Resistive instabilities in Hall current plasma discharge

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Plasma perturbations in the acceleration channel of a Hall thruster are found to be unstable in the presence of collisions. Both electrostatic lower-hybrid waves and electromagnetic Alfvén waves transverse to the applied electric and magnetic field are found to be unstable due to collisions in the $E\times B$ electron flow. These results are obtained assuming a two-fluid hydrodynamic model in slab geometry. The characterisitic frequencies of these modes are consistent with experimental observations in Hall current plasma thrusters.
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I. INTRODUCTION

Hall thrusters are now being tested and seriously considered for propelling the next generation of spacecraft. One of the important issues for the successful operation of the Hall current plasma thrusters is the presence of plasma oscillations. This issue has been recognized since the earliest investigations (see, for example, Ref. 1). Apart from their interactions with the power processing circuits, these oscillations play an important role in controlling the transport, conduction and mobility in Hall thrusters, directly affecting the thruster performance.

Careful consideration must also be given to possible interactions with the satellite electromagnetic systems used for communication and navigation. The possible influence of thruster plumes on communication signal propagation is of considerable interest.[2] Nonetheless, the electromagnetic interference, produced by the thruster during normal operation, is rarely studied and its driving mechanisms are not sufficiently understood.

For example, the model of hydrodynamic plasma instabilities, usually considered in the literature,[3] despite its general agreement with the experimental data,[4] does not explain all of the observations. Azimuthally propagating waves in the acceleration zone of the thruster are observed[5] even when the instability condition given in Ref. 3 is not satisfied.

In this paper we study two-dimensional plasma perturbations in a Hall current plasma thruster using two-fluid hydrodynamic theory. Instead of looking at the gradient-driven instabilities, we identify and consider the destabilizing effects associated with electron collisions. Despite the widely recognized importance of electron collisions for thruster operation, most of the models for plasma oscillations do not include collisional terms. Yet, we do find oscillations characteristic of Hall thrusters to be destabilized precisely by electron collisions.

We consider also electromagnetic waves. Perturbations of an electromagnetic nature, namely Alfvén waves, were not considered in Ref. 3. The characteristic plasma frequency and velocity scales found will then apply only for a small fraction of possible Hall thruster configurations and operation regimes. The limitations of the electrostatic-only approach were pointed out in Ref. 6; however, the stability analysis of Alfvén waves cannot be found in the available literature.

The resistive instability discussed in this paper is in fact a special case of the more general process wherein an instability is driven by coupling to a dissipative process. Such effects have been well studied in the literature, such as plasma flow in the presence of a resistive wall.[7] In the present case, although the driving mechanism for the instability is different from the resistive-wall instability,[8] it is clear that the instability will occur only due to the interaction of the wave with the electron $E\times B$ flow in the presence of electron collisions.

This paper is organized as follows: In Sec. II, we write the model equations. In Sec. III, we derive the dispersion relation for unstable electrostatic modes, namely lower-hybrid waves. In Sec. III, the growth rate for unstable electromagnetic modes is found. Section IV offers a dicussion and summary of our results.

II. MODEL EQUATIONS

Let us consider Hall thruster two-component plasma, consisting of ions and electrons immersed in the magnetic field B_0, such that on the scale of the device electrons are magnetized, while ions are unmagnetized:

$$\rho_e \ll L \ll \rho_i . \tag{1}$$

In order to simlify the problem, we neglect the variations of electric field and plasma density in the radial direction. We also neglect the axial component of the magnetic field. Such a purely radial magnetic field in the thruster channel will diverge with its magnitude being inversely proportional to the radius. Therefore the drift velocity of the electrons will be proportional to the radial position in the channel. However, the angular velocity of the electrons around the channel will be constant. Therefore there will be no shear in the electron flow. Such characteristic of the electron flow is usually called the isodrift flow. Thus we can simplify the problem to purely two-dimesional by neglecting the channel curvature and considering slab geometry. For simplicity in the following analysis, we also neglect all plasma inhomogeneities, assuming variations of both density and magnetic field along the channel to be small.

The ion motion is governed by the following set of fluid equations:

$$\frac{\partial N_i}{\partial t}+\nabla(\vec{v}_i N_i)=0, \tag{2}$$

$$\frac{\partial \vec{v}_i}{\partial t}+(\vec{v}_i\cdot\nabla)\vec{v}_i=\frac{e}{M}\vec{E}. \tag{3}$$

The zeroth order solution is the axial flow of unmagnetized ions being accelerated by the electric field in the channel according to

$$v_0\frac{dv_0}{dx}=\frac{e}{M}E_0. \tag{4}$$

The electron motion is accordingly governed by the set of continuity and momentum equations:

$$\frac{\partial N_e}{\partial t}+\nabla(\vec{v}_e N_e)=0, \tag{5}$$

$$\frac{\partial \vec{v}_e}{\partial t}+(\vec{v}_e\cdot\nabla)\vec{v}_e=-\frac{e}{m}\left(\vec{E}+\frac{1}{c}\vec{v}_e\times\vec{B}\right)-\nu_e\vec{v}_e. \tag{6}$$

In the zeroth order, assuming $\nu_e \ll \Omega_e$ the electrons move in the $\hat{y}$ direction with the drift velocity,

$$u_0=-c\frac{E_0}{B_0}. \tag{7}$$

The linearized system for the small perturbations of ion and electron densities and velocities can then be written as follows:

$$\frac{\partial n_i}{\partial t}+\vec{v_0}\frac{\partial n_i}{\partial x}+n_0\nabla\cdot\vec{v}_i=0, \tag{8}$$

$$\frac{\partial \vec{v}_i}{\partial t}+v_0\frac{\partial \vec{v}_i}{\partial x}=\frac{e}{M}\vec{E}_1, \tag{9}$$

$$\frac{\partial n_e}{\partial t}+\vec{u_0}\frac{\partial n_e}{\partial y}+n_0(\nabla\cdot\vec{v}_e)=0, \tag{10}$$

$$\frac{\partial \vec{v}_e}{\partial t}+u_0\frac{\partial \vec{v}_e}{\partial y}=-\frac{e}{m}\left(\vec{E}_1+\frac{1}{c}\vec{v}_e\times\vec{B_0}+\frac{1}{c}\vec{u_0}\times\vec{B}\right)-\nu_e\vec{v}_e. \tag{11}$$

III. ELECTROSTATIC LOWER-HYBRID WAVES

Let us now consider electrostatic perturbations in the presence of electron collisions. Then the perturbation of electric field is determined by the perturbation of the potential $\vec{E}_1=-\nabla\phi$. Considering oscillatory perturbations $A \propto A_0\exp(i\omega t-i\vec{k}\vec{r})$, the fluid equations for the two species will yield the following solutions for density perturbations:

$$n_i=\frac{k^2 e N_{i0}}{M(\omega-k_x v_0)^2}\phi, \tag{12}$$

$$n_e=\frac{k^2 e N_{e0}}{m\Omega_e^2}\phi-i\frac{eN_0\nu_e k^2}{m(\omega-k_y u_0)\Omega_e^2}\phi. \tag{13}$$

We have introduced here gyrofrequencies and plasma frequencies for ions and electrons $\Omega_{i,e}=eB_0/m_{i,e}c$ and $\omega_{i,e}^2=4\pi e^2 N_{e0}/m_{i,e}$ respectively. Also we assume

$$\Omega_i \ll \omega \ll \Omega_e, \tag{14}$$

which is mostly valid for the oscillations, observed in Hall thrusters, where ion and electron gyrofrequencies are typically of the order of 10^4 and 10^9 Hz, respectively.

Now we can substitute the obtained density perturbations into Poisson's equation

$$-k^2\phi=4\pi e(n_e-n_i), \tag{15}$$

which will yield the following dispersion relation:

$$1-\frac{\omega_i^2}{(\omega-k_x v_0)^2}+\frac{\omega_e^2}{\Omega_e^2}-\frac{\omega_e^2}{\Omega_e^2}\frac{i\nu_e}{(\omega-k_y u_0)}=0. \tag{16}$$

This is the dispersion relation for the lower-hybrid waves, modified to include collisions of the rotating electrons.

Consider now waves propagating along the $\hat{y}$ direction, so that $k_x=0$, which, in real thruster geometry, corresponds to azimuthally propagating waves. Under the assumption

$$\omega \ll |k_y u_0|, \tag{17}$$

which will be discussed later, the solutions for the dispersion relation (16) can be obtained as follows:

$$\omega^2=\frac{\omega_i}{1+\dfrac{\omega_e^2}{\Omega_e^2}+\dfrac{i\nu_e\omega_e^2}{k_y u_0\Omega_e^2}}, \tag{18}$$

where the last term in the denominator in the right-hand side (RHS) is small, therefore the final solution is

$$\omega \approx \pm\omega_{LH}\left(1-\frac{i\nu_e\omega_e^2}{2k_y u_0(\omega_e^2+\Omega_e^2)}\right). \tag{19}$$

Here the lower-hybrid frequency is defined as

$$\omega_{LH}^2=\frac{\omega_i^2\Omega_e^2}{\omega_e^2+\Omega_e^2}. \tag{20}$$

We have two modes, one of which will be unstable with the growth rate

$$\gamma=\frac{\nu_e}{2k_y u_0}\frac{\omega_e^2}{(\omega_e^2+\Omega_e^2)}\omega_{LH}\approx\pm\omega_{LH}\frac{\nu_e}{2k_y u_0}. \tag{21}$$

Let us return to the assumption (17). In a typical Hall plasma thruster with the maximum applied magnetic field 200 Gs, thruster channel diameter 10 cm and characteristic plasma density $10^{12}\,\mathrm{cm}^{-3}$, the lower-hybrid frequency $\omega_{LH}\sim 10^7$ Hz can indeed be neglected compared to $k_y u_0$, which even for the smallest values of k_y allowed by azimuthal symmetry will be at least order of magnitude larger. At the same time, a similar order of magnitude estimate for the increment of this instability, using effective electron collision frequency $\nu_e \sim 10^6\ \mathrm{s}^{-1}$, yields $\gamma \sim 10^6\ \mathrm{s}^{-1}$.

IV. ELECTROMAGNETIC WAVES

For electromagnetic waves, it is no longer valid to represent the perturbations as functions of the electric potential only. The density and velocity perturbation can, however, be related to the electric field. The ion equations then remain almost unchanged:

$$v_{ix} = i \frac{1}{(\omega - k_x v_0)} \frac{e}{M} E_x, \tag{22}$$

$$v_{iy} = i \frac{1}{(\omega - k_x v_0)} \frac{e}{M} E_y, \tag{23}$$

$$n_i = \frac{eN_0}{M(\omega - k_x v_0)^2}(k_x E_x + k_y E_y). \tag{24}$$

While for the electrons from Eqs. (10) and (11), we obtain

$$v_{ex} = \frac{1}{\Omega_e} \frac{e}{m} E_y - \frac{i(\omega - k_y u_0 - i\nu_e)}{\Omega_e^2} \frac{e}{m} E_x, \tag{25}$$

$$v_{ey} = -\frac{1}{\Omega_e} \frac{e}{m} E_x - \frac{i(\omega - k_y u_0 - i\nu_e)}{\Omega_e^2} \frac{e}{m} E_y + u_0 \frac{B}{B_0}, \tag{26}$$

$$n_e = \frac{i(\omega - k_y u_0 - i\nu_e)}{\Omega_e^2(\omega - k_y u_0)} \frac{eN_0}{m}(k_x E_x + k_y E_y) - \frac{eN_0}{m\Omega_e \omega}(k_x E_y - k_y E_x). \tag{27}$$

We have used here the Maxwell's equation relating perturbations of electric and magnetic field, which in our case can be written as

$$-i\omega \vec{B} = -c\hat{z}(k_x E_y - k_y E_x). \tag{28}$$

The perturbation of the current $\vec{j}$ can be written as

$$j_x = e[n_i v_0 + N_0(v_{ix} - v_{ex})], \tag{29}$$

$$j_y = e[-n_e u_0 + N_0(v_{iy} - v_{ey})]. \tag{30}$$

The plasma dielectric tensor ϵ_{ik} can then be written as

$$D_i = \epsilon_{ik} E_k = E_k \delta_{ik} + \frac{4\pi}{i\omega} j_i(E_k). \tag{31}$$

The components of the dielectric tensor can then be found to be

$$\epsilon_{xx} = 1 + \frac{(\omega - k_y u_0)\omega_e^2}{\omega \Omega_e^2} - \frac{\omega_i^2}{(\omega - k_x v_0)^2}, \tag{32}$$

$$\epsilon_{xy} = -i \frac{\omega_e^2}{\omega \Omega_e} - \frac{\omega_i^2 k_y v_0}{\omega(\omega - k_x v_0)^2}, \tag{33}$$

$$\epsilon_{yy} = 1 - \frac{\omega_i^2}{\omega(\omega - k_x v_0)} + \frac{\omega_e^2}{\Omega_e^2} + \frac{\omega_e^2}{\Omega_e^2} \frac{i\nu_e}{(\omega - k_y u_0)}, \tag{34}$$

$$\epsilon_{yx} = i \frac{\omega_e^2}{\Omega_e \omega} + \frac{\omega_e^2 k_x u_0}{\omega \Omega_e^2} + \frac{\omega_e^2 k_x u_0}{\omega \Omega_e^2} \frac{i\nu_e}{(\omega - k_y u_0)}. \tag{35}$$

The wave equation

$$\left(k^2 \delta_{ij} - k_i k_j - \frac{\omega^2}{c^2} \epsilon_{ij}\right) E_j = 0 \tag{36}$$

yields the general notation for the dispersion relation, which can be written as

$$k_x^2 \epsilon_{xx} + k_y k_x(\epsilon_{xy} + \epsilon_{yx}) + k_y^2 \epsilon_{yy} + \frac{\omega^2}{c^2}(\epsilon_{xx}\epsilon_{yy} - \epsilon_{xy}\epsilon_{yx}) = 0. \tag{37}$$

If we once again consider only ''azimuthal'' propagation $k_x = 0$, $k = k_y$, we have

$$\frac{k^2 c^2}{\omega^2} = \epsilon_{xx} - \frac{\epsilon_{xy}\epsilon_{yx}}{\epsilon_{yy}}. \tag{38}$$

We now substitute the notations for plasma dielectric tensor (32)–(35) to obtain the dispersion relation

$$\frac{k^2 v_A^2}{\omega^2} \frac{\omega_i^2}{\Omega_i^2} = 1 - \frac{\omega_i^2}{\omega^2} + \frac{\omega_i^2}{\omega_{LH}^2} \frac{(\omega - k u_0 - i\nu_e)}{\omega} - \frac{\omega_i^4}{\Omega_i^2} \frac{\omega_{LH}^2}{(\omega^2 - \omega_i^2)\omega_{LH}^2 + \omega^2 \omega_i^2 \left(1 - \frac{i\nu_e}{\omega - k u_0}\right)}. \tag{39}$$

Here we have introduced the Alfvén velocity

$$v_A^2 = \frac{B_0^2}{4\pi N_0 M}, \tag{40}$$

and simplified (20) to

$$\omega_{LH}^2 = \frac{\omega_i^2 \Omega_e^2}{\omega_e^2 + \Omega_e^2} \simeq \Omega_i \Omega_e. \tag{41}$$

We seek the solution of (39) in the form of unstable wave with the frequency $\omega = \omega_r - i\gamma$, assuming $\gamma \ll \omega_r$. After some calculations using the estimate $\Omega_i \ll \omega, \omega_i, \omega_{LH}$, we obtain the solution as the Alfvén wave

$$\omega_r \approx k v_A, \tag{42}$$

which will be unstable with the growth rate

$$\gamma = \frac{\nu_e}{2} \frac{v_A}{u_0} \frac{\omega_r^2}{\omega_{LH}^2}. \tag{43}$$

For the typical value of Alfvén velocity in Hall thrusters $v_A \sim 5 \times 10^6$ cm/s, the frequency of these waves will be $\omega_r \sim 10^6$ Hz and the growth rate $\gamma \sim 10^4\ \mathrm{s}^{-1}$.

V. DISCUSSION AND CONCLUSIONS

We obtained linearly unstable solutions for azimuthally propagating waves in Hall thrusters. The purely azimuthal electrostatic lower-hybrid wave is shown to be driven unstable due to the resistive coupling to the electron drift flow, rather then, as commonly thought, driven unstable only by density and magnetic field gradients. The Alfvén wave is also shown to be unstable due to the electron collisions.

For typical Hall thruster parameters, the electrostatic instability will occur for azimuthally propagating lower-hybrid waves with frequencies $\sim 10^7$ Hz and the electromagnetic instability for Alfvén waves with frequencies $\sim 10^6$ Hz. While the growth rate for Alfvén waves are much smaller, on the time scales associated with the steady state thruster operation, both of these modes can become quite significant. It would be the nonlinear saturation mechanism, not considered here, rather than the linear growth rate, that would determine the relative importance of these mechanisms.

We also note, that for the obtained frequency ranges and modes, the hydrodynamic description is quite appropriate. There is no collisionless damping or excitation in the direction transverse to the applied magnetic field, and for the direction parallel to the magnetic field kinetic effects could be neglected if $\omega \gg k_z v_{Te}$. This condition is easily satisfied for the considered waves and thruster operation parameters.

Plasma oscillations in these frequency ranges were observed in multiple experimental studies of Hall thrusters.[9] It remains, however, to distinguish experimentally the resistive mechanism of instability of the modes considered here from the mechanism of instability due to field or density gradients. Observations of the instability in regions clearly gradient free would support the theories advanced here.

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