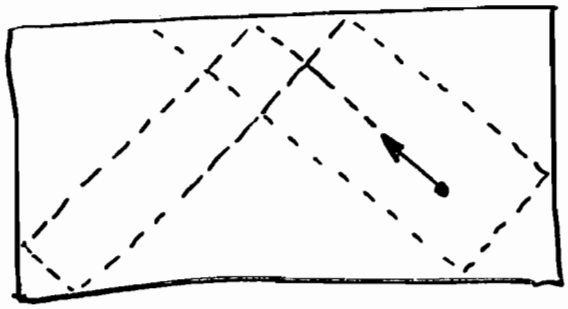
Many simple systems are chaotic

Frictionless billiard balls on a billiard table. [Macintosh program – John Cary Univ. of Colorado.]



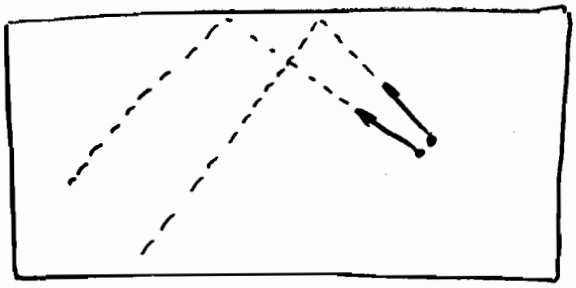
Rectangular table – regular motion

ball is restricted to traveling in one of 4 directions

Billiard chaos slides from
Charles Karney, PPPL (~1993)

Start two balls close to each other

(ignore collisions between balls)



separation of balls grows slowly

Two characteristics of regular motion

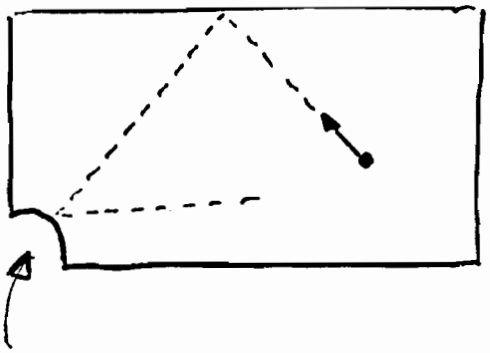
- motion is restricted

- errors grow slowly

  — prediction is possible

  (for a fairly long time)

Take a "bite" out of table



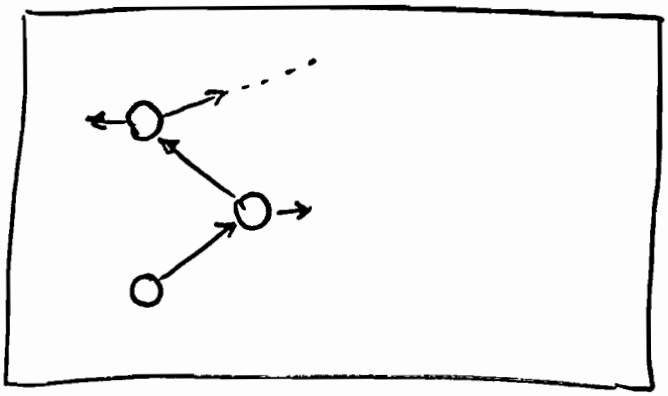
makes motion irregular

Two characteristics of irregular motion

- no restriction on motion —
  every point is visited at every angle
- errors grow quickly (exponentially)
  - errors (in initial placement or unevenness of surface) eventually swamp the motion
  - (long term) prediction is impossible

The growth of errors is very rapid

frictionless billiard table with several colliding balls

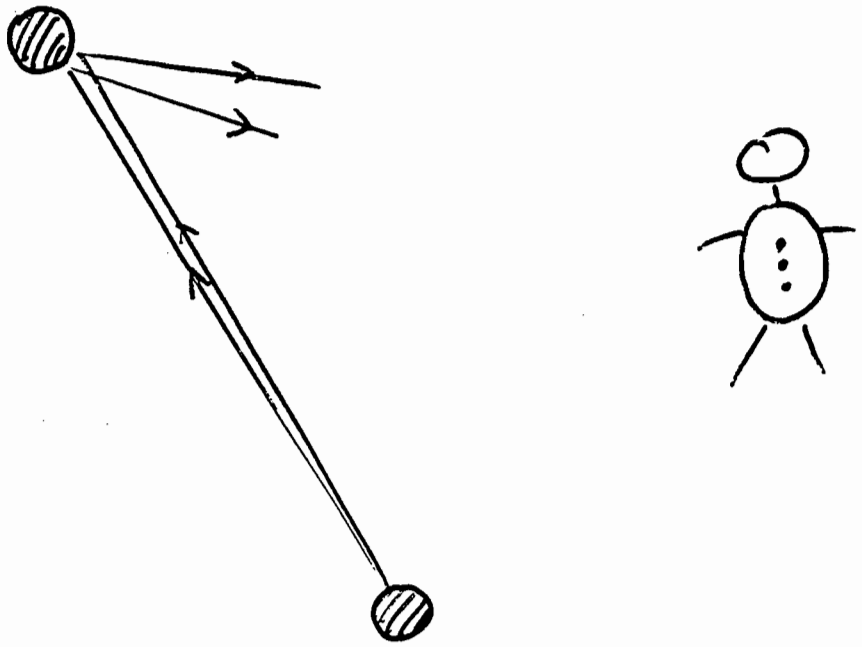


Knowing initial positions and speeds of balls, try to predict their positions after several collisions.

THIS IS IMPOSSIBLE!

(See M.V. Berry, in Topics in Nonlinear Dynamics, S. Jorna, Ed., (AIP, 1978)).

Suppose your friend enters the room to observe your experiment.
He will gravitationally attract the billiard balls



* [This will completely alter their motion after about 10 collisions.

The same effect prevents weather forecasts beyond about 1 week.

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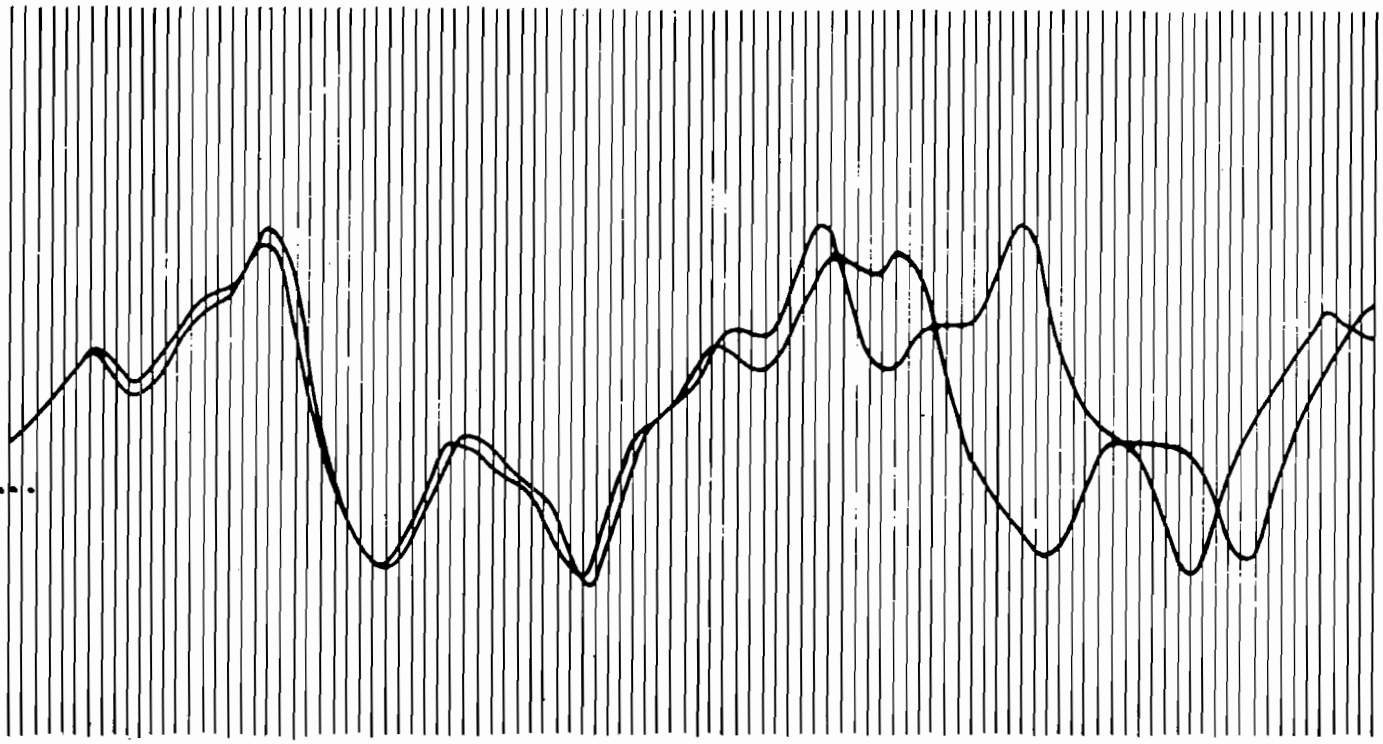
* Homework: Prove this! Hints: ① Gravitational Constant $G = 6.67 * 10^{-11}$ Newton$-m^2/kg^2$, ② Consider all balls fixed except for one, ③ Don't try to be precise, just get order-of-magnitude effects + scaling...

Consider a large collection of air molecules at room temperature and pressure. Can show that the gravitational force due to an electron located at the edge of the universe is enough to make the trajectories completely different after about ~60 collisions.

Exercise 5.1 in Statistical Mechanics summary chapter of

T. Padmanabhan, Theoretical Astrophysics, Vol. I: Astrophysical Processes.



HOW TWO WEATHER PATTERNS DIVERGE. From nearly the same starting point, Edward Lorenz saw his computer weather produce patterns that grew farther and farther apart until all resemblance disappeared. (From Lorenz's 1961 printouts.)

From Gleick's Chaos (1987)

practical purposes, the cycles would be predictable—and eventually uninteresting. To produce the rich repertoire of real earthly weather, the beautiful multiplicity of it, you could hardly wish for anything better than a Butterfly Effect.

The Butterfly Effect acquired a technical name: sensitive dependence on initial conditions. And sensitive dependence on initial conditions was not an altogether new notion. It had a place in folklore:

> *"For want of a nail, the shoe was lost;*
> *For want of a shoe, the horse was lost;*
> *For want of a horse, the rider was lost;*
> *For want of a rider, the battle was lost;*
> *For want of a battle, the kingdom was lost!"*

In science as in life, it is well known that a chain of events can have a point of crisis that could magnify small changes. But chaos meant that such points were everywhere. They were pervasive. In systems like the weather, sensitive dependence on initial conditions was an inescapable consequence of the way small scales intertwined with large.

GWH clarification: No matter how hard the butterflies flap their wings, they can't make it snow in NYC in July. What chaos does do is change the phase of events or give a different sampling, e.g. make it rain 6 times in July instead of 7, or make it rain on July 28 instead of July 30. A corrollary: although chaos means that one can not predict the precise weather at 3 p.m. exactly 3 months from now, one can try to predict the climate, the statistics of the weather.

Quote from George Herbert (1593–1633)

Quoted in J. Gleick, Chaos, Making a New Science, (Viking, New York, 1987)

J.C. Maxwell, Matters and dynamics (1877):
"There is a maxim in studying the physical phenomena, that is, 'Same phenomena result in same.' This is true in all cases.
And there is another maxim, 'Similar phenomena result in similar.' This is appropriate for many phenomena too, but not all."

James Clerk Maxwell (1831-1879)

Maxwell's Eqs - Unified electrical & magnetic forces
Maxwellian distribution - "bell-shaped curve" important in thermodynamics & theory of gases.
Maxwell was the first scientist to understand chaos *

"No one, I suppose, would assign to free will a more than infinitesimal range. No leopard can change his spots, nor can any one by merely wishing it, or, as some say, *willing* it, introduce discontinuity into his course of existence... In the course of this our mortal life we more or less frequently find ourselves in a *physical* or moral watershed, where an imperceptible deviation is sufficient to determine into which of two valleys we *shall* descend."

From an essay concerning the debate between determinism & free will, delivered at Cambridge Univ, 1873. Quoted by Hunt & Yorke, Nonlinear Science Today 3, p. 1 (1993).

"the existance of unstable conditions renders impossible the prediction of future events, if our knowledge of the present state is only approximate, & not accurate..."