The possibility of high amplitude driven contained modes during ion Bernstein wave experiments in the tokamak fusion test reactor

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Extremely high quasilinear diffusion rates for energetic beam ions can be deduced from mode conversion experiments in the Tokamak Fusion Test Reactor (TFTR) [K. M. McGuire, H. Adler, P. Alling *et al.*, Phys. Plasmas **2**(6), 2176 (1995)]. A comparison of the experimental loss rates with the theoretical prediction for the interaction of energetic ions with mode converted ion Bernstein waves showed the theory to underpredict the diffusion coefficient by a factor of 30–70. An anomalously high diffusion coefficient might enhance the advantageous channeling of energetic alpha particle energy in a tokamak reactor. Resolving this discrepancy is thus of importance from the standpoint of practical interest in an improved tokamak reactor as well as from the standpoint of academic interest in basic wave–particle theory. A mechanism is proposed for this accelerated diffusion involving the excitation of a contained mode, possibly similar to that used in explaining the ICE (ion cyclotron emission) phenomenon, near the edge of a tokamak. *© 2000 American Institute of Physics.* [S1070-664X(00)03107-4]

I. INTRODUCTION

During mode converted ion Bernstein wave (MCIBW) experiments in the Tokamak Fusion Test Reactor (TFTR),[1] substantial evidence of interaction of IBWs with fast particles was observed.[2–4] These experiments also presented an opportunity for experimentally gauging the prospects of alpha channeling in a fusion reactor.[5] In the alpha channeling process, one or a spectrum of waves is utilized to diffuse quasilinearly the highly energetic alpha particles produced as a byproduct of deuterium–tritium (DT) fusion reactions simultaneously in both energy and position.[6] If the energy so transfered from the alpha particles to the imposed wave field damps on the fuel ion population, the double benefit of extracting the alpha particle ''ash'' from the reactor as well as ''channeling'' the extracted energy to fuel ion heating would be achieved. Since the IBW is both highly dispersive (and so may resonate with a possibly broad distribution of alpha particles) and has been shown to damp on ions in slightly deuterium rich DT plasmas,[4,7] this wave would be an excellent tool for the channeling process.[8,9]

In the TFTR MCIBW experiments, energetic deuterium beam ions could be imagined as surrogates for the alpha particles in interacting with the IBW (albeit that the beam ions were being heated as opposed to cooled), while wall-mounted charged particle scintillation detectors—built to detect fusion alpha particles—measured the loss rates and distribution of the ejected beam ions.[10,11] A theoretical study of the wave–particle interaction reproduced in some detail the general power scaling and distribution (in poloidal angle, velocity space pitch angle, and energy) of lost beam ions as a function of time. However, the absolute magnitude of the losses was seriously underpredicted.[12] On the surface, this appears to bode well for the potential radio-frequency (rf) power requirements of alpha channeling in any future reactor; however, any reactor-scale extrapolations of course require a thorough understanding of the loss enhancement mechanism.

Elucidating the physical mechanism for such a diffusion enhancement poses a distinctly mysterious conundrum in the field of wave–particle interactions. Whereas the case of anomalously weak diffusion might be generally explained on the basis of poor wave coupling to the plasma or a host of other inefficiencies, the case of anomalously strong interaction requires the introduction of some new effect which either amplifies the wave field or otherwise accelerates the diffusion process. In addition, any candidate explanation is challenged to match the four dimensional signature of the data (in energy, pitch angle, poloidal angle, and time) recorded by the lost alpha detectors. Indeed, fully plausible explanations are difficult to find. The central result of this paper is that, among several potential mechanisms for diffusion enhancement, only one is both plausible and not inconsistent with experimental observations.

In the context of mode conversion processes in plasmas, some attention has recently been focused on the resonant cavity or global eigenmode aspect of ion cyclotron frequency waves in tokamaks. In one case, the precise standing wave pattern resulting from the reflection of a launched fast wave (FW) between the low and high field side cold plasma cutoffs of a tokamak has been shown to influence mode conversion efficiency to an IBW.[13] Other work has studied the growth of eigenmodes near the tokamak edge as a mechanism for the enhancement of edge ion cyclotron emission (ICE).[14–16]

Along similar lines, a mechanism is proposed here by which the excitation of a global-type eigenmode oscillation to high amplitude and its subsequent interaction with a population of energetic particles might explain the unexpectedly

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large ion losses observed in TFTR. While an exact treatment of the behavior of the plasma and its geometry requires a numerical implementation, a simplified, analytically tractable model for a region of the tokamak plasma behaving as a resonant cavity is used to demonstrate that the generation of high amplitude localized wave fields is possible for typical experimental parameters. From this field amplitude, an estimate is made of the resulting quasilinear diffusion coefficient which indicates it to be on the order of that inferred from the experimental data. The distribution in poloidal angle and velocity of losses induced by such an eigenmode are also discussed.

The goal of this work is to show that there exists an explanation for the anomalously large diffusion coefficient that is plausible. What we have not done is to formulate a detailed model on the basis of this plausible explanation. Such a model would be valuable; through a detailed comparison with the experimental results, it could put to rest all questions concerning the physics at play in the unusual observations on TFTR. Such a model and comparison are beyond the scope of the work. However, the sole plausible explanation which we are left with—namely the postulated existence of a high amplitude contained mode—is quite remarkable, and should serve as an impetus for further study.

The organization of this paper is as follows: Section II contains a review of the IBW and fast ion experiments in TFTR. Section III describes the conundrum posed by the observed fast ion diffusion rate. Section IV discusses several candidate explanations for this diffusion enhancement. Sections V and VI summarize example eigenmode theories developed by Monakhov *et al.*[13] and Coppi *et al.*[14] Section VII describes the eigenmode model proposed for explaining the observed IBW–fast ion interaction in TFTR. Section VIII gives a numerical estimate of the diffusion rates implied by this mode. Section IX discusses other implications of the presence of this mode. Section X considers possible implications of such a mode for alpha channeling, and Sec. XI offers concluding remarks.

II. TFTR D-BEAM EXPERIMENTS

The results of the ^{3}He MCIBW experiments in TFTR were initially reported in Ref. 10 and interpreted in Ref. 12. In investigating these lost particle signals, the identification of the loss mechanism first requires identification of the lost species: a charged fusion product (CFP), an accelerated beam ion, or a species of the background plasma. The lost alpha scintillation detectors distributed poloidally below the outboard midplane of TFTR (the expected intersection point for an exiting fusion alpha particle with the reactor wall) yielded information on the poloidal distribution of the lost species, its velocity-space pitch angle, and its gyroradius. However, no information was provided on the mass or charge (and hence identity) of the lost ion. Other characteristic dependencies of the loss rate were then enlisted for this identification. The experiments which witnessed significant losses were characterized by the parameters: $B_T=4.8$ T, $I_p=1.4\,\mathrm{MA}$, $n_e(0)=5\times10^{19}\,\mathrm{m}^{-3}$, $n_{^3\mathrm{He}}/n_e\geq0.15$, $T_e(0)=7$ keV, $P_{\mathrm{NB}}=0-10\,\mathrm{MW}$, $P_{\mathrm{rf}}=3-5\,\mathrm{MW}$, and $f=43\,\mathrm{MHz}$. Since the plasma was composed of D, ^{4}He, and varying fractions of ^{3}He, and was heated with D neutral beams, the potential energetic lost species included DD fusion tritons, protons, or ^{3}He ions; D^{3}He fusion protons or alpha particles; accelerated D beam ions; or rf-induced tail ions from the bulk plasma distributions.[10,12]

The fact that the magnitude of the total ion losses depended sensitively on the magnetic field strength and on the ^{3}He fraction in the plasma (both of which determine the location of the MC layer) suggested that the losses were indeed induced by the MCIBW. The distribution in gyroradius of the lost particles (being both broader and peaked at a higher value than would be expected for first orbit CFP losses) further implied that the lost ions were being heated due to wave interaction. The failure to detect any magnetic signature of an energetic tail distribution rendered unlikely the bulk species being the source of lost ions, leaving only CFP and beam ions as possibilities. Comparison of shots in which no neutral beams were present, shots in which T beams were utilized in place of D beams, and shots with alternately co- or counter-injected D beams revealed that the presence of counter-going D beam ions was necessary for the observation of significant losses. Since CFP losses would have been observed irrespective of the beam type or direction, the lost species was confirmed to be D beam ions. The added characteristics of the persistence of the fast ion losses throughout an rf pulse lasting much longer than the beam-induced neutron rate (to which CFP losses should be correlated) and that the abundance of D beam ions should be a factor of $\sim10^4$ greater than that of any CFP are further corroborations that the losses are indeed heated beam ions.[10,12]

III. THE DIFFUSION RATE CONUNDRUM

With confirmation of the identity of the lost species, data from counter-going ''beam-blip'' experiments with varying rf power levels were studied to investigate the loss rate's dependence on IBW intensity. Specifically, an energy diffusion coefficient of $D_\varepsilon\simeq25\,\mathrm{MeV}^2/\mathrm{s}$ was inferred for the lost beam deuterons.[10] A theoretical estimate of this diffusion coefficient was made by Herrmann under the assumption that ions diffuse quasilinearly in the presence of the IBW.[12] Considering the rapid variation of the poloidal projection of the IBW wave vector to be the dominant contributer to the autocorrelation time of the diffusion process, a stationary phase calculation of the deuteron energy step due to interaction with the IBW yields a coefficient of

$$D_\varepsilon\simeq\frac{\langle|\Delta\epsilon|^2\rangle}{\tau_b}$$
$$\simeq\frac{1}{\tau_b}(Z|e|\Phi_0(\mathbf{r}_{g.c.}))^2J_n^2(k_\perp\rho)\frac{2\pi\omega^2}{\left|\dfrac{v_\parallel}{JB}\dfrac{\partial}{\partial\theta}(k_\parallel v_\parallel+n\Omega_D)\right|}. \tag{1}$$

Here Φ_0 is the IBW electrostatic potential, J is the magnetic coordinates' Jacobian, n is the harmonic number of the interaction, and the bounce time between resonances for a particle orbiting on a flux surface is approximately

$$\tau_b \simeq qR/v_\parallel .$$

For a typical TFTR mode conversion experiment with $P_{\mathrm{rf}} = 2\,\mathrm{MW}$, $B_T = 5.0\,\mathrm{T}$, and $n_{^3\mathrm{He}}/n_e = 0.15$, 1D (one-dimensional) ray tracing of the IBW yielded the parameters (for a 250 keV deuteron with a pitch angle of $\lambda = -0.63$ at the MC layer)

$$\tau_b \simeq 10^{-5} \ \mathrm{s}, \quad \omega t_{\mathrm{res}} \simeq 20,$$

$$J_1^2(k_\perp \rho) \simeq 0.1 (\text{averaging over } k_\perp \rho),$$

$$\Phi_0 \simeq 0.3 \ \text{statvolt},$$

$$\Delta\varepsilon \simeq 0.5 \ \mathrm{keV},$$

so that the diffusion coefficient is roughly

$$D_\varepsilon \simeq 0.025 \ \mathrm{MeV}^2/\mathrm{s},$$

a value orders of magnitude below that implied by the data.[12]

Herrmann confirmed this result through a numerical simulation of the quasilinear diffusion of beam ions by IBWs (including the effects of collisions and a realistic magnetic geometry): While the distribution in pitch angle and poloidal angle of the predicted losses and the general scaling of the loss rate with rf power appeared to be accurate over the range of $P_{\mathrm{rf}} = 1.5 - 3.5\,\mathrm{MW}$, the magnitude of the loss rate was under-predicted by a factor of 30–70.[12] Grounded in the well established mechanisms of quasilinear diffusion and ray tracing of the IBW and further approximately reproducing the four-dimensional distribution of losses, this much more accurate investigation served only to underscore the perplexing nature of the discrepancy in loss magnitude. Explanation of the anomalous diffusion then requires some unanticipated mechanism by which the diffusive effect of the wave field is substantially enhanced.

IV. POTENTIAL MECHANISMS FOR DIFFUSION ENHANCEMENT

As discussed by Herrmann,[12] only four candidate explanations for this enhanced diffusion seem reasonable: (i) that the interaction of fast ions with the IBW could display long-range correlations and become an effectively super-diffusive process, (ii) that the magnitude of the electric field intensity in the interaction region could have been underestimated and with it the diffusion coefficient, (iii) that an error in the ray tracing of the IBW could have lead to an underestimation of the resonance time of a particle with the wave and hence of the diffusion coefficient analogous to (ii), or (iv) that an internal eigenmode of the plasma was excited to high amplitude resulting in an enhanced quasilinear diffusion coefficient. Noting that with the correlation of n interactions of energy step $\Delta\varepsilon$ occurring at time increments Δt the energy diffusion coefficient is enhanced according to $D_\varepsilon \rightarrow (n\Delta\varepsilon)^2/(n\Delta t) = nD_\varepsilon$, (i) seems unlikely as a diffusion enhancement by a factor of 30–70 would require the correlation of an equal number of interactions. Since the particles are orbiting in an inhomogeneous plasma which would imprint its variations and fluctuations on the IBW field and are additionally traveling toroidally through a sequence of resonance points, it seems very improbable that phase correlations could be sustained through as many as 30 wave–particle resonances. Further, even if the long-range correlation of several fast ion–IBW interactions were deemed plausible, this condition alone would not necessarily guarantee enhanced particle diffusion. The added condition that the nearby fast ion phase portrait be free of diffusion-impeding Kolmogorov–Arnold–Moser surfaces would also have to be satisfied. In the case of (ii), the wave intensity has been verified with both ray tracing and a full-wave code. Despite the calculated value for the electric field intensity still being only approximately correct, any error is unlikely to be large enough to account for the discrepancy in D_ε of more than an order of magnitude. Similarly for (iii), an error in the estimated resonance time by a factor ($\sim$6) sufficient to yield a factor of 30–70 error in D_ε seems unlikely. With elimination of these other possibilities, the excitation of an internal eigenmode (iv) remains as the most plausible, if remarkable, mechanism for diffusion enhancement. Two examples of such internal eigenmodes are described in the following two sections.

V. GLOBAL EIGENMODES IN MCIBW THEORY

Monakhov *et al.*[13] employed a resonant cavity perspective in discussing the importance of standing wave effects in (and a resonant cavity perspective for interpreting) MCIBW experiments in Tore Supra and other large tokamaks. For the launch of a FW from the low field side of a tokamak, the mode conversion layer has traditionally been described in cold plasma wave theory by using the Budden model in which asymptotic wave solutions are matched to a cutoff-resonance doublet ($n_\parallel^2 = L$ and $n_\parallel^2 = S$, respectively in the notation of Stix[17]) or a cutoff-resonance-cutoff triplet[18] (now including the $n_\parallel^2 = R$ high field side density cutoff). By adding to this model the low field side density cutoff (also at $n_\parallel^2 = R$) and recognizing the possibility for a low field incident wave to reflect from both the high and low field cutoffs in the limit of weak damping, a standing wave pattern can be seen to form between these cutoffs across the tokamak minor cross section.

Developed for the study of ion cyclotron emission, the 1D full-wave VICE code solves the Vlasov–Maxwell system of equations for a currentless slab plasma with a Hamiltonian action-angle formulation for particle orbits and wave–particle interactions.[19] Though neglecting 2D (two-dimensional) effects, the code is accurate to third order in the ion Larmor radius and permits solutions on meshes sufficiently fine to resolve an IBW in a large tokamak. Using VICE, Monakhov *et al.* explored the properties of the tokamak resonant cavity in the form of wave quality factor and wave energy density as a function of the location of the mode conversion layer, etc. The mode conversion efficiency to an IBW was optimized by simultaneously aligning an antinode of the global wave field pattern with the mode conversion layer and by locating the conversion layer sufficiently far from any cyclotron layers. Alternately, the mode conversion efficiency is minimized by arranging the conversion layer to coincide with a node of the standing wave field. Wave quality factors of $Q\sim 500$ and energy density amplifications factors for the trapped FW of ~ 100 were found under ideal conditions for this ''plasma cavity.''[13]

VI. THEORY OF CONTAINED MODES

Coppi *et al.*[14,15] and Gorelenkov *et al.*[16] have explained the enhanced radiation at harmonics of the edge ion cyclotron frequencies in tokamaks (ion cyclotron emission or ICE) as a result of the formation of contained modes on the outboard edge of the device: In the cylindrical limit for a cold plasma, the plasma wave equation $\nabla\times\nabla\times\mathbf{E}=(\omega^2/c^2)\underline{\underline{\epsilon}}\cdot\mathbf{E}$ may be reduced to a single equation for the poloidal electric field E_θ

$$\left(\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}+\frac{1}{R_0^2}\frac{\partial^2}{\partial\phi^2}+\frac{\omega^2}{c_A^2}\frac{n(r)}{n_0}\right)E_\theta=0, \qquad (2)$$

with c_A the Alfvén speed on axis, n_0 the on axis plasma density, and $n(r)$ the radially varying density profile. By the usual Fourier *ansatz* $E_\theta(r,\theta)=\tilde{E}(r)\exp i(m\theta-n\phi)$, Eq. (2) may be rewritten in terms of an effective potential

$$\left(\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r}-V(r,\omega)\right)\tilde{E}=0, \qquad (3)$$

where

$$V(r,\omega)\equiv\frac{m^2}{r^2}+\frac{n^2}{R_0^2}-\frac{\omega^2}{c_A^2}\frac{n(r)}{n_0}.$$

Physically, for certain values of n, m, ω, and c_A, the perturbation E_θ can be trapped in the potential well $V(r,\omega)$ formed by the density (or local Alfvén velocity) dependence of $\underline{\underline{\epsilon}}$ and the $k_\theta^2=m^2/r^2$ contribution to $\nabla\times\nabla\times$.

Expanding about the minimum of this effective potential at $r=r_0$

$$V\simeq V(r_0)+\tfrac{1}{2}V''(r_0)(r-r_0)^2, \qquad (4)$$

and introducing the local variable x, mode width Δ, radial mode number s, and scaled field amplitude $E(x)$ according to

$$E(x)\equiv\sqrt{r}\tilde{E}(r),$$

$$x\equiv\frac{\sqrt{2}}{\Delta}(r-r_0),$$

$$V(r_0)\equiv-\frac{2s+1}{\Delta^2},$$

$$V''(r_0)\equiv\frac{2}{\Delta^4},$$

the amplitude equation [Eq. (3)] may be cast in the convenient form of a parabolic cylinder equation

$$\frac{d^2E}{dx^2}+\left(s+\frac{1}{2}-\frac{x^2}{4}\right)E=0. \qquad (5)$$

For selected eigenfrequencies (corresponding to the condition that s be an integer), quantum harmonic oscillator-type solutions satisfying the boundary condition $E(\pm\infty)=0$ are found

$$\tilde{E}(r)=\frac{\tilde{E}_0}{\sqrt{r}}H_s\left(\frac{r-r_0}{\Delta}\right)\exp\left(-\frac{(r-r_0)^2}{2\Delta^2}\right), \qquad (6)$$

where H_s is the sth Hermite polynomial. Neglecting the $k_\parallel=n/R_0$ contribution to $V(r,\omega)$ and assuming a parabolic density profile $n=n_0(1-r^2/a^2)^\nu$, the localization radius and radial Gaussian width of these eigenmode oscillations to lowest order in the small parameter Δ/a are

$$\frac{r_0^2}{a^2}\simeq\frac{1}{1+\nu}-2(2s+1)\frac{\Delta^2}{a^2},$$

$$\frac{\Delta^2}{a^2}\simeq\frac{1}{m}\sqrt{\frac{\nu}{2(1+\nu)^3}}.$$

For the case of a finite inverse aspect ratio ϵ_0, the mode is additionally localized poloidally on the outboard midplane of the tokamak with a Gaussian width

$$\eta^2\simeq\frac{\Delta}{r_0}\sqrt{\frac{2}{2s+1}\left(1+\frac{1}{\epsilon_0}\right)}.$$

The excitation of such a contained eigenmode by energetic nuclei at an edge cyclotron frequency is believed to explain observed ICE spectra.

A more accurate expression for the mode localization radius $r_0(m,\nu)$ (as will be needed below) may be obtained by considering the equation from which this minimum is calculated

$$0=V'(r_0)=-2\frac{m^2}{r_0^3}+2\nu\frac{\omega^2}{c_A^2}\frac{r_0}{a^2}\left(1-\frac{r_0^2}{a^2}\right)^{\nu-1}.$$

Introducing the normalized quantities $z\equiv r_0^2/a^2$ and $\gamma\equiv a^2\omega^2/c_A^2$, the transformed equation

$$z^2(1-z)^{\nu-1}=\frac{m^2}{\nu\gamma},$$

may be solved iteratively to any desired accuracy by defining the function

$$f(z)\equiv\frac{m}{\sqrt{\nu\gamma}}(1-z)^{1-\nu/2}.$$

Beginning from the approximation $z\simeq 1/(1+\nu)$, then

$$r_0(m,\nu)\simeq a\sqrt{f\left(f\left(\cdots f\left(\frac{1}{1+\nu}\right)\cdots\right)\right)}. \qquad (7)$$

Notably, Eq. (7) is seen to be exact for the case $\nu=1$, while for the typical case $m/\sqrt{\nu\gamma}\ll 1$

$$|f'(z)|_{z=1/1+\nu}=\left|\frac{1-\nu}{2}\frac{m}{\sqrt{\nu\gamma}}(1-z)^{-1+\nu/2}\right|_{z=1/(1+\nu)}<1,$$

implying the convergence of the iteration. The mode width Δ may be found by substituting the above r_0 into the expression

$$\Delta \equiv [2/V''(r_0)]^{1/4},$$

with the know form of $V''(r)$.

The eigenmodes described by Monakhov *et al.* and Coppi *et al.* differ in crucial respects relating to quasilinear diffusion. The eigenmodes of Monakhov *et al.* are principally global modes of the FW in contrast to the highly localized and distinct contained eigenmodes of Coppi *et al.* Though Monakhov *et al.* demonstrate that standing-wave modes of large amplitude may be excited, the large wavelengths typifying these modes ($\sim\lambda_{FW}$) will likely not violate the conservation of a particle's magnetic moment μ and so these modes would not induce diffusion in perpendicular energy. The high poloidal mode numbers typical of the modes of Coppi *et al.*, however, could lead to violation of μ conservation and hence energy diffusion. In light of these characteristics, we evaluate the realizability of what appears to be the most probable scenario for diffusion enhancement: Namely that, during at least some of the TFTR MCIBW experiments, localized eigenmodes of the type described by Coppi *et al.* (excited by a passing FW or the FW antenna itself) were driven to high amplitude resulting in significant increases in energetic particle losses above expectations for simple IBW–induced diffusion.

VII. A MODEL FOR CONTAINED MODE EXCITATION

To simulate the case of a driven contained mode of the type described by (6) but with a now arbitrary driving frequency (i.e., s not an integer), the solution domain would naturally be extended from the single domain for the case of eigenmodes within the plasma to two domains: a domain representing the vacuum region outside of the plasma in which a driving antenna would be located and a domain representing the plasma itself. In the plasma region, (5) would be solved as above but for the more general case of noninteger s and that solution subsequently matched to the inhomogeneous solution of the vacuum wave equation in the neighborhood of the antenna. However, the expansion (4) of the confining potential is valid only for a narrow domain about the localization radius r_0 (the only region in which the eigenmode solution takes on nonzero values) and not into the core of the plasma near $r=0$ (where the driven solution may be of significant amplitude). Therefore, the domain representing the plasma is further divided into two subdomains: One about $r=r_0$ in which the parabolic approximation of $V(r)$ is valid and one in which the m^2/r^2 term dominates the potential with the dielectric contribution being approximately constant. The undetermined constant in the approximate form of $V(r)$ for the inner domain is adjusted to insure continuity of $V(r)$ at the boundary between inner and outer regions, chosen arbitrarily to be at the location $r_1 \equiv r_0 - \Delta$. The approximate form of the potential is then

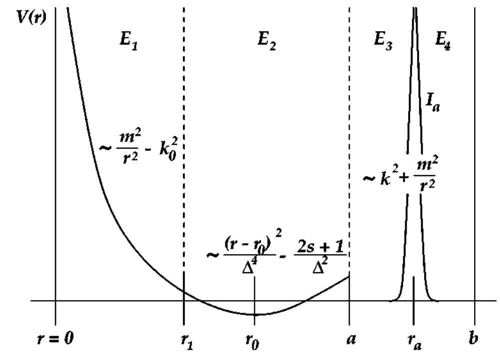

FIG. 1. Piece-wise approximate contained mode potential.

$$V(r) = \begin{cases} -k_0^2 + \dfrac{m^2}{r^2}, & 0<r<r_1 \\ -\dfrac{2s+1}{\Delta^2} + \dfrac{(r-r_0)^2}{\Delta^4}, & r_1<r<a, \\ \dfrac{m^2}{r^2} + k^2, & a<r<b \end{cases} \tag{8}$$

with

$$k_0 \equiv \sqrt{\frac{m^2}{r_1^2} + \frac{2s}{\Delta^2}}.$$

Here, a is the radius of the plasma and b the radius of the (assumed) perfectly conducting vacuum vessel. This approximate potential with the division of the plasma and the vacuum regions each into two subregions (splitting the vacuum domain according to $r>r_a$ and $r<r_a$ with r_a the location of the antenna) is shown in Fig. 1.

Since the injected waves are evanescent in the vacuum region for typical choices of n, the vacuum solution can be written in terms of the modified Bessel functions I_m and K_m when the constant k is chosen to be real

$$k \equiv \sqrt{\frac{n^2}{R_0^2} - \frac{\omega^2}{c^2}}.$$

The two independent solutions for arbitrary (noninteger) s in the parabolic region of the potential—generalizing from the previous integral-s Hermite function solutions—are

$$y_1(x) \equiv {}_1F_1\left(-\frac{s}{2}, \frac{1}{2}; \frac{x^2}{2}\right) \exp\left(-\frac{x^2}{4}\right),$$

$$y_2(x) \equiv x\, {}_1F_1\left(\frac{1-s}{2}, \frac{3}{2}; \frac{x^2}{2}\right) \exp\left(-\frac{x^2}{4}\right),$$

where ${}_1F_1$ is the confluent hypergeometric (Kummer) function[20] and as above $x(r) \equiv \sqrt{2}(r-r_0)/\Delta$. Solutions in the four domains can then be directly written as

$$E_1(r) = \alpha J_m(k_0 r),$$

$$E_2(r) = \beta \frac{y_1(x(r)) + \gamma y_2(x(r))}{\sqrt{r}},$$

$$E_3(r)=\frac{4\pi\omega r_a}{c^2}\frac{I(m)I_m(kr_a)}{\phi(r_a)+\delta}[\phi(r_a)-\phi(b)]\times[K_m(kr)+\delta I_m(kr)],$$

$$E_4(r)=\frac{4\pi\omega r_a}{c^2}I(m)I_m(kr_a)[K_m(kr)-\phi(b)I_m(kr)].$$

With an antenna current of the form $\mathcal{I}_a=i\hat{\theta}I(m)\delta(r-r_a)$, the boundary conditions linking the four domains are

$$E_1(r_1)-E_2(r_1)=E_2(a)-E_3(a)=0,$$

$$E_1'(r_1)-E_2'(r_1)=E_2'(a)-E_3'(a)=0,$$

$$E_3(r_a)-E_4(r_a)=E_4(b)=0,$$

$$E_3'(r_a)-E_4'(r_a)=-\frac{4\pi\omega}{c^2}I(m),$$

and $E_1'(0)=0$ or $E_1(0)=0$ for m even or odd.

The tedious but straightforward algebra of matching together these separate solutions according to the given boundary conditions yields values for the multiplying constants

$$\alpha=\beta\frac{y_1(x(r_1))+\gamma y_2(x(r_1))}{\sqrt{r_1}J_m(k_0r_1)}, \tag{9}$$

$$\beta=\frac{\sqrt{a}A}{y_1(x(a))+\gamma y_2(x(a))}\frac{\phi(a)+\delta}{\phi(r_a)+\delta}, \tag{10}$$

$$\gamma=\frac{y_1'(x(r_1))+(k_0\Delta/\sqrt{2})(\Omega(r_1)+(1/2r_1))y_1(x(r_1))}{(k_0\Delta/\sqrt{2})(\Omega(r_1)+(1/2r_1))y_2(x(r_1))-y_2'(x(r_1))}, \tag{11}$$

with the auxiliary quantities

$$A=\frac{4\pi\omega r_a}{c^2}I(m)[\phi(r_a)-\phi(b)]I_m(kr_a)I_m(k_a),$$

$$\eta=\frac{kI_m'(ka)}{I_m(ka)}\frac{y_1(x(a))+\gamma y_2(x(a))}{(\sqrt{2}/\Delta)y_1'(x(a))-(1/2a)y_1(x(a))+\gamma((\sqrt{2}/\Delta)y_2'(x(a))-(1/2a)y_2(x(a)))},$$

$$\delta=\frac{\eta\phi'(a)-\phi(a)}{1-\eta},\quad \phi(r)=\frac{K_m(kr)}{I_m(kr)},\quad \phi'(r)=\frac{K_m'(kr)}{I_m'(kr)},\quad \Omega(r)=\frac{J_m'(k_0r)}{J_m(k_0r)}.$$

The salient features of this solution are shown by the different plotted modes in Fig. 2. Noting that for a typical TFTR MCIBW experiment, the toroidal antenna current density is approximately $I\sim1.0$ amp/cm (assuming nearly all of the power to be in the dominant toroidal mode),[21] fields of ~1.0 statvolt/cm or larger are possible.

The mode amplitude in the localization region increases with decreasing n and m (corresponding to the deepening of the confining potential) and varies sharply with the density peakedness parameter ν. As reflected in Eq. (10) for the field amplitude in region 2, the condition $\phi(r_a)+\delta=0$ represents a resonance for the driven contained mode. Figure 3 shows β as a function of ν for representative mode numbers in the range of realistic peakedness of $0.4<\nu<1.0$ for these discharges. Prominent resonances are seen for $m<20$ in the neighborhoods of $\nu\simeq0.4$ and 0.75.

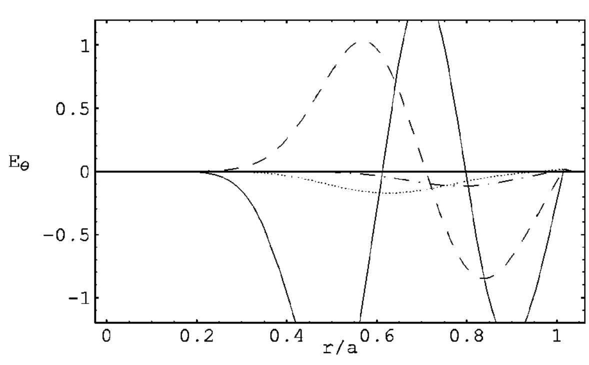

FIG. 2. Contained mode amplitudes in units of statvolt/cm for a current density of 1.0 amp/cm on the antenna with $(n,m,\nu)=(-36, 10, 0.75)$ (dotted), $(-36, 10, 0.5)$ (dashed), $(-18, 10, 0.75)$ (dot–dashed), and $(-18, 10, 0.5)$ (solid).

VIII. QUASILINEAR DIFFUSION IN A CONTAINED MODE

With these values for the electric field amplification, an estimate may be made of a quasilinear diffusion coefficient. Like IBW-induced diffusion, a particle drifting on a flux surface is imagined to enter the wave localization region once every bounce time τ_b, receive an energy increment $\Delta\varepsilon$, and so diffuse according to $D_\varepsilon\simeq\langle|\Delta\varepsilon|^2\rangle/\tau_b$. Experimentally, the majority of detected losses were witnessed at the passing-

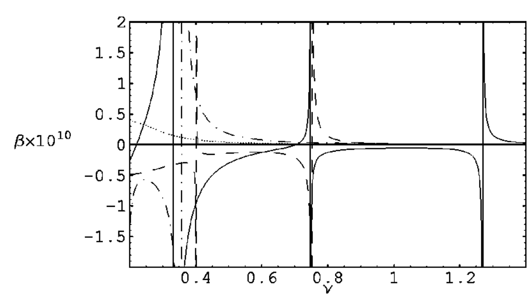

FIG. 3. β plotted as a function of the density peakedness parameter ν for $(n,m)=(-36, 15)$ (dotted), $(-36, 10)$ (dashed), $(-22, 15)$ (dot–dashed), and $(-22, 10)$ (solid).

trapped boundary of velocity space suggesting that the primary loss mechanism is the conversion of a passing particle to an unconfined trapped particle by the wave field. A similar mechanism is envisioned for the case considered here, whereby a passing beam ion would resonate with the wave field at some point in the poloidal plane until being converted to a trapped particle and promptly lost. From the examples plotted in Fig. 2, it is seen that, unlike the narrowly localized eigenmodes of the ICE mechanism, the driven eigenmodes extend across an appreciable segment of the minor radius. Such spreading makes possible the resonant interaction and subsequent diffusion of a beam ion near the plasma core with the edge of the excited wave field. The condition for resonance overlap, $\omega_b \geqslant (0.2) k\,\delta v$, where ω_b is the particle bounce frequency in the wave field and δv is the spacing in phase velocity between adjacent modes,[22,23] is also approximately satisfied for high amplitude modes of this type and makes valid the quasilinear diffusion mechanism.

As in the electrostatic case of IBW-induced diffusion, the energy increment given to the particle by the electromagnetic wave may be calculated as

$$\Delta\varepsilon = Z|e|\int_{-\infty}^{\infty} dt\,\mathbf{v}\cdot\mathbf{E}. \tag{12}$$

Describing the particle's gyro-orbit by

$$\rho = |\rho|[(\hat{\mathbf{e}}\times\hat{\mathbf{b}})\cos\Omega t + \hat{\mathbf{b}}\times(\hat{\mathbf{e}}\times\hat{\mathbf{b}})\sin\Omega t],$$

$$\dot{\rho} = \Omega\hat{\mathbf{b}}\times\rho,$$

and the impinging wave field by

$$\mathbf{E} = E_0\hat{e}\exp iS + \text{c.c.},$$

$$S \equiv \int^{x} d\mathbf{x}'\cdot\mathbf{k}(x') - \omega t,$$

the approximations of $\hat{e}\cdot\nabla S \simeq 0$ and $\hat{\mathbf{b}}\cdot\hat{\mathbf{e}} \simeq 0$ for an electromagnetic FW then yield

$$\mathbf{v}\cdot\mathbf{E} \simeq \dot{\rho}\cdot(E_0\hat{\mathbf{e}}\exp iS + \text{c.c.}). \tag{13}$$

Expanding the wave phase as

$$S \simeq S(g.c.) + \rho\cdot\nabla S = S(g.c.) + \rho|\nabla S|\sin\alpha,$$

so that

$$\exp i(S\pm\Omega t) \simeq \exp i(S(g.c.)\pm\Omega t) \times \sum_{-\infty}^{\infty} J_n(\rho|\nabla S|)\exp in\alpha,$$

a stationary phase calculation yields

$$\int_{-\infty}^{\infty} dt\,\exp i(S(g.c.)\pm\Omega t + in\alpha) \sim \sqrt{i\frac{2\pi}{W_{\pm}}}\exp i(S(g.c.)\pm\Omega t + n\alpha)\bigg|_{S_0,t_0}, \tag{14}$$

where

$$W_{\pm} \equiv \frac{d^2}{dt^2}(S(g.c.)\pm\Omega t + n\alpha)\bigg|_{S_0,t_0} \simeq \mathbf{v}(g.c.)\cdot\nabla(k_{\parallel}v_{\parallel} + (n\pm 1)\Omega)|_{S_0,t_0},$$

and the resonant phase S_0 and time t_0 are defined by

$$0 = \frac{d}{dt}(S(g.c.)\pm\Omega t + n\alpha)\bigg|_{S_0,t_0} \simeq \mathbf{v}(g.c.)\cdot\nabla S - \omega + (n\pm 1)\Omega|_{S_0,t_0}.$$

This last condition effectively selects the harmonic number n of the interaction.

A straightforward calculation for a particle circulating on a flux surface yields

$$W_{+} \gg W_{-} \simeq \epsilon_0\frac{k_{\parallel}v_{\perp}^2}{2qR_0},$$

so that, on averaging over S_0 and t_0 or equivalently $S(g.c.)$ and α, Eq. (12) with Eqs. (13) and (14) gives

$$\langle|\Delta\varepsilon|^2\rangle \simeq \frac{\pi}{W_{-}}(Z|e|v_{\perp}E_0)^2 J_1^2(\rho|\nabla S|) \simeq 2\pi\frac{qR_0}{\epsilon_0 k_{\parallel}}(Z|e|E_0)^2 J_1^2(k_{\perp}\rho).$$

Taking $\tau_b = qR_0/v_{\parallel}$ and utilizing values for a representative counter-going 100 keV D beam ion which resonates with the $(n,m) = (-18,10)$ mode near $\theta \simeq 1.8$ rad in a plasma with $\nu \simeq 0.4$,

$$k_{\parallel} \simeq 0.08\ \text{cm}^{-1},\quad k_{\perp} \simeq 0.3\ \text{cm}^{-1},$$

$$\rho \simeq 1.1\ \text{cm},\quad v \simeq 3.1\times 10^8\ \text{cm/s},\quad \lambda \simeq -0.5,$$

$$E_0 \simeq 1.0\ \text{statvolt/cm},$$

then finally

$$D_{\varepsilon} \simeq 2\pi\frac{v\lambda}{\epsilon_0 k_{\parallel}}(Z|e|E_0)^2 J_1^2(k_{\perp}\rho) \sim 100\ \text{MeV}^2/\text{s}, \tag{15}$$

on the order of the experimentally inferred value of 25 MeV^2/s.[10] Though this example can be taken to imply no more than the order of magnitude of possible diffusion rates in contained modes (addressing only a single particle of a given pitch and energy interacting with a single wave for an approximate value of ν), a plausible mechanism for diffusion enhancement is nonetheless identified. It should also be noted that due to the extent of the mode in the poloidal angle θ, it is possible for the particle resonance condition [Eq. (18)] to continue to be satisfied as particles diffuse from $\varepsilon = 100$ keV to 2 MeV by the variation in the local gyrofrequency from the inboard to the outboard side of the tokamak. From the effective inverse squared resonance time W_{-}, it may also be shown that the particle rotates no more than a few degrees poloidally while in resonance with the wave ($\tau_b \gg \tau_{\text{res}}$). This condition justifies the straight-orbit approximation and the treatment of the diffusion process as a local phenomenon independent of the spatial extent and amplitude variation of the wave field, so long as the particle satisfies the resonance condition within the wave region.

IX. DISCUSSION

Many aspects of the fast ion losses observed during TFTR MCIBW experiments had been explained in the context of IBW physics, particularly the distribution of losses in energy, pitch-angle, and poloidal angle. While a contained mode has been shown to be a potential explanation for the fast ion loss rate, its presence could conceivably be inconsistent with other aspects of the observed losses. Clearly, if a contained mode is to be a plausible theory, it must be shown to agree with these other loss characteristic.

Firstly, note that the diffusion paths of fast ions through the (ε,μ,P_ϕ) constants-of-motion space in a tokamak would be identical for interaction with an electrostatic IBW or with an electromagnetic FW, namely governed by the relations

$$\frac{dP_\phi}{d\varepsilon}=\frac{n}{\omega}, \tag{16}$$

$$\frac{d\mu}{d\varepsilon}=\frac{Z|e|n}{m_D c\omega}, \tag{17}$$

where P_ϕ is the particle's canonical angular momentum, μ its magnetic moment, ϵ its energy, and n the cyclotron harmonic number for the interaction. Since the diffusion paths are identical for these two scenarios, the only distinguishing effect on the loss characteristics (i.e., where in the constants-of-motion space a particle crosses a loss boundary) would result from the different wave fields interacting with fast ions in different regions of the constants-of-motion space, so that particles effectively ''begin'' diffusing at different locations in that space. Noting that the dominant contribution to the population of lost particles is from counter-going beam ions crossing the passing-trapped boundary to be promptly lost, the configuration of (ϵ,μ,P_ϕ)-space is such that the particle population interacting with a contained mode would likely lead only to a potentially larger number of particles being lost and at slightly larger poloidal angles (closer to the inner midplane) than the analogous population for interaction with the MCIBW at the same energy and pitch-angle. A realistic numerical simulation is required to verify the impact of FW-induced diffusion on the lost particle distribution. Nonetheless, because of Eqs. (16) and (17), it appears plausible that the distribution of losses in energy, pitch-angle, and poloidal angle for FW interaction would be at least similar to those seen from numerical simulations of interaction with an IBW which themselves were in approximate agreement with experimental observations.

Other features of the TFTR MCIBW experiments which had previously been regarded as characteristic of the IBW can also be explained in the context of a contained mode. Particularly, the co-going deuteron beam losses observed during mode converted current drive (MCCD) experiments and used to infer the ''flipping'' of the sign of the IBW's $k_\parallel$ are similarly consistent with the existence of a contained mode. Initially it had been interpreted that the extremely large transverse wave number k_x attained by the IBW as it propagated from the MC layer coupled with the large aspect ratio expression for $k_\parallel$,

$$k_\parallel \simeq \frac{n}{R_0}+\epsilon_0\frac{k_x}{q}\sin\theta,$$

made possible a sign change in $k_\parallel$ from the launched $n<0$ for certain poloidal angles θ. With a positive $k_\parallel$, the wave–particle resonance condition

$$k_\parallel v_\parallel=\omega-n\Omega_D, \tag{18}$$

could be satisfied by co-going $(v_\parallel>0)$ deuterons near the MC layer $(\omega-\Omega_D>0)$ resulting in their ejection. The high poloidal mode numbers typical of a contained mode, however, also permit components of the mode spectrum to flip the sign of the $k_\parallel$ and interactions with co-going particles to be observed. It should be recalled that since for low energy particles $P_\phi \propto -\psi_p$, Eq. (16) requires a particle to interact with a wave of $n<0$ if it is to be both heated $(d\epsilon>0)$ and ejected $(dP_\phi<0)$. In the case of the ''beam-blip'' experiments in which only counter-going losses $(v_\parallel<0)$ were detected, the more negative wave toroidal mode number used in these IBW heating cases ($n\simeq-36-43$ as opposed to the $n\simeq-23-25$ typical of MCCD experiments) precludes the flipping of $k_\parallel$ for a contained mode just as for an IBW. Thus, the observation of wave–particle resonance with strictly counter-going ions comports with the existence of a contained mode.

Similarly, what was seen as a sensitive dependence of the loss rate on the location of the IBW MC layer via variations in B_T and the ^{3}He concentration may alternatively be interpreted as variations in the parameters governing the contained mode amplitude. Since both of these quantities determine the local Alfvén speed, a rapid passage through resonance of a contained mode due to varying plasma parameters, as suggested by Fig. 3, could potentially result in the factor of 10 increase in losses observed with the increase of the ^{3}He fraction from 0.15 to 0.20. In contrast, numerical simulations of IBW–beam ion interaction demonstrated at most a factor of 4 increase in losses with a step of 0.05 in the ^{3}He fraction—and then only in progressing from concentrations of 0.10–0.15. Noting that the amplitude of a contained mode is an extremely sensitive function of the density profile shape and Alfvén speed, for a fixed FW frequency, a particular contained mode $(n,\ m)$ might go rapidly in or out of resonance due to very slight variations of the plasma profile or magnetic field. Indeed, in many of the experiments in which MCCD and IBW heating were explored, a contained mode may never have been excited to significant amplitude such that the interpretation of these experiments is not changed.

X. IMPLICATIONS FOR ALPHA CHANNELING

For alpha channeling to be effective in any future reactor, a significant fraction of an alpha particle's energy must be extracted in less than the collisional slowing-down time τ_s of the particle on electrons. Making the reasonable assumption that the same diffusion coefficient governs both particle heating and cooling and utilizing the parameters from Sec. III for estimating such a coefficient for an IBW, the effective channeling requirement of $D_\varepsilon \simeq (3.5\,\mathrm{MeV})^2/\tau_s$ with $\tau_s \simeq 0.1\,\mathrm{s}$ then implies the only marginally feasible reactor power requirement of $P_{\mathrm{rf}} \simeq 100\,\mathrm{MW}$. The experimentally observed loss enhancement then suggests a requirement of $P_{\mathrm{rf}} \ll 100\,\mathrm{MW}$, one of the necessary if not sufficient requirements for alpha power channeling.

However, if the modes discussed above are responsible for the loss enhancement and an attempt is then made to apply them in a channeling scenario, there are several caveats which must be recognized. The question of the intersection of the region in which the modes achieve significant amplitude with the orbits of energetic particles must firstly be accurately resolved. In a reactor-sized plasma much larger than that of TFTR, it may be that an edge contained mode is well removed from any energetic particles confined in the core making such modes ineffective for particle diffusion. The tailoring of the excitation of such modes effectively to cool as well as extract alpha particles from the plasma is a further challenge. In the case of the TFTR IBW experiments, only the heating of particles was suggested; the capability of such a quasilinear diffusion process for particle cooling must be convincingly established. Finally, it must be evaluated whether the alpha particle power extracted relative to the power requirements in driving such modes could make for an energetically efficient channeling scheme.

XI. CONCLUSIONS

The anomalously large diffusion rate of energetic ions from TFTR IBW experiments is difficult to explain by conventional ideas in quasilinear diffusion. The data is essentially four-dimensional, and detailed enough to rule out insufficient explanations. Instead, we are left with only one remarkable conjecture that appears to be plausible, namely, the excitation of an internal eigenmode by the IBW which subsequently interacts with the beam ions.

The driving of edge eigenmodes (modes similar to those described in relation to the ICE phenomenon) by the FW antenna may produce large amplitude standing waves across the outer half of a tokamak plasma. The resulting enhancement of the wave quasilinear diffusion coefficient has been shown to be on the order of the value inferred experimentally. It is also noted that the loss characteristics for particles from such an edge contained mode are not dissimilar from those experimentally observed.

It is remarkable to imagine that the tokamak plasma is essentially resonantly ringing from the IBW wave excitation. The assumption at zeroth order of a quiescent plasma is considerably challenged by this picture. The conjecture, however, has explanatory force. More to the point, conventional explanations are not at all satisfactory.

There are, of course, a number of caveats. Many simplifications have been made in this model which could be improved. Primarily, consideration should be given to more realistic modeling of the plasma and its geometry, likely involving a numerical treatment of wave coupling, mode excitation, and the medium's dielectric response including mode damping. The outstanding question of whether a contained mode wave field could access a comparable number of beam ions in a region of the plasma distinct from the location of the IBW and so with its enhanced diffusion coefficient realistically generate the enhanced losses observed is likewise only approachable numerically. The trajectory of a beam ion diffusing from 100 keV fully to 2 MeV by interacting with a complete spectrum of waves (not only the single wave considered in the above estimate of D_ε) could only be followed by such a technique. Consideration of similar experiments in other tokamaks with plasma (and hence mode localization) volumes of varying sizes could also be illuminating. The case of a loss enhancement by a factor of ~ 10 due to varying concentrations of ^{3}He is a potent example of the sensitivity of such a mode to experimental conditions.

Despite these inaccuracies, a driven contained mode remains the most plausible mechanism for the observed diffusion enhancement. In particular, none of the characteristics of the distribution of losses clearly contradicts the possibility of a contained mode mechanism, while no other candidate mechanism appears as fully to explain the observations. The existence of such high amplitude modes would clearly be a remarkable addition to the already diverse phenomenology of tokamak plasmas.

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