AS552 General Plasma Physics II
May 9, 2001

Problem Set #8 (due May 16, 2001)

Kinetic derivation of η_i instability:

The fluid derivation of the η_i mode found an instability even if the higher order polarization drift is neglected and only the $\vec{E} \times \vec{B}$ drift is kept. (The polarization drift leads to $\mathcal{O}(b_s) \sim \mathcal{O}(k_\perp^2 \rho_s^2)$ corrections that are important for drift waves but are not needed for the simplest η_i mode at long wavelength.) Thus it should be possible to demonstrate the existence of this instability using the drift kinetic equation for ions

$$\frac{\partial f}{\partial t} + \left(v_\parallel \hat{b} + \frac{c}{B^2} \vec{E} \times \vec{B} \right) \cdot \frac{\partial f}{\partial \vec{x}} + \frac{q}{m} E_\parallel \frac{\partial f}{\partial v_\parallel} = 0,$$

and using quasineutrality and a simple adiabatic Boltzmann response for electrons. [For drift waves, where the $\mathcal{O}(b_s)$ corrections are needed, one usually uses the full "gyrokinetic" equation to get the FLR corrections.]

Assuming $\omega / k_\parallel \gg v_{ti} = \sqrt{T_i / m_i}$, expand the linearized drift-kinetic ion response to obtain the perturbed ion density response to order $(k_\parallel v_{ti} / \omega)^2$. Obtain the governing dispersion relation (or eigenvalue equation), and show that instabilities driven by ∇T_i can occur.

(You can probably borrow some of your results from the previous homework for the kinetic electron response, just changing the charge and mass. However, in the last homework problem you found the kinetic electron response in the $\omega / k_\parallel \ll v_t$ limit, while here you will be working in the opposite limit for ions.)