# Fast Compression of Laser Beams to Highly Overcritical Powers

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Laser beams can be strongly compressed in a plasma by stimulated Raman backscattering in a time short compared to the time scale for filamentation instabilities to develop. Such a compression should make feasible multi-MJ multi-exawatt-laser pulses technologically challenging by other means. The compression efficiency can reach nearly 100% at the Langmuir wave breaking limit. [S0031-9007(99)09280-7]

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Achievable laser intensities have increased remarkably during recent years mainly due to the method of chirped pulse amplification (CPA). Petawatt ($\mathrm{PW} = 10^{15}$ W) powers and kJ energies have been attained by stretching, amplifying, and then recompressing laser pulses [1]. Certain modern applications, however, require even higher powers and energies. An example of such an application is the fast igniter approach to inertial confinement fusion [2]. Higher power pulses could have applications for radiography of dense materials. Important applications are already envisioned for exawatt ($\mathrm{EW} = 10^{18}$ W) MJ pulses. The higher total power in the pulse presents a practical difficulty to CPA techniques, because very large gratings would be needed to handle large power and heat loads.

The rationale of the CPA technique of stretching and recompressing is to avoid nonlinear effects which tend to spoil the beam quality. These nonlinear effects appear in the refraction coefficient at power densities $\sim\mathrm{GW/cm^2}$. Longitudinal recompressing by $\sim 10^3$ times allows CPA to reach power densities $\sim\mathrm{TW/cm^2}$, without focusing in vacuum. For kJ 0.5 psec pulse, the cross section of the amplifier and compressor must be $\sim 2000\ \mathrm{cm^2}$, so that $\sim$50-cm-diameter beam and $\sim$1-m-diameter gratings are required [1]. A higher recompression factor of $10^4$ would allow CPA to get a 10 kJ, 0.5 psec pulse using gratings about a meter in diameter. For higher energies, larger, more expensive gratings are required.

This Letter examines an alternative to the CPA approach, wherein the laser beams are amplified to power densities much higher than a $\mathrm{GW/cm^2}$, even without stretching-recompressing and focusing outside the amplifying medium. The medium capable of bearing such high power densities and heat loads is plasma, but intense pulses in plasmas are subject to numerous instabilities. In particular, the modulational instability can lead to filamentation of the laser beam followed by ponderomotive expulsion of electrons from the filaments. The critical power for such a cavitation is $P_{\mathrm{cr}} \approx 17(\omega/\omega_p)^2$ GW, where $\omega$ is laser frequency and $\omega_p$ is plasma frequency [3]. Larger $P_{\mathrm{cr}}$ occurs for larger $\omega/\omega_p$, but then the laser-plasma coupling becomes poor and hence the compression efficiency decreases. For a reasonably large frequency ratio, say $\omega/\omega_p = 10$, one has $P_{\mathrm{cr}} \approx 1.7$ TW. Highly overcritical powers are needed, but instabilities stand in the way of reaching those powers.

The difficulties could be overcome by ultrafast compression of laser beams, such that highly overcritical laser powers are attained before the instabilities develop. Stimulated Raman backscattering in plasma is an interesting candidate for the ultrafast compression mechanism. Experiments, carried out in liquids, showed that this process can rapidly deplete the pump [4]. An important feature of stimulated Raman backscattering, noted in [4], is that the pumped pulse grows to amplitudes much higher than that of the pump. This is because the pulse continually encounters and absorbs unperturbed regions of the pumping beam and the medium.

Consider then (see Fig. 1) a short laser seed amplified to a high power within a plasma layer, thin enough that destructive instabilities have no time to develop. The duration of the pump has to be at least $2L/c$, where $L$ is the width of the plasma slab, and $c$ is the speed of light. By properly shaping the plasma layer, the amplified pulse could be focused to reach even higher intensity on the target.

The simplest mathematical model for the amplification process is given by the classic equations for resonant 3-wave interaction. These were investigated in many physical contexts, in particular, for stimulated Brillouin and Raman scattering in optical fibers, liquids, gases, and plasma. The equations are integrable by the inverse scattering method, which can be employed to determine the large-time asymptotic behavior of well-localized wave packets [5]. For amplification/compression purposes, however, an advanced evolution of a short pulse inside a relatively long pumping beam is of particular interest. This intermediate asymptotic behavior can be approximated by a quasi-self-similar attractor solution, a so-called "$\pi$ pulse," which fully depletes the pumping beam while undergoing amplification and contraction (see Fig. 2).

During the advanced nonlinear stage of the interaction, the basic equations can be reduced to the single sine-Gordon equation that has quasi-self-similar attractor solutions. The amplitude and energy of the amplified pulse increase approximately proportionally to the length of the already absorbed part of the pumping beam (i.e., to the

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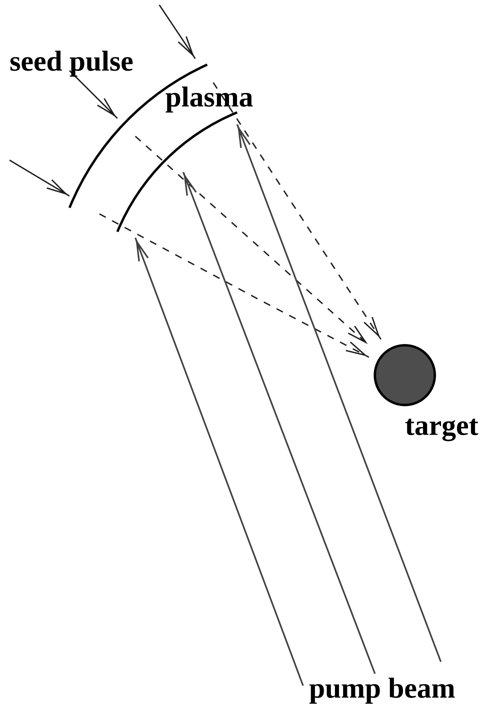


FIG. 1. A conceptual scheme for obtaining ultrahigh laser powers by means of Raman backscattering in a plasma.

propagation distance), while the pulse width decreases inversely proportionally to that length [6].

The $\pi$-pulse regime of stimulated Raman backscattering can be initiated in a plasma by injecting a seed pulse which is down-shifted from the pumping beam by the plasma frequency. The resonantly excited Langmuir wave then backscatters the pump into the pulse. For laser frequencies much higher than the plasma frequency, the group velocities of both lasers are nearly equal to the speed of light, and their dispersion is small and can be neglected. The major events take place at the location of the short pulse, which is nearly fixed in variable $\zeta = z/c + t$. The evolution of the lasers and Langmuir wave envelopes due to the resonant 3-wave interaction is described by the following equations:

$$a_t + 2a_\zeta = \omega_p f g, \qquad f_t + f_\zeta = -\omega a g^*/2, \quad (1)$$

$$g_t = -\omega_p f^* a. \quad (2)$$

Here indices denote the respective derivatives, $a$ and $g$ are the envelopes of the vector potentials of the pump and pulse, respectively, measured in the units of $m_e c^2/e \approx 5 \times 10^5$ V, and $f$ is the electrostatic field of the Langmuir wave, normalized to $m_e c \omega_p/e = c\sqrt{4\pi m_e n_e} \approx \sqrt{n_e[\text{cm}^{-3}]}$ V/cm. The duration of the pulse is considered to be larger than $\omega_p^{-1}$. Both lasers are circularly polarized. Self-nonlinearities of lasers and Langmuir wave are neglected. Plasma ions are assumed to be immobile.

During the linear stage of the backscattering instability [7], the originally small and narrow counterpropagating seed pulse is amplified and broadened. Its maximum moves with speed equal to the half of the speed of light in vacuum, so that the distance between the maximum and the front of the pulse ($\zeta = 0$) is increasing.

The nonlinear stage begins when the pump depletion becomes noticeable. During the advanced nonlinear stage, the pulse absorbs all the pump energy it encounters. The pulse amplitude and inverse width increase $\propto t$, while the maximum moves with a superluminous speed, approaching the front. As the $\zeta$ distance between the pulse front and maximum becomes much smaller than the total distance passed $t$, the $t$ derivatives of wave envelopes in Eqs. (1) become negligible in comparison with $\zeta$ derivatives. The integration of Eqs. (1) with neglected $t$ derivatives and boundary conditions $a \to a_0$, $f \to 0$ at $\zeta \to -\infty$ gives for real (up to the constant phase factor $\arg g = -\arg f =$ const) functions

$$a = a_0 \cos(u/2), \qquad f = \sqrt{\omega/\omega_p}\, a_0 \sin(u/2), \quad (3)$$

$$g = -u_\zeta/\sqrt{\omega \omega_p}. \quad (4)$$

The substitution of these formulas into Eq. (2) for $g$ leads to the sine-Gordon equation

$$u_{\tau\zeta} = \sin u, \qquad \tau \equiv t\omega\omega_p a_0^2/2. \quad (5)$$

The scaling properties of the most advanced part of the pulse front (which is small and evolves according to the linear theory) and of the sine-Gordon equation suggest that the attractor solution can be approximated by a self-similar function $u(\zeta, \tau) = \tilde{u}(\xi)$, $\xi \equiv 2\sqrt{\zeta\tau}$, that satisfies the ordinary differential equation

$$\tilde{u}_{\xi\xi} + \tilde{u}_\xi/\xi = \sin(\tilde{u}). \quad (6)$$

Its solution depends on a single parameter $\tilde{u}(+0) = \epsilon \ll 1$, which is determined by the integrated amplitude of the initial seed: $\epsilon = \sqrt{\omega\omega_p} \int d\zeta\, g(t = 0, \zeta)$. The dependence of the advanced-stage solution on $\epsilon$ is only logarithmic. That is why the solution of (5) can be approximated

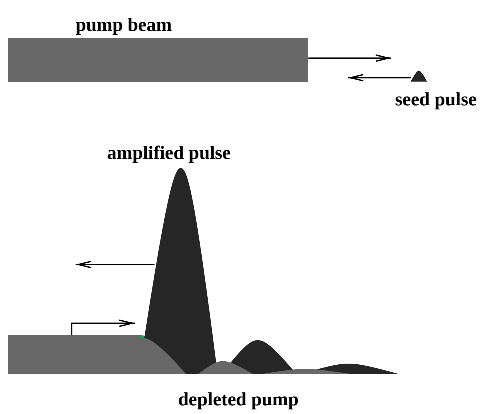


FIG. 2. The 1-dimensional 3-wave model of amplification via stimulated Raman backscattering. The multispiked $\pi$ pulse fully depletes the pump. The fraction of the pump energy, contained in the leading spike, is determined by the initial amplitude of the seed pulse integrated over its width.

by the solution of (6) even for time-dependent $\epsilon$. [Note that the effective $\epsilon$ can be time dependent due to nonlinear effects occurring within the width of initial seed pulse and thus affecting the integration domain of $g(t = 0, \zeta)$ in $\epsilon$.]

The function $\tilde{u}$ oscillates while approaching its limit value $\pi$ at $\xi \to \infty$, so that "$\pi$ pulse" represents, in fact, a wave train. Because of the smallness of $\epsilon$, the leading spike is close enough to the classical $2\pi$-pulse solution of the sine-Gordon equation, $\tilde{u} \approx 4\arctan(\epsilon e^{\xi}/4\sqrt{2\pi\xi})$. The $\tilde{u}_\xi$ maximum is located at $\xi_M \approx \ln(4\sqrt{2\pi\xi_M}/\epsilon)$. Taking into account a small deviation from the $2\pi$ pulse, one can show that the leading spike contains $I_1 \approx 4/(\xi_M + 2)$ portion of the total $\pi$-pulse energy. This simple asymptotic ($\epsilon \to 0$) formula is applicable even for $\epsilon = 0.1$, giving $I_1 \approx 0.53$ in excellent agreement with the numerical solution of Eq. (6), as shown in Fig. 3.

The time of amplification (and, therefore, the amount of pump energy available for compression) is limited by instabilities that may arise from noise.

For the Raman backscattering of the pumped pulse of duration $T$, the Stokes component, down-shifted by the plasma frequency, experiences about $gT\sqrt{\omega\omega_p}$ exponentiations. This factor does not change in the pumping process and remains of the order of 5. A more precise calculation for the leading spike confirms that the amplitude of the Stokes component is amplified only by a factor $\sim\exp(\pi\sqrt{2}) \approx 85$ times, making this channel of energy loss relatively benign.

The near-forward Raman scattering has a smaller growth rate but more time to develop (since the scattered light moves together with the pulse). Of interest here is the amplification exponent in the weakly coupled regime of short-pulse instability, $\sim g\sqrt{Tt\omega_p^3/\omega}$ [8]. For the exactly forward Raman scattering there is an additional reduction factor $\sqrt{\omega_p/\omega}$ [9]. This reduction is caused by the excitation of the anti-Stokes component. The near-forward Raman scattering limits the time of pulse amplification by $t < t_{\rm fw} = \frac{\Lambda_{\rm fw}}{\omega_p a_0}\sqrt{\frac{\xi_M\omega}{2\pi\omega_p}}$, where $\Lambda_{\rm fw}$ is the number of exponentiations allowed for the instability.

Alternatively, the limit on amplification may arise from the modulational instability, caused by the relativistic nonlinearity of the amplified pulse. For $P \gg P_{\rm cr}$, the growth rate is $\sim\omega_p^2 g^2/2\omega$. According to the above formulas for $g$, the pulse amplification time cannot exceed $t_{\rm md} = \frac{\xi_M^{2/3}\Lambda_{\rm md}^{1/3}}{2^{1/3}\omega_p a_0^{4/3}}$, where $\Lambda_{\rm md}$ is the number of exponentiations which can be tolerated for modulations.

The maximum energy to which the pulse may be amplified before either instability develops increases with the pump amplitude $a_0$ like $a_0^2\min(t_{\rm fw}, t_{\rm md})$. The ratio $t_{\rm fw}/t_{\rm md} \propto a_0^{1/3}$ also increases with $a_0$. Therefore, the modulational instability is relatively more important for larger $a_0$. These scalings apply so long as there is no Langmuir wave breaking, i.e., $a_0 < a_{\rm br} \approx \frac{1}{4}(\frac{\omega_p}{\omega})^{3/2}$. For $a_0 \approx a_{\rm br}$, one has $t_{\rm md}/t_{\rm fw} \sim \pi^{1/2}(2\xi_M)^{1/6}(4\Lambda_{\rm md})^{1/3}/\Lambda_{\rm fw}$, so that the times $t_{\rm md}$ and $t_{\rm fw}$ differ just by a logarithmic factor, and $t_{\rm md}$ may already be smaller than $t_{\rm fw}$.

For $a_0 \approx a_{\rm br}$ the Langmuir wave breaks near the first maximum of its envelope, which is close to the leading maximum of the amplified pulse (see Fig. 3). This prevents the reverse scattering of the pulse into the pumping beam, so that all secondary solitons in the $\pi$-pulse wave train can be suppressed. Then the fraction of the pump energy $I_1$, scattered into the leading spike, approaches 1. This conclusion is supported by a 1-dimensional-particle code simulation (see Fig. 4).

In this regime, the length of amplification, the final duration of compressed pulse, and the pulse energy are, respectively, $ct_{\rm md} \sim 5\xi_M^{2/3}\Lambda_{\rm md}^{1/3}\frac{c}{\omega_p}(\frac{\omega}{\omega_p})^2$, $T_M \sim 8(\frac{\xi_M}{2\Lambda_{\rm md}})^{1/3}\frac{1}{\omega_p}$, and $w_M \sim \frac{\xi_M^{2/3}\Lambda_{\rm md}^{1/3}}{\lambda[\mu{\rm m}]}\frac{kJ}{{\rm cm}^2}$. Hence, $T\omega_p > 1$ even in the most compressed state. Note that the final pulse energy

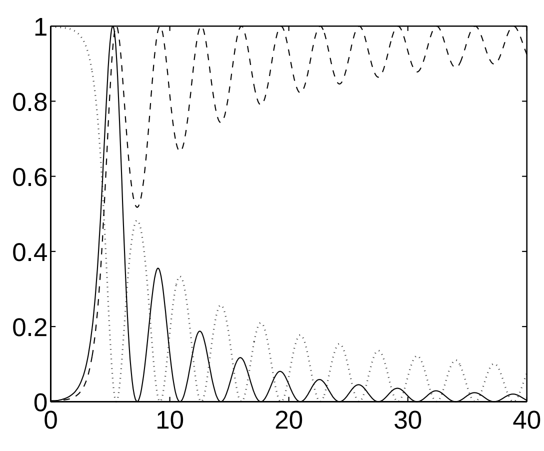

FIG. 3. Normalized energy densities {pumped pulse, $[u_\xi/\max(u_\xi)]^2$—solid line; Langmuir wave, $[\sin(u/2)]^2$—dashed line; and pump, $[\cos(u/2)]^2$—dotted line} as functions of $\xi = 2\sqrt{\zeta\tau}$ for $\epsilon = 0.1$.

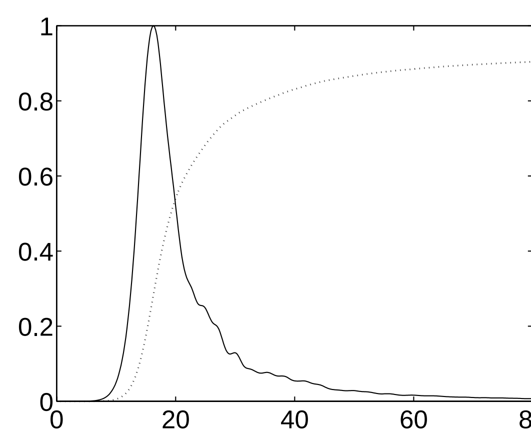

FIG. 4. The pumped pulse normalized energy density (solid line) and depletion (dotted line) versus $\zeta$ for $\omega/\omega_p = 10$ (i.e., $a_{\rm br} \approx 0.008$), $a_0 = 0.009$, $ct = 0.44$ mm, and the initial integrated amplitude of Gaussian seed pulse $\epsilon = 0.1$. A single-spiked signal is formed with almost 80% of the pump energy (6 kJ/cm$^2$ for 1 $\mu$m laser) in the spike. The code is described in [10].

does not depend on plasma frequency. For a 1-$\mu$m-wavelength laser and realistic values of the logarithmic factors $\xi_M$ and $\Lambda_{\rm md}$, the pulse energy reaches 5–8 kJ per cm$^2$. If also $\omega/\omega_p = 10$ (which corresponds to the plasma concentration $n \approx 10^{19}$ cm$^{-3}$), then $ct_{\rm md} \sim 0.8\xi_M^{2/3}\Lambda_{\rm md}^{1/3}$ mm $\sim$ 3–6 mm, and $T_M \sim$ 30–40 fs for realistic $\xi_M$ and $\Lambda_{\rm md}$, corresponding to a power density of about 200 PW/cm$^2$.

For even larger pump amplitude, $a_0 \gg a_{\rm br}$, the Langmuir wave breaks much before the pump is completely backscattered, which reduces the pump depletion and efficiency of amplification. Ahead of the breaking, $\zeta < \zeta_{\rm br}$, the linear theory is valid, so that the wave-breaking point can be calculated from Eqs. (3): $2\sqrt{\tau\zeta_{\rm br}} \equiv \xi_{\rm br} \approx \ln(\sqrt{2\pi\xi_{\rm br}}\, u_{\rm br}/\epsilon)$, where $u_{\rm br} \approx 2a_{\rm br}/a_0 \ll 1$. The relative depletion of the pump energy at the breaking point, $\alpha_{\rm br} \approx u_{\rm br}^2/4 \approx (a_{\rm br}/a_0)^2 \ll 1$, is small. The pulse envelope $g$ reaches there the space maximum, $g \sim \omega_p^2 a_0 t/2\omega\xi_{\rm br}$. Then the amplification time is restricted by $t < t_{\rm md}^{\rm br} = \frac{2\omega\xi_{\rm br}^{2/3}\Lambda_{\rm md}^{1/3}}{\omega_p^2 a_0^{2/3}} = \frac{\xi_{\rm br}^{2/3}\Lambda_{\rm md}^{1/3}}{2^{1/3}\omega_p a_0^{2/3} a_{\rm br}^{2/3}}$, because of the modulational instability. The pulse amplitude reached at $t = t_{\rm md}^{\rm br}$ is $g_M \sim (\Lambda_{\rm md} a_0/\xi_{\rm br})^{1/3}$.

The early Langmuir wave breaking for $a_0 \gg a_{\rm br}$ indicates that the short-scale ($\sim\lambda/2$) plasma electric field is not very important in this regime. Under such conditions the Raman backscattering can be dominated by Compton backscattering on individual electrons. There is experimental evidence of the transition from the stimulated Raman to Compton scattering via breaking of plasma waves [11]. The individual electron bounce frequency in the ponderomotive potential of the laser beat wave is $\omega_b = 2\omega\sqrt{a_0 g}$. For $g \sim g_M$, it follows that $\omega_b = 2\omega(\Lambda_{\rm md}/\xi_{\rm br})^{1/6} a_0^{4/3} = \omega_p(\Lambda_{\rm md}/4\xi_{\rm br})^{1/6} \times (a_0/2a_{\rm br})^{4/3} \gg \omega_p$. Such a regime of Compton backscattering was considered recently for ultrashort and intense initial seed pulses [12]. The current study indicates that it may be possible to enter this regime starting from a small-intensity seed pulse. When the logarithmic factor $\xi_{\rm br}$ is replaced by 1 (which is appropriate for a strong and short initial seed pulse), the above estimate for $g$ in the advanced Langmuir breaking regime gives $g \sim \omega_p^2 a_0 t/2\omega$ which agrees with formula 3 of [12]. The pulse duration in this regime is estimated as $\pi/\omega_b$. Then, the pulse energy ($\propto g^2/\omega_b$) evaluated at $t = t_{\rm md}^{\rm br}$ does not depend on $a_0$. Since the involved pump energy is $\propto a_0^2 t_{\rm md}^{\rm br} \propto a_0^{4/3}$, the efficiency in this strongly kinetic regime is $\sim(a_{\rm br}/a_0)^{4/3} \ll 1$.

To summarize, below the Langmuir wave breaking, $a_0 \ll a_{\rm br}$, the pump is completely depleted and the highest pumped pulse energy achievable before filamentation occurs increases with the pump amplitude $a_0$. For $a_0 \gg a_{\rm br}$, the efficiency is small. The most favorable regime for the pulse amplification is the near-threshold regime $a_0 \sim a_{\rm br}$, combining the highest achievable energy with the highest possible efficiency of amplification. Large energy fluxes, which can be sustained in this regime without causing beam filamentation, enable a substantial reduction of the sizes and costs of ultrahigh-energy laser amplifiers. Alternatively, with beam cross sections of the order used by CPA, the powers and energies would substantially increase. For instance, ten beams each with cross sections of 200 cm$^2$ would carry a total energy of 10–15 MJ with a total power of 300–400 EW.

Detailed 3D numerical calculations are required to verify quantitatively the analytical estimates presented here. But even if the amplification time turns out to be somewhat smaller than that given by the analytical treatment, the pulse would still be amplified to much higher energies than in currently employed schemes.

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