle source and the spatial transport. The first two processes can be described adequately, as done in writing Eq. (2), but the physics of α-particle transport is still uncertain, both theoretically and experimentally. Suppose that the transport of α-particles is diffusive in nature and can be described by a diffusion coefficient of the form $\vec{\vec{D}} = \vec{\vec{D}}(\vec{v})$. The kinetic equation governing the α-particle distribution can then be written as in Eq. (2) as

$$\begin{aligned} \frac{\partial}{\partial t} f(\vec{v}, \vec{r}, t) &= -\frac{\partial}{\partial \vec{v}} \cdot \vec{S} + \frac{\dot{n}_s(\vec{r})}{4\pi v_\alpha^2}\,\delta(v - v_\alpha) \\ &\quad + \nabla\cdot\vec{\vec{D}}(\vec{v})\cdot\nabla f \end{aligned} \tag{28}$$

where $\dot{n}_s(\vec{r})$ is the source density, where the last term accounts for the spatial diffusion and where the flux $\vec{S}$ now represents collisional flux only. For diffusion out of a cylinder, we are interested in the distribution of α-particles as a function of the perpendicular velocity direction and as a function of radial distance r; hence, we define

$$F(v_\perp, r, t) \equiv \int_{-\infty}^{+\infty} f(\vec{v}, \vec{r}, t)\,dv_\parallel dz \tag{29}$$

where $\hat{z}$ is the ignorable direction along the cylinder axis.

There are a number of diffusion coefficients with very different parametric dependences that have been proposed for α-particle transport [13, 14], and certain toroidal effects might occur even in the wave–particle interaction [15]. The general case will be difficult to consider, but some insight may be gained by studying a simpler, analytically tractable problem. Let us, for simplicity, restrict our attention to cylindrical geometry and, further, to a diagonal diffusion coefficient, such that the radial component is a function of perpendicular velocity only, i.e. of the form $\overleftrightarrow{D}(\vec{v}) = D_r(v_\perp)\hat{r}\hat{r} + D_z(\vec{v})\hat{z}\hat{z}$. (Such a diffusion might arise, for example, if the typical radial step size in a collision were the α-particle gyroradius.) Then, we can integrate over Eq. (28) to derive

$$\frac{\partial}{\partial t} F(v_\perp, r, t) = \frac{\nu}{v_\perp} \frac{\partial}{\partial v_\perp} v_\perp^2 F + \frac{D_r(v_\perp)}{r} \frac{\partial}{\partial r} r \frac{\partial}{\partial r} F + \frac{\dot{n}(r)}{2\pi v_\alpha} (v_\alpha^2 - v_\perp^2)^{-1/2} H(v_\alpha - v_\perp) \tag{30}$$

where $\dot{n}(r)$ is the axially integrated source density, i.e.

$$\dot{n}(r) \equiv \int dz\, \dot{n}_s(\vec{r})$$

In the steady state, $\partial/\partial t = 0$, we can cast Eq. (30) into an inhomogenous diffusion equation for $\Psi = v_\perp^2 F$. Using the new variable u,

$$u(v_\perp) \equiv -\int_0^{v_\perp} \frac{D(s)}{\nu s} ds \tag{31}$$

we can write

$$\frac{\partial}{\partial u} \Psi(u, r, t) = \frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial}{\partial r} \Psi + S(u)\dot{n}(r) \tag{32}$$

where we defined

$$S(u) = \frac{v_\perp^2}{2\pi v_\alpha D(v_\perp)} (v_\alpha^2 - v_\perp^2)^{-1/2} \tag{33}$$

Equation (32) is an inhomogeneous diffusion equation with constant coefficient, in cylindrical geometry, to be solved with homogeneous initial and boundary conditions. It has the well known Green's function solution

$$G(u - u', r - r') = \frac{I_0(rr'/2(u - u'))}{2(u - u')} \times \exp(-(r^2 + r'^2)/4(u - u'))\, H(u - u') \tag{34}$$

where I_0 is a modified Bessel function. The function G solves the heat equation in a cylinder

$$\frac{\partial}{\partial u} G - \frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial}{\partial r} G = \delta(u - u') \frac{1}{r} \delta(r - r') \tag{35}$$

Using Eq. (35), it is possible to estimate the damping by α-particles born on different flux surfaces. In a tokamak, the concentration of α-particle births is largest in the centre, where the percentage of energetic α-particles would also be correspondingly largest. The α-particles would diffuse away from the centre as they slowed down. Their contribution to wave damping off the centre would then be in the form of less energetic particles, so that the wave–particle resonance would be more easily circumvented.

The key simplifying assumption in deriving this Green's function is in the assumed form of the α-particle transport, especially in the assumption that the radial diffusion coefficient depends on $v_\perp$ only. This assumption completely decouples the parallel velocity from the perpendicular velocity space dynamics, allowing us to integrate Eq. (28) over the parallel direction to find an equation for $F(v_\perp, r, t)$. In the absence of this decoupling, such a large simplification appears unlikely, and, although a Green's function still exists (since the equation is linear), it is not likely to be so simple.

To examine a special case of particular interest, suppose the α-particles are all born on the axis $r = 0$, i.e.

$$\dot{n}_s(r) = \frac{\dot{N}}{2\pi r} \delta(r)$$

then

$$\Psi(r, u) = \dot{N} \int_{u_\alpha}^{u} du' \frac{\exp(-r^2/4(u - u'))}{4\pi(u - u')} S(u') \tag{36}$$

where $u_\alpha \equiv u(v_\alpha)$. Note that we have defined u to be negative, and the range of interest is $0 > u > u_\alpha$.

Suppose, further, the special case of a radial diffusion coefficient of the form $D_r(v_\perp) = \tau_\alpha v_\perp^2$. Then we can perform the integral in Eq. (36). First, note that from Eq. (31)

$$u(v_\perp) = -\frac{\tau_\alpha}{\nu} \int_0^{v_\perp} v_\perp dv_\perp = -\frac{\tau_\alpha}{2\nu} v_\perp^2 \tag{37}$$

so that the source S becomes

$$S(u) = \frac{1}{2\pi v_\alpha \sqrt{2\nu\tau_\alpha}} (u - u_\alpha)^{-1/2} \tag{38}$$

Substituting now for the source in Eq. (36), we have

$$\Psi(r, u) = \frac{1}{4\pi^2 v_\alpha \sqrt{2\nu\tau_\alpha}} \times \int_{u_\alpha}^{u} \frac{\exp(-r^2/4(u - u'))}{2(u - u')} (u' - u_\alpha)^{-1/2} du'$$

$$= \frac{1}{4\pi^2 v_\alpha \sqrt{2\nu\tau_\alpha}}$$

$$\times \int_0^{u-u_\alpha} \frac{\exp(-r^2/4x)}{2x} (u - u_\alpha - x)^{-1/2} dx$$

$$= \frac{\Gamma(1/2)}{8\pi^2 v_\alpha \sqrt{2\nu\tau_\alpha \pi(u - u_\alpha)}}$$

$$\times \exp(-r^2/8(u - u_\alpha)) K_0 \left(\frac{r^2}{8(u - u_\alpha)}\right) \tag{39}$$

where K_0 is a modified Bessel function.

From Eq. (39), the conclusion can be drawn that there is damping but never amplification of the wave, both near and far from the source at the axis. This can be seen by substituting for u and Ψ to obtain

$$F(v_\perp, r) \equiv \dot{N} \frac{\Gamma(1/2)}{8\pi^2 \tau_\alpha v_\alpha v_\perp^2 \sqrt{\pi(v_\alpha^2 - v_\perp^2)}}$$

$$\times \exp\left(-r^2/4\frac{\tau_\alpha}{\nu}(v_\alpha^2 - v_\perp^2)\right) K_0 \left(\frac{r^2}{4\frac{\tau_\alpha}{\nu}(v_\alpha^2 - v_\perp^2)}\right) \tag{40}$$

In this case, there is clearly no wave growth since $F(v_\perp, r)$ is a monotonically decreasing function of $v_\perp$ for all r.

A very interesting question is what diffusion coefficient can result in local amplification of the wave. In other words, although the spatially integrated distribution function is monotonically decreasing and hence Landau damping, there may be radii at which the local distribution function is inverted and hence Landau amplifying. It would appear that if high $v_\perp$ but low $v_\parallel$ α-particles diffuse most quickly, then there may be an inversion in $v_\perp$ space at large radial distances from the source. Such a diffusion coefficient actually violates the condition under which Eq. (39) was derived, but one might also suppose that the parallel dependence here is inconsequential. It turns out that finding the class of diffusion coefficients that allow or disallow growth is more challenging than one might have expected, and we must leave as an open question such proofs for all but the simple case we considered, namely a radial diffusion coefficient of the form $D_r(v_\perp) = \tau_\alpha v_\perp^2$.

5. CONCLUSIONS

Several issues have become apparent in the effort to implement in tokamak reactors current drive by lower hybrid waves. In this work we have expanded upon the key issues related to the α-particle environment. Here, we summarize our findings and address what further experimental effort might point to circumventing the problems that we have outlined.

As discussed by several authors, the damping by α-particles is significant enough to prevent efficient current drive in the plasma centre. In Sections 2 and 3 we supported the conclusions of these authors with an analytical calculation of power dissipation that is both precise and simple. Our observation was that the problem of α-particle damping could actually be posed precisely over the region of interest in tokamaks as a one-dimensional problem in velocity space. In addition we derived a 4/5 law for power dissipation in the asymptotic limit of high power waves.

Given the necessity for avoiding α-particles at any power, what is first needed is an account of the α-particle distribution as a function of both minor radius and velocity. In Section 4, we derived an integral equation for the α-particle distribution for a certain class of spatial diffusion. As energetic α-particles diffuse from the plasma centre, we can expect them also to slow down, which alleviates the requirement on the lower hybrid wave phase velocity. In addition, away from the tokamak centre, the plasma is less dense, so the perpendicular lower hybrid wave phase velocity is in any event faster and more likely to avoid the α-particles. It may also be the case that the α-particle distribution as a function of radius could develop anisotropies in velocity space. This would mean that, in principle, there could be local amplification of the lower hybrid waves.

In view of the above, the best scenario for current drive might occur if there were spatial diffusion of lower hybrid current carriers to produce current in the tokamak centre with power dissipated off the centre. What is needed, however, is firmer empirical evidence of both the radial transport of the fast electrons and the radial transport of the confined α-particles as they slow down. Also, because of the critical role played by the wave phase velocity, which governs the resonant interaction, a more precise study of the perpendicular index of refraction in toroidal geometry may be useful. Only with such empirical evidence can we evaluate the possibility of maintaining current drive by lower hybrid waves in a tokamak D–T reactor.

What are the necessary experiments? The physics of α-particle transport is under study for other reasons, particularly energy confinement. Although our problem presents some peculiarities, in particular with respect to anisotropic distributions, the physics of α-particle transport may have to await burning experiments. On the other hand, probably a great deal more effort could be made to understand the physics of fast electron transport. Bounds on the radial transport of these electrons might be inferred through global considerations, such as the rate of current production.

An alternative that may give more precise information on electron transport would be to measure with radial resolution signatures of the fast electrons, such as the bremsstrahlung [16, 17] or the synchrotron radiation [18, 19]. Such measurements would reveal more if, at the same time, an effort were made to localize in space, and possibly in time, the power dissipation. The advantage of localizing in time is that the transport process might be followed directly if time resolved measurements could be made. A second advantage is that, in the absence of spatial resolution, it might be possible to infer from the time resolved measurements certain plasma parameters, including those governing the spatial transport. One consideration in localizing in time the power deposition is that the transport itself may be affected by the concomitant induced electric field. Under steady power conditions, however, a steady state can be achieved with non-local current production exactly in the same way as with local production.

In summary, a number of concrete issues need to be clarified before definitive statements can be made on using lower hybrid current drive to provide all the required non-inductive current in a D–T tokamak reactor. One possibility of efficient current production by this means might rely upon the deposition of lower hybrid power somewhat off the centre, with subsequent penetration of the plasma centre by the fast, superthermal electrons.

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