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Non-existence of normal tokamak equilibria with negative central current

G. W. Hammett*, S. C. Jardin, and B. C. Stratton
Princeton Plasma Physics Laboratory
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Recent tokamak experiments employing off-axis, non-inductive current drive have found that a large central current hole can be produced. The current density is measured to be approximately zero in this region, though in principle there was sufficient current drive power for the central current density to have gone significantly negative. Recent papers have used a large aspect-ratio expansion to show that normal Magnetohydrodynamic (MHD) equilibria (with axisymmetric nested flux surfaces, non-singular fields, and monotonic peaked pressure profiles) can not exist with negative central current. We extend that proof here to arbitrary aspect ratio, using a variant of the virial theorem to derive a relatively simple integral constraint on the equilibrium. However, this constraint does not, by itself, exclude equilibria with non-nested flux surfaces, or equilibria with singular fields and/or hollow pressure profiles that may be spontaneously generated.

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I. INTRODUCTION

Tokamaks with reversed central magnetic shear (and thus low core current density) are of interest for at least two reasons: 1) internal transport barriers associated with reduced turbulence are often observed in them, leading to improved energy and particle confinement; and 2) they are the natural result of high beta operation and high bootstrap current fraction used to reduce non-inductive current drive requirements for steady state operation. Both of these features could make reversed magnetic shear operation attractive for a tokamak reactor.

Recent experiments on the Joint European Torus (JET)[1,2,3,4] and the Japan Atomic Energy Research Institute Tokamak-60 Upgrade (JT-60U)[5,6,7] have pushed the core current density to very low values using off-axis, non-inductive current drive. Large central current holes (regions of nearly zero current density) are produced because off-axis, non-inductive current drive in the same direction as the Ohmic current induces a back electromotive force inside the non-inductive current drive radius that decreases the core current density.

An interesting feature of current hole discharges is that the core current density is approximately zero (within Motional Stark Effect diagnostic measurement errors), even though there is often sufficient current drive power that the core current could in principal go significantly negative[2,3] (negative relative to the direction of the total plasma current). Recent non-linear toroidal resistive MHD (Magnetohydrodynamic) simulations[2,8] predict that current hole discharges undergo rapid $n = 0$ reconnection events (axisymmetric sawteeth) that clamp the core current near zero. More generally, this reconnection occurs whenever the current density profile is such that the rotational transform, ι, goes to zero on any surface in the plasma (this includes the case where the current density on-axis is positive, but the current profile goes sufficiently negative somewhere off-axis that the total current enclosed by some flux surface vanishes). Reduced MHD simulations in cylindrical geometry have also shown that $n = 0$ resistive kink instabilities can clamp the core current density at zero when it attempts to go negative.[9] Breslau *et al.*[8] and Stratton *et al.*[2] stated that a second-order, large aspect ratio expansion of the MHD equations indicates that a normal toroidal equilibrium is not possible if ι crosses through 0 at some radius. (They also stated that a more general proof is needed, which we provide here.) A recent paper by Chu and Parks[10] used a second order aspect ratio expansion to prove that a normal equilibrium with a peaked pressure profile is not possible with negative core current. They extended the analysis to provide matching conditions at the boundary of a central region with no current and no pressure gradient, showing explicitly that current hole equilibria are theoretically possible (with zero, but not negative, current).

This paper extends some of these results to arbitrary aspect ratio, employing a relatively simple constraint based on a version of the virial theorem to show that a "normal" toroidal MHD equilibrium (with axisymmetric nested flux surfaces around a single magnetic axis, non-singular continuous fields, and a monotonic peaked pressure profile) is not possible with negative core current. Or more generally, a normal equilibrium is not possible if the toroidal current enclosed by any flux surface goes negative relative to the direction of the total plasma current, so that there is an $\iota = 0$ surface somewhere in the plasma where the poloidal field vanishes (the null surface). Though the starting point of this analysis is based on well-known equations, they are often specialized to large aspect ratio or simplified geometry, while the present analysis is more general.

However, the virial constraint does not necessarily eliminate the possibility of more exotic equilibria, such as with non-nested flux surfaces with islands, or with singular fields and/or off-axis peaks in the pressure profiles

*Electronic mail: hammett@princeton.edu

that may be spontaneously generated by the plasma near the null surface. Some examples are considered here. In this paper we investigate the consequences of only one integral constraint on equilibria, while there can be other constraints that further limit the types of theoretically possible or experimentally realizable equilibria[10].

The non-existence of normal equilibria with negative core current, and/or the rapid axisymmetric sawteeth that are predicted to occur if the enclosed current goes negative, may also explain the results of other experiments, such as the low efficiency seen in some electron cyclotron counter current drive experiments[6,11].

II. DERIVATION

The MHD equilibrium equation $\nabla p = \boldsymbol{j} \times \boldsymbol{B}/c$ can be written as

$$0 = -\nabla \left(p + \frac{B^2}{8\pi} \right) + \frac{1}{4\pi} \left(\boldsymbol{B} \cdot \nabla \right) \boldsymbol{B} \tag{1}$$

One common use of the virial theorem is to take the inner product of this equation with the position vector $\boldsymbol{x}$ and integrate over all space to show that an isolated MHD equilibrium can not exist by itself (unless there are physical coils or gravity to provide overall force balance).[12,13] Here we use a version of the virial theorem that can be used to derive the Shafranov shift[14], by focusing on radial force balance of axisymmetric equilibria in cylindrical coordinates (R, Z, ϕ). Taking the inner product of Eq. (1) with $\boldsymbol{R} = R\hat{\boldsymbol{R}}$, the radial vector in cylindrical geometry, and integrating over space out to some flux surface of volume V, gives

$$0 = -\int dV \boldsymbol{R} \cdot \nabla \left(p + \frac{B^2}{8\pi} \right) + \frac{1}{4\pi} \int dV \boldsymbol{R} \cdot \left(\boldsymbol{B} \cdot \nabla \right) \boldsymbol{B} \tag{2}$$

For the second integral we use the identity $\boldsymbol{R} \cdot (\boldsymbol{B} \cdot \nabla) \boldsymbol{B} = \nabla \cdot (\boldsymbol{B}\boldsymbol{R} \cdot \boldsymbol{B}) - B_R^2 - B_\phi^2$. The integral of $\nabla \cdot (\boldsymbol{B}\boldsymbol{R} \cdot \boldsymbol{B})$ vanishes because $\boldsymbol{B} \cdot d\boldsymbol{S} = 0$ on a flux surface. The first integral can be integrated by parts using $\nabla \cdot \boldsymbol{R} = 2$, so that Eq. (2) becomes

$$\begin{aligned} 0 &= -p(\rho) \int d\boldsymbol{S} \cdot \boldsymbol{R} - \int d\boldsymbol{S} \cdot \boldsymbol{R} \frac{B^2}{8\pi} \\ &\quad + 2 \int dV \left(p + \frac{B^2}{8\pi} \right) - \frac{1}{4\pi} \int dV \left(B_R^2 + B_\phi^2 \right) \\ &= -p \int d\boldsymbol{S} \cdot \boldsymbol{R} - \int d\boldsymbol{S} \cdot \boldsymbol{R} \frac{B^2}{8\pi} + 2 \int dV \left(p + \frac{B_Z^2}{8\pi} \right), \end{aligned} \tag{3}$$

where $p = p(\rho)$ is the pressure at the surface labeled by ρ enclosing the volume $V(\rho)$, and B_Z is the vertical magnetic field. For the first surface integral we can use Gauss' theorem to write $\int d\boldsymbol{S} \cdot \boldsymbol{R} = \int dV \nabla \cdot \boldsymbol{R} = 2V$. For the second surface integral, we use $d\boldsymbol{S} = 2\pi R \hat{\boldsymbol{\phi}} \times d\vec{\ell}$, where $d\vec{\ell}$ is a poloidal path length element along the surface, to write $d\boldsymbol{S} \cdot \boldsymbol{R} = 2\pi R^2 \hat{\boldsymbol{Z}} \cdot d\vec{\ell}$ so $\int d\boldsymbol{S} \cdot \boldsymbol{R} = 2\pi \oint R^2 \hat{\boldsymbol{Z}} \cdot d\vec{\ell} = 2\pi \oint R^2 dZ$. Since the toroidal field $B_\phi \propto 1/R$ in a flux surface, the B_ϕ^2 contribution to this surface integral vanishes, and we have

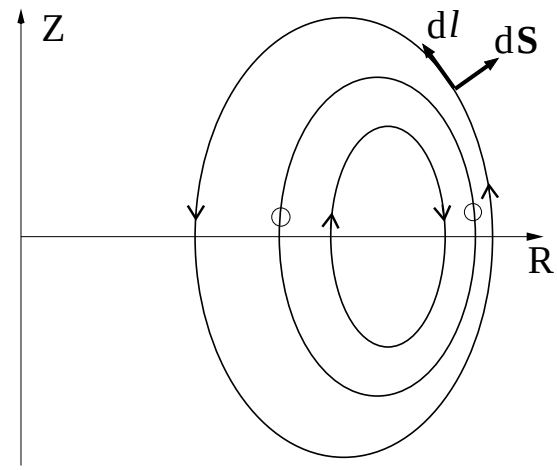

FIG. 1: Sketch of hypothetical equilibrium with nested flux surfaces and negative central current (relative to the total current), so that the poloidal field points clockwise near the axis, counterclockwise near the edge, and is zero on a flux surface in between. A normal equilibrium is not possible in this case (with a normal peaked pressure profile)

$$0 = -p(\rho) - \frac{1}{V} \oint d\vec{\ell} \cdot \hat{\boldsymbol{Z}} \pi R^2 \frac{B_{pol}^2}{8\pi} + \langle p \rangle + \left\langle \frac{B_Z^2}{8\pi} \right\rangle \tag{4}$$

where $\langle \ldots \rangle = \int dV \ldots / \int dV$ denotes a volume average, and $B_{pol}^2 = B_R^2 + B_Z^2$ is the poloidal field strength squared. The poloidal field can be written as $\boldsymbol{B}_{pol} = \nabla\phi \times \nabla\psi = (\hat{\boldsymbol{\phi}} \times \nabla\rho)(\partial\psi/\partial\rho)/R$, where ρ is a flux surface label. [While ψ is also constant on a flux surface, there can be two surfaces with the same value of ψ in the presence of negative central current, so it is convenient to choose another flux surface label ρ, such as based on the enclosed volume or toroidal flux, to maintain monotonic labeling.] If the toroidal current near the magnetic axis is in the opposite direction as the total plasma current, then the poloidal field must reverse direction somewhere and there must be a null surface on which the poloidal magnetic field is everywhere zero, as shown in Fig. 1. Another way to see this is to note that the poloidal field is related to the enclosed toroidal current by $4\pi I_\phi(\rho)/c = \oint d\vec{\ell} \cdot \boldsymbol{B}_{pol} = (\partial\psi/\partial\rho) \oint d\vec{\ell} \cdot \hat{\boldsymbol{\phi}} \times \nabla\rho/R$, so $B_{pol} \propto \partial\psi/\partial\rho = 0$ on any flux surface that encloses zero toroidal current. This is also the flux surface on which the rotational transform $\iota = 0$ (corresponding to the safety factor $q = \infty$). [These arguments assume that B_{pol} is continuous and finite, we consider a singular exception in the next section.]

On such a flux surface where the poloidal field vanishes, the second term of Eq. (4) vanishes and we are left with

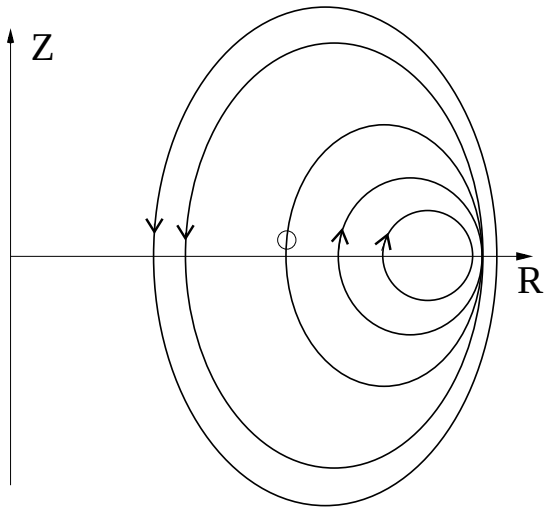

FIG. 2: Ideal MHD formally allows singular solutions where the poloidal field vanishes almost everywhere on a flux surface, so that the enclosed current $\propto \oint d\vec{\ell} \cdot \boldsymbol{B}_{pol} = 0$ but $\oint d\ell B_{pol}^2$ is finite. Such an equilibrium could then in principle satisfy the integral force balance Eq. (4). In this case, adjacent flux surfaces approach one another at one point where the poloidal field becomes infinite, but this is an integrable singularity with finite energy. [All of these sketches are intended only to illustrate topology and are not precise.]

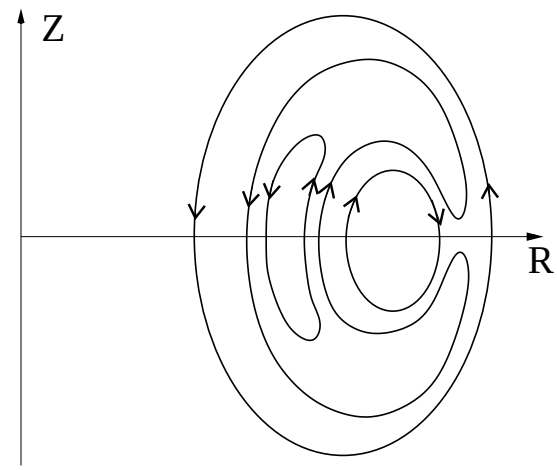

FIG. 3: An equilibrium with this topology of non-nested flux surfaces is not ruled out by the integral force balance Eq. (4). Note that the toroidal current in the inner part of the plasma is in the reverse direction from the total plasma current.

constraint

$$0 = -p + \langle p \rangle + \left\langle \frac{B_Z^2}{8\pi} \right\rangle \tag{5}$$

Since the last two terms are positive definite, the only way this equation can possibly be satisfied is if the pressure at this flux surface, p, is larger than the volume-averaged pressure inside that flux surface, $\langle p \rangle$. I.e., the pressure profile must be hollow at least in some region, and can not be a monotonically decreasing function of ρ at all radii as usual pressure profiles do.

This is in agreement with the result of Chu and Parks[10], who also found that a normal equilibrium with a negative central current is not possible if the pressure profile is monotonically decreasing. These earlier results used a second-order large aspect ratio ordering while our derivation is valid for arbitrary aspect ratio. In other ways, their calculation goes beyond ours, as they have investigated additional constraints that can further limit the class of accessible equilibria.

III. POSSIBLE ALTERNATE SOLUTIONS

Here we consider several possible alternate solutions for satisfying force balance in equilibria. Each differs from "normal" equilibria in a different way.

The argument in the previous section applies rigorously only for nested flux-surfaces where the fields are continuous and finite. Within the framework of ideal MHD, in principle there could be a singular poloidal field on a flux surface such that $\oint d\vec{\ell} \cdot \boldsymbol{B}_{pol} = 0$ so that this surface encloses zero toroidal current, but $\int d\vec{\ell} \cdot \hat{\boldsymbol{Z}} \pi R^2 B_{pol}^2/(8\pi)$ is still finite and can contribute to Eq. (4) so that integrated force balance can be satisfied. An example of such a field might be the limiting case $B_{pol} \propto |\nabla\psi| \sim C \exp(-\ell^2/w^2)/\sqrt{w}$. Then in the limit as $w \to 0$ we have zero toroidal current enclosed while still giving a finite contribution to the second term of Eq. (4) (due to a singularity in the field chosen to be at $\ell = 0$ in this example). Although B_{pol} is becoming infinite at some point on the flux surface, it is an integrable singularity containing a finite amount of energy, and so could formally be considered as an admissible solution of ideal MHD. Since the spacing between two nearby flux surfaces labeled by poloidal flux ψ_1 and ψ_2 is given by $\Delta \approx (\psi_2 - \psi_1)/|\nabla\psi|$ (except where second derivatives have to be considered), there will be flux surfaces with finite separation at some places which will approach one another at the singular point where $|\nabla\psi| \propto B_{pol} \to \infty$. The topology of this configuration is illustrated in Fig. 2. [This sketch is intended only to illustrate the topology of a possible solution which satisfies the integral force balance constraint, Eq. (4). An actual detailed solution that would satisfy force balance locally at all points is left for future work, and may involve highly distorted flux-surface shapes with boundary layers[15,16,17].] Note that not only is the poloidal field infinite at the singular point, it must also flip signs from $+\infty$ to $-\infty$ in the limit as the singular point is approached radially from opposite directions. Of course this singular configuration is strongly susceptible to magnetic reconnection when resistivity is included, consistent with the interpretation of rapid reconnection observed in simulations[2,8]. If finite resistivity is included, then the singularity in the field is smoothed out and loss of equilibrium can help drive

reconnecting flows.

Another possible way of satisfying integrated force balance with negative central current, while keeping the field finite and continuous, is if the flux surfaces are non-nested. An example is shown in Fig. 3, which is similar to the intermediate configurations observed during some toroidal simulations of axisymmetric reconnection in negative central current plasmas (for example, Fig. 11 of Ref. 2, though in other cases they see islands with higher poloidal mode numbers). This case has an axisymmetric islandand the poloidal field is non-zero almost everywhere (except at the two magnetic axes and at the X-point) and so in principle can be arranged to give a negative contribution to the second term of Eq. (4) to satisfy integrated force balance. This is related to the role in a normal equilibrium of the Shafranov shift, which provides a larger value of $R^2 B_{pol}^2$ on the outer part of a flux surface than on the inner part, so that the second term in Eq. (4) is negative. The X-point of a non-nested equilibrium might not be on the low-field side, and another possible equilibrium might be obtained by rotating Fig. 3 by 180° and shifting the spacing between flux surfaces so that the integral of $R^2 B_{pol}^2$ on the outer part of the flux surface is again larger than on the inner part. One or the other of these configurations may be an unstable equilibrium and prefer to flip to the other orientation. [Takizuka[18] earlier proposed another possible non-nested equilibria with negative central current, involving $(m = 2, n = 0)$ islands, while the example we discussed here has an $(m = 1, n = 0)$ island.]

Another way of thinking about ideal MHD equilibria is to modify the time-dependent ideal MHD equations to include viscosity and parallel thermal conduction while retaining the ideal Ohm's law[13]. Since viscosity should eventually damp the velocity $\boldsymbol{u}$ to zero, and parallel thermal conduction will lead to $\boldsymbol{B} \cdot \nabla p = 0$, the dissipative terms vanish in a stationary steady state and the solution is also an ideal MHD equilibrium. One can then start with any arbitrary initial configuration of the magnetic field (which can be assumed to be nested flux surfaces) with arbitrary initial profiles, as functions of toroidal flux Φ, for the rotational transform $\iota(\Phi)$ and the adiabatic parameter $\mu(\Phi) = p/n^\Gamma$ (where $p(\Phi)$ is the pressure profile, $n(\Phi)$ is the density profile, and Γ is the ratio of specific heats). Since this initial condition is not necessarily an equilibrium, flows will be driven and the plasma will move about, perhaps oscillating for a while. But it seems reasonable to assume that the viscosity will eventually damp out the oscillations and the plasma will settle into an equilibrium configuration while conserving $\iota(\Phi)$ and $\mu(\Phi)$. (The motions are assumed to be constrained to be axisymmetric to find such an equilibria. This approach to equilibria of course does not address the issue of stability, and these equilibria might then be unstable to symmetry-breaking perturbations.)

This was the logic that motivated the flux-conserving tokamak equilibria concepts[13] that showed that there is formally no equilibrium limit on the pressure in a tokamak, since as the pressure increases, the Shafranov shift and the current can also increase to provide sufficient poloidal magnetic field on the outboard side to provide force balance.

Presumably this procedure would also find an equilibrium even if the rotational transform changed sign so that there was a null flux surface where $\iota = 0$. In some cases with certain initial conditions, it might be possible for the plasma to spontaneously adjust flux surfaces near the null ι surface to produce a local peak in the pressure profile that can satisfy Eq. (5). [We have focussed on the consequences of only one integral constraint that rules out "normal" equilibria with negative central current, and there can be other constraints that would further limit the practical accessibility of such non-monotonic pressure equilibria[10].] The more typical case is probably that the equilibrium that is approached will have a singular structure, such as in Fig. 2, in order to satisfy Eq. (4). [This is similar to studies showing that the nonlinear saturation of an internal kink mode approaches a neighbouring equilibrium state with singular currents[15,16].] These singular or near-singular states will be subject to strong reconnection if a small amount of resistivity is introduced, and the change in topology may dominate what happens. Realistic evaluations of what happens may depend on fully including various dissipation mechanisms (thermal, momentum, and particle anisotropic transport driven by small scale turbulence and collisional effects, as well as resistivity, current drive, heating and loss mechanisms). We leave detailed investigation of these issues to other work.

IV. RELATION TO THE SHAFRANOV SHIFT

For completeness, we show the relation of Eq. (4) to usual expressions for the Shafranov shift[12,13]. The second term of Eq. (4) can be written as

$$T_2 = -\frac{\pi}{V} \oint d\vec{\ell} \cdot \hat{\boldsymbol{Z}} R^2 \frac{B_{pol}^2}{8\pi} = -\frac{\pi}{V} \frac{(\partial\psi/\partial\rho)^2}{8\pi} \oint d\vec{\ell} \cdot \hat{\boldsymbol{Z}} |\nabla\rho|^2 \tag{6}$$

At this point, many previous calculations specialize to a large aspect ratio expansion and/or to specified shapes for the flux surfaces. For example, assume shifted circular flux surfaces with $R(\rho, \theta) = R_0 - \Delta(\rho) + \rho\cos\theta$ and $Z(\rho, \theta) = \rho\sin\theta$, where ρ has now been chosen to be the minor radius of the flux surface, and Δ is the Shafranov shift. It can be shown that $|\nabla\rho|^2 = 1/(1 - \Delta'\cos\theta)^2$, where $\Delta' = d\Delta/d\rho$. Defining $\partial\psi/\partial\rho = B_p(\rho)(R_0 - \Delta)$, we have $B_{pol} = B_p[R_0 - \Delta]/[R(1 - \Delta'\cos\theta)]$ (this would be exact if the flux surfaces really were shifted circles), and T_2 becomes

$$T_2 = -\frac{B_p^2}{8\pi} \frac{R_0 - \Delta}{r} \frac{\Delta'}{(1 - \Delta')^{3/2}} \tag{7}$$

where we have used $\int d\theta \cos\theta/(1-\Delta'\cos\theta)^2 = 2\pi\Delta'/(1-\Delta')^{3/2}$. Inserting this into Eq. (4) yields

$$\frac{\Delta'}{(1-\Delta')^{3/2}} = \frac{r}{R_0-\Delta}\frac{8\pi}{B_p^2}\left(\langle p\rangle - p + \left\langle \frac{B_Z^2}{8\pi}\right\rangle\right) \quad (8)$$

Evaluating this in the large aspect ratio limit at the plasma edge where $p = 0$ gives the familiar form $\Delta' = (r/R_0)(\beta_{pol} + \ell_i/2)$, where β_{pol} is the poloidal beta and ℓ_i is the internal inductance per unit length. [Note that $\ell_i/2 = \langle B_Z^2\rangle/B_p^2(a) = \langle B_{pol}^2 cos^2(\theta)\rangle/B_p^2(a) = \langle B_{pol}^2\rangle/2B_p^2(a)$, so that the factor of 1/2 comes from the fact that only the vertical magnetic field contributes to the numerator in the shift.] The nonlinear form of the left-hand side of Eq. (8) has the nice property of insuring that the flux surfaces are well behaved and don't cross ($|\Delta'| < 1$) for arbitrarily high β_{pol}, though this equation only rigorously applies if the flux surfaces remained shifted circles, which breaks down at high beta.

V. SUMMARY

We have presented a relatively simple integral constraint on toroidally axisymmetric MHD equilibrium that shows that a normal equilibrium (with nested magnetic flux surfaces, non-singular fields, and a peaked pressure profile that falls monotonically with radius) can not exist if the toroidal current inside any flux surface is negative relative to the total plasma current. This generalizes previous results[2,8,10] to arbitrary aspect ratio.

However, the integral constraint does not necessarily prevent negative central current equilibria with non-nested or singular magnetic flux surfaces. Possible examples of this are shown in Figs. 2 and 3. A plasma with nested flux surfaces and negative central current that is initially out of equilibrium could presumably move towards an equilibrium, though it seems most likely that this new equilibrium would have singular or near-singular fields and thus would be subject to strong reconnection[2,8] if finite resistivity is introduced, changing the topology. There can also be other constraints that limit the accessible class of alternate equilibria[10]. One might be able to understand the structure of some of these possible solutions in the vicinity of the null surface as a boundary layer analysis of a shock-like solution. But a realistic evaluation of such scenarios would require including finite cross-field transport, viscosity, resistivity, and FLR effects. We leave these issues to future work. Other interesting questions to consider are whether such "non-normal" MHD equilibria are stable to ideal and/or resistive MHD modes and/or are experimentally accessible.

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NOTE: After submitting this paper for publication, we became aware of recent related work by Martynov *et al.*[19], who study island equilibria with negative central current in much more detail.

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