A Simple Phase-Mixing Paradigm

Consider simple 1-D kinetic Eq. for $f(z,v_{||},t)$

(Carl Oberman reminded me of this view of Landau damping)

$$\frac{\partial f}{\partial t} + \mathrm{v}_{||}\frac{\partial f}{\partial z} = 0$$

(solutions of this simple equation are Green's functions for more complicated problems that could include E fields etc. on RHS)

Exact solution: $f\left(z, \mathrm{v}_{||}, t\right) = f_0\left(z - \mathrm{v}_{||}t, \mathrm{v}_{||}\right)$

Start w/ Maxwellian w/ spatial density perturbation:

$$f_0 = (1 + a\sin(k_{||}z))\, f_M(\mathrm{v}_{||}) = f_M(\mathrm{v}_{||}) + \underbrace{\mathrm{Im}[ae^{ik_{||}z} f_M(\mathrm{v}_{||})]}_{\delta f\,(t=0)}$$

$$\delta f \propto e^{ik_{||}(z - \mathrm{v}_{||}t)} e^{-\mathrm{v}_{||}^2/(2\mathrm{v}_t^2)}$$

At any fixed $\mathrm{v}_{||}$, f oscillates in time with $\omega = k\,\mathrm{v}_{||}$ and no damping. However, any v-integral of f will exponentially decay in time:

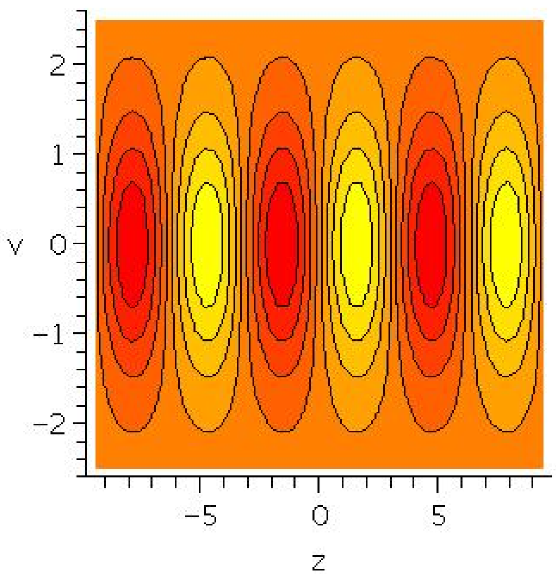
$$\delta n(z,t) = \int d\mathrm{v}_{||}\delta f \quad \propto \; e^{ik_{||}z} \underbrace{\int d\mathrm{v}_{||}}_{mixing} \underbrace{e^{-ik_{||}\mathrm{v}_{||}t}}_{phases} e^{-\mathrm{v}_{||}^2/(2\mathrm{v}_t^2)}$$

$$\boxed{\delta n(z,t) = ae^{ik_{||}z} e^{-k_{||}^2 \mathrm{v}_t^2 t^2/2}}$$

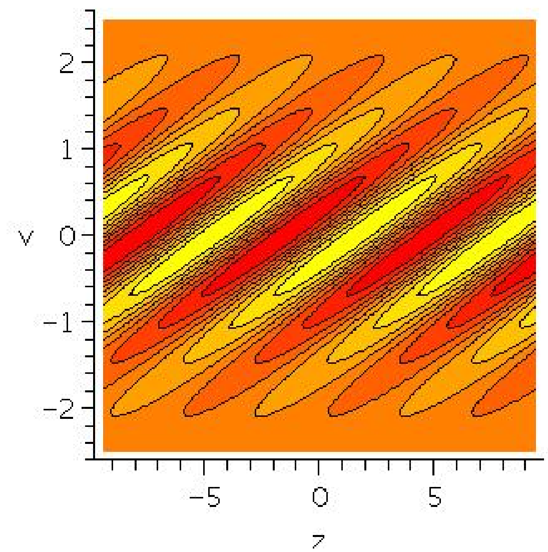
reversible Hamiltonian dynamics can appear irreversible if there is "coarse-graining"

Phase-mixing -> fluid moments of *f* decay in time

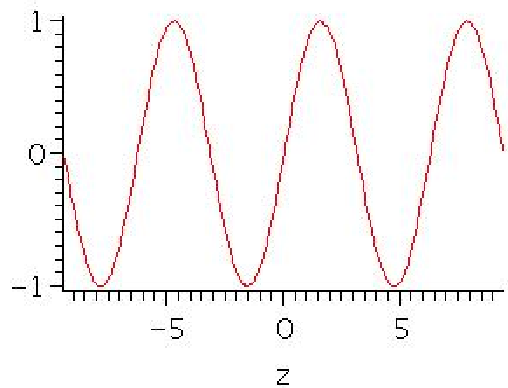
δf(z,v)
t=0



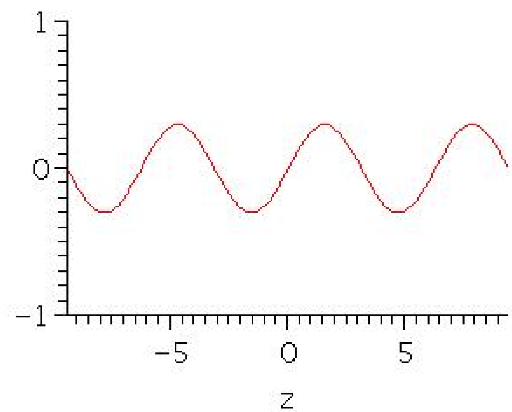
δf(z,v)
t=5



δn(z)
t=0



δn(z)
t=1.5

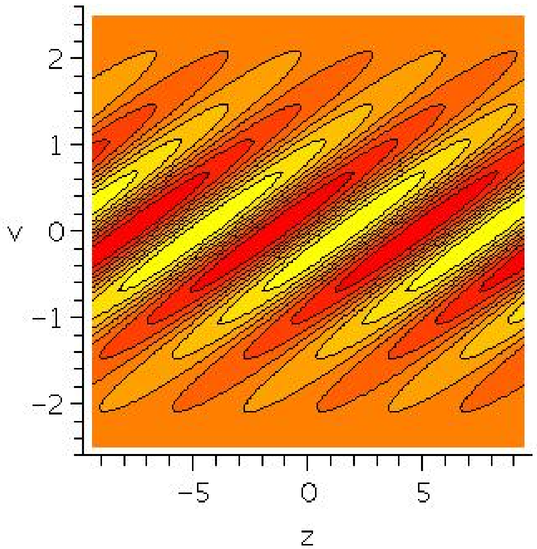


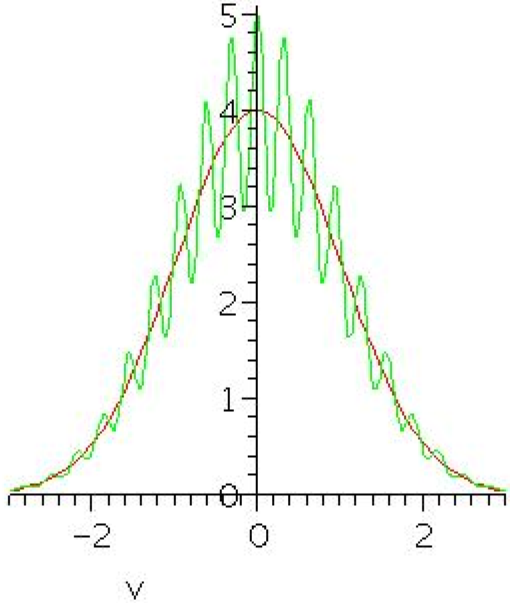
Fluid closure approximations need to introduce damping at rate ~ $|k_{||}|\ v_t$

$$\delta n(z,t) = a \sin(k_{||} z) e^{-k_{||}^2 v_t^2 t^2 / 2}$$

Phase-mixing -> very fine scales in velocity easily wiped out by a small amount of collisions

f(z,v)
t=5





At late times, $\delta f = exp(- i\, k_{||}\, v\, t\,)\, f_M(v)$ is very oscillatory in *v*

Collisions dominate at time $\tau \sim (\, 3\, /\, \nu\, v_t^2\, k_{||}^2)^{1/3}$

$$C(f) \approx \nu\, \mathrm{v}_t^2 \frac{\partial^2 f}{\partial \mathrm{v}^2} \approx -\nu\, \mathrm{v}_t^2 k_{||}^2 t^2 f$$

Full resolution in velocity requires:

$$\Delta \mathrm{v}_{||} / \mathrm{v}_t \sim \left(\nu / 3 k_{||} \mathrm{v}_t\right)^{1/3}$$

$$\sim \left(\nu_* / 3\right)^{1/3} (a / R)^{1/2} \sim 0.08$$

($k_{||} \sim 1/(qR)$ ITER $\nu_* \sim 0.008$)

Low collisionality dynamics can be simulated on an even coarser velocity grid using hypercollisions & hyperdiffusion, to damp small velocity and spatial scales