Existence of core localized toroidicity-induced Alfvén eigenmode

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The core-localized toroidicity-induced Alfvén eigenmode (TAE) is shown to exist at finite plasma pressure due to finite aspect ratio effects in tokamak plasma. The new critical beta for the existence of the TAE mode is given by $\alpha \approx 3\epsilon + 2s^2$, where $\epsilon = r/R$ is the inverse aspect ratio, s is the magnetic shear and $\alpha = -Rq^2 d\beta/dr$ is the normalized pressure gradient. In contrast, previous critical α is given by $\alpha \approx s^2$. In the limit of $s \ll \sqrt{r/R}$, the new critical α is greatly enhanced by the finite aspect ratio effects. © *1995 American Institute of Physics.*

The toroidicity-induced Alfvén eigenmode (TAE)[1,2] exists in a tokamak plasma due to toroidal mode coupling and finite magnetic shear. The mode frequency is located inside a continuum gap induced by toroidicity. Recently, it has been shown[3–5] that there is a critical pressure gradient above which the TAE mode no longer exists. In the limit of small shear, the critical value is given by $\alpha = \alpha_c = s^2/(1+s)$, where $s = rdq/qdr$ is the magnetic shear, $\alpha = -Rq^2 d\beta/dr$ represents the product of magnetic field curvature and plasma pressure gradient with R being the major radius, r being the plasma radius, q being the safety factor and β being the plasma toroidal beta. This result implies that the TAE mode should not exist in the core of tokamak plasma where the values of α and s are comparable and small for typical parameters. In particular, $\alpha/s \gg 1$ asymptotically when r goes to zero. However, recent numerical results[6] indicate that a TAE mode, which is localized at a single gap, does exist in the core of the tokamak plasma, where $\epsilon = r/R \sim 0.1$ and $s \sim \alpha \sim \epsilon$. We call this type of TAE mode the core-localized TAE mode. These core-localized modes are particularly susceptible to destabilization by fusion alpha particles in a tokamak plasma since the density profile of the alpha particles is sharply peaked at the center of plasma.[6]

In this work, we will show that the effects of finite aspect ratio change the critical α qualitatively using the high-n ballooning mode equation. The previous results,[3,4] based on the standard $s-\alpha$ model ballooning mode equation,[7] is only valid in the limit of $\epsilon \ll s^2$. We will show that our new critical α reduces to $\alpha_c = 3\epsilon$ in the limit of $s \ll \sqrt{\epsilon} \ll 1$. In the following, we will derive an analytic expression for critical α by taking into account the effects of finite aspect ratio.

We start with the following ballooning mode equation:

$$\frac{\partial}{\partial\theta}[G_1(\theta)+G_2(\theta)h^2(\theta)]\frac{\partial}{\partial\theta}\Phi + \Omega^2[G_1(\theta)+G_2(\theta)h^2(\theta)] \times (1+4\epsilon\cos\theta)\Phi + \alpha[\cos\theta + h(\theta)\sin\theta]\Phi = 0 \tag{1}$$

where $h(\theta) = s\theta - \bar{\alpha}\sin\theta$, $G_1(\theta) = 1 - 2(\epsilon + \Delta')\cos\theta$, $G_2(\theta) = 1 + 2\Delta'\cos\theta$, $\bar{\alpha} = \alpha + 2\epsilon - 2(1-s)\Delta'$, $\Delta' = \epsilon(1/4 + \beta_p)$ is the radial derivative of the Shafranov shift, and Ω is the mode frequency normalized to the Alfvén frequency. The poloidal beta is defined as $\beta_p = 8\pi(\langle p\rangle - p)/B_p^2$ where $\langle\ \rangle$ denotes a volume average.

Equation (1) is derived for a large aspect ratio, low-beta tokamak plasma by using the Shafranov shifted circle flux coordinates.[8] We note that the previous work[3,4] neglects the ϵ term in $G_1(\theta)$ and $G_2(\theta)$ and assumes $\alpha \gg \epsilon$. In this limit, the Eq. (1) reduces to the standard $s-\alpha$ model ballooning equation:[7]

$$\frac{\partial}{\partial\theta}[1+h^2(\theta)]\frac{\partial}{\partial\theta}\Phi + \Omega^2[1+h^2(\theta)](1+4\epsilon\cos\theta)\Phi + \alpha[\cos\theta + h(\theta)\sin\theta]\Phi = 0. \tag{2}$$

Comparing Eqs. (1) and (2), the new terms in Eq. (1), of order $O(\epsilon)$, come from the intrinsic toroidicity and pressure-induced Shafranov shift. We will show below that these ϵ terms in G_1 and G_2 can not be neglected when ϵ is comparable or larger than s^2.

To make analytic progress, we make the following transformation $\Phi \to (1/\sqrt{F})\Phi$ with $F = G_1(\theta) + G_2(\theta)h^2(\theta)$, Eq. (1) then becomes

$$\frac{\partial^2}{\partial\theta^2}\Phi + \left[\Omega^2(1+4\epsilon\cos\theta) - \frac{F''}{2F} + \frac{(F')^2}{4F^2} + \frac{\alpha(\cos(\theta) + h(\theta)\sin\theta)}{F}\right]\Phi = 0 \tag{3}$$

where the prime denotes the derivative with respect to θ. Assuming $\epsilon \ll 1$ and $s \sim \alpha \sim O(\epsilon)$, we expand Eq. (3) to the second order of ϵ. The resulting equation is given by

$$\frac{\partial^2}{\partial\theta^2}\Phi + \Omega^2(1+4\hat{\epsilon}\cos\theta)\Phi + \left[\frac{H_1(\theta)}{1+s^2\theta^2} - \frac{H_2(\theta)}{(1+s^2\theta^2)^2}\right]\Phi = 0 \tag{4}$$

where the functions H_1 and H_2 are defined as

$$H_1(\theta) = (\alpha - \delta)\cos\theta - 2\alpha\Delta'\cos^2\theta + (2\delta\Delta' + \alpha\bar{\alpha} - \bar{\alpha}^2)\sin^2\theta, \tag{5}$$

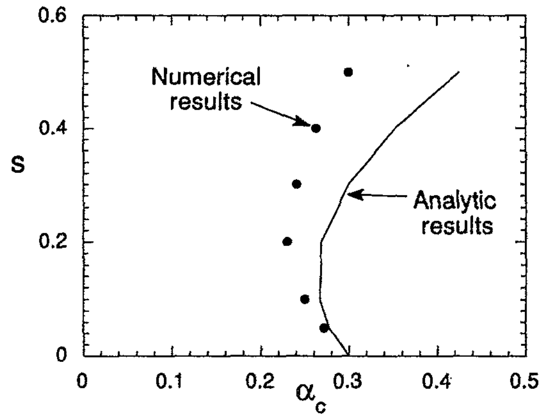

FIG. 1. The critical α value, α_c, as a function of shear s obtained with $\epsilon=0.1$ near the center of plasma.

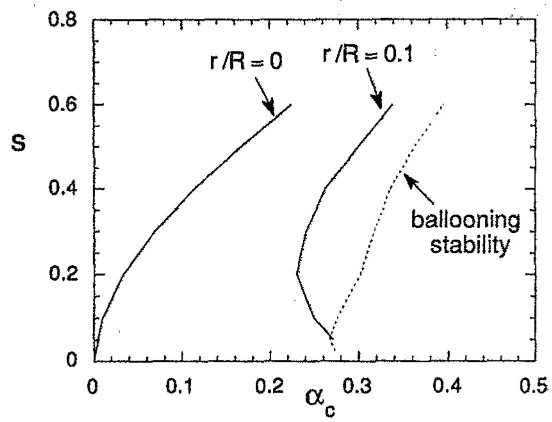

FIG. 2. The critical α for the TAE as a function of shear s from both the previous results (solid line marked with $r/R=0$) and the new results (solid line marked with $r/R=0.1$). The ballooning first stability boundary is also shown (dotted line).

$$H_2(\theta)=(s-\bar{\alpha}\cos\theta)^2+2\delta(\delta-\alpha)\cos^2\theta - (\delta^2+2\bar{\alpha}^2-2\alpha\bar{\alpha})\sin^2\theta \tag{6}$$

with $\hat{\epsilon}\approx\epsilon+\Delta'$ and $\delta=\epsilon+2\Delta'$.

Following Fu and Cheng,[3] a dispersion relation for the TAE mode frequency can be derived by solving Eq. (4) asymptotically. The result is given by

$$\sqrt{\frac{\Omega^2-\Omega_-^2}{\Omega_+^2-\Omega^2}}=\frac{\pi}{4s}(1+\epsilon+s)(\alpha_c-\alpha) \tag{7}$$

where Ω_- and Ω_+ are the lower bound and the upper bound of the Alfvén continuum gap respectively. The critical α for the existence of TAE mode, α_c, is given by

$$\alpha_c=\frac{(1+\epsilon)(\epsilon+2\Delta')+s^2-2(\epsilon-\Delta')s}{1+\epsilon+s}. \tag{8}$$

In the limit of $\epsilon\to 0$ and $\Delta'\to 0$, the critical value given in Eq. (8) reduces to the previous results.[3–5] It should be noted that Δ' is dependent on the poloidal beta which is proportional to α. This implicit dependence can be made explicit in the core of plasma where we can expand the plasma pressure near $r=0$. Thus, we have the following relations near the center of plasma for typical pressure profile of form $p\propto(1-r^2)^b$ with b being a constant: $\alpha=4\epsilon\beta_p$ and $\bar{\alpha}=\delta=3\epsilon/2+\alpha/2$. The critical α then reduces to

$$\alpha_c=\frac{3\epsilon(1+\epsilon)+2s^2-3\epsilon s}{1+\epsilon+s}. \tag{9}$$

It is instructive to note that in the limit of zero shear, the critical alpha is simply given by $\alpha_c=3\epsilon$ which corresponds to $\beta_{pc}=3/4$. Thus, the finite aspect ratio effects greatly enhance the critical beta for small shear. It is also interesting to note that in the limit of $\epsilon\to 0$, the critical α is a factor two larger than the previous results.[3–5] This factor of 2 comes from the pressure-induced Shafranov shift.

The critical α, given by Eq. (9), has been confirmed by solving Eq. (1) numerically. Figure 1 shows that the analytic results for the critical α (solid line) agree well with the numerical results (solid dots). For comparison, we have shown in Fig. 2 both the previous results (solid line marked with $r/R=0$) and our new results (solid line marked with $r/R=0.1$). As a reference, the ballooning mode first stability boundary is also shown (dotted line). We observe that the effects of finite aspect ratio makes the critical α much closer to the first stability boundary for small value of shear. (It was shown previously[3] that the critical α for TAE mode approaches the first stability boundary asymptotically for large values of shear.) Although our results are based on shifted-circle model equilibria, we conjecture that the critical α for TAE mode is also close to the ballooning first stability limit in general equilibria, and that the TAE can generally exist as long as the tokamak plasma is ballooning stable.

Finally, the analytic critical α in the limit of $s^2\ll\epsilon$, $\alpha_c=3\epsilon$, has been confirmed in both magnitude and scaling by numerical calculations using a global stability code NOVA.[6]

After this work was completed, we learned that another core localized mode also exists near the upper tip of the continuum gap.[9] The critical beta value for this mode is approximately given by $\alpha_c=3\epsilon-2s^2$. It should be pointed out that this mode exists purely due to the finite aspect ratio effects.

In conclusion, we have shown that the core-localized TAE mode exists at finite plasma pressure in tokamaks due to finite aspect ratio effects. The critical beta for the existence of TAE mode in the core region of plasma is given by $\beta_p=3/4$ in the limit of $s\ll\sqrt{r/R}\ll 1$.

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