# Ultra-powerful compact amplifiers for short laser pulses*

V. M. Malkin,† G. Shvets, and N. J. Fisch
*Department of Astrophysical Sciences, Princeton University, Princeton, New Jersey 08544*

(Received 18 November 1999; accepted 8 February 2000)

Laser energies and powers, significantly much higher than available now through the most advanced chirped pulse amplifiers, might be achieved in much smaller devices. The working medium in such devices is plasma, capable of tolerating ultrahigh laser intensities within times shorter than it takes for filamentation instabilities to develop. The ultrafast amplification mechanism that outruns filamentation instabilities is the transient Raman backscattering of a laser pump in plasma. In principle, this mechanism is fast enough to reach nearly relativistic pumped pulse intensities, like $10^{17}\,\mathrm{W/cm}^2$ for $\lambda = 1\ \mu$m wavelength radiation. Such a nonfocused intensity would be $10^5$ times higher than currently available. This mechanism also produces complete pump depletion. Many amplifiers with expensive and fragile meter-size gratings might then be replaced by a single amplifier comprised of a 1 cm size plasma layer. Raman instabilities of the pump to noise, as the pump traverses plasma layer towards the seed pulse, can be suppressed by detuning the resonance appropriately, even as the desired amplification process persists with high efficiency due to nonlinear resonance broadening. Moreover, since the peak intensity scales like $1/\lambda^2$, even much higher laser intensities might become feasible when appropriate x-ray pump lasers are developed.
 [S1070-664X(00)96605-9]

## I. INTRODUCTION

Laser intensities inside conventional amplifiers are limited to gigawatts ($\mathrm{GW}=10^9\,\mathrm{W}$) per $\mathrm{cm}^2$, above which nonlinear modification of the material refraction index causes unacceptable distortions of the laser pulses. This limitation kept the peak laser intensity nearly constant for almost 20 years until the chirp pulse amplification (CPA) technique was invented in middle–late 1980s (see, for instance, Ref. 1). The CPA reaches terawatt ($\mathrm{TW}=10^{12}\,\mathrm{W}$) per $\mathrm{cm}^2$ intensities by means of longitudinal compression of laser pulses *after* their amplification. The compression of specially prepared chirped pulses is usually accomplished by two parallel diffraction gratings. The preparation includes stretching of the seed pulse by a matching stretcher (usually consisting of two antiparallel diffraction gratings) and then amplification by a high-quality broad-bandwidth amplifier. To get petawatt ($\mathrm{PW}=10^{15}\,\mathrm{W}$) laser power (necessary, say, for the fast igniter scenario of inertial fusion, Ref. 2) having $\mathrm{TW/cm}^2$ output intensities, one needs about meter-diameter gratings. To extrapolate this technique to the exawatt ($\mathrm{EW}=10^{18}\,\mathrm{W}$) power range, hundreds of such gratings would be required. The gratings are expensive, can be damaged by intense light, and yet high-quality broad-bandwidth amplifiers are needed. Moreover, there is a technological limit to the intensity that can be concentrated onto a final material surface, whether it be a mirror or a grating, employed for focusing the output pulse outside the compressor. This limit will be in the range of tens of $\mathrm{TW/cm}^2$.

It can be imagined, however, that both the focused and nonfocused ultrahigh laser intensities might be most naturally produced by amplifying plasma layers. An extremely fast amplification mechanism in the plasma is needed for this, since intense pulses in plasma are subject to very fast filamentation instabilities. The major idea here is to reach ultrahigh laser intensities inside a plasma-based amplifier in a time short compared to the growth time for filamentation instabilities. Analysis of transient Raman backscattering in plasma as a possible mechanism for such a fast compression of a laser pump indicates that $10^{17}\,\mathrm{W/cm}^2$ intensities of $\lambda = 1\ \mu$m-wavelength radiation might be achieved. The corresponding fluences would be about $5\,\mathrm{kJ/cm}^2$. These intensities and fluences are huge compared to what might be the technological limit for output pulses reflected from a material surface, such as occurs for CPA. Yet, $1/\lambda^2$ times higher intensities and $1/\lambda$ times higher fluences might be achieved for shorter-wavelength lasers.

Note that output pulses intensities given above for both the CPA and plasma based fast compressors are nonfocused, i.e., taken before the transverse focusing subsequent to the compression. The focused intensities are substantially higher; for instance, focusing of a 10 cm-diameter pulse to 10 $\mu$m increases the intensity $10^8$ times. The total power, however, is not changed by focusing.

A conceptual scheme for obtaining ultrahigh laser powers by means of transient Raman backscattering in a plasma is illustrated by Fig. 1. A short laser seed is amplified by a nearly counterpropagating laser pump, up-shifted by the plasma frequency $\omega_p$, to a high power within an undercritical ($\omega_p \ll \omega \equiv 2\pi c/\lambda$) plasma layer, thin enough that destructive instabilities have insufficient time or distance to develop. The duration of the pump has to be at least $2L/c$, where $L$ is the width of the plasma slab, and $c$ is the speed of light. By properly shaping the seed, pump and plasma layer,

*Paper DI24 Bull. Am. Phys. Soc. **44**, 88 (1999).
†Invited speaker.

 

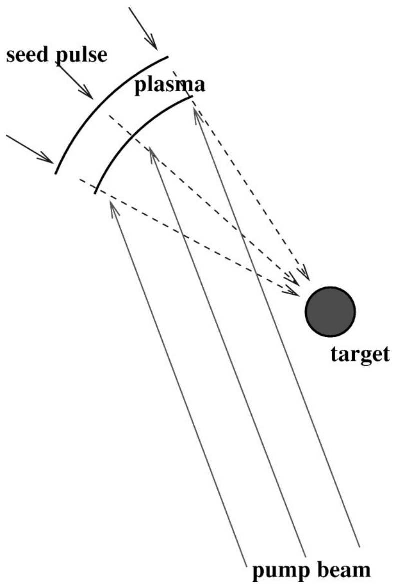


FIG. 1. A conceptual scheme for obtaining ultrahigh laser powers by means of transient Raman backscattering in a plasma. A short seed pulse is injected into an underdense plasma layer where it is amplified by a nearly counter-propagating long laser pump. The seed is down-shifted from the pump by the plasma frequency, so that the lasers are coupled in the plasma through the resonantly excited Langmuir wave. The coupling can be strong enough even in a highly underdense plasma, where each of the lasers propagates like in the vacuum before meeting the second laser. This is because the backward Raman coupling in a highly underdense plasma is much (by twice the laser-to-plasma frequency ratio) stronger than the forward Raman coupling. By appropriate shaping of the lasers and plasma layer, the ultraintense output pulse can be focused, without contacting a material surface, to reach extreme intensity on the target.

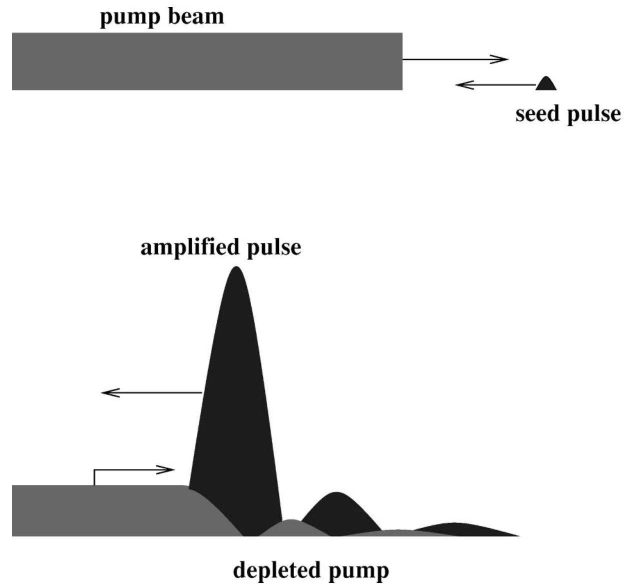


FIG. 2. Schematic of laser amplification via transient stimulated Raman backscattering. The originally small seed grows and completely consumes the incident pump. The seed, even though exceeding the pump intensity, still consumes the pump. This is because a Langmuir wave is needed for the back energy transfer process from the seed to the pump (to up shift the seed frequency to that of the pump), but the seed encounters all the time fresh plasma layers where Langmuir waves are absent. For ultrashort seed pulses, the plasma heating within the pulse duration is negligible.

the amplified pulse might be focused to reach extreme intensity at the target.

The transient backward Raman amplification process is shown schematically in Fig. 2. An initially small seed pulse grows, consuming all the incident pump energy. The momentum is transfered to the relatively low-frequency (and hence low-energy) Langmuir wave.

## II. HISTORICAL BACKGROUND

Stimulated Raman backscattering was observed first in liquids.[3] Experiments showed that this process can rapidly deplete the pump. An important feature of stimulated Raman backscattering, also noted in Ref. 3, is that the pumped pulse grows to intensities much higher than that of the pump. This is because the pulse continually encounters unperturbed regions of the medium and absorbs new regions of the pumping beam. In the absence of the medium excitations, the pumped pulse, being even more intense than the pump, cannot return energy back to the higher-frequency pump.

It was realized soon that the process can be used to convert energy stored in a long laser beam into a much shorter laser pulse. The possibility was broadly considered in connection with excimer laser pulse compression for inertial confinement fusion systems. An early review of backward Raman compression of excimer lasers for inertial fusion, Ref. 4, outlined major advantages and problems of conventional backward Raman compressors–amplifiers. Among the advantages are very modest requirements to the pump laser quality in such devices, due to the averaging of fluctuations in the pump intensity over the pumped pulse path.

The major drawbacks to conventional backward Raman compressors–amplifiers appear to be associated with the forward Raman scattering instability of both the pump and pumped lasers. In gases, the forward instability usually has a higher growth rate than the backward one. The unfavorable forward-to-backward asymmetry of Raman gain imposes severe limitations on efficiency of the laser pulse compression by backward Raman scattering. The most critical limitation comes from the forward Raman scattering of the pumped pulse which intensity exceeds that of the pump. The corresponding parasitic signal is the first forward Stokes component with respect to the pumped pulse, that is the second backward Stokes component with respect to the pump.

This second Stokes radiation of Ref. 4 has been recently suppressed by using Raman gas mixtures to arrange for the second backward Stokes to be under transient conditions, while the first backward Stokes is amplified under stationary conditions (see Refs. 5 and 6). In these experiments, KrF laser pulses of 249 nm wavelength and energies 0.1 J and 10 J were compressed from 20 ns to 67 ps and 150 ps with the estimated efficiency of 22% and 27%, respectively. This technique, however, cannot be extrapolated to the much more intense ($10^6$) and much shorter ($10^{-3}$) duration pulses through the methods proposed here.

Interestingly, several early ideas in Raman compression that apparently were not appreciated for their intended applications, or at least not pursued very vigorously, may now be adapted to the high power applications considered here. In particular, it was noted in Ref. 7 that the forward-to-backward Raman gain ratio is small in plasma. The favorable forward-to-backward Raman gain ratio could alleviate the problem of backward second Stokes (yet not removing it, because the forward-scattered signal propagates together with the parent laser and so has more distance to grow than does the backscattered signal which quickly passes through a short pulse). However, the proposal Ref. 7 apparently required too high a plasma homogeneity for maintaining pulse amplification, because of a very narrow resonance between the lasers and the Langmuir wave. The resonance might be broadened by using a preheated plasma having larger Landau damping of the resonant Langmuir wave. However, the larger damping then reduces the Langmuir wave amplitude and hence the Raman gain. As a result, the estimated amplification length, even in a dense plasma, is too large (meters), with still severe requirements on the plasma homogeneity.

Our proposal for ultrapowerful Raman backward amplifiers does employ the favorable forward-to-backward Raman gain ratio in plasma, but relates to the very different amplification regimes associated with ultrashort ultraintense pulses. The large frequency bandwidth of such ultrashort pulses makes the amplification resonance robust enough to tolerate reasonable fluctuations in plasma frequency (and hence density). Also, the large intensities make electron motion in the laser field nonperturbative, in contrast to regimes of Ref. 7. Thermal motion can be neglected so that a cold plasma approach is valid. Both the Landau and collisional Langmuir wave damping are negligible. Note that, for circularly polarized lasers, there is an effective nonlinear suppression of the Langmuir wave collisional damping. Electrons in the field of a circularly polarized laser move in circles with velocity proportional to the vector-potential of the laser field. The electron–ion collision frequency for these electrons is inversely proportional the cube of their velocity. For a very intense pumped laser pulse, the collision frequency of the rapidly oscillating electrons is small in the pulse location, so that Langmuir wave damping is negligible there. The Langmuir wave then reaches higher amplitude and provides stronger coupling between the pump and pumped lasers. (Note that for linearly polarized lasers, this effect on the Langmuir wave damping is less pronounced, because the electron velocity in laser fields then has stagnation points where the collision frequency is high.)

The fact that Langmuir wave damping is negligible within the pulse duration implies that so-called transient regimes of Raman backscattering must be considered, rather than the stationary regimes of Ref. 7, where the Langmuir wave is damped. The equations describing transient Raman backscattering—an important special case of the resonant three-wave interaction—are applicable to many physical contexts, including, classical laser amplifiers employing two-level inverse-populated systems, and stimulated Brillouin scattering in optical fibers, gases and plasma. These equations are integrable by the inverse scattering method, which can be employed to determine the large-time asymptotic behavior of well-localized wave packets (see Ref. 8). For amplification (compression) purposes, however, an advanced evolution of a short pulse inside a relatively long pumping beam is of particular interest.

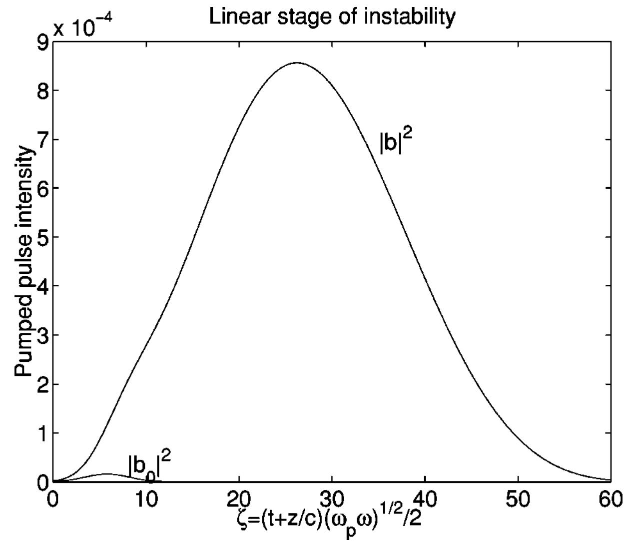


FIG. 3. During the linear stage of the pump backscattering instability, the seed-pulse stretches; its front moves at the vacuum light speed $-c$, while its maximum moves with a half of this speed $-c/2$. The maximum increases with the peak growth rate for the monochromatic wave instability $a_0\sqrt{\omega_p\omega/2}$. The linear stage ends when the area under the pulse envelop $\int b\, d\zeta$ becomes of the order of 1. At that point the pump depletion becomes significant.

For an exactly resonant interaction, this intermediate asymptotic behavior is approximated well by a quasi-self-similar attractor solution, a so-called "$\pi$-pulse," which fully depletes the pumping beam while undergoing amplification and contraction (see Ref. 9). The pulse maximum moves with a super-luminous speed, approaching the front. Note here the contrast to the linear stage of the backscattering instability (see Ref. 10), illustrated by Fig. 3, when the originally small and narrow seed pulse is not just amplified but also broadened, having its maximum moving with the speed $c/2$, so that the distance between the maximum and the front of the pulse increases with the speed $c/2$. That is why the pumped pulse may be broader than the initial seed even after substantial nonlinear contraction, as shown in Fig. 2.

The $\pi$-pulse regime for ultrapowerful plasma-based Raman backward amplifiers was proposed and analyzed in Ref. 11. It was shown that the pulse can, in principle, be amplified with 100% efficiency to huge intensities and fluences (see Table I) within a time short compared to the time scale for

TABLE I. Example of the pump and output pulse parameters scaling with laser wavelength for a fixed laser-to-plasma frequency ratio and the largest allowed amplification length, near the threshold of Langmuir wave breaking.

| | | | | |
|---|---|---|---|---|
| Laser wavelength ($\mu$m) | 1/40 | 1/4 | 1 | 10 |
| Pump duration (ps) | 1.25 | 12.5 | 50 | 500 |
| Pump intensity (W/cm$^2$) | $1.6\times10^{17}$ | $1.6\times10^{15}$ | $10^{14}$ | $10^{12}$ |
| Pump vector-potential ($a_0$) | 0.006 | 0.006 | 0.006 | 0.006 |
| Laser-to-plasma frequency ratio | 12 | 12 | 12 | 12 |
| Concentration of plasma (cm$^{-3}$) | $1.1\times10^{22}$ | $1.1\times10^{20}$ | $7\times10^{18}$ | $7\times10^{16}$ |
| Linear $e$-times growth length (cm) | 0.000 43 | 0.0043 | 0.013 | 0.13 |
| Total amplification length (cm) | 0.018 | 0.18 | 0.7 | 7 |
| Output pulse duration (fs) | 1 | 10 | 40 | 400 |
| Output pulse fluence (kJ/cm$^2$) | 160 | 16 | 4 | 0.4 |
| Output pulse intensity (W/cm$^2$) | $1.6\times10^{20}$ | $1.6\times10^{18}$ | $10^{17}$ | $10^{15}$ |

filamentation instabilities to develop. Note that this regime of fast compression by a resonant three-wave interaction is complementary to the interesting regime of laser amplification via Compton backscattering in a more rarefied plasma where wave breaking occurs and where the pump depletion can be neglected (see Ref. 12). This so-called superradiant regime is mathematically similar to superradiant amplification in free electron lasers (see Ref. 13).

It is the high efficiency of the transient Raman backscattering in a dense plasma that allows one to outrun filamentation instabilities of the pumped pulse. On the other hand, because of this high efficiency, the Raman backscattering of the pump by thermal Langmuir waves may lead to premature pump depletion, as the pump traverses the plasma layer towards the seed pulse. It appears, however, that appropriate detuning can suppress this unwanted Raman backscattering of pump by noise, while not suppressing the desirable seed pulse amplification, as shown in Ref. 14.

This very useful feature of the detuning effect is remarkable, since the linear Raman backscattering instability of the pump (responsible for noise amplification) has a larger growth rate than its nonlinear counterpart (responsible for the useful amplification of seed laser pulse). However, the linear instability has a narrower frequency bandwidth. In the nonlinear regime, the pumped pulse duration decreases inversely proportional to the pulse amplitude. The stronger the pulse, the faster the pump depletion. Thus, the pulse frequency bandwidth increases like the pulse amplitude, so that the nonlinear instability, as it grows, can tolerate larger and larger external detuning from the backscattering resonance. The same bandwidth broadening slows down the nonlinear instability by increasing the effective internal detuning.

The arranged detuning can suppress also Raman near-forward scattering of the pumped pulse, which is one of the major factors limiting the time and hence the maximum intensities, for pulse amplification in the fast laser compressor. (Exactly forward Raman scattering in plasma is suppressed anyway by the extra factor $\sqrt{\omega_p/\omega}$, because of the coupling with the anti-Stokes component, see for instance Refs. 15 and 16.) To suppress Raman near-forward scattering of the pumped pulse, the plasma frequency gradient has to be larger than the pulse bandwidth gradient. To preserve the useful amplification process, the plasma frequency detuning can be partially compensated by the pump frequency chirp.

From Fig. 4, it can be seen that the total external detuning, $\delta\omega_{\text{detuning}} \equiv \delta\omega$ (which governs the pump backscattering), can be made less than the plasma frequency detuning $\delta\omega_{\text{plasma}} \equiv \delta\omega_p$ (which governs the pumped pulse forward scattering). The larger plasma frequency gradient $\omega_p'$ can suppress Raman near-forward scattering of the pumped pulse into the Stokes pulse down shifted by the plasma frequency. The detuning gradient $\delta\omega'$ can be selected to suppress the pump Raman backscattering by noise.

Interestingly, the idea of pump chirping for suppressing forward scattering in gas-based backward Raman amplifiers was put forth 20 years ago,[17] where it was envisioned that both the pump and pumped lasers could be chirped, while varying Raman frequencies via the Zeeman effect. This idea, however, was apparently not pursued. Here the effect of detuning is considerably more advantageous, because it can be used together with the nonlinear filtering effect.

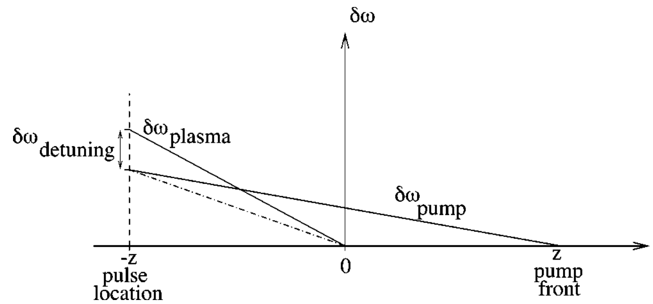


FIG. 4. A conceptual scheme of detuned Raman backward amplification. The detuning $\delta\omega_{\text{plasma}} \equiv \delta\omega_p$, caused in the seed-pulse location by plasma density gradient, depends on the distance from the seed to the plasma boundary $-z$. The detuning $\delta\omega_{\text{pump}} \equiv \delta\omega_a$, caused in the seed-pulse location by pump chirp, depends on the distance from the seed to the pump front $-2z$. Note that the chirp can be used to compensate partially the detuning effect of the density gradient. A smaller detuning can be used to stabilize the pump to noise without disruption of the desired amplification process (nonlinear filtering effect), while a larger plasma density gradient can suppress Raman near-forward scattering of the seed pulse. The combined effect of plasma density gradient and pump chirp on external detuning in Raman backward amplifier is described by the detuning gradient $\delta\omega' = \omega_p' - 2\omega_a'$, where primes signify derivatives of plasma and pump frequencies at the pulse location over distances from locations of exact resonance $(-z)$ and pump front $(-2z)$, respectively. The detuning gradient is characterized below by dimensionless parameter $q = 2(\omega_p' - 2\omega_a')c/\omega_p\omega_a a_0^2$.

## III. BASIC EQUATIONS

A slightly detuned transient three-wave interaction in plasma Raman backward amplifier can be described by equations

$$a_t + c a_z = \omega_p f b, \quad b_t - c b_z = -\omega_p f^* a,$$
$$f_t + i\,\delta\omega f = -\omega a b^*/2, \quad \delta\omega = \omega_p + \omega_b - \omega_a. \tag{1}$$

Here $a$ and $b$ are vector-potential envelopes of the pump and pulse, respectively, in units of $m_e c^2/e \approx 5\times10^5$ V, and $f$ is the envelope of the Langmuir wave electrostatic field $\vec{E} = E\vec{e}_z$ in units of $m_e c\,\omega_p/e = c\sqrt{4\pi m_e n_e} \approx \sqrt{n_e[\text{cm}^{-3}]}$ V/cm, defined by formulas

$$\begin{aligned}(A_x + iA_y)e/m_e c^2 &= a e^{i(k_a z - \omega_a t)} + b e^{i(k_b z - \omega_b t)},\\ Ee/m_e c\,\omega_p &= f e^{i[(k_a - k_b)z - \omega_p t]} + \text{c.c.};\end{aligned} \tag{2}$$

$A_x$ and $A_y$ are components of the real vector-potential $\vec{A}$ in the plane transverse to the propagation direction $z$; for the pump propagating in the positive and the seed pulse in the negative direction, like in Fig. 2, $k_a = \sqrt{\omega_a^2 - \omega_p^2}/c$ and $k_b = -\sqrt{\omega_b^2 - \omega_p^2}/c$; $\omega_p$, $\omega_b$, and $\omega_a$ are the plasma, laser-seed and laser-pump frequencies, and $\delta\omega$ is the detuning from the three-wave resonance; and subscripts $t$ and $z$ denote time and space derivatives. The pulse duration is larger than $\omega_p^{-1}$. Both lasers are circularly polarized. Self-nonlinearities of lasers and Langmuir wave are neglected. Plasma ions are assumed to be immobile. The Langmuir wave group velocity is neglected in comparison with the speed of light. For $\omega_b \gg \omega_p$, we approximate $\omega_a \approx \omega_b = \omega$ and $k_a \approx -k_b \approx \omega/c$, retaining the exact frequencies only in calculating the detuning

$\delta\omega$. For an inhomogeneous plasma, $\omega_p$ is a function of the distance passed by pumped pulse $-z$, and for a chirped pump, $\omega_a$ is a function of the distance to the pump front $ct-z$.

For further analysis, it is convenient to count time from the seed-laser front arrival to a given point $-z$ and to rescale variables,

$$\zeta=(t+z/c)\sqrt{\omega\omega_p}/2, \quad \tau=-z\sqrt{\omega\omega_p}/c, \qquad f=\overline{f}\sqrt{\omega/\omega_p}, \quad \delta\omega=\sqrt{\omega\omega_p}\,\delta/2, \tag{3}$$

so that Eqs. (1) take the form

$$a_\zeta-a_\tau=\overline{f}b, \quad b_\tau=-\overline{f}^*a, \quad \overline{f}_\zeta+i\overline{f}\delta=-ab^*. \tag{4}$$

The detuning caused by the plasma density variation is a function of $z=-c\tau/\sqrt{\omega\omega_p}$, while the detuning produced by the pump chirp is a function of the distance to the pump front $ct-z=2c(\tau+\zeta)/\sqrt{\omega\omega_p}$. For $\zeta\ll\tau$, i.e., $ct-z\ll-z$, the chirp-produced detuning and, hence, the total detuning may be considered to be a function of just $\tau$. When this function is smooth enough, it can be linearized near the resonance $\tau=0$, so that $\delta\approx\delta'\tau$. This approximation is used below. It covers the whole region of interest during the most important nonlinear amplification stage when the pumped pulse contracts and is well localized in the domain $\zeta\ll\tau$. As for the linear stage of the pump instability, the above approximation fully covers the case of detuning caused by just a plasma density gradient, and can be easily generalized to cover fully the chirp-produced detuning as well without a noticeable change in major conclusions. As seen from the above formulas, $\delta'=2(\omega_p'-2\omega_a')c/\omega_p\omega_a\equiv qa_0^2$ with $q$ defined as in Fig. 4.

Note that, mathematically, the small varying parts of $\omega_p$ and $\omega_a$ could have been included in the respective envelope definitions, which would apparently reduce the basic equations (1) to the same form as for zero frequency detuning. However, this approach would complicate the boundary conditions at large $-z$ or $ct-z$. It is interesting that such a transformation, removing the *frequency* detuning from Eqs. (1) and (2), differs from the well-known transformation of Ref. 18, which removes the *wavelength* detuning from equations for the nearly resonant three-wave interaction. Indeed, when the group velocity of one wave is zero, the transformation of Ref. 18 is reduced just to a *time-independent* phase shift for this wave. Such a shift apparently cannot remove the *frequency* detuning from the basic equations. It indicates that the general case of *frequency* detuning is not covered by the transformation of Ref. 18. Also, the linear theory for parametric instabilities in inhomogeneous medium, suggested in Ref. 19, is not directly applicable to the *frequency* detuned interaction. The underlying physical reason for these differences is as follows. The *wavelength* detuned three-wave interaction involves primarily wave field evolution in a fixed space location, so that the group velocity of the material wave is important. In contrast, for amplification of short laser pulses, the wave field evolution in the pulse location moving with the speed of light is primarily interesting. The propagation of the material wave, which is the Langmuir wave in the above context, with group velocity small in comparison to speed of light, is no longer important; rather new Langmuir waves are excited all the time in new locations of the amplified pulse. The new moving-location dynamics may affect dramatically the fixed-location dynamics, since the amplified pulse may completely deplete the pump and hence prevent its further access to locations already passed by the pulse, even before the pump instability in these fixed locations develops.

## IV. LINEAR STAGE OF THE PUMP INSTABILITY

During the linear stage of the backscattering instability, when the pump depletion is negligible, $a\approx a_0=\text{const}$, the solution of (4) with $\delta=qa_0^2\tau$ can be obtained by Laplace transformation and written as

$$b(\zeta,\tau)=\frac{\partial}{\partial\zeta}\int d\zeta_1\, G(\zeta-\zeta_1,\tau)b(\zeta_1,0),$$
$$G(\zeta,\tau)=\frac{1}{2\pi i}\int_C\frac{dp}{p}\exp\left[p\eta+\frac{i}{q}\ln\left(1-\frac{iq}{p}\right)\right], \tag{5}$$
$$\eta\equiv a_0^2\zeta\tau,$$

where the integration contour $C$ in the complex plane $p$ encompasses in the positive direction singularities at $p=0$ and $p=iq$.

For $|q|\sqrt{\eta}\ll1$, the Green's function $G$ reduces to that in a uniform plasma, where $q=0$ and $G=I_0(2\sqrt{\eta})$, in agreement with the linear theory of Ref. 10 for backscattering in uniform media. Here $I_0$ is the modified Bessel function. In the domain $\eta\gg1$ (but still $|q|\sqrt{\eta}\ll1$, which is possible when $|q|\ll1$), one has $G\approx\exp(2\sqrt{\eta})/2\sqrt{\pi\sqrt{\eta}}$. In original variables, $\eta=a_0^2\omega\omega_p(t+z/c)(-z)/2c$, so that maximum $G$ is reached at $z=-ct/2$ (i.e., it moves with a half of speed of light) and increases with the peak growth rate for the monochromatic wave instability $a_0\sqrt{\omega\omega_p/2}$. The pulse quickly stretches, since the front is moving with the speed of light. This dynamic is illustrated by Fig. 3.

The effect of detuning becomes noticeable at $|q|\sqrt{\eta}\sim1$, when the backscattering instability makes about $|q|^{-1}$ exponentiations. For $|q|\ll1$, the Green's function (5) can be evaluated by the method of steepest descent. It increases exponentially with $\eta$ up to the point $\eta_M=4/q^2\gg1$. In the domain $\eta\gg1$, but $\eta_M-\eta\gg1$, $G$ can be approximated by the formula

$$G=\frac{\exp[\sqrt{\eta(1-\eta/\eta_M)}+(\sqrt{\eta_M}+i)\operatorname{arcctg}\sqrt{\eta_M/\eta-1}-i\eta/\sqrt{\eta_M}]}{2\sqrt{\pi\sqrt{\eta(1-\eta/\eta_M)}}}. \tag{6}$$

For $\eta \ll \eta_M$, Eq. (6) reduces to the exact resonance case. At the applicability limit $\eta_M - \eta \sim 1$, $|G|$ attains its maximum value, $\max_\eta |G| \sim \exp(\pi/|q|)$. For larger $\eta$, $|G|$ drops abruptly. Hence, the maximum amplification factor for the integrated amplitude, $u \equiv \int^\zeta d\zeta\, b(\zeta,\tau)$, of a small narrow seed $b$ is $\sim \exp(\pi/|q|)$. If the initial value of the integrated amplitude $u$ is much smaller than $\exp(-\pi/|q|)$, it remains smaller than 1, as well as the pump depletion does, so that such a small seed never reaches the nonlinear stage of instability. This determines the threshold for the detuning gradient $q$ for stabilizing the pump in a backward amplifier in the presence of small noise.

## V. NONLINEAR STAGE OF THE PULSE AMPLIFICATION

Consider the nonlinear evolution of a seed pulse with initial integrated amplitude $u$ larger than $\exp(-\pi/|q|)$, which is sufficient to deplete pump before making $\pi/|q|$ exponentiations. In the nonlinear stage of amplification, the pump depletion scale decreases, while the amplification time increases, since a fixed pump acts relatively more slowly on larger signals. When the $\zeta$ scale becomes much smaller than the $\tau$-scale, the term $|a_\tau| \ll |a_\zeta|$ in (4) can be neglected. Then the resulting set of nonlinear equations has a new symmetry, allowing the self-similar substitution

$$a(\zeta,\tau) = a_0 \tilde{a}(\eta), \quad \overline{f(\zeta,\tau)} = a_0 \tilde{f}(\eta), \qquad b(\zeta,\tau) = a_0^2 \tau \tilde{b}(\eta), \quad \eta = a_0^2 \tau \zeta, \tag{7}$$

so that the scaled functions satisfy the nonlinear ordinary differential equation (ODE)

$$\tilde{a}_\eta = \tilde{f}\tilde{b}, \quad \tilde{f}_\eta + iq\tilde{f} = -\tilde{a}\tilde{b}^*, \quad (\eta\tilde{b})_\eta = -\tilde{a}\tilde{f}^* \tag{8}$$

with the initial condition $\tilde{a} \to 1$ at $\eta \to +0$. The solution depends on a single parameter, say $\tilde{b}(+0) = \epsilon_1$. The quantity $\epsilon_1$ coincides in fact with the integrated amplitude of a narrow initial seed pulse that moves ahead of the similarity domain not belonging to it. The substitution (7) can be done there as well, but with $\tilde{b}$ dependent of both $\eta$ and $\tau$. Taking into account that $u \equiv \int^\zeta d\zeta\, b(\zeta,\tau) = \int^\eta d\eta\, \tilde{b}$, it is easy to get the equation for $u$ in the similarity domain. For small $\eta$, when one can set $\tilde{a} = 1$ in the two last Eqs. (8), one gets $(\eta u_\eta)_\eta + iq\eta u_\eta = u$, with $u(+0) = u_\eta(+0) = \epsilon_1$. Note that for such a narrow seed pulse, $u$ is proportional to the Green's function for the linear problem, $u = \epsilon_1 G$, as long as the approximation $\tilde{a} = 1$ is valid, i.e., the pump depletion is negligible.

To analyze Eqs. (8) for arbitrary pump depletion, it is useful to rewrite them in real form introducing real amplitudes and phases by $\tilde{a} = Ae^{i\alpha}$, $\tilde{b} = Be^{i\beta}$, $\tilde{f} = Fe^{i\phi}$, to obtain a set of six real first-order equations,

$$\begin{aligned} &A_\eta = FB\cos\Delta, \quad \alpha_\eta = FB\sin\Delta/A, \\ &F_\eta = -AB\cos\Delta, \quad \phi_\eta = AB\sin\Delta/F - q, \\ &(\eta B)_\eta = -AF\cos\Delta, \quad \beta_\eta = AF\sin\Delta/B\eta, \\ &\Delta \equiv \beta + \phi - \alpha. \end{aligned} \tag{9}$$

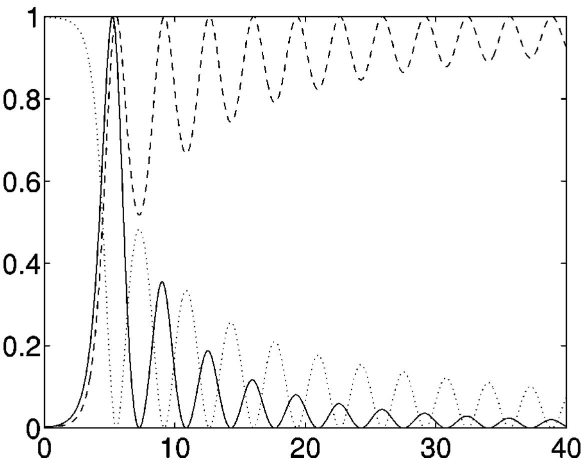


FIG. 5. Normalized energy densities of the pumped pulse: $(U_\xi/\max(U_\xi))^2$, solid line, Langmuir wave; $(\sin(U/2))^2$, dashed line, and pump $(\cos(U/2))^2$, dotted line, as functions of $\xi = 2Ca_0\sqrt{\zeta\tau}$ for $\epsilon = 0.1$ and $q = 0$. After the pulse passes, the pump is completely depleted, and just the relatively low-frequency (and hence low-energy) Langmuir wave remains. More than half of the pump energy goes into the leading spike of the $\pi$-pulse wavetrain.

These six equations can be reduced to a set of just two real first-order equations. Reduction of order by two is due to the invariance of (9) to the two-parameter group of constant phase-shifts that do not change $\Delta$. An extra reduction by one, is due to the integral of (9), $A^2 + F^2 = C^2 = \text{const}$, corresponding to conservation of the joint number of the pump and Langmuir quanta in the 3-wave decay interaction, and allowing one to introduce function $U$, such that

$$A = C\cos(U/2), \quad F = C\sin(U/2). \tag{10}$$

To proceed with the order reduction, the identity $(\eta^2 B^2 \beta_\eta)_\eta = (\eta^2 B^2)_\eta q/2$ can be derived from Eqs. (9). Taking into account regularity of all fields at $\eta \to +0$, it follows $\beta_\eta = q/2$ for all $\eta$'s. Thus, the amplified pulse acquires a linear frequency chirp equal to half of the external detuning gradient, while the central pulse frequency drift compensates for a half of the external detuning.

Substitution of (10) and the extra integral $\beta_\eta = q/2$ in Eq. (9) gives a closed set of two first order ordinary differential equations (ODE),

$$\begin{aligned} &U_\eta = -2B\cos\Delta, \\ &2(\eta B)_\eta = -C^2 \sin U \cos\Delta, \\ &q\eta B = C^2 \sin U \sin\Delta. \end{aligned} \tag{11}$$

The initial value of $U$ is $U(+0) = -2\,\text{arctg}\,\epsilon_1 \equiv \epsilon$ [since $A(+0) = 1$ and $F(+0) = -B(+0) = -\epsilon_1$].

For $q = 0$, when one can take $\Delta = 0$, Eqs. (11) are equivalent to the second order ODE $(\eta U_\eta)_\eta = C^2 \sin U$. With $\eta = \xi^2/4C^2$, it reduces to Eq. (6) of Ref. 11, $U_{\xi\xi} + U_\xi/\xi = \sin U$, corresponding to the exactly resonant interaction. The exactly resonant solution is shown in Fig. 5.

For $q \neq 0$, the equations can be put in the form

$$\begin{aligned} &U_\eta = -c^2 \sin(2\Delta)\sin U/q\eta, \\ &\Delta_\eta = 2\,c^2 \sin^2\Delta \cos U/q\eta - q/2, \\ &\Delta(+0) = 0, \quad U(+0) = \epsilon. \end{aligned} \tag{12}$$

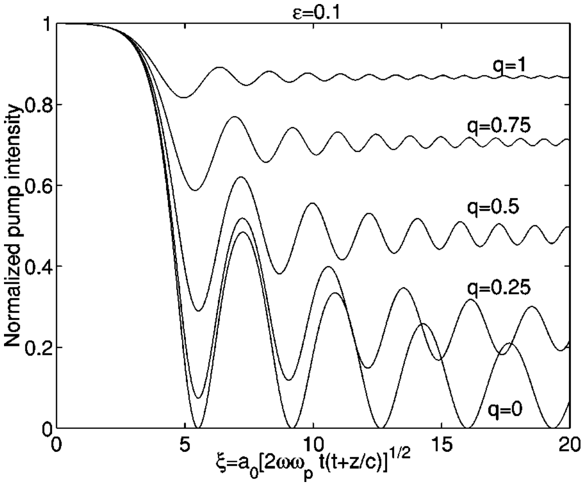


FIG. 6. The pump depletion is smaller for larger detuning gradients $|q|$. Yet, the depletion can be large enough for a moderately small detuning gradient; about 80% for $|q|=0.25$.

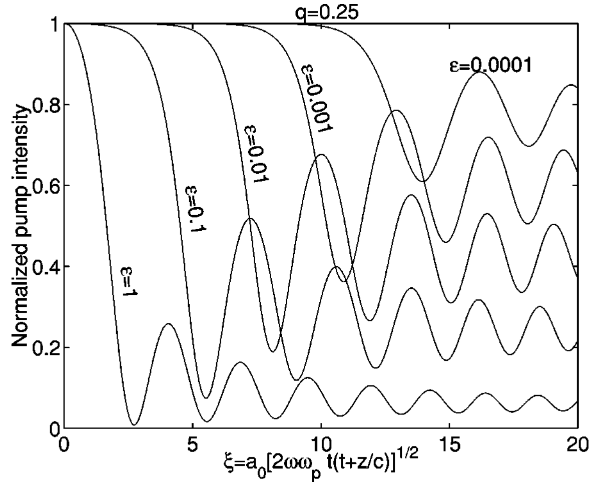


FIG. 7. The pump depletion is smaller for smaller integrated seed amplitudes $|\epsilon|$. Very small seeds hardly deplete the pump, which indicates the pump stability to noise. A moderately small seed can consume nearly all the pump energy, resulting in a nonlinear filtering effect.

The limit $q\to 0$ has a peculiarity. For $q\neq 0$, $U$ varies only inside $(0,\pi)$ interval, while for $q=0$, $U\equiv U^0$ varies inside $(0,2\pi)$ interval and tends to the limit $\pi$ at $\xi\to\infty$ oscillating around this value (''$\pi$-pulse'' solution). The points where $U^0=\pi$ are not zeros of $B$, but are rather close to maxima of the pulse intensity $B^2$. For small $q\neq 0$, one can neglect variation of $B\approx B_*$ near a point $\eta=\eta_*$ where $U^0=\pi$ and, integrating there (11), to get $\tilde{U}\equiv\pi-U\approx|B_*|\sqrt{q^2\eta_*^2/C^4+4(\eta-\eta_*)^2}$. The function has a minimum $\tilde{U}_*\approx|B_*q|\eta_*/C^2$ at $\eta_*$. The corresponding $\sin\Delta\approx\tilde{U}_*/\tilde{U}$ tends to zero as $q\to 0$ outside a narrow ($|\eta-\eta_*|\leqslant q\eta_*$) vicinity of $\eta_*$. In the outer domain, the solution is close to the $\pi$-pulse up to the terms $\propto q^2$.

The pump intensity has a minimum $A_*^2\approx C^2\tilde{U}_*^2/4\approx B_*^2q^2\eta_*^2/4C^2$ at $\eta_*$. The zero-order values of $B_*$ and $\eta_*$ can be taken from the $\pi$-pulse solution. For $\epsilon\ll 1$, the integrated amplitude of the leading spike of the $\pi$-pulse wavetrain is close to the classical $2\pi$-pulse solution of the Sine–Gordon equation, $U^0\approx 4\,\mathrm{arctg}(\epsilon e^{\xi}/4\sqrt{2\pi\xi})$.[11] The point $U^0=\pi$ is located at $\xi_*\approx\ln(4\sqrt{2\pi\xi_*}/\epsilon)$ ($\eta_*=\xi_*^2/4$). Calculating a small deviation from the $2\pi$ pulse, one can show that the pulse intensity is $B_*^2\approx 4/(\xi_*+1)^2$. Then, $A_*^2\approx q^2\xi_*^4/16(\xi_*+1)^2$. This simple asymptotic ($q\to 0$, $\epsilon\to 0$) formula agrees with numerical results presented in Fig. 6. For $q=0.25$, $\epsilon=0.1$, both the analytical and numerical solutions give $A_*^2\approx 8\%$. Even for $q=0.5$, $\epsilon=0.1$, the agreement is still reasonable: $A_*^2\approx 33\%$ analytical versus $A_*^2\approx 29\%$ numerical.

Numerical solution of Eqs. (12) confirms the linear theory prediction that detuning suppresses the pump instability to noise: as seen from Figs. 6 and 7, very small seeds virtually do not deplete the pump. Yet, it is possible to maintain a high efficiency of the useful amplification process that starts from a moderately small initial seed and can tolerate a large enough detuning gradient $q$.

After the pumped pulse passes, there is no further depletion. The final pump depletion depends on parameters $\epsilon$ and $q$ and may take any value in the (0,1) interval. It corresponds to $U(+\infty)$ taking any value in the $(0,\pi)$ interval. Thus, all ''less-than-$\pi$-pulses'' can appear in detuned Raman amplifiers, in contrast to the well-known exactly resonant case where just $\pi$ pulses appear.[9] Figure 8 demonstrates that the self-similar solution is an attractor. It also shows that the final pump depletion (say, 76% for $\epsilon=0.1$, $q=0.25$, or 52% for $\epsilon=0.1$, $q=0.5$) can be somewhat increased by working near the threshold of the Langmuir wave breaking (see below). The near-threshold breaking occurs near the leading maximum of the pumped pulse intensity, which prevents the pulse energy from scattering back to the pump. It suppresses the second and further spikes in the amplified pulse wavetrain.

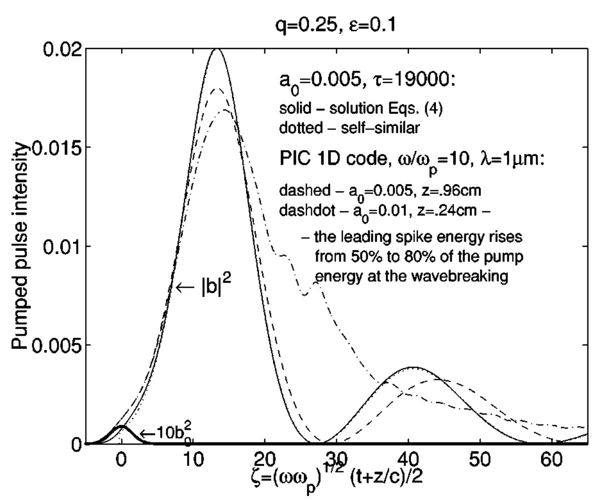


FIG. 8. Coincidence of the solid and dotted lines indicates that the self-similar solution is an attractor for Eq. (4) with small and narrow initial seed pulse (small spike at $\zeta=0$, which the intensity $b_0^2$ is multiplied by 10 to make it distinguishable). The dashed and dashed–dotted lines, based by 1D PIC simulation, show that the Langmuir wave breaking (occurring at $a_0\approx 0.008$ for $\omega/\omega_p=10$) raises the total energy extraction from the pump and suppresses secondary spikes.

## VI. EVALUATION OF THE OUTPUT PULSE PARAMETERS

The amplification time is mainly limited by the pumped pulse instabilities that may arise from noise.

The largest growth rate is associated with the Raman backscattering instability, but the Stokes backscatter quickly passes through the short pulse, making this channel of energy loss relatively benign. Indeed, for the pumped pulse of duration $T$, the backward Stokes component experiences about $bT\sqrt{\omega\omega_p}$ exponentiations. This factor does not change in the pumping process and remains of the order of 5. A more precise calculation for the leading spike of the $\pi$-pulse wavetrain confirms that the amplitude of the Stokes component is amplified only by a factor $\sim\exp(\pi\sqrt{2})\approx 85$ times.

The near-forward Raman scattering has a smaller growth rate but more time to develop, since the scattered light moves together with the pulse. Of interest here is the amplification exponent in the weakly coupled regime of short-pulse instability, $\sim b\sqrt{Tt\omega_p^3/\omega}$ (Ref. 16) [which is $(\omega_p/\omega)\sqrt{t/T}$ times the backscattering exponent]. The near-forward Raman scattering limits the time of pulse amplification by $t<t_{fw} =(\Lambda_{fw}/\omega_p a_0)\sqrt{\xi_*\omega/2\pi\omega_p}$, where $\Lambda_{fw}$ is the number of exponentiations allowed for the instability.

Alternatively, the limit on amplification may arise from the modulational instability, caused by the relativistic electron nonlinearity of the amplified pulse. For highly overcritical powers considered here, the peak growth rate is $\sim\omega_p^2 b^2/2\omega$ (see, for instance, Ref. 20). According to the above formulas for $b$, the pulse amplification time cannot exceed $t_{md}=\xi_*^{2/3}\Lambda_{md}^{1/3}/2^{1/3}\omega_p a_0^{4/3}$, where $\Lambda_{md}$ is the number of exponentiations which can be tolerated for modulations.

The time allowed for amplification before either instability develops is larger when $\omega_p$ is smaller, i.e., when plasma is more rarefied. However, to avoid the Langmuir wave breaking which dramatically reduces coupling of the pump and pumped lasers, the condition $\omega_p>\omega(4a_0)^{2/3}$ should be satisfied. Thus, the most favorable regime occurs near the wavebreaking limit where $t_{md}=\xi_*^{2/3}\Lambda_{md}^{1/3}/2^{5/3}\omega a_0^2$, $t_{fw} =\xi_*^{1/2}\Lambda_{fw}/4\sqrt{2\pi}\omega a_0^2$. As can be seen, these $t_{md}$ and $t_{fw}$ coincide up to logarithms. For reasonable values of the logarithmic factors, the allowed amplification distance can be evaluated as $z_{\mathrm{ampl}}=c\min(t_{md},t_{fw})\sim\lambda/4a_0^2$. The respective output pulse fluence $w$ is independent of the pump intensity, $w=I_1(2\pi a_0^2/\lambda^2)z_{\mathrm{ampl}}(m_e c^2/e)^2$, where $I_1$ is the relative pump depletion. A reasonable estimate is $w\sim(1/2\lambda)\ J/\mathrm{cm}$. The output pulse duration and intensity can be evaluated as $T\sim 6/\omega_p\sim 1.3a_0^{-2/3}\lambda[\mu\mathrm{m}]\mathrm{fs}$ and $W=(w/T) \sim(40a_0^{2/3}/\lambda^2)\ \mathrm{GW}\sim(30b_M^2/\lambda^2)\ \mathrm{GW}$, where $b_M\sim 1.2a_0^{1/3}$ is the output pulse amplitude.

For even more rarefied plasma, $\omega_p\ll\omega(4a_0)^{2/3}$, the Langmuir wave breaks much before the pump is completely backscattered, which reduces the pump depletion and efficiency of amplification. The early Langmuir wave breaking indicates that the short-scale ($\sim\lambda/2$) plasma electric field is not very important in this regime. Under such conditions the Raman backscattering is dominated by Compton backscattering on individual electrons. There is experimental evidence[21] of the transition from the stimulated Raman to Compton scattering via breaking of plasma waves. Theoretical estimates[11] indicate that the Raman backward amplification regime turns then into a strongly kinetic (''superradiant'') amplification regime of Ref. 12, and that the efficiency of the pump energy transfer to the pumped pulse decreases then with decrease in the plasma density.

## VII. PROPOSED EXPERIMENTS

The assertions of this paper rely only on rather transparent well accepted nonlinear equations for elementary three wave interactions. Nonetheless, to proceed with confidence, experimental verification not only of the final result, but also of intermediate stepping stones, is necessary. The following sequence of increasingly ambitious experiments is one such path:

(1) First, to verify experimentally the linear theory of a seeded Raman backscattering instability of a laser pump in plasma. Note that the theory of the instability growing from a thermal noise has already been validated experimentally in Ref. 22.
(2) Second, to observe onset of the pump depletion and stop of the pumped pulse stretching.
(3) Third, to observe the $\pi$-pulse self-contraction regime for the pulse pumped through the stimulated Raman backscattering in plasma.
(4) Fourth, to verify the amplification length limit imposed by filamentation instabilities.
(5) Fifth, to verify the pump stabilization to premature backscattering in a noisy plasma layer through an appropriate detuning.
(6) Sixth, to verify the pumped pulse stabilization to Raman near-forward scattering through the nonlinear filtering effect.
(7) Seventh, to verify the suppression secondary spikes and the leading spike enhancement in the Langmuir near-wavebreaking regime.
(8) Eighth, to verify the predicted scalings for the output pulse parameters with the laser wavelength.

Numerical examples are given in Table I. Note that just a very modest 10 GW power (corresponding to a 100$\lambda$-diameter beam for which the diffraction length still exceeds the amplification length) is sufficient for all the basic principle-of-proof experiments.

## VIII. CONCLUSION

Self-similar attractor solutions are found for the frequency detuned three-wave interaction. The solutions generalize the classical $\pi$-pulse regime solution for the exactly resonant three-wave interaction, a problem of broad interest.[9] Similarly, the results of this work are anticipated to be applicable to a broad range of phenomena.

A new integral of the detuned equations is identified which shows that the amplified pulse acquires the frequency shift exactly equal to the half of the imposed detuning, and the chirp exactly equal to half the imposed detuning gradient.

A new scheme of the fast laser compression in plasma is proposed capable of raising nonfocused output intensities of laser amplifiers to $10^{17}$ W/cm$^2$ at 1 $\mu$m wavelength, which exceeds by $\sim 10^5$ times nonfocused outputs of the most advanced chirp pulse amplification techniques. Focusing of the new enhanced outputs might produce laser pulses with ultra-relativistic intensities, not achievable in currently employed schemes. This will enable laboratory studies of basic matter properties in quite a new parameter range. For instance, it might be possible to enhance substantially the recent experiments on photonuclear physics reported in Ref. 23.

A nonlinear filtering effect is identified which enables the selective suppression of unwanted Raman instabilities by a combination of detuning and nonlinear effects. It makes robust to noise the technologically simplest fast compression scheme in which the laser pump propagates towards the seed-pulse precisely through the plasma layer where the amplification then occurs.

Note that the limit on the intensity of nonfocused output pulse in the proposed scheme scales with the laser wavelength like $1/\lambda^2$. Hence, this scheme might be even more advantageous in x-ray range, when appropriate x-ray laser pump lasers are developed (see, e.g., Ref. 24). For instance, a 1/40 $\mu$m-wavelength output pulse of Table I with diameter 10 cm, if focused to a 40 wavelength diameter, would reach an intensity of $10^{30}$ W/cm$^2$. At such an intensity, the laser field produces vacuum breakdown.

For the scheme advanced here, the pump quality in fast compressors may be very modest, since fluctuations are averaged over the amplification distance. All the assertions here really follow from the simple backward Raman scattering equations in the short pulse transient regime. Although these equations are well accepted and relied upon within the literature, a sequence of experiments both to verify the scheme that we propose as well as its key building blocks is indicated.

## ACKNOWLEDGMENT

The work is supported by the United States Department of Energy Contract No. DE-FG030-98DP00210.

[1] G. A. Mourou, C. P. J Barty, and M. D. Perry, Phys. Today **51**, 22 (1998); M. D. Perry *et al.*, Opt. Lett. **24**, 160 (1999).
[2] M. Tabak *et al.*, Phys. Plasmas **1**, 1626 (1994).
[3] M. Maier, W. Kaiser, and J. A. Giordmaine, Phys. Rev. Lett. **17**, 1275 (1966); Phys. Rev. **177**, 580 (1969).
[4] J. R. Murray, J. Goldhar, D. Eimerl, and A. Szoke, IEEE J. Quantum Electron. **QE-15**, 342 (1979).
[5] H. Nishioka, K. Kimura, K. Ueda, and H. Takuma, IEEE J. Quantum Electron. **QE-29**, 2251 (1993).
[6] E. Takahashi, Y. Matsumoto, I. Matsushima, I. Okuda, Y. Owadano, and K. Kuwahara, Fusion Eng. Des. **44**, 133 (1999).
[7] C. E. Capjack, C. R. James, and J. N. McMullin, J. Appl. Phys. **53**, 4046 (1982).
[8] V. E. Zakharov and S. V. Manakov, Sov. Phys. JETP **42**, 842 (1975); D. J. Kaup, A. Reiman, and A. Bers, Rev. Mod. Phys. **51**, 275 (1979).
[9] G. L. Lamb, Jr., Phys. Lett. A **29**, 507 (1969); Rev. Mod. Phys. **43**, 99 (1971); *Elements of Soliton Theory* (Wiley, New York, 1980); V. E. Zakharov, JETP Lett. **32**, 589 (1980); S. V. Manakov, *ibid.* **35**, 237 (1982); V. A. Gorbunov, V. B. Ivanov, S. B. Papernyi, and V. R. Startsev, Izv. Akad. Nauk SSSR, Ser. Fiz. **48**, 1580 (1984) [Bull. Acad. Sci. USSR, Phys. Ser. **48**, 120 (1984)]; J. Coste and C. Montes, Phys. Rev. A **34**, 3940 (1986).
[10] D. L. Bobroff and H. A. Haus, J. Appl. Phys. **38**, 390 (1967).
[11] V. M. Malkin, G. Shvets, and N. J. Fisch, Phys. Rev. Lett. **82**, 4448 (1999).
[12] G. Shvets, N. J. Fisch, A. Pukhov, and J. Meyer-ter-Vehn, Phys. Rev. Lett. **81**, 4879 (1998).
[13] R. Bonifacio, L. De Salvo, P. Pierini, N. Piovella, and C. Pellegrini, Phys. Rev. Lett. **73**, 70 (1994), and references therein.
[14] V. M. Malkin, G. Shvets, and N. J. Fisch, Phys. Rev. Lett. **84**, 1208 (2000).
[15] W. B. Mori, C. D. Decker, D. E. Hinkel, and T. Katsouleas, Phys. Rev. Lett. **72**, 1482 (1994); A. S. Sakharov and V. I. Kirsanov, Phys. Rev. E **49**, 3274 (1994).
[16] T. M. Antonsen and P. Mora, Phys. Rev. Lett. **69**, 2204 (1992).
[17] J. A. Caird, IEEE J. Quantum Electron. **QE-16**, 489 (1980).
[18] A. H. Reiman, A. Bers, and D. J. Kaup, Phys. Rev. Lett. **39**, 245 (1977); A. Reiman, Rev. Mod. Phys. **51**, 311 (1979).
[19] M. N. Rosenbluth, Phys. Rev. Lett. **29**, 565 (1972).
[20] A. G. Litvak, Zh. Éksp. Teor. Fiz. **57**, 629 (1969) [Sov. Phys. JETP **30**, 344 (1970)]; C. Max, J. Arons, and A. B. Langdon, Phys. Rev. Lett. **33**, 209 (1974); Guo-Zheng Sun, E. Ott, Y. C. Lee, and P. Guzdar, Phys. Fluids **30**, 526 (1987).
[21] M. J. Everett *et al.*, Phys. Rev. Lett. **74**, 1355 (1995).
[22] C. Rousseaux, G. Malka, J. L. Miquel, F. Amiranoff, S. D. Baton, and Ph. Mounaix, Phys. Rev. Lett. **74**, 4655 (1995).
[23] K. W. D. Ledingham *et al.*, Phys. Rev. Lett. **84**, 899 (2000); T. E. Cowan *et al.*, *ibid.* **84**, 903 (2000).
[24] D. Korobkin, A. Goltsov, A. Morozov, and S. Suckewer, Phys. Rev. Lett. **81**, 1607 (1998); B. R. Benware, C. D. Macchietto, C. H. Moreno, and J. J. Rocca, *ibid.* **81**, 5804 (1998); J. J. Rocca, Rev. Sci. Instrum. **70**, 3799 (1999).