ELSEVIER

Physica A 305 (2002) 287–296

PHYSICA A

www.elsevier.com/locate/physa

Quantum corrections to the distribution function of particles over momentum in dense media

A.N. Starostin[a,*], A.B. Mironov[a], N.L. Aleksandrov[b], N.J. Fisch[c], R.M. Kulsrud[c]

[a] *Troitsk Institute for Innovation and Fusion Research, Moscow region, 142092 Troitsk, Russia*
[b] *Moscow Institute of Physics and Technology, Moscow region, Russia*
[c] *Princeton Plasma Physics Laboratory, Princeton University, NJ, USA*

Abstract

A simple derivation of the Galitskii–Yakimets distribution function over momentum is presented. For dense plasmas it contains the law $\sim p^{-8}$ as a quantum correction to the classical Maxwellian distribution function at large momenta. The integral equation for the width of the spectral distribution of kinetic Green functions is analyzed. The asymptotic behavior of the quantum corrections to the distribution function of particles is expressed via the Fourier transform of the wave function in the external potential. It is shown that the asymptotic power law for the distribution function over momentum is also correct for a non-equilibrium at the external electrical and laser fields. © 2002 Elsevier Science B.V. All rights reserved.

Keywords: Distribution function; Green function; Lorentz gas; Self-energy; Density matrix

1. Introduction

In classical statistics the commutativity of kinetic and potential energy operators leads to the Maxwell distribution function over momentum even for the strong interparticle interaction. On account of the effect of the quantum uncertainty the distribution function has a non-maxwellian form and contains a power-law tail [1]. This causes a non-exponential temperature dependence of the rates of threshold inelastic processes [2]. In this paper, we show simple ways of quantum corrections to the distribution function derivation with help of Green function and one-particle density matrix.

* Corresponding author. Fax: +7-095-334-5128.
E-mail addresses: staran@triniti.ru (A.N. Starostin), alek@neq.mipt.ru (N.L. Aleksandrov), fisch@pppl.gov (N.J. Fisch).

0378-4371/02/$ - see front matter © 2002 Elsevier Science B.V. All rights reserved.
PII: S0378-4371(01)00677-X

2. Expansion in series of $\hbar^2$

In 1932 E. Wigner, G.E. Uhlenbeck, L. Gropper have calculated quantum corrections to the classical distribution function over momentum [3]. In calculations they made use of fact that the distribution function over momenta can be derived by integration of function $I(p,q)$ over coordinates q.

$$I = \frac{1}{V^N}\exp\left(-\frac{\mathrm{i}}{\hbar}\sum_i p_i q_i\right)\exp(-\beta\hat{H})\exp\left(\frac{\mathrm{i}}{\hbar}\sum_i p_i q_i\right), \tag{1}$$

$$\hat{H} = \sum_i \frac{\hat{p}_i^2}{2m} + U(q_1,\dots,q_N) = -\frac{\hbar^2}{2}\sum_i \frac{1}{m_i}\frac{\partial^2}{\partial^2 q_i^2} + U(q_1,\dots,q_N)\,. \tag{2}$$

As a result of such calculations they have the one-particle distribution function

$$\mathrm{d}w_{p_i} = const\,\exp\left\{-\frac{p_i^2}{2m_i}\left[1-\frac{\hbar^2}{12T^3 m_i}\left\langle\left(\frac{\partial U}{\partial q_i}\right)^2\right\rangle\right]\right\}\mathrm{d}p_i \tag{3}$$

that differs from the Maxwellian distribution in "effective temperature"

$$T_{eff} = T + \frac{\hbar^2}{12 m_i T^2}\left\langle\left(\frac{\partial U}{\partial q_i}\right)^2\right\rangle\,. \tag{4}$$

Angle brackets mean averaging with help of classical Gibbs distribution.

3. Inclusion of quantum degeneracy effects

It is possible to obtain the corrected distribution function (3), (4) using the Green functions method. We will assume that there is no interaction between the particles, they only interact with heavy almost immobile impurities; N is the number of the impurity atoms, $\{\mathbf{R}_m\}$ is the totality of their coordinates, $U(\mathbf{r}) = \sum_m U_0(\mathbf{r}-\mathbf{R}_m)$ describes the interaction of the particle of mass m with impurities. This is the so-called Lorentz gas model [4].

Calculating the half-sum of the equations for the retarded Green function G^R we have

$$\frac{1}{2}\left[\mathrm{i}\hbar\frac{\partial}{\partial t_1} - \mathrm{i}\hbar\frac{\partial}{\partial t_2} + \frac{\hbar^2}{2m}\Delta_1 + \frac{\hbar^2}{2m}\Delta_2 - U(\mathbf{r_1}) - U(\mathbf{r_2})\right]$$
$$\times G^R(\mathbf{r_1},t_1,\mathbf{r_2},t_2) = \delta(\mathbf{r_1}-\mathbf{r_2})\delta(t_1-t_2)\,. \tag{5}$$

In new coordinates $\tau = t_1 - t_2$, $T = (t_1+t_2)/2$, $\xi = \mathbf{r_1}-\mathbf{r_2}$, $\mathbf{R} = (\mathbf{r_1}+\mathbf{r_2})/2$ after the expansion of $U(\mathbf{R}\pm\xi/2)$ in the region close to $\mathbf{R}$ an equation for the Fourier transform of the Green function

$$G^R(E,\mathbf{p},\mathbf{R}) = \int \exp\left(\mathrm{i}E\tau - \mathrm{i}\frac{\mathbf{p}\xi}{\hbar}\right) G^R(\xi,\tau,\mathbf{R},T)\,\mathrm{d}\tau\,\mathrm{d}\xi \tag{6}$$

will be written as follows:

$$\left[E+\frac{\hbar^2}{8m}\Delta_{\mathbf{R}}-\varepsilon_p-U(\mathbf{R})+\frac{\hbar^2}{8}\frac{\partial^2}{\partial p_i\partial p_j}\frac{\partial^2 U}{\partial \mathbf{R}_i\mathbf{R}_j}\right]G^R=1\,. \tag{7}$$

The last two terms in square brackets are the consequence of $U(\mathbf{R}\pm\rho/2)$ expansion in series near $\mathbf{R}$, $\varepsilon_p=p^2/2m$ is the kinetic energy.

To find an expansion in terms of $\hbar^2$ we have to substitute $G^R=G_0^R+\hbar^2G_1^R$. After the separation of different order of $\hbar$ terms

$$G_0^R=\frac{1}{E-\varepsilon_p-U+\mathrm{i}\delta}\,, \tag{8}$$

$$G_1^R=-\frac{1}{4m}\frac{\Delta U}{(E-\varepsilon_p-U+\mathrm{i}\delta)^3}-\frac{1}{4m}\frac{(\Delta U)^2}{(E-\varepsilon_p-U+\mathrm{i}\delta)^4}$$
$$-\frac{1}{4m^2}p_ip_j\frac{\partial^2 U}{\partial \mathbf{R}_i\mathbf{R}_j}\frac{1}{(E-\varepsilon_p-U+\mathrm{i}\delta)^4}\,. \tag{9}$$

The distribution function over momentum can be calculated as [1]

$$\tilde{f}(\mathbf{p})=\int\frac{\mathrm{d}E}{\pi}\,n(E)\,\mathrm{Im}\,G^R\,. \tag{10}$$

Here $n(E)$ are the occupation numbers of particles with "energy" E. In the non-degenerated case $n(E)\sim\exp(-E/T)$. By means of tilde we note that the distribution function over momentum was found for the certain impurities spatial distribution, i.e., $U(\mathbf{r})$ in (5) implies $U(\mathbf{r},\{\mathbf{R}_m\})$, where $\{\mathbf{R}_m\}$ are coordinates of all impurity atoms. Therefore, it is necessary to average $\tilde{f}(\mathbf{p}_i,\{\mathbf{R}_m\})$ over $\{\mathbf{R}_m\}$. Under the assumption that interactions do not affect the statistical homogeneity, after all the integrations we obtain (3), (4), but angle brackets now (and hereinafter) mean the averaging over the impurities locations. The advantage of this approach is an opportunity to calculate the corrections not only to the Maxwell distribution function but also to take the particles quantum identity into account. In this case

$$n(E)=\frac{1}{\mathrm{e}^{(E-\mu)/T}\pm1} \tag{11}$$

and

$$f(\mathbf{p},\mathbf{R})=\int\mathrm{d}E\,n(E)\delta(E-\varepsilon_p-U(\mathbf{R}))$$
$$-\frac{\hbar^2}{8m}\Delta U\int\mathrm{d}E\,n(E)\frac{\partial^2}{\partial E^2}\,\delta(E-\varepsilon_p-U(\mathbf{R}))$$
$$+\frac{\hbar^2}{24m}(\nabla U)^2\int\mathrm{d}E\,n(E)\frac{\partial^3}{\partial E^3}\,\delta(E-\varepsilon_p-U(\mathbf{R}))$$
$$+\frac{\hbar^2}{24m^2}p_ip_j\frac{\partial^2 U}{\partial\mathbf{R}_i\partial\mathbf{R}_j}\int\mathrm{d}E\,n(E)\frac{\partial^3}{\partial E^3}\,\delta(E-\varepsilon_p-U(\mathbf{R}))\,. \tag{12}$$

After the integration over $\mathbf{R}$ we will obtain the momentum distribution.

4. One-particle density matrix approach

Another way of calculating the quantum corrections to the distribution function which leads to (3), (4) is employing the one particle density matrix

$$\rho(\mathbf{r_1},\mathbf{r_2}) = \frac{1}{Z}\sum_i e^{-\beta E_i}\psi_i(\mathbf{r_1})\psi_i^*(\mathbf{r_2}) . \tag{13}$$

Differentiating (13) with respect to β

$$\begin{aligned}\frac{\partial\rho(\mathbf{r_1},\mathbf{r_2})}{\partial\beta} &= \left(-\frac{\hbar^2}{2m}\Delta_1 + U(\mathbf{r_1})\right)\rho(\mathbf{r_1},\mathbf{r_2}) \\ &= \left(-\frac{\hbar^2}{2m}\Delta_2 + U(\mathbf{r_2})\right)\rho(\mathbf{r_1},\mathbf{r_2}) ,\end{aligned} \tag{14}$$

$$\frac{\partial\rho(\mathbf{r_1},\mathbf{r_2})}{\partial\beta} = \frac{1}{2}\left(-\frac{\hbar^2}{2m}\Delta_1 + U(\mathbf{r_1}) - \frac{\hbar^2}{2m}\Delta_2 + U(\mathbf{r_2})\right)\rho(\mathbf{r_1},\mathbf{r_2}) \tag{15}$$

and making change of variables $\xi = \mathbf{r_1} - \mathbf{r_2}$, $\mathbf{R} = (\mathbf{r_1} + \mathbf{r_2})/2$ for the Fourier transform of the density matrix

$$\rho(\mathbf{p},\mathbf{R}) = \int \exp\left(-i\frac{\mathbf{p}\xi}{\hbar}\right)\rho(\mathbf{R},\xi)\,d\xi . \tag{16}$$

We have an equation

$$\begin{aligned}\frac{\partial\rho(\mathbf{p},\mathbf{R})}{\partial\beta} = \int \exp\left(-i\frac{\mathbf{p}\xi}{\hbar}\right)\Bigg[&-\frac{\hbar^2}{8m}\Delta_R - \frac{\hbar^2}{2m}\frac{\partial^2}{\partial\xi^2} + U(\mathbf{R}) \\ &+ \frac{\xi_i\xi_j}{8}\frac{\partial^2 U}{\partial R_i \partial R_j}\Bigg]\rho(\mathbf{R},\xi)\,d\xi\end{aligned} \tag{17}$$

with the boundary condition $\rho = 1$ at $\beta = 0$. Keeping in mind to find an expansion in terms of $\hbar^2$, solution is seeking in the form of $\rho(\mathbf{p},\mathbf{R}) = e^{-\beta\varepsilon_p - \beta U(\mathbf{R})}\chi$, where $\chi = 1 + \hbar^2\chi_1$ and $\chi_1 = 0$ at $\beta = 0$.

$$\frac{\partial\chi_1}{\partial\beta} = -\frac{\beta^2}{8m^2}p_i p_j\frac{\partial^2 U}{\partial\xi_i\partial\xi_j} - \frac{\beta^2}{8m}(\nabla U)^2 , \tag{18}$$

$$\tilde{f}(\mathbf{p}) = \int \rho(\mathbf{p},\mathbf{R})\,d\mathbf{R} . \tag{19}$$

Since $U(\mathbf{r})$ implies $U(\mathbf{r},\{\mathbf{R}_m\})$, repeating previous section reasoning, after all integrations we obtain (12) again.

It is possible to show that for electrons interacting with neutral atoms via potential $U(\mathbf{R}) \sim -\alpha e^2/R^4$, where α is the atom polarizability, the quantum corrections to the distribution function are important for atoms density $n_a \sim 3 \times 10^{23}\ \mathrm{cm}^{-3}$.

5. Tails of the momentum distribution function

The quantum uncertainty results in that the distribution function over momentum contains a power-law tail. Here we will show the simple way to derive such non-exponential distribution with help of the one-particle density matrix within the described model. It is hard to obtain such results using expansion in series of $\hbar^2$ [3] for fixed value of momentum.

Writing (19) in the explicit form

$$\tilde{f}(\mathbf{p}) = \frac{1}{ZV}\sum_i \int \mathrm{d}\mathbf{R} \int \mathrm{d}\xi \exp\left(-\frac{i}{\hbar}\mathbf{p}\xi - \beta E_i\right) \times \psi_i(\mathbf{R}+\xi/2)\psi_i^*(\mathbf{R}-\xi/2)\,, \tag{20}$$

$$Z = \sum_i \exp(-\beta E_i) \tag{21}$$

and expressing the wave function via its Fourier transformation

$$\tilde{f}(\mathbf{p}) = \frac{1}{ZV}\sum_i \mathrm{e}^{-\beta E_i} \int \mathrm{d}\mathbf{R} \int \mathrm{d}\xi \exp\left(-\frac{i}{\hbar}\mathbf{p}\xi\right) \times \iint \frac{\mathrm{d}\mathbf{p_1 p_2}}{(2\pi)^6} \exp\left(\frac{i}{\hbar}\mathbf{p_1}(\mathbf{R}+\xi/2)\right)\psi_i(\mathbf{p_1}) \times \exp\left(-\frac{i}{\hbar}\mathbf{p_2}(\mathbf{R}-\xi/2)\right)\psi_i^*(\mathbf{p_2})\,. \tag{22}$$

We have

$$\tilde{f}(\mathbf{p}) = \frac{1}{ZV}\sum_i \mathrm{e}^{-\beta E_i}|\psi_i(\mathbf{p})|^2\,. \tag{23}$$

In accordance with selected model ψ_i is the solution of the Schroedinger equation

$$\psi_i^{(+)}(\mathbf{p}) = (2\pi\hbar)^3\delta(\mathbf{p}_i - \mathbf{p}) + \frac{4\pi F(\mathbf{p},\mathbf{p}_i)}{p_i^2 - p^2 + \mathrm{i}0}\,, \tag{24}$$

$$F(\mathbf{p},\mathbf{p}_i) = -\frac{m}{(2\pi)^4}\int U(\mathbf{p}-\mathbf{p}')\psi_i^{(+)}(\mathbf{p}')\,\mathrm{d}\mathbf{p}'\,. \tag{25}$$

In the Born approximation we get

$$\psi_i(\mathbf{p}) = (2\pi\hbar)^3\delta(\mathbf{p}-\mathbf{p}_i) + \frac{2m}{p_i^2 - p^2 + \mathrm{i}0} \times \sum_j \exp\left(\frac{i}{\hbar}(\mathbf{p}-\mathbf{p}_i)\mathbf{R}_j\right)U_0(\mathbf{p}-\mathbf{p}_i)\,, \tag{26}$$

where $U_0(\mathbf{p})$ is the Fourier transform of $U_0(\mathbf{r})$. Substituting (26) into (23) and then averaging over the impurities locations for large values of p we obtain

$$f(\mathbf{p}) \sim \hbar \frac{N}{V} \frac{|U_0(\mathbf{p})|^2}{\varepsilon_p^2} . \tag{27}$$

Further more, under the assumption that for the short range potential the scattering amplitude (25) can be represented as a sum of scattering amplitudes of particle in the presence of single impurity atom $F^{(1)}(\mathbf{p}_i, \mathbf{p})$.

$$f(\mathbf{p}) \sim \hbar \frac{N}{V} \frac{1}{\varepsilon_p^2} \int |F^{(1)}(\mathbf{p}_i, \mathbf{p})|^2 \, \mathrm{d}\Omega . \tag{28}$$

The same result may be expressed via the Fourier transform of the wave function of particle in the presence of single impurity atom

$$f(\mathbf{p}) \sim \hbar \frac{N}{V} \int \left| \int \exp\left(-\frac{\mathrm{i}\mathbf{p}R}{\hbar} \right) \psi_i^{(+)}(\mathbf{r}) \, \mathrm{d}\mathbf{r} \right|^2 \mathrm{d}\Omega . \tag{29}$$

As we are interested in studying of a dense media when the impurity atoms locations can be strongly correlated we can average (23) taking the Boltzman statistics into account. It leads to the appearance of the additional multiplier in (28) which characterizes such correlations by means of the correlation function g.

$$f(\mathbf{p}) \sim \hbar \frac{N}{V} \frac{1}{\varepsilon_p^2} \int |F^{(1)}(\mathbf{p}_i, \mathbf{p})|^2 S(\mathbf{p}) \, \mathrm{d}\Omega , \tag{30}$$

$$S(\mathbf{p}) = 1 + \frac{N}{V} \int (g(\mathbf{R}) - 1)) \exp\left(-\frac{\mathrm{i}\mathbf{p}R}{\hbar} \right) \mathrm{d}\mathbf{R} . \tag{31}$$

In (28)–(30) $p_i \sim \sqrt{2mT} \ll p$. From (27) we have for $U(\mathbf{r}) \sim \delta(\mathbf{r})$ at large values of p $f(\mathbf{p}) \sim \hbar n/p^4$—the result of Belyakov [4] , for $U(\mathbf{r}) \sim 1/r$ $f(\mathbf{p}) \sim \hbar n/p^8$ as was shown by Galitskii and Yakimets [1], in the relativistic case of a short-range potential $f(\mathbf{p}) \sim \hbar n/p^6$. Because we are looking at the asymptotical form of $f(\mathbf{p})$ at large momenta it is possible to use Born approximation for the scattering amplitude.

Refusing the assumption about the additivity of the scattering amplitudes it is possible to write an equations set that gives an opportunity to calculate the multiple scattering amplitude for the system of short-range potentials [5] numerically.

$$f_N = \sum_j \left[f_j \exp\left(\frac{i}{\hbar} (\mathbf{p}_i - \mathbf{p}) \mathbf{z}_j \right) + \sum_{k \neq j} f_j \frac{\exp(\mathrm{i} p_i R_{jk}/\hbar)}{R_{jk}} Q_k \right] , \tag{32}$$

$$Q_j = f_j \exp\left(\frac{i}{\hbar} \mathbf{p}_i \mathbf{z}_j \right) + \sum_{k \neq j} f_j \frac{\exp(\mathrm{i} p_i R_{jk}/\hbar)}{R_{jk}} Q_k , \tag{33}$$

$$f_N = \sum_j Q_j \exp\left(\frac{i}{\hbar} \mathbf{p} \mathbf{z}_j \right) . \tag{34}$$

Calculated in such way f_N is the amplitude of multiple scattering over the N centers. The distribution function at large values of momenta one can obtain by averaging f_N over the centers locations.

6. Kinetic Green functions method

In general case the generalized distribution function $f(E,\mathbf{p})$ can be obtained with help of Kadanoff–Baym [6], Keldysh diagram technique [2,7]

$$f(E,\mathbf{p}) = -\mathrm{i}\,\frac{n(E)}{2\pi}\,(G^R(E,\mathbf{p}) - G^A(E,\mathbf{p}))\,, \tag{35}$$

$$G^A(E,\mathbf{p}) = (G^R(E,\mathbf{p}))^* \tag{36}$$

and the distribution function over momentum is given by $f(\mathbf{p}) = \int f(E,\mathbf{p})\,\mathrm{d}E$ (see also (10)).

In case of low density ideal Fermi gas

$$f(E,\mathbf{p}) = n(E)\delta(E - \varepsilon_p)\,. \tag{37}$$

But for non-ideal gas

$$f(E,\mathbf{p}) = n(E)\,\frac{1}{\pi}\,\frac{\mathrm{Im}\,\Sigma^R(E,\mathbf{p})}{(E - \varepsilon_p - \mathrm{Re}\,\Sigma^R(E,\mathbf{p}))^2 + (\mathrm{Im}\,\Sigma^R(E,\mathbf{p}))^2}\,. \tag{38}$$

Later on we will denote $\mathrm{Im}\,\Sigma^R(E,\mathbf{p})$ as $\gamma(E,\mathbf{p})$ and $\mathrm{Re}\,\Sigma^R(E,\mathbf{p})$ as $\Delta(E,\mathbf{p})$; Σ^R is the retarded self-energy operator.

$$f(E,\mathbf{p}) = n(E)\delta_\gamma(E,\mathbf{p}) \tag{39}$$

$$\delta_\gamma(E,\mathbf{p}) = \frac{1}{\pi}\,\frac{\gamma(E,\mathbf{p})}{[E - \varepsilon_p - \Delta(E,\mathbf{p})]^2 + \gamma^2(E,\mathbf{p})} \tag{40}$$

The power-law tails in the distribution function over momentum appears by reason of γ is not equal to zero. The particular form of γ and Δ is defined by a nature of the interparticle interactions.

As it was shown in Ref. [8], within the described model an equation for γ can be written as

$$\gamma(E,\mathbf{p}) = \frac{\pi n}{\hbar^4}\int |U(\mathbf{p} - \mathbf{p_1})|^2\delta_\gamma(E,\mathbf{p_1})\,\frac{\mathrm{d}^3 p_1}{(2\pi)^3} \tag{41}$$

In case of Δ is not neglected Eq. (41) should be solved consistently with [6]

$$\Delta(E,\mathbf{p}) = \mathbf{P}\int \frac{\gamma(x,\mathbf{p})}{x - E}\,\mathrm{d}x + \Sigma^{HF}(\omega,\mathbf{p}) \tag{42}$$

The principal value of the integral is denoted by means of $\mathbf{P}$.

From (38) under the approximating assumption that δ_γ can be replaced by the delta-function in (41) it is possible to obtain (28), (29). The approximation $\gamma(E,\mathbf{p}) \sim \hbar n\sigma_t(\mathbf{p})\sqrt{2E/m}$ is more correct than $\gamma \sim \hbar n\sigma_m(\mathbf{p})v$ used in Refs. [2,7,9,10]. Here σ_t is the total scattering cross section, σ_m is the momentum transfer cross section.

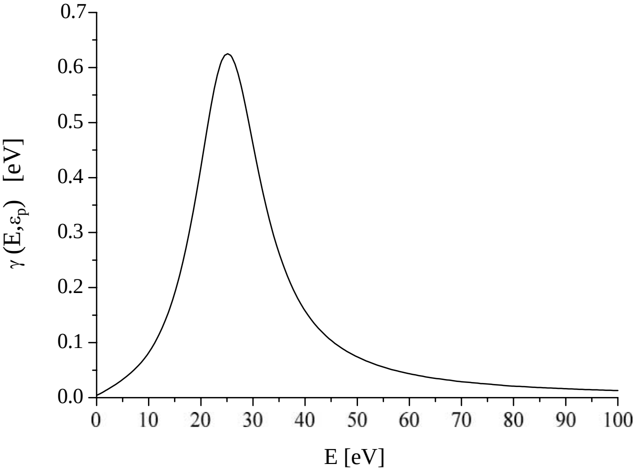

Fig. 1. The spectral width $\gamma(E,\varepsilon_p)$ at the fixed $\varepsilon_p = 25$ eV.

In Ref. [8], Eq. (41) was solved numerically for the Debye interaction potential. Fig. 1 displays the solution of (41) $\gamma(E,\varepsilon_p)$ for electrons interacting with heavy impurities in media of density $n = 10^{21}$ cm^{-3} and temperature $T = 10^4$ K at the fixed value of momentum $\varepsilon_p = 25$ eV.

This solution was used to calculate the thermonuclear fusion rates. As it was shown in Ref. [2] the generalized expression for the threshold process rate k_{ij} is given by

$$k_{ij} = A \int_0^{\infty} \int_0^{\infty} \int_{-\infty}^{+\infty} \mathrm{d}E\, \mathrm{d}\mathbf{p}\, \mathrm{d}\mathbf{p}' |f_{ij}(\mathbf{p},\mathbf{p}')|^2 \delta_\gamma(E,\mathbf{p}) n(E,\mathbf{p}) \times \delta_\gamma(E \pm I, \mathbf{p}')[1 - n(E \pm I, \mathbf{p}')] \,. \tag{43}$$

Here A is the normalization factor which can be found from the condition that (43) tends to its classical value at $\gamma \to 0$, $n(E,\mathbf{p})$ is the generalized population numbers [7], "+" and "−" correspond to release and absorption of energy I respectively, f_{ij} is the scattering amplitude of the process $i \to j$ outside of the mass shell. For the Sun interior the approximate ratio of the "quantum" and classical rates of fusion reaction between species i and j is

$$\frac{k_{ij}^{quantum}}{k_{ij}^{classical}} = \frac{2 \times \sqrt{2} \times 3^{19/2} \times 5!}{\pi^{7/6} \times 2^{17/3}} \frac{\hbar N_A e^4 \rho}{\sqrt{m_p}} \sum_l \frac{X_l Z_i^2 Z_l^2}{A_l A_{il}^{1/2}} \left(\frac{A_{il}}{A_{ij}}\right)^2 \frac{\mathrm{e}^{\tau_{ij}}}{\tau_{ij}^9} \frac{1}{T^{7/3}} \,, \tag{44}$$

$$\tau_{ij} = 3\left(\frac{\pi}{2}\right)^{2/3} \left(\frac{E_G^{ij}}{T}\right)^{1/3} \,, \tag{45}$$

$$E_G^{ij} = 4\frac{m_p}{m_e} R_y Z_i^2 Z_j^2 A_{ij} = 10^2 Z_i^2 Z_j^2 A_{ij}(\mathrm{keV}) \,. \tag{46}$$

Here ρ is the total density, $A_{ij}=A_iA_j/(A_i+A_j)$, X_l is the mass fraction of the background species, Z_i is the state of species j, m_p and m_e are the electron and the proton masses, respectively. These results are more correct in comparison with those obtained in Ref. [10,11] with help of simplified expression for γ.

In Ref. [8] the distribution function of electrons in a strong laser field was calculated with help of such crude estimate of γ. Provided that γ depends not only on momentum p but also on E and for the Debye potential its asymptotic form at large values of p is $\gamma(E,\mathbf{p})\sim\sqrt{E}/\varepsilon_p^2$ we can obtain the tail $f(\mathbf{p})\sim p^{-6.5}$. Really, as it was stayed in Ref. [9] the generalized populations numbers of electrons in a strong laser field $E=E_L\exp(-\mathrm{i}\omega_L t)$ are $n(E,\mathbf{p})\sim\exp\left(-\frac{m\omega_L^2E^2}{e^2E_L^2\varepsilon_p}\,p^q\right)$. So the tail is

$$f(\mathbf{p})\sim\int\frac{n(E,\mathbf{p})\gamma(E,\mathbf{p})}{\varepsilon_p^2}\,\mathrm{d}E\sim\int\exp\left(-\frac{m\omega_L^2E^2}{e^2E_L^2\varepsilon_p}\,p^q\right)\frac{\sqrt{E}}{\varepsilon_p^4}\,\mathrm{d}E$$
$$\sim p^{(-26+3q)/4}\,, \qquad (47)$$

where q is measure of collisions inelasticity, $v_u/v_m\sim p^q$, v_u,nu_m are the energy and momentum exchange collisions frequencies [8].

7. Conclusions

The simple Lorentz gas model is proposed to calculate the quantum distribution function over momentum in dense media. The asymptotic form of the distribution function at large value of momenta is expressed via the scattering amplitude or Fourier transform of the scattered over the single center particle wave function. A non-linear integral equation for the width of the generalized distribution function over “energy” and momentum Lorentz profile is presented and simple analytic approximations for $\gamma(E,\mathbf{p})$ are derived. The influence of power-law tails in the distribution function over momentum on the reaction rates is presented for the nuclear fusion rates in the Sun interior.

Acknowledgements

This work was partially supported by the Russian Foundation for Basic Research (RFBR) (99-0218176, 00-15-96539).

References

[1] V.M. Galitskii, V.V. Yakimets, Zh. Eksp. Teor. Fiz. 51 (1966) 957 (Sov. Phys. JETP 24 (1967) 637).
[2] N.L. Aleksandrov, A.N. Starostin, J. Exp. Theor. Phys. 86 (1998) 903 (Zh. Eksp. Teor. Fiz. 113 (1998) 1661).
[3] E. Wigner, Phys. Rev. 40 (1932) 749.
[4] A.A. Abrikosov, L.P. Gor’kov, I.E. Dzyaloshinskii, Methods of Quantum Field Theory in Statistical Physics, Prentice-Hall, Englewood Cliffs, New York, 1963.

[5] L.L. Foldy, Phys. Rev. 67 (1945) 107.
[6] G. Baym, L. Kadanoff, Quantum Statistical Mechanics. Green's Function Methods in Equilibrium and Nonequilibrium Problems, Benjamin, New York, 1962.
[7] A.N. Starostin, N.L. Aleksandrov, Phys. Plasmas 5 (1998) 2127.
[8] A.N. Starostin, N.L. Aleksandrov, A.B. Mironov, M.V. Schipka, Contrib. Plasma Phys. 41 (2001) 299.
[9] A.N. Starostin, N.L. Aleksandrov, A.M. Konchakov, A.M. Okhrimovskyy, M.V. Shchipka, Contrib. Plasma Phys. 39 (1999) 93.
[10] A.N. Starostin, V.I. Savchenko, N.J. Fisch, Phys. Lett. A 274 (2000) 64.
[11] V.M. Galitskii, A.B. Migdal, Zh. Eksp. Teor. Fiz. 34 (1958) 139.