Particle Noise-Induced Diffusion & Its Effect on ETG Simulations

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Acknowledgments: S. Cowley, B. Cohen, W. Dorland, F. Jenko, J. Krommes, M. Kotschenreuther

Outline

• Back-of-the-envelope estimate of the spectrum of potential fluctuations due to a discrete number of gyrokinetic particles. Within a factor of 2 of the more precise calculation based on an extension of Krommes' 1993 using the classic fluctuation-dissipation theorem or test-particle superposition principle.

• Agrees very well with Dimit's gyrokinetic PIC ETG simulations (with no free parameters!)

• Renormalized calculation of noise-induced ExB diffusion, D_{noise}, for a test-particle in this spectrum of random potential fluctuations.

• Theory determines D_{noise} if given rms particle weights.
Can monitor $k_\perp^2 D_{noise}$ to insure it is $\ll \gamma_{lin}$ for a reliable simulation.

• Toy model for interaction of Noise and Turbulence, useful for predicting results if one doesn't have access to weights. Also explains how noise can increase transport in some cases (ITG) and decrease it in other cases (ETG).

Jenko & Dorland found ETG turbulence >> ITG turbulence (in Gyro-Bohm units)

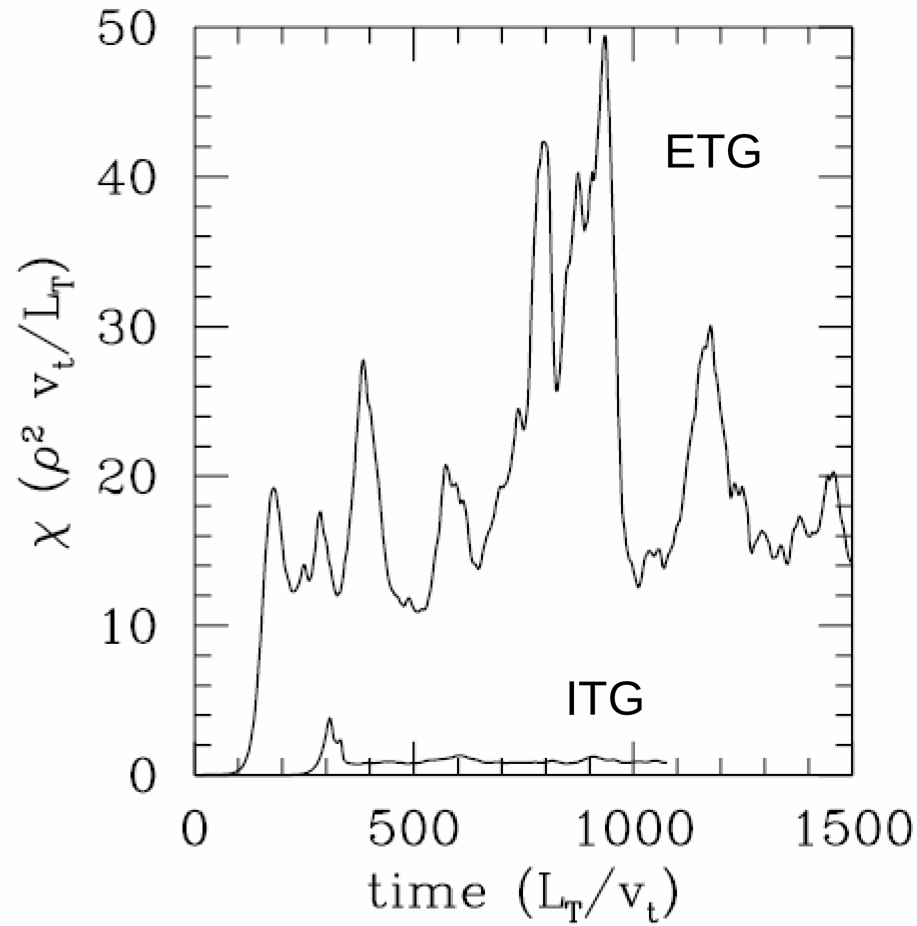

FIG. 1. χ_e^{ETG} (upper curve) and χ_i^{ITG} (lower curve) for similar parameters.

(Dorland & Jenko 2000, see also Jenko & Dorland 2002: with larger box, Lx=512 ρ, report χ_e = 13)

ETG eddies are radially extended streamers

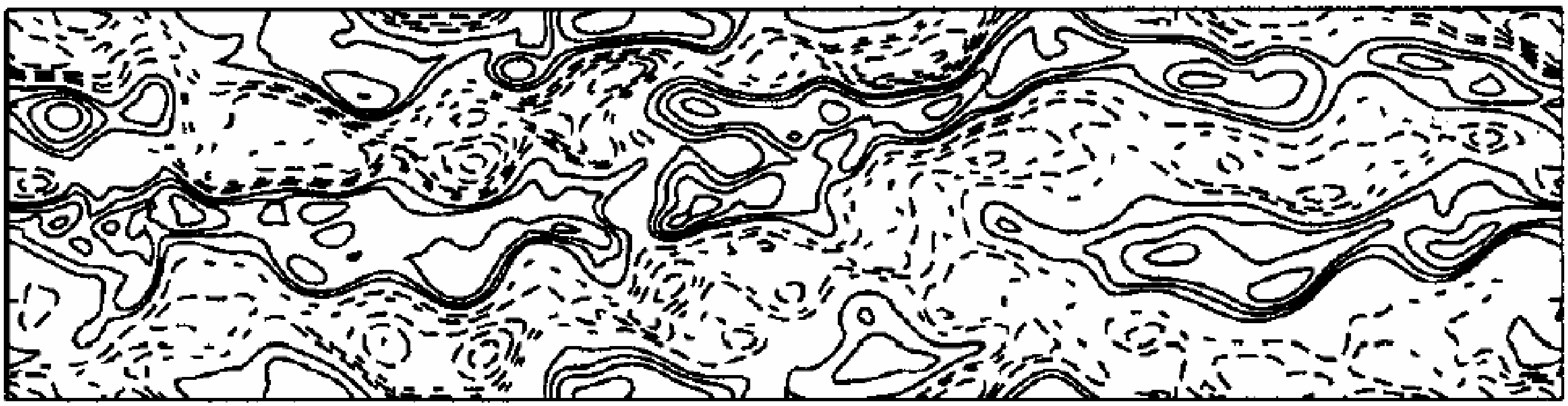

FIG. 2. Characteristic ϕ contours in the outboard x-y plane. This snapshot was taken at the end of the ETG run shown in Fig. 1. The figure is $256\rho_e \times 64\rho_e$.

(Jenko & Dorland 2000)

Key ITG/ETG Difference: different adiabatic response to zonal flows

ITG turbulence, adiabatic electron response:

$$n_e = n_i$$

$$n_{e0}\frac{e}{T}\left(\Phi - \langle\Phi\rangle\right) = \int d^3v\, J_0 f_i - n_{i0}\left(1 - \Gamma_0(k_\perp \rho_i)\right)\frac{e}{T}\Phi$$

Flux-surface averaged potential, electrons adiabatic because $k_{||}=v_{te} >> \omega$
don't respond to zonal flows ($k_{||}=0$, pure E_r).

ETG turbulence, adiabatic ion response:

$$n_i = n_e$$

$$n_{i0}\frac{e}{T}\Phi = \int d^3v\, J_0 f_e + n_{ei0}\left(1 - \Gamma_0(k_\perp \rho e)\right)\frac{e}{T}\Phi$$

Ions adiabiatic because $k_\perp\rho_i >> 1$. Ions CAN shield zonal flows.
↓ zonal flows --> streamers elongate --> transport ↑

Detailed secondary/tertiary instability analysis includes this, explains ITG/ETG saturation level differences, scalings (Rogers, Dorland, Jenko papers)

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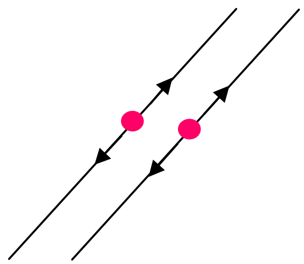
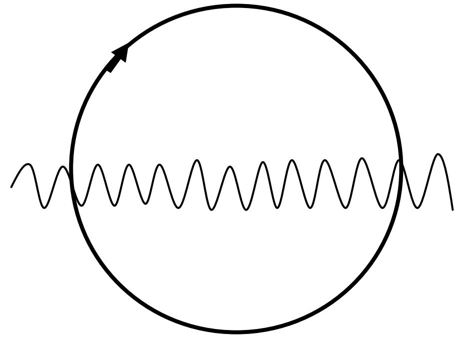


electrons ***don't*** respond to zonal flows ($k_{||}$=0, pure E_r).
since electrons are adiabatic because $k_{||}v_{te} >> \omega$

ETG turbulence, adiabatic ion response:



Ions do shield zonal flows for ETG
Since ions are adiabatic because $k_{\perp}\rho_i >> 1$.
↓ zonal flows --> streamers elongate --> transport ↑

Detailed secondary/tertiary instability analysis includes this, explains ITG/ETG saturation level differences, scalings (Rogers, Dorland, Jenko papers)

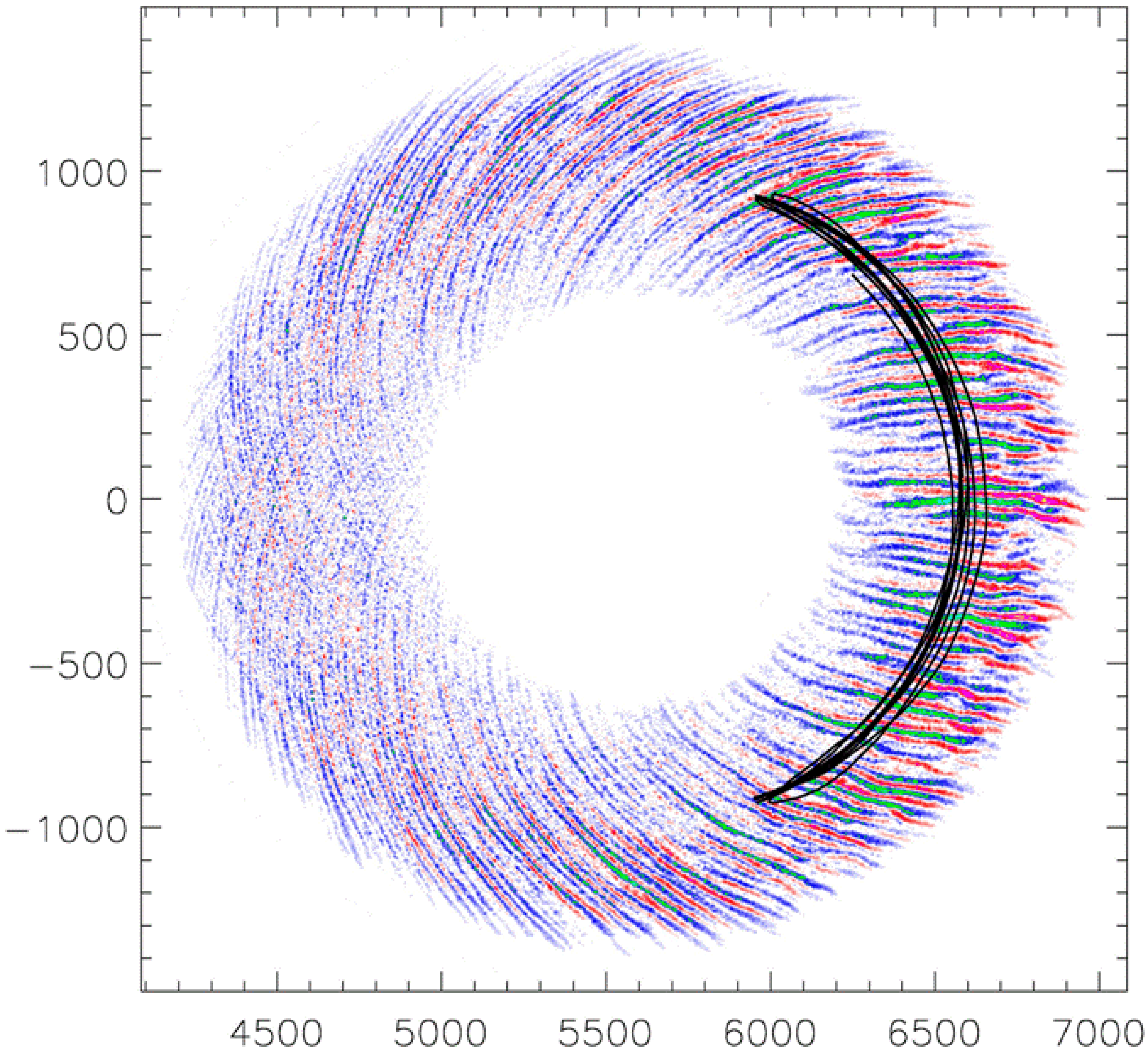
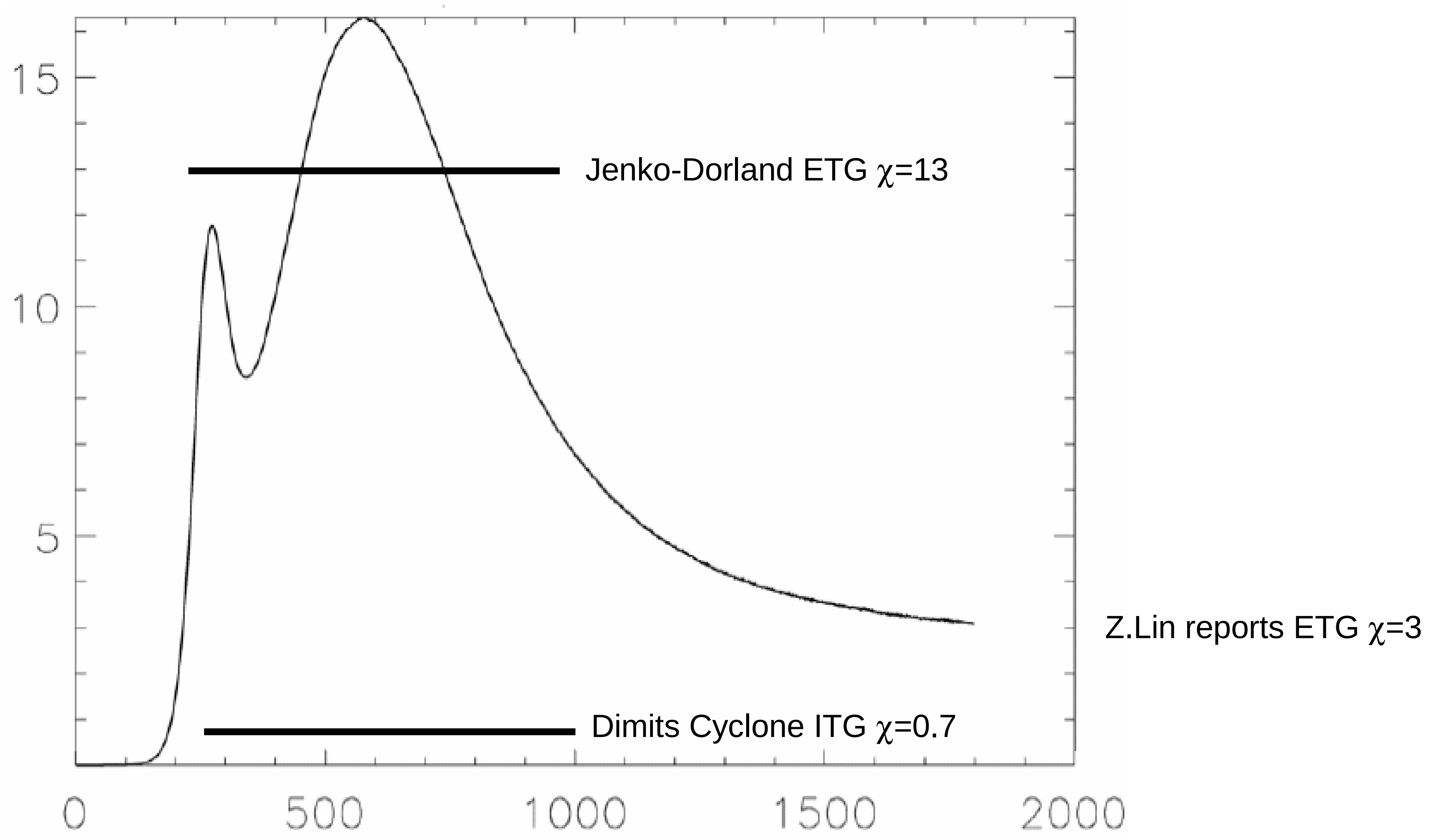


Fig. of etg streamers
from Z. Lin global PIC
simulations IAEA 2004



From Z.Lin's IAEA 2004 slides (at URL below). Believed to be $\chi_{etg}(t)$. Initial large values of χ_{etg} comparable to Jenko-Dorland 2002 χ_{etg} ~ 13. Ignoring initial "transient", reported result is χ_{etg} ~ 3. Scanned 5 to 20 particles/cell.

http://www.cfn.ist.utl.pt/20IAEAConf/presentations/T5/2T/5_H_8_4/Talk_TH_8_4.pdf

Krommes' Calculation of Noise Spectrum

Krommes' 1993 calculation of the gyrokinetic noise spectrum uses the classic fluctuation-dissipation theorem, and shows equivalent results from the test-particle superposition principle (shielded test particles can be treated as independent). (see also W.W. Lee 1987, classic paper by A.B. Langdon 1979)

Krommes' calculation used shielding by linear dielectric from gyrokinetic equation in a slab, uniform plasma. Hu & Krommes 94 extended to δf.

We have extended Krommes' test-particle superposition calculation to:

- Treat one species as adiabatic instead of with particles.

- Include factors for finite-size particle shape S (accounts for interpolation of particle charge to grid, and forces from grid to particles) & S_{filt} factor for explicit filtering of Φ. Important for quantitative comparisons.

- Use a renormalized dielectric, including a $k_\perp^2 D_{NL}$ term on the non-adiabatic part of the shielding cloud, and including random walks in the test particle trajectories instead of assuming straight-line trajectories. Affects frequency spectrum of fluctuations, but not the frequency-integrated k spectrum.

Simple Estimate of Noise: Randomly Positioned Particles

Fourier conventions:

$$\Phi(\vec{x}) = \frac{1}{V}\sum_{\vec{k}} e^{i\vec{k}\cdot\vec{x}}\tilde{\Phi}_{\vec{k}}$$

$$\tilde{\Phi}_{\vec{k}} = \int_V d^3x\, e^{-i\vec{k}\cdot\vec{x}}\Phi(\vec{x})$$

Adiabatic electrons + ion polarization density = ion guiding center contribution

Gyrokinetic Poisson Eq:

$$n\frac{e\Phi}{T} + n(1-\Gamma_0)\frac{e\Phi}{T} = S_{filt}S\int d^3\mathrm{v}\, J_0\,\delta f$$

$$= S_{filt}S\sum_i w_i J_{0i}\delta(\vec{x}-\vec{x}_i)$$

$$\frac{e\tilde{\Phi}_k}{T} = \frac{S_{filt}S}{n(2-\Gamma_0)}\sum_i w_i J_{0i} e^{-i\vec{k}\cdot\vec{x}_i}$$

$$\left\langle \left|\frac{e\tilde{\Phi}_k}{T}\right|^2 \right\rangle = \frac{S_{filt}^2 S^2}{n^2(2-\Gamma_0)^2}\sum_i w_i^2 J_{0i}^2$$

Simple Estimate of Noise Spectrum (cont.)

$$\left\langle \left| \frac{e\tilde{\Phi}_k}{T} \right|^2 \right\rangle = \frac{S_{filt}^2 S^2}{n^2 (2-\Gamma_0)^2} \sum_i w_i^2 J_{0i}^2$$

$$= \frac{S_{filt}^2 S^2 N}{n^2 (2-\Gamma_0)^2} \frac{1}{N} \sum_i w_i^2 J_{0i}^2$$

$$= \frac{V^2 S_{filt}^2 S^2}{N (2-\Gamma_0)^2} \left\langle w^2 \right\rangle \Gamma_0$$

$$\left\langle \left| \frac{e\Phi_{noise}}{T} \right|^2 \right\rangle = \left\langle \left| \frac{e\Phi(\vec{x})}{T} \right|^2 \right\rangle = \frac{1}{V^2} \sum_k \left| \frac{e\tilde{\Phi}_k}{T} \right|^2 = \frac{\left\langle w^2 \right\rangle}{nV} \sum_k \frac{S_{filt}^2 S^2 \Gamma_0}{(2-\Gamma_0)^2}.$$

$$\approx \frac{\left\langle w^2 \right\rangle}{n(2\pi)^3} \int d^3k \, \frac{S_{filt}^2 S^2 \Gamma_0}{(2-\Gamma_0)^2} = \frac{\left\langle w^2 \right\rangle}{nV_{smooth}}.$$

Compare spectra from ETG simulations with noise theory w/ no free parameters

Dimits pg3eq gyrokinetic PIC code filter and particle shape factor (corresponding to Nearest Grid Point interpolation):

$$S_{filt} = \frac{\exp(-(ak_y\Delta y)^8)}{[1+(ak_x\Delta x)^8][1+(ak_z\Delta z)^8]} \qquad S = \frac{\sin(k_x\Delta x/2)}{k_x\Delta x/2}\frac{\sin(k_y\Delta y/2)}{k_y\Delta y/2}\frac{\sin(k_z\Delta z/2)}{k_z\Delta z/2}$$

Z. Lin's GTC code uses linear interpolation, giving the shape factor S below.
If GTC uses a 3-point binomial filter that would be:

$$S_{filt} = \cos^2(k_x\Delta x/2)\cos^2(k_y\Delta y/2)\cos^2(k_z\Delta z/2)$$

$$S = \left[\frac{\sin(k_x\Delta x/2)}{k_x\Delta x/2}\frac{\sin(k_y\Delta y/2)}{k_y\Delta y/2}\frac{\sin(k_z\Delta z/2)}{k_z\Delta z/2}\right]^2$$

Renormalized Dielectric Shielding

Nonlinear gyrokinetic Eq. (uniform slab, electrostatic):

$$\frac{\partial \delta f}{\partial t} + \mathrm{v}_{\|} \hat{b} \cdot \nabla \delta f + \frac{c}{B} \hat{b} \times \nabla J_0 \Phi \cdot \nabla \delta f = -\mathrm{v}_{\|} \left(\hat{b} \cdot \nabla J_0 \frac{q\Phi}{T} \right) F_{Max,0}$$

If ExB velocity is small-scale random fluctuations, treat as random walk diffusion:

$$\frac{\partial \delta f}{\partial t} + \mathrm{v}_{\|} \hat{b} \cdot \nabla \delta f \qquad - D_{NL} \nabla_{\perp}^2 \delta f \quad = -\mathrm{v}_{\|} \left(\hat{b} \cdot \nabla J_0 \frac{q\Phi}{T} \right) F_{Max,0}$$

Better renormalization (Catto 78): nonlinearity affects only non-adiabatic part of δf:

$$\frac{\partial \delta f}{\partial t} + \mathrm{v}_{\|} \hat{b} \cdot \nabla \delta f - D_{NL} \nabla_{\perp}^2 \left(\delta f - F_{Max.0} J_0 \frac{q\Phi}{T} \right) = -\mathrm{v}_{\|} \left(\hat{b} \cdot \nabla J_0 \frac{q\Phi}{T} \right) F_{Max,0}$$

Preserves the form of the Fluctuation-Dissipation Theorem, insures no nonlinear damping of a thermal equilibrium solution (the adiabatic solution) (Catto78, Krommes81, Krommes02).

Detailed Calculation of Noise-Spectrum Incl. Self-Shielding

Potential induced by shielded test particle density ρ_{ext}:

$$\Phi = \frac{4\pi q \rho_{ext}}{k^2 \varepsilon(\vec{k},\omega)}$$

$$\varepsilon\left(\vec{k},\omega\right) = \frac{k_D^2}{k^2}\left[\frac{T}{T_a}\left(1-\delta_{k_\parallel}\right) + 1 - \Gamma_0 + S_{filt} d_\parallel S^2 \left\langle J_0^2 \right\rangle \left(1 + \zeta Z(\zeta + i\zeta_D)\right)\right]$$

Gyrokinetic dielectric shielding including simple renormalized D_{NL} model of nonlinear effects on shielding cloud and test-particle random walk trajectory, $\zeta_D = k_\perp^2 D_{NL}/(|k_{||}|\ v_t\ 2^{1/2})$. Integrating $<|\Phi_k|^2>(\omega)$ over all ω gives a result independent of D_{NL}. To preserve this feature of the Fluctuation-Dissipation Theorem, important to apply renormalized $k_\perp^2 D_{NL}$ only to the non-adiabatic part of δf (Catto78, Krommes81, Krommes02). Resulting k spectrum:

$$\left\langle \left| \frac{e\tilde{\Phi}_{noise,k}}{T} \right|^2 \right\rangle = \frac{V^2 \left\langle w^2 \right\rangle}{N} \frac{S_{filt}^2 S^2 \left\langle J_0^2 \right\rangle}{\left[\frac{T}{T_a}\left(1-\delta_{k_\parallel}\right) + 1 - \Gamma_0 + \underbrace{S_{filt} d_\parallel S^2 \left\langle J_0^2 \right\rangle}_{\text{Only difference from simple random-particle spectrum.}}\right]\left[\frac{T}{T_a}\left(1-\delta_{k_\parallel}\right) + 1 - \Gamma_0\right]}$$

Only difference from simple random-particle spectrum. $< \Phi_k^2>$ only 50% lower at low $k_\perp$ (adiabatic electrons already got half of shielding), equal at high $k_\perp$ (ion shielding vanishes)

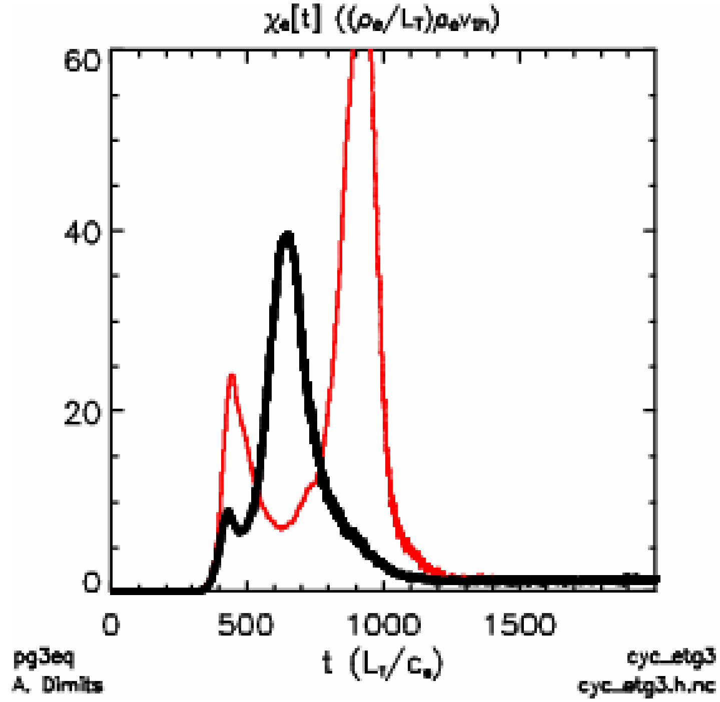
Note: In the following plots comparing the noise formula on the last page with <$\Phi^2(k_x,k_y)$> spectra from Dimits gyrokinetic PIC simulations, there are no free parameters.

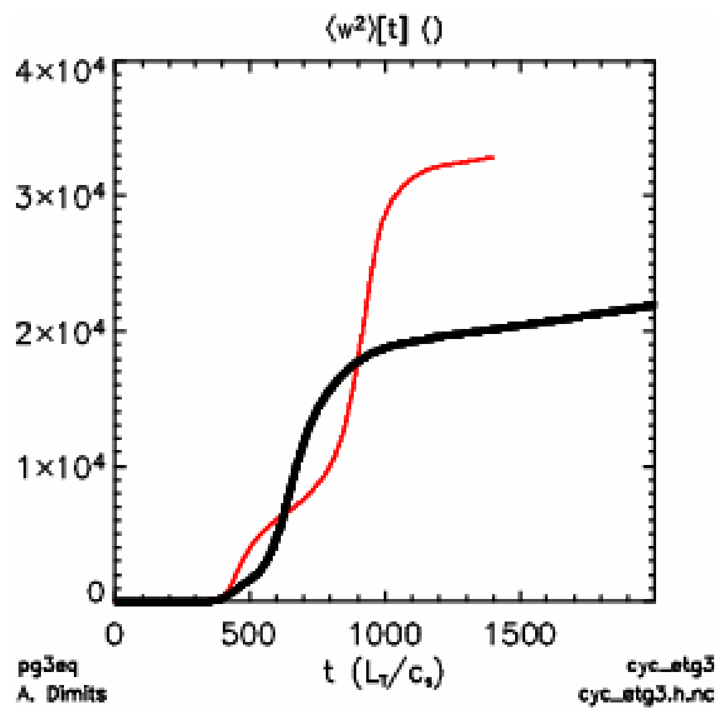
Detail: Nevin's GKV diagnostic is the $k_\perp$ spectrum at z=0, so have to sum the expression for the noise Φ_k^2 spectrum over all k_z:

$$\left\langle \left| \frac{e\tilde{\Phi}_{k_x,k_y}(z=0)}{T} \right|^2 \right\rangle = \frac{1}{L_z^2} \sum_{k_z} \left\langle \left| \frac{e\tilde{\Phi}_k}{T} \right|^2 \right\rangle$$

Discrete Particle Noise can sometimes be an issue for Particle-in-cell simulations

- Red curve: 16 particles/cell, 256 x 64 x 32
- Black curve: 8 particles/cell 128 x 128 x 64
- The mean square particle weight $<w^2>(t)$ measures discrete particle noise
- Entropy Theorem of Lee & Tang:
- χ_e is larger (in GK-units, $\rho^2 v_{th}/L_T$) Discrete particle noise a greater issue for ETG that ITG?
- Discrete particle noise looks important for *t > 700 L_T/v_{th}* in PG3EQ simulations





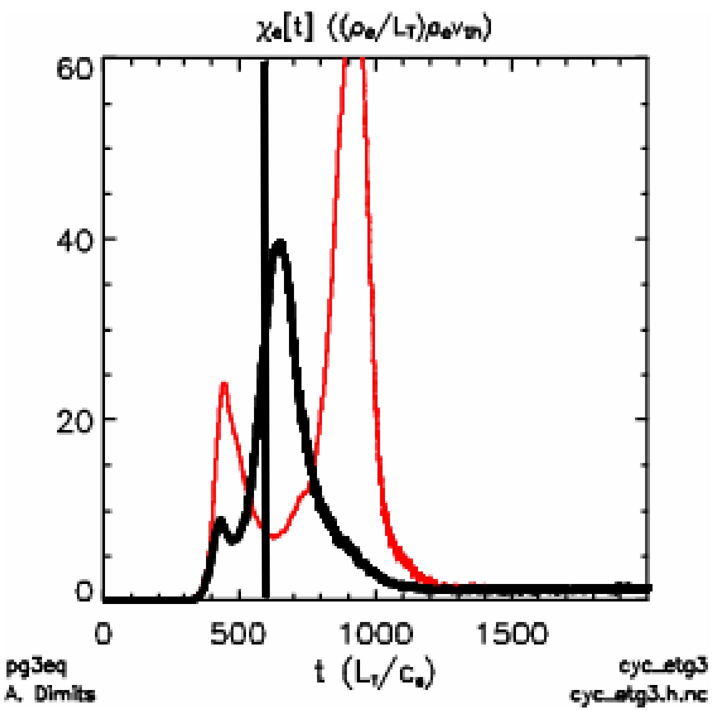
Total & Noise part of ϕ spectrum $S_\phi(k)$ at t=

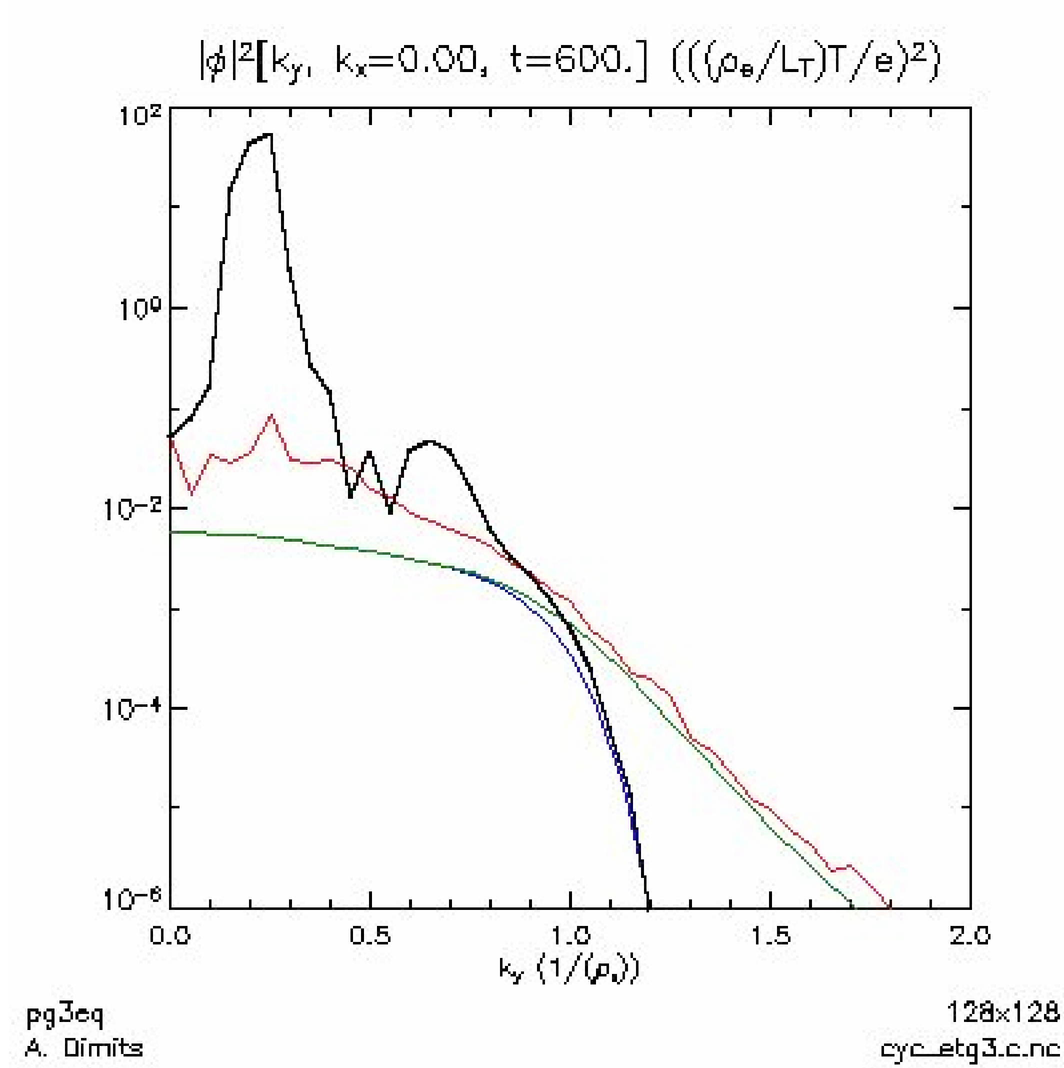
$S_\phi(k)$ from simulation:

- $S_\phi(k_x=0, k_y\ \ )$ —black
- $S_\phi(k_x\ \ , k_y=0)$ —red/brown

$S_\phi(k)$ from noise:

- $S_{noise}(k_x=0, k_y\ \ )$ —blue
- $S_{noise}(k_x\ \ , k_y=0)$ —green





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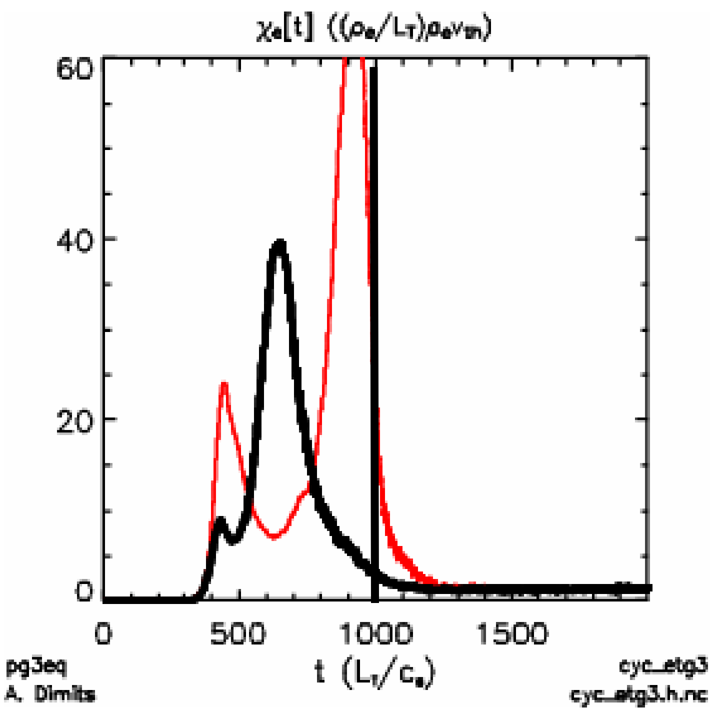

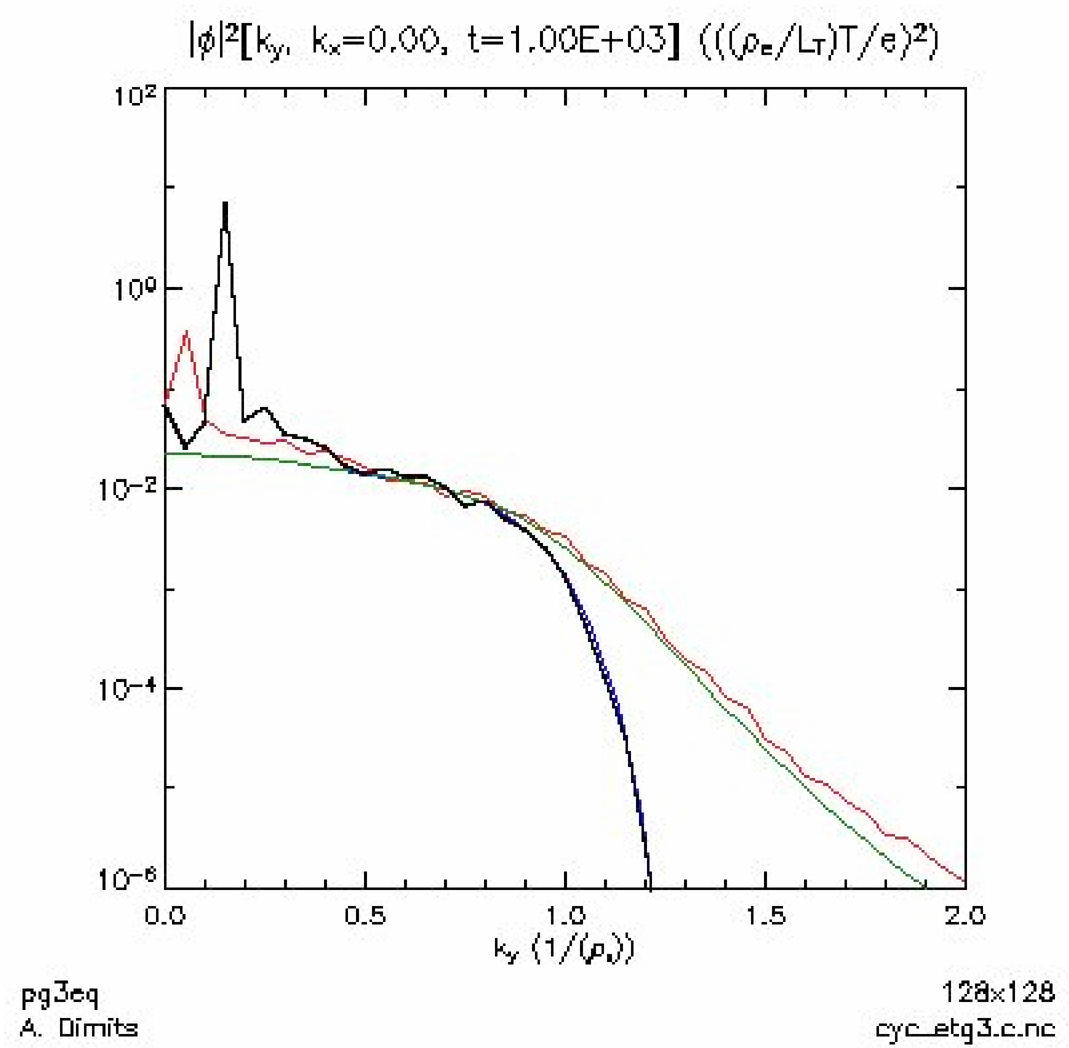

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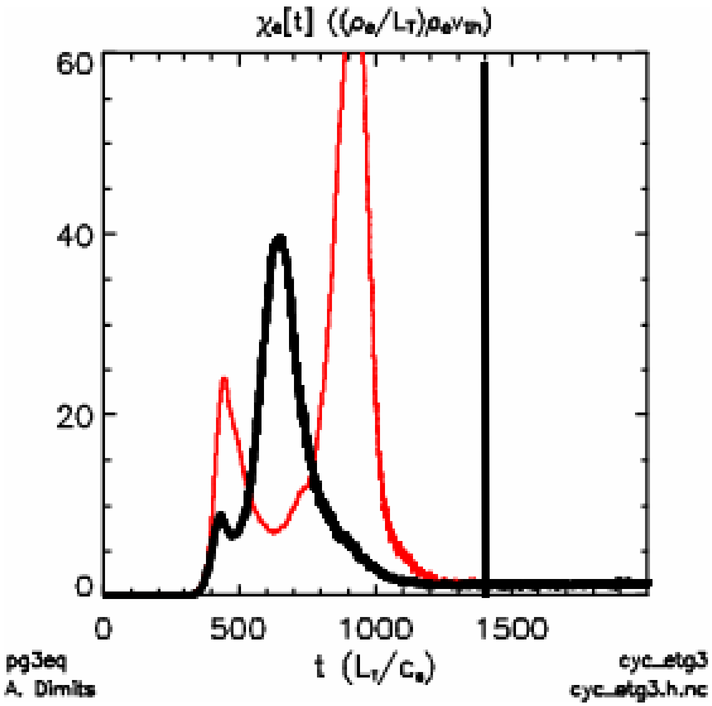

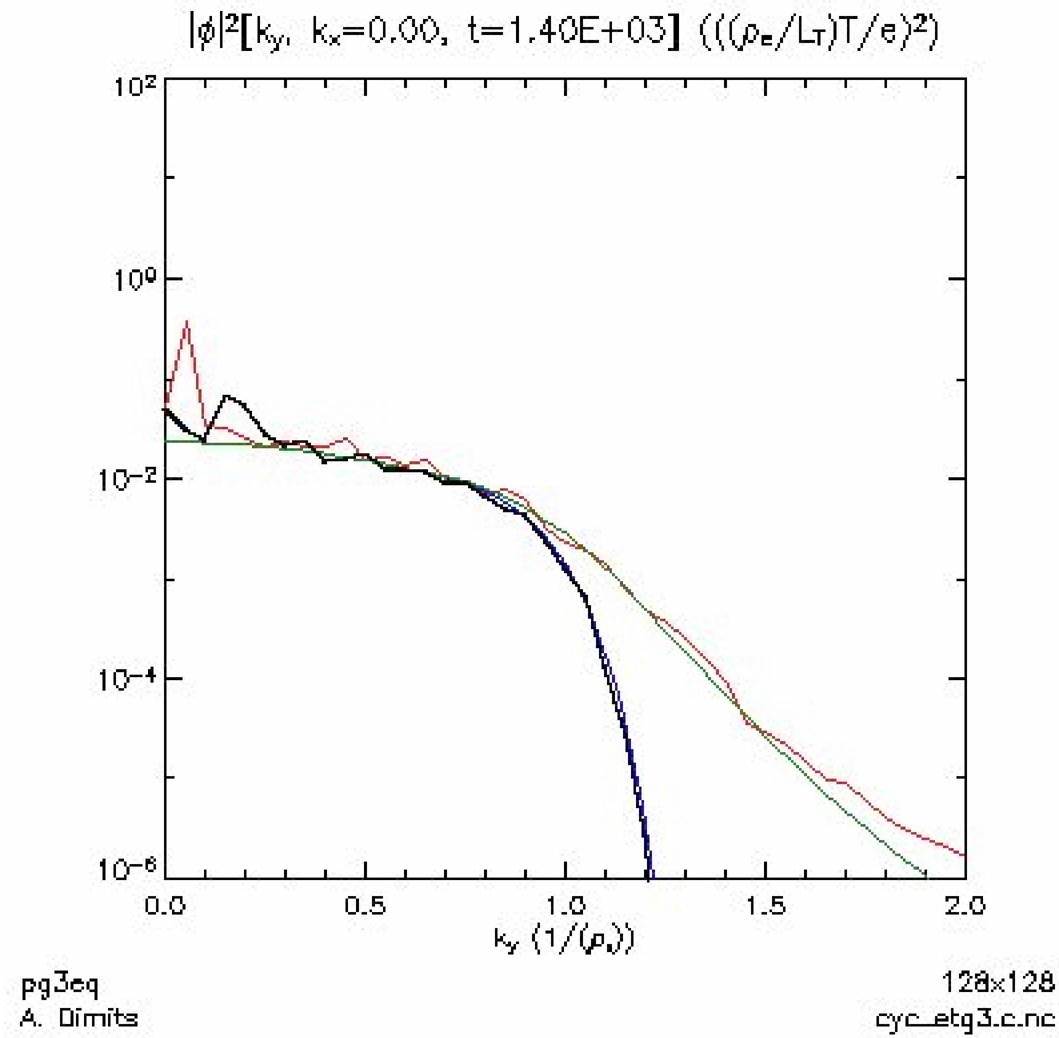

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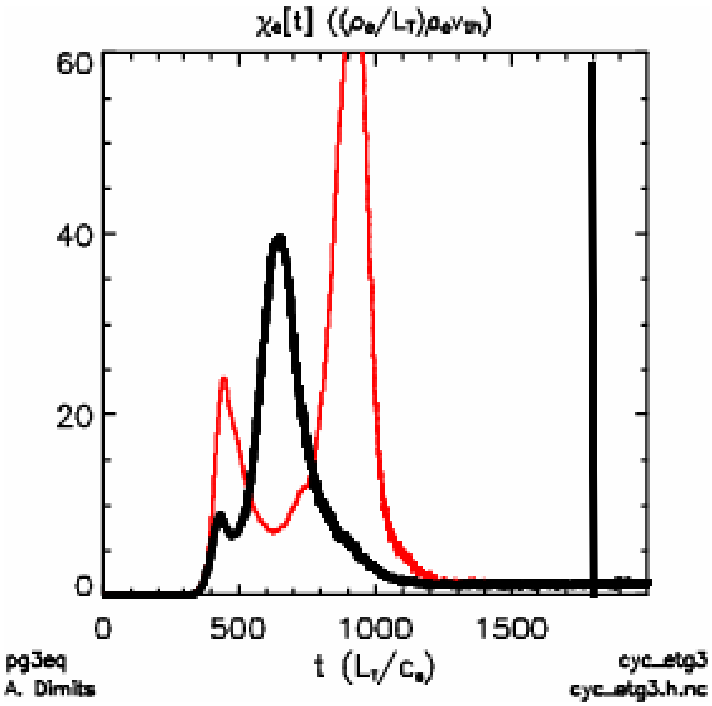
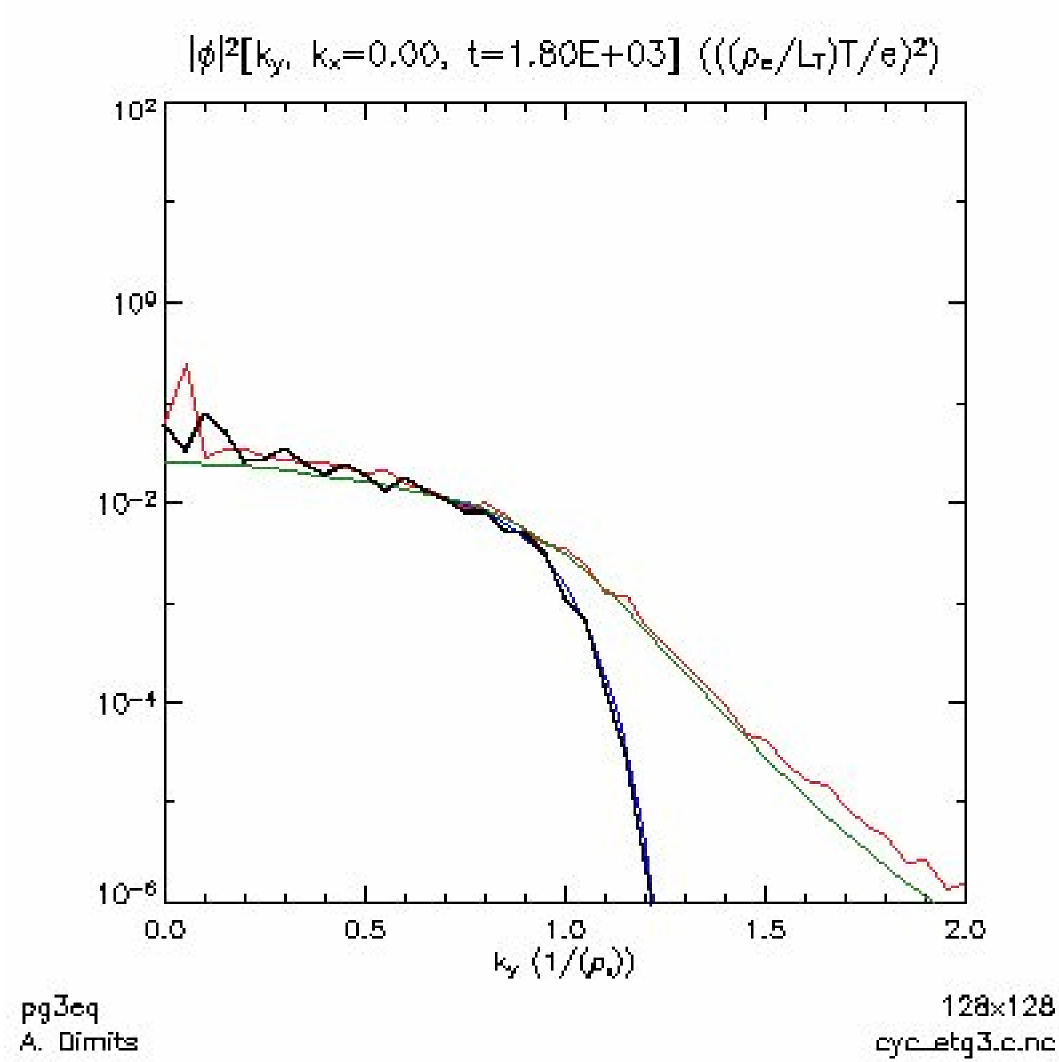




$1/(D_{noise}\, k_\perp^2) \sim 1/((2/3)\ 2\ 0.05^2) \sim 300$

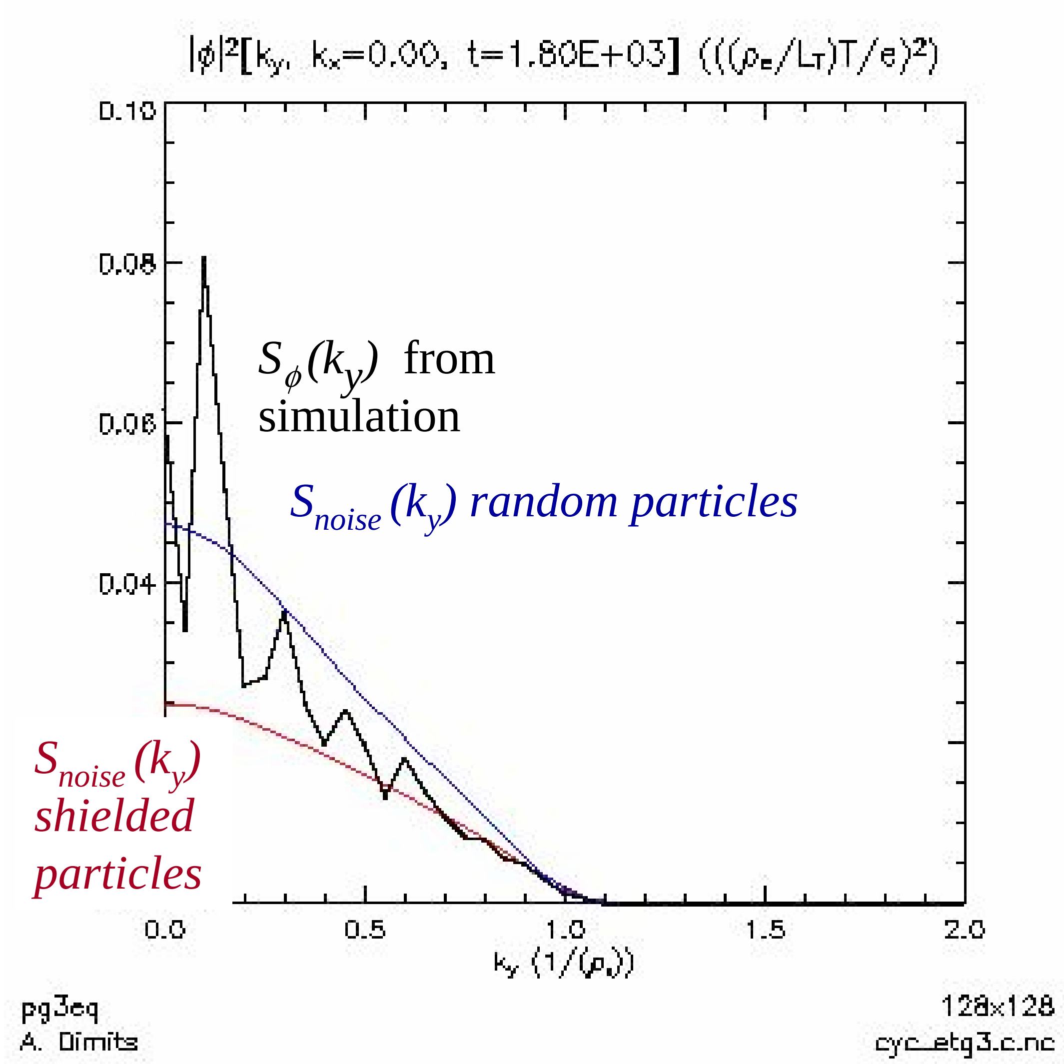
Noise calculation including shielding more accurate.

Simple noise calculation assuming randomly located particles is at most a factor of 2 higher than noise from test-particle superposition principle, including shielding cloud of other particles.

The two noise calculations approach each other for $k_y \rho_e >> 1$, where FLR makes shielding ineffective.

Simple noise from random particles slightly overpredicts observed spectrum.

Noise calculated including shielding from linear gyrokinetic dielectric fits observations better at $k_y \rho_e > 0.5$, provides lower bound on observation.



Why Particle Weights Grow in Time

$$\frac{\partial f}{\partial t} + \left(\mathrm{v}_{\|} + \mathrm{v}_{ExB}\right)\cdot \nabla f + \frac{q}{m} E_{\|} \frac{\partial f}{\partial \mathrm{v}_{\|}} = 0$$

$$\frac{Df}{Dt} = 0$$

Clever δf algorithm to reduce noise: f = smooth f_0 + particles δf

$$\delta f = \sum_j w_j(t)\,\delta(x - x_j(t))\,\delta(\mathrm{v} - \mathrm{v}_j(t))$$

$$w_j = \frac{\delta f}{f} = \frac{f - f_0}{f}$$

f = constant along a particle's trajectory. But as particle moves to position where local f_0 is different than the f where the particle started, the weight grows to represent the difference.

$$\frac{dw_{rms}^2}{dt} = \frac{2\chi_{tot}}{L_T^2}$$

entropy balance in steady state
W.W. Lee & W. Tang 88

Renormalization of Noise-induced test-particle diffusion

$$D_{noise} \approx \left\langle \mathrm{v}_{ExB}^2 \right\rangle \tau_c \propto \frac{1}{V^2} \sum_{\vec{k}} k_y^2 J_0^2 \,|\, \phi_{noise,k} \,|^2 \, \frac{1}{|\, k_\parallel \,|\, \mathrm{v}_t}$$

(in simple limits)

$$\to \frac{1}{V^2} \sum_{\vec{k}} k_y^2 J_0^2 \,|\, \phi_{noise,k} \,|^2 \, \frac{1}{|\, k_\parallel \,|\, \mathrm{v}_t + k_\perp^2 D_{noise} + \nu_{turb}}$$

$$\propto \frac{L_z}{V^2} \sum_{k_x, k_y} k_y^2 J_0^2 \,|\, \phi_{noise,k} \,|^2 \, \log\left(\frac{|\, k_{\parallel,\max} \,|\, \mathrm{v}_t + k_\perp^2 D_{noise} + \nu_{turb}}{k_\perp^2 D_{noise} + \nu_{turb}} \right)$$

Test-particle diffusion coefficient has a logarithmic divergence in the correlation time if integration is over straight-line trajectories. Use standard trick of treating trajectories as stochastic random walks consistent with diffusion. (Singularity also resolved now that I've included renormalized nonlinearities in calculation of noise spectrum.) Include also model of effect of large-scale turbulence on small scales as turbulent shearing rate ν_{turb} (but results insensitive to this).

(More accurate calculation possible by integrating test particle trajectory over 2-time 2-point correlation function from spectrum of $<|\Phi_{noise,k}|^2>(\omega)$. Integrals doable with variations of contour integrals used for Kramers-Kronig relations and Fluctuation-Dissipation theorem.)

Noise-induced test-particle diffusion

Integrate over properly weighted (ω,k) spectrum of noise fluctuations to find test-particle diffusion coefficient. Used a renormalized propagator to resolve a logarithmic divergence in the correlation time. (Have also included est. of turbulent shearing decorrelation, etc.)

$$D_{noise} \propto \left\langle \left(\frac{e\Phi_{noise}}{T} \right)^2 \right\rangle \frac{a_\parallel}{L_T} \frac{b_{\max}^2}{\log(1+2b_{\max})} \log\left(1 + c_0 \frac{(1+b_{\max})}{(2+b_{\max})} \frac{k_{\parallel\max} \mathrm{v}_{ti}}{D_{noise} k_{\perp\max}^2} \right)$$

Note: in large D_{noise} limit, this gives $D_{noise} \sim \Phi_{noise} \sim w_{rms}$, not $\sim w_{rms}^2$. Essential to get linear growth of D_{noise} as seen in Dimits' particle # scan.

$$\frac{dw_{rms}^2}{dt} \propto (D_{turb} + D_{noise}) \sim w_{rms}$$

$$\frac{dw_{rms}}{dt} \propto const.$$

Noise-induced test-particle diffusion

$$D_{noise} \propto \left\langle \left(\frac{e\Phi_{noise}}{T} \right)^2 \right\rangle \frac{a_{\parallel}}{L_T} \frac{b_{\max}^2}{\log(1+2b_{\max})} \log\left(1 + c_0 \frac{1+b_{\max}}{2+b_{\max}} \frac{k_{\parallel\max} \mathrm{v}_{ti}}{D_{noise} k_{\perp\max}^2} \right)$$

But it is also important to keep the logarithmic corrections so that in the low D_{noise} limit, $D_{noise} \sim \Phi_{noise}^2 \sim w_{rms}^2$ which gives a much smaller D_{noise} (otherwise noise would dominate from near the beginning of the simulation).

Physics Basis & Caveats

Test particle superposition principle: Particle correlations accounted for by dielectric shielding. Test particles with their shielding clouds can be assumed random and independent.

Used gyrokinetic plasma dielectric in uniform plasma limit to calculate shielding. For a turbulent system, should really use a renormalized dielectric that accounts for turbulent decorrelation. But present calculation perhaps not too bad because noise spectrum is flat and D_{noise} dominated by high $k_\perp$ and high $k_{||}$, where $k_{||} v_t \gg J_0 \omega_*$ so plasma inhomogeneities perhaps not too important.

On the other hand, there might be some interesting modifications in some case, perhaps in weak turbulence regimes or where noise is just about to dominate over the turbulence, where there is a weakly damped normal mode that might be strongly excited by noise.

A few analytic approximations made in velocity integrals, FLR effects, etc. to get a semi-analytic result. In process of redoing calculation more accurately.

Discrete Particle Noise can sometimes be an issue for Particle-in-cell simulations

- Red curve: 16 particles/cell, 256 x 64 x 32
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- The mean square particle weight <w^2>(t) measures discrete particle noise
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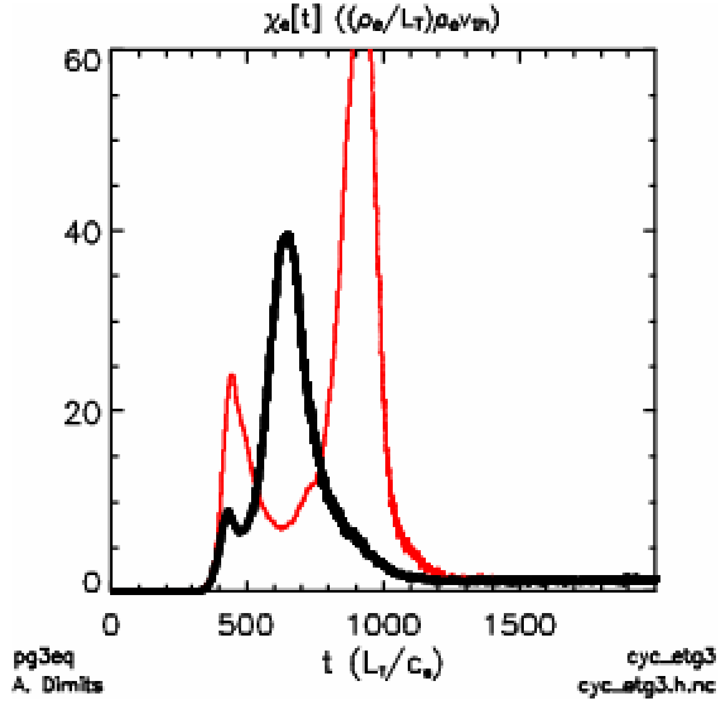
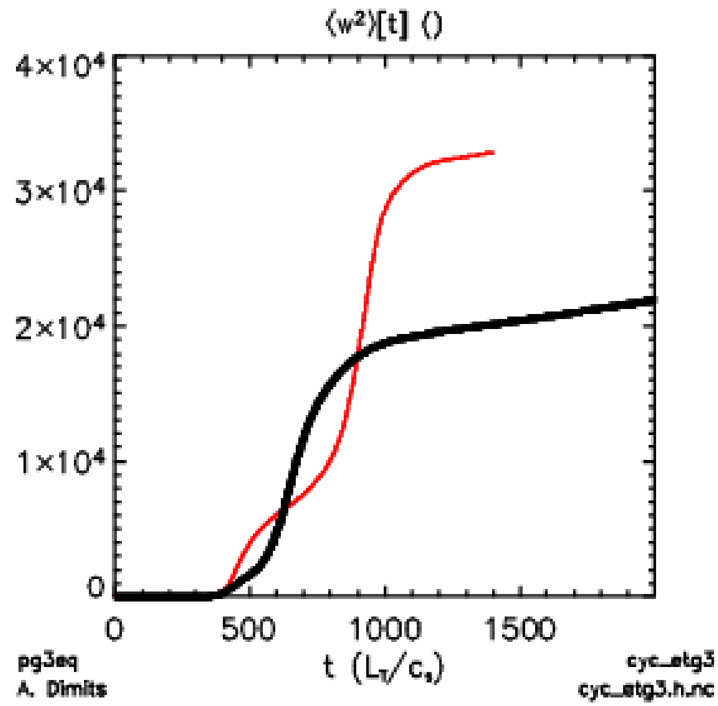




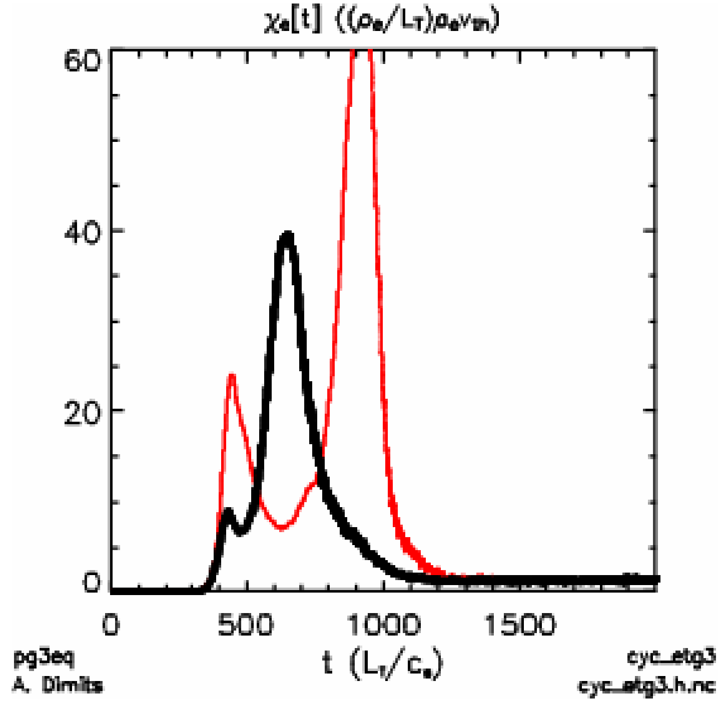
Comparison of actual $\chi_e(t)$ with model of mean χ_e (t) and χ_{noise} (t) (given actual $<w^2>(t)$ to calc $\chi_{noise}(t)$)

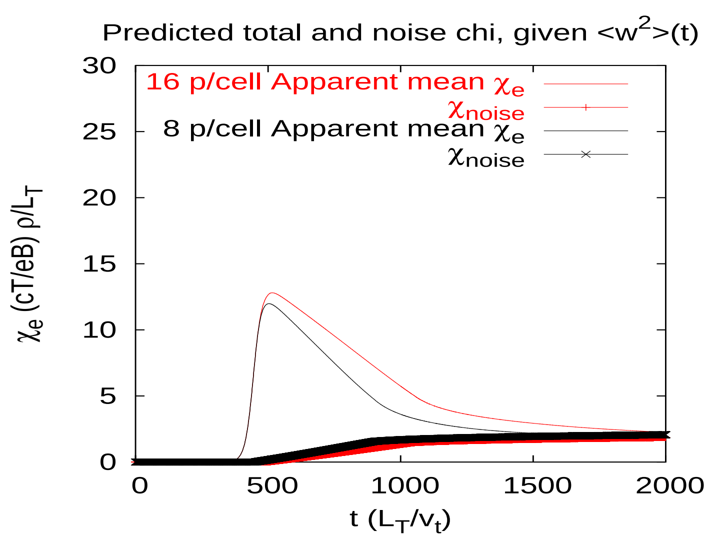
Given $<w^2>(t)$, can calculation $\chi_{noise}(t)$ (lower panel), and use that to predict apparent mean $\chi_e(t)$ (in $< w^2 >=0$ limit, would recover Jenko & Dorland's χ_e~13).

Actual χ_e (t) fluctuates around this due to initial transient overshoots, or occasional formation of long streamers. Jenko & Dorland find variability drops as box size increases.

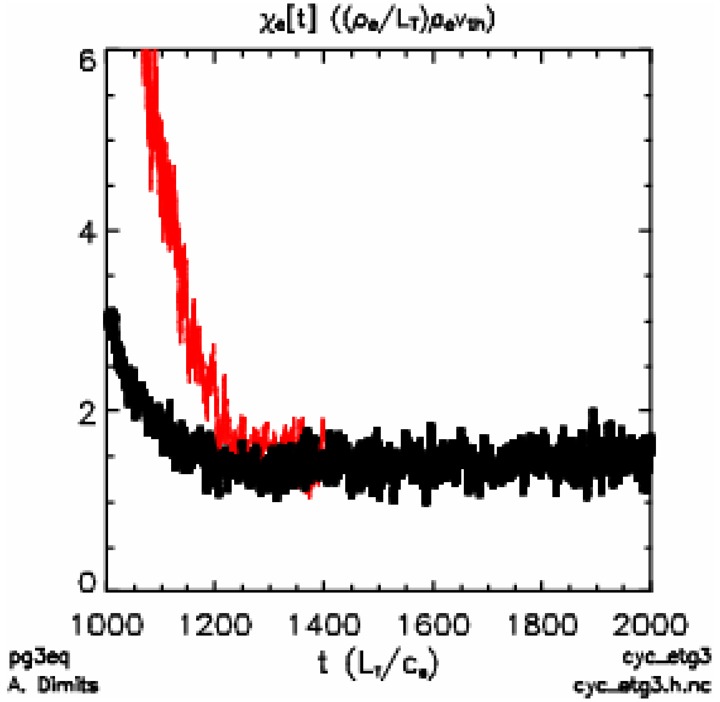
Speed of transition from turbulence-dominated state to noise-dominated state depends on # of particles, filtering, and size of transient overshoots that drive $< w^2 >$ harder. Slower transitions might occur in Z. Lin's simulations which average over a large number of eddies and so should show less variability.

Working on improved model of shearing of noise by turbulence, may sharpen transition between states.





Comparison of actual χ_e (t) with model of mean χ_e (t) and χ_{noise} (t) (given actual <w^2>(t) to calc χ_{noise}(t)) (zoom on later stages)



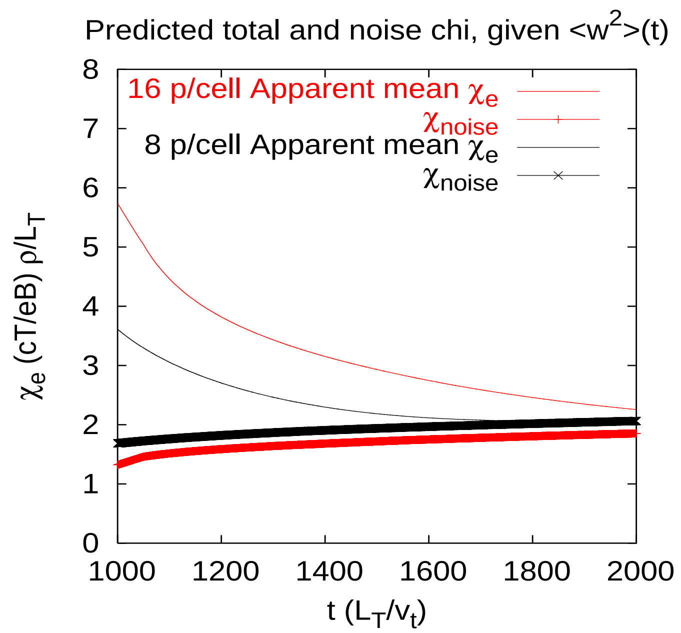
Given <w^2>(t), can calculation χ_{noise} (t)

Final $\chi_{noise} \sim 2$, gives

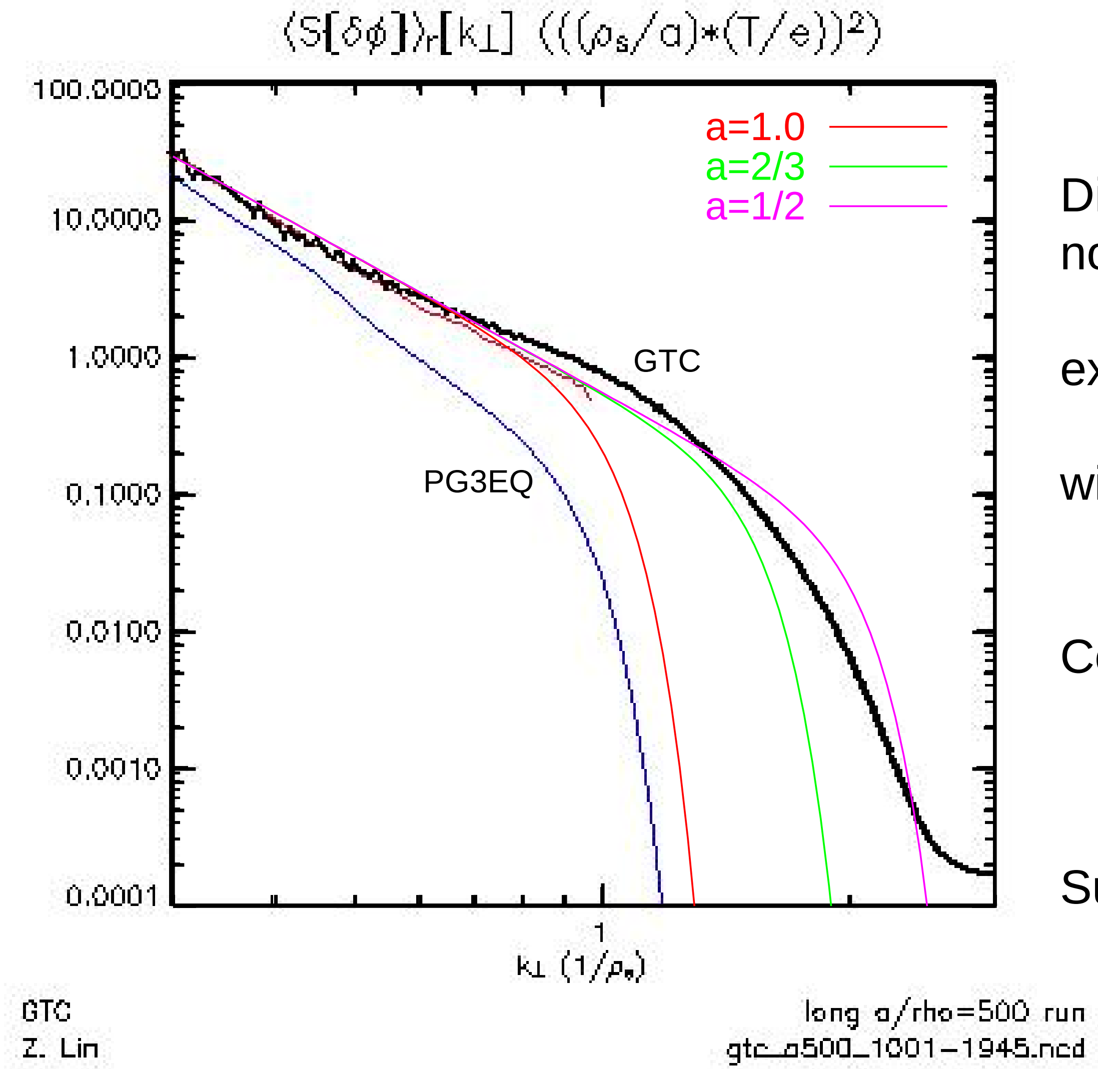
$D_{noise}\, k_\perp^2 \sim (2/3)\, 2\; 0.3^2 \sim 0.12$
(at $k_\perp = 0.3$) is much larger than
the growth rate of the fastest growing mode.

Even at $k_\perp \sim 0.15$, $D_{noise}\, k_\perp^2 \sim 0.03$
is greater than growth rate at $k_y \sim 0.15$.

Thus one expects turbulence to be nearly suppressed, and most of the apparent chi is actually due to noise



Estimate of Particle Filtering Parameters



Dimits PG3EQ uses spectral / non-local filter

$\exp(-(ak_y\Delta y)^4)/(1+(ak_x\Delta x)^4)/(1+ak_z\Delta z)^4$

with a=1

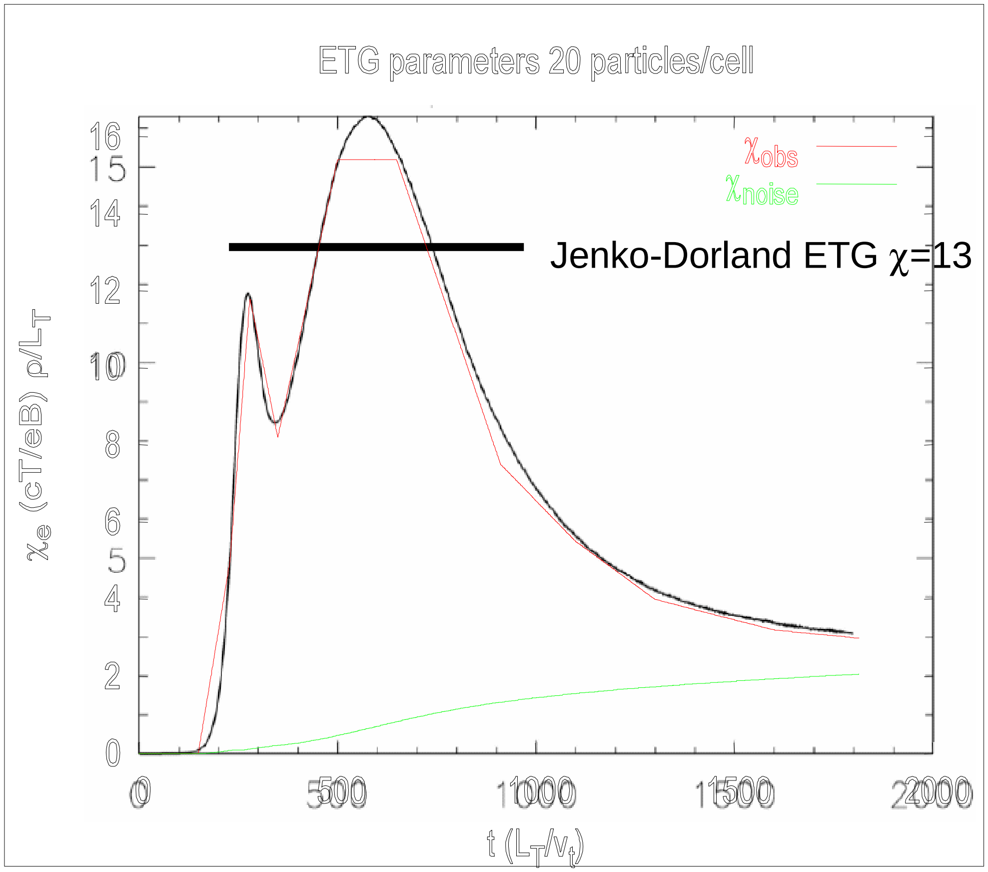
Comparison of GTC spectra with

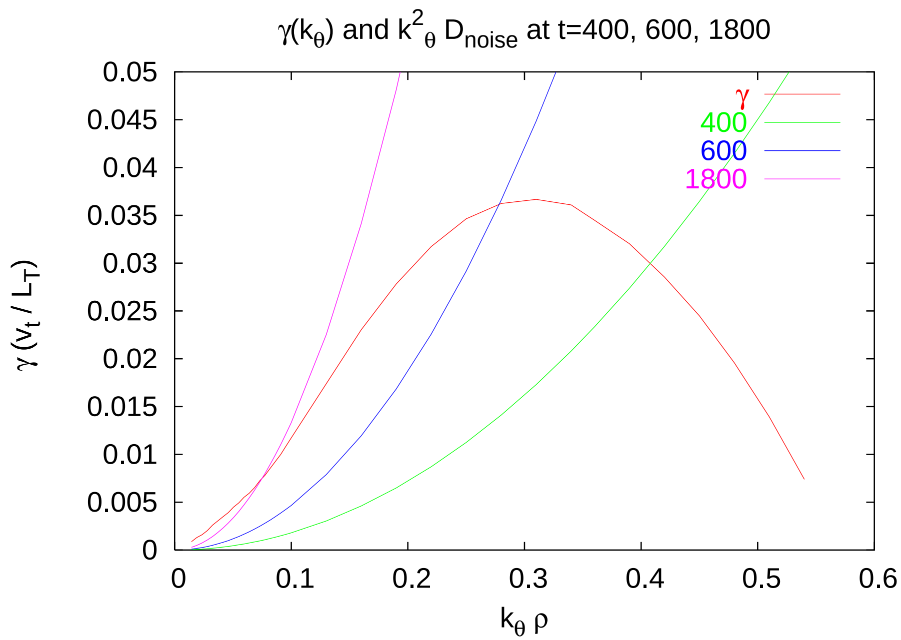
$S \sim (1/k^{3.3}) \exp(-(ak_y\Delta y)^8)$

Suggests filter width a~2/3 roughly.

Data for ITG spectra from Z. Lin & A. Dimits provided to W. Nevins for a paper showing similarities between codes for ITG turbulence. Small differences at low k not significant, depend on time averaging.

$D_{\text{noise}}\, k_\perp^2$ >~ γ by end of Z. Lin ETG simulation →Turbulence suppressed





Growth rate for Cyclone base case with s-alpha geometry.
$\gamma\uparrow$ ~20% with real circles.

Believed to be χ_{etg}(t). from Z.Lin's IAEA 2004 slides
http://www.cfn.ist.utl.pt/20IAEAConf/presentations/T5/2T/5_H_8_4/Talk_TH_8_4.pdf

Toy model of interaction of turbulence and noise

$$\frac{dD_{turb}}{dt} = 2\left(\gamma_{lin} - k_{r0}^2 D_{turb} - (k_{r0}^2 + k_{y0}^2) D_{noise}\right) D_{turb}$$

Steady state solution: $$D_{turb} = \frac{\gamma_{lin} - k_{\perp 0}^2 D_{noise}}{k_{r0}^2}$$

Normalized ETG turbulence larger than ITG turbulence modeled by longer ETG streamers (lower k_{r0}). Large ETG anisotropy $k_{\perp 0}^2 / k_{r0}^2 >> 1$ means $D_{tot} = D_{turb} + D_{noise}$ can drop as D_{noise} increases.

$$D_{noise} \sim \frac{w_{rms}}{\sqrt{N_{cell}}}$$

D_{noise} grows as
avg particle weight w_{rms} grows in time

$$\frac{dw_{rms}^2}{dt} = \frac{2\chi_{tot}}{L_T^2} = \frac{3(D_{turb} + D_{noise})}{L_T^2}$$

Particle noise is larger for ETG
Because of larger initial values of D_{turb}
(entropy balance, W.W. Lee & Tang)

Toy model of interaction of turbulence and noise

Steady state ("slow manifold") solution: $$D_{turb} = \frac{\gamma_{lin} - k_{\perp 0}^2 D_{noise}}{k_{r0}^2}$$

Dorland & Jenko found normalized ETG turbulence is much larger than ITG turbulence, understood in terms of secondary instabilities and differences in zonal flow dynamics (Rogers et al., Jenko & Dorland 2002), which leads to ETG streamers being much more elongated (lower k_{r0}) than ITG streamers.

Model this here just by using lower k_{r0} for ETG than for ITG. I.e., choose k_{r0} so that true $D_{turb} \sim \gamma_{lin} / k_{r0}^2$. (Braginskii heat flux $\chi_{turb} \equiv (3/2)\, D_{turb}$)

Large ETG anisotropy $k_{\perp 0}^2 / k_{r0}^2 >> 1$ means $D_{tot} = D_{turb} + D_{noise}$ can drop as D_{noise} increases.

Alternative interpretation

$$\frac{dD_{turb}}{dt} = 2\left(\gamma_{lin} - k_{r0}^2 D_{turb} - (k_{r0}^2 + k_{y0}^2) D_{noise}\right) D_{turb}$$

$$\frac{dD_{turb}}{dt} = 2\left(\gamma_{lin} \quad - \alpha D_{turb} \quad - (\alpha + \beta) D_{noise}\right) D_{turb}$$

Alternatively, just think of two fitting coefficients α & β to parameterize expected nonlinear decorrelation / saturation. Fit α & β for ITG to Dimits simulations. Fit α for ETG to Jenko/Dorland simulations. Use to predict noise effects on PIC ETG simulations.

Model parameters used

Determine k_{r0} to give true $D_{turb} = \gamma_{lin} / k_{r0}^2$, with γ_{lin} =0.038 v_t/L_T
(Braginskii heat flux $\chi_{turb} = (3/2)\ D_{turb}$)

Dimits/Cyclone ITG χ_{turb} = 2.2 (cT/eB) ρ/L_n
= 0.71 (cT/eB) ρ/L_T gives $k_{r0}\rho = 0.28$

Jenko & Dorland 2002 ETG $\chi_{turb} \sim 13$ (cT/eB) ρ/L_T

Using Fig.4 of Dimits et al. 2000, extrapolate from r/R=0 to r/R=0.18 to slight enhancement $\chi_{turb} \sim 17$ (cT/eB) ρ/L_T. Gives $k_{r0}\rho = 0.058$ and radial correlation length $1/k_{r0} \sim 17\ \rho$ (roughly comparable to Jenko's observed ETG eddies, and still much shorter than Z. Lin's very extended eddies.)

Z. Lin et al IAEA 2004 ETG χ_{turb} = 3.2 (cT/eB) $\rho/L_T = 4.5 \times \chi_{ITG}$

For both ITG/ETG, $k_{y0}\rho \sim 0.15$ (typical downshift from fastest growing mode)

Possible Effects of geometry

Z. Lin's simulations using real circular geometry (including finite aspect ratio effects on grad(B) drifts which are neglected in simple s-alpha geometry). Usually enhances growth rates and turbulence level over s-alpha result.

If R in following is interpreted as local $R=R_0+r$, in bad curvature region:

$$D_{turb} \approx \frac{cT}{eB}\frac{\rho}{R}\left(\frac{R}{L_T} - \left(\frac{R}{L_T}\right)_{crit}\right) \approx \frac{cT}{eB}\frac{\rho}{R_0+r}\left(\frac{R_0+r}{L_T} - 4\right)$$

Then extrapolate from J&D s-alpha result to Lin geometry to get a further 20% enhancement, to expected $\chi_{turb} \sim 20\ (cT/eB)\ \rho/L_T$.

However, will use expected ETG $\chi_{turb} = 17\ (cT/eB)\ \rho/L_T$, and $k_{r0}\rho = 0.058$, in model to predict noise effects.

ETG eddies are radially extended streamers

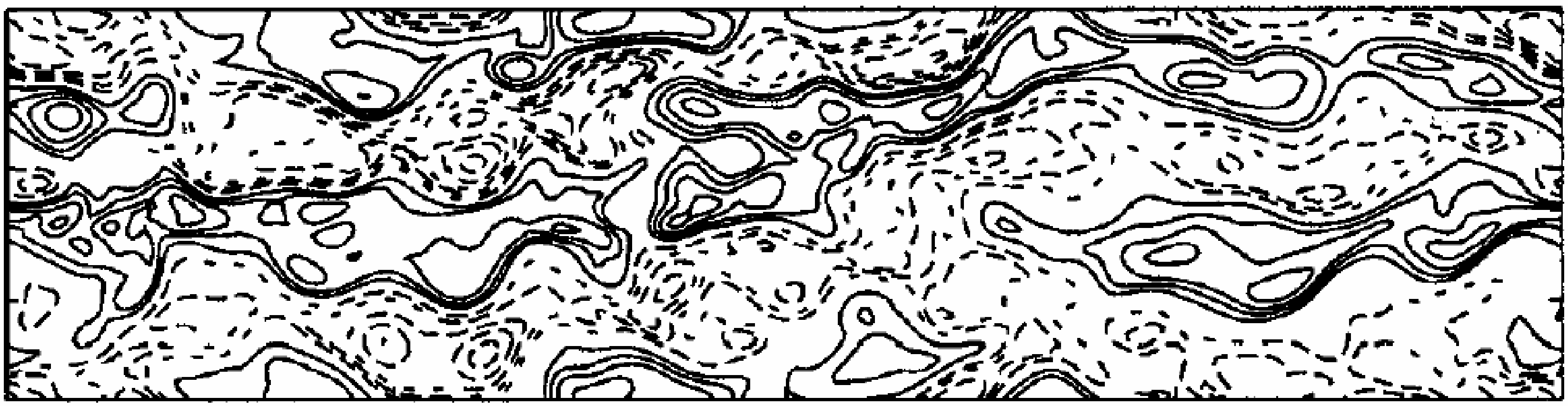
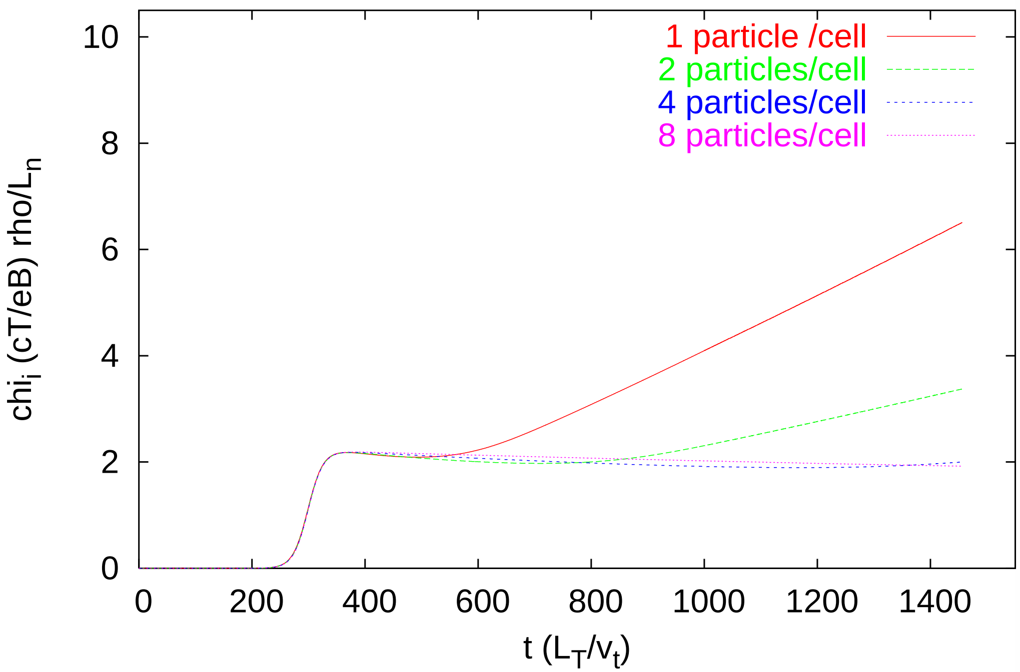


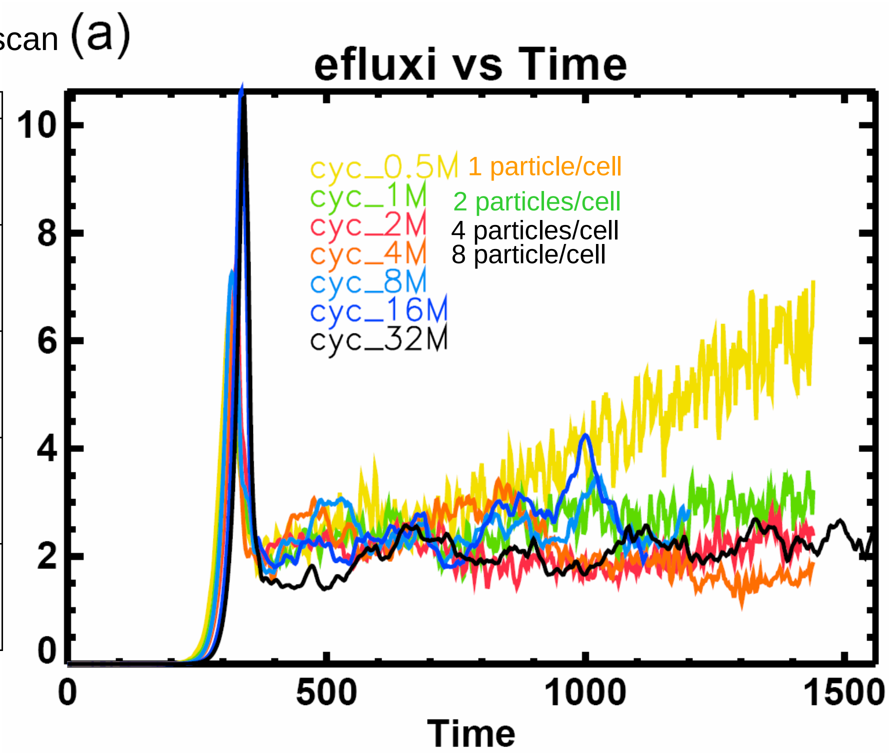
FIG. 2. Characteristic ϕ contours in the outboard x-y plane. This snapshot was taken at the end of the ETG run shown in Fig. 1. The figure is $256\rho_e \times 64\rho_e$.

(Jenko & Dorland 2000)

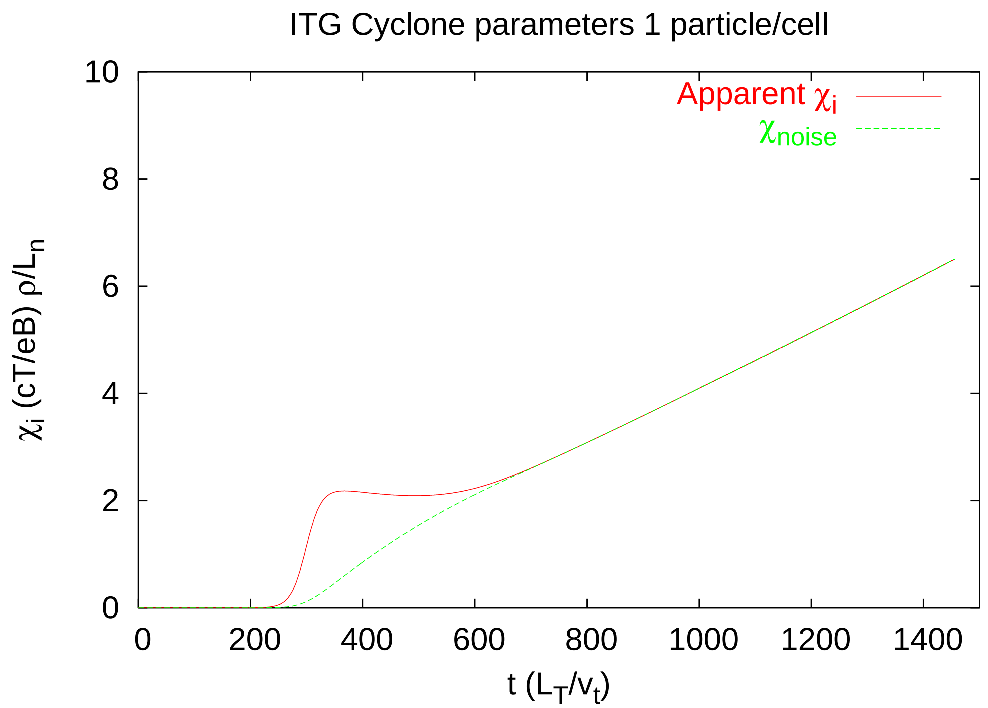
Model of noise-induced diffusion agrees well with particle scan in ITG simulation (Dimits 98/2000)

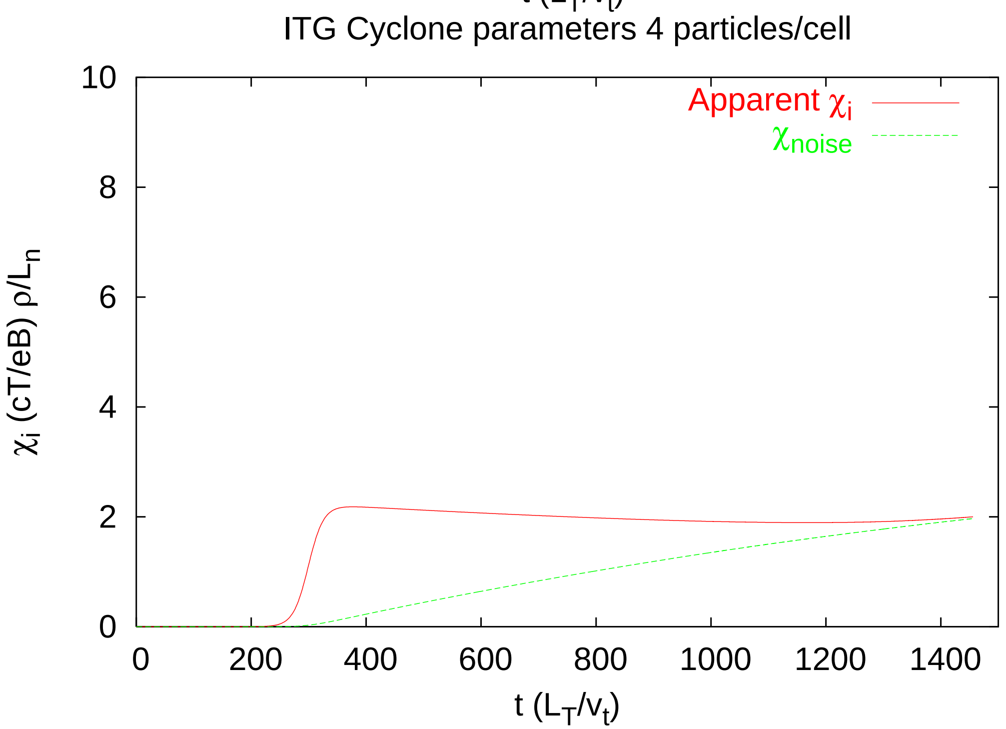
Apparent chi_i (incl. noise) for ITG Cyclone parameters, particle scan





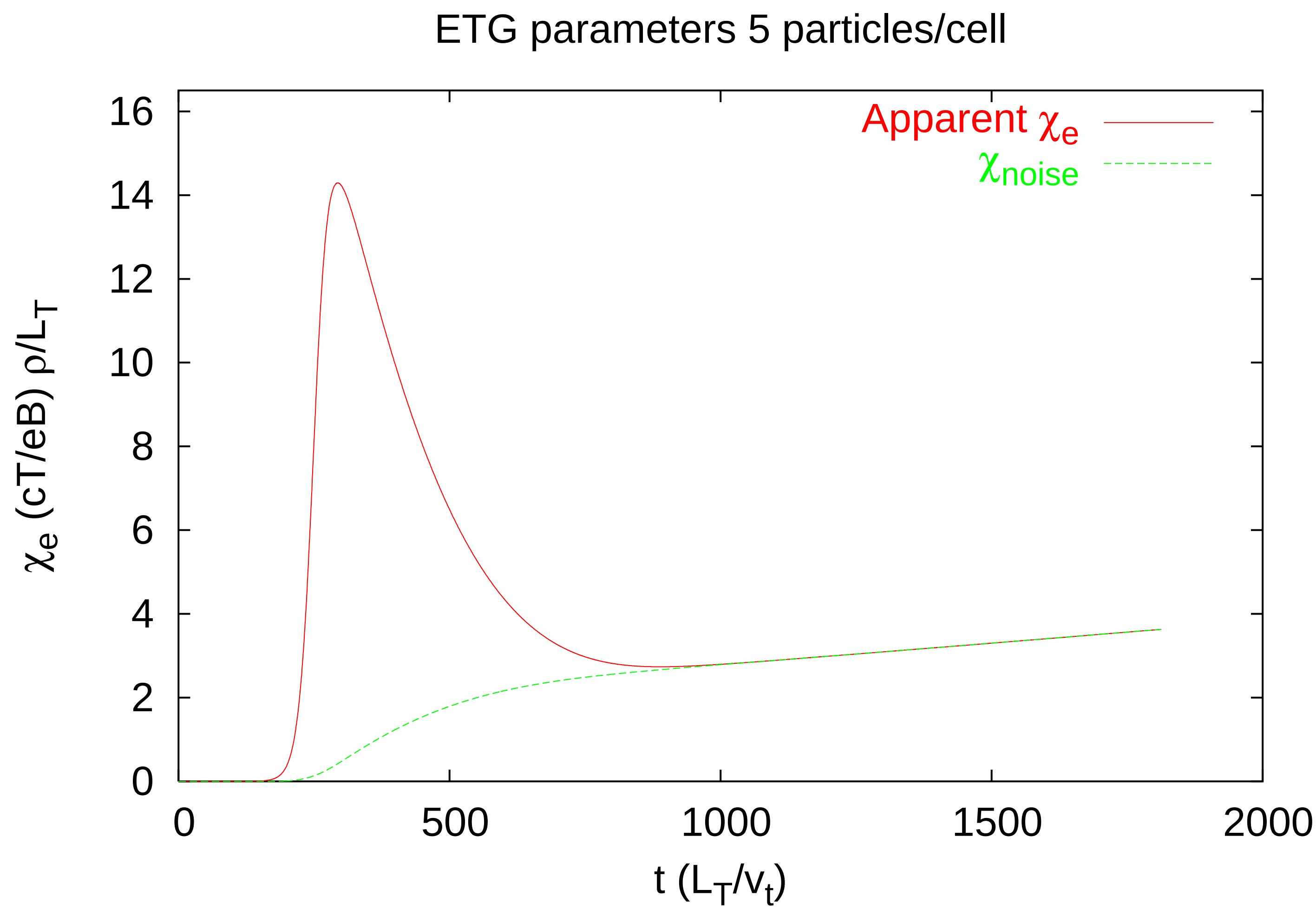
One adjustable order-unity coefficient in model for D_{noise} (renormalized decorrelation time only roughly estimated at present), fit to match Dimits' 1 particle/cell case





Model tracks effects of noise on particle scan for ITG turbulence

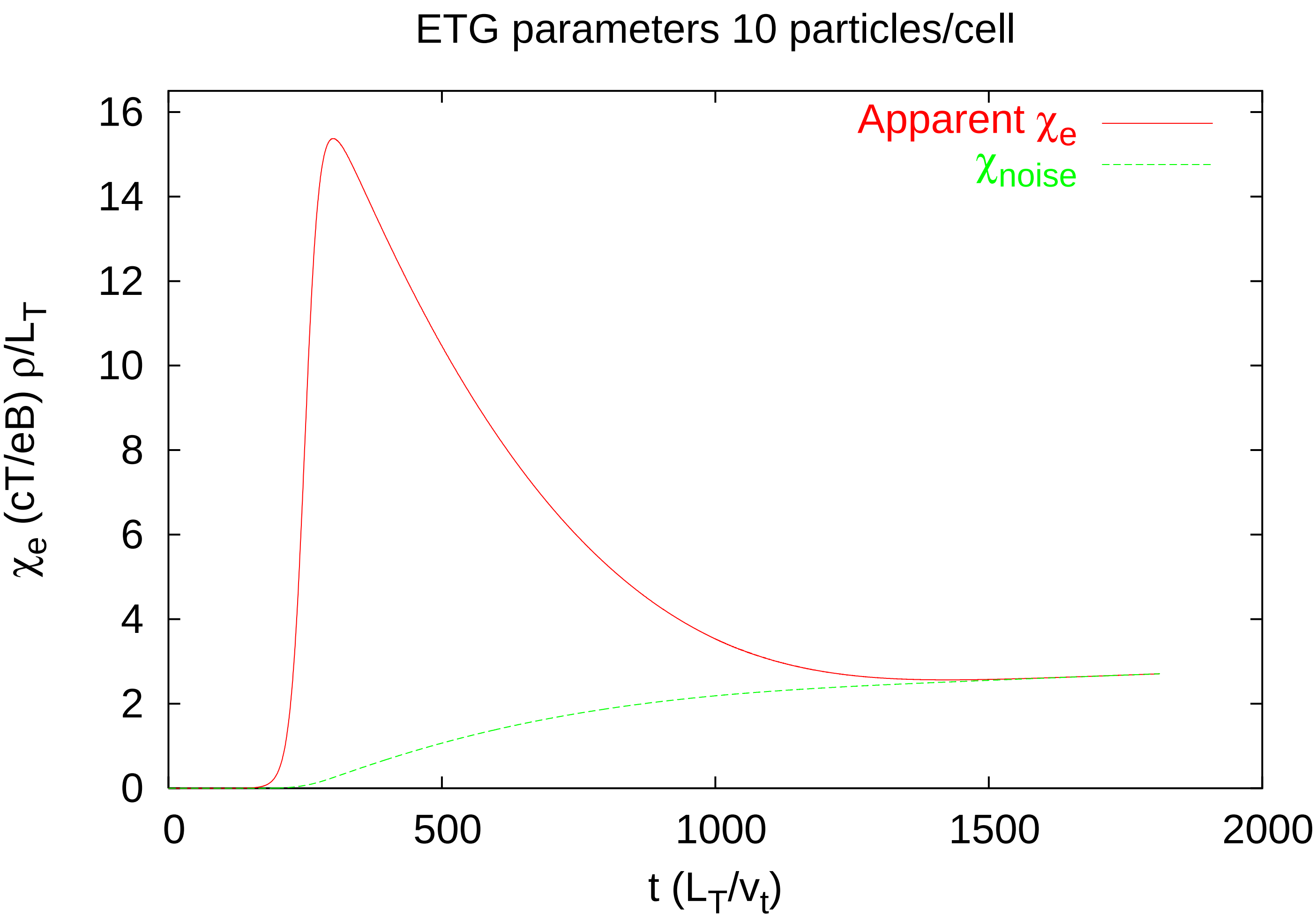
Model predicts significant suppression of ETG turbulent χ due to noise, for 5, 10, and even 20 particles per cell. (χ=17 w/o noise)



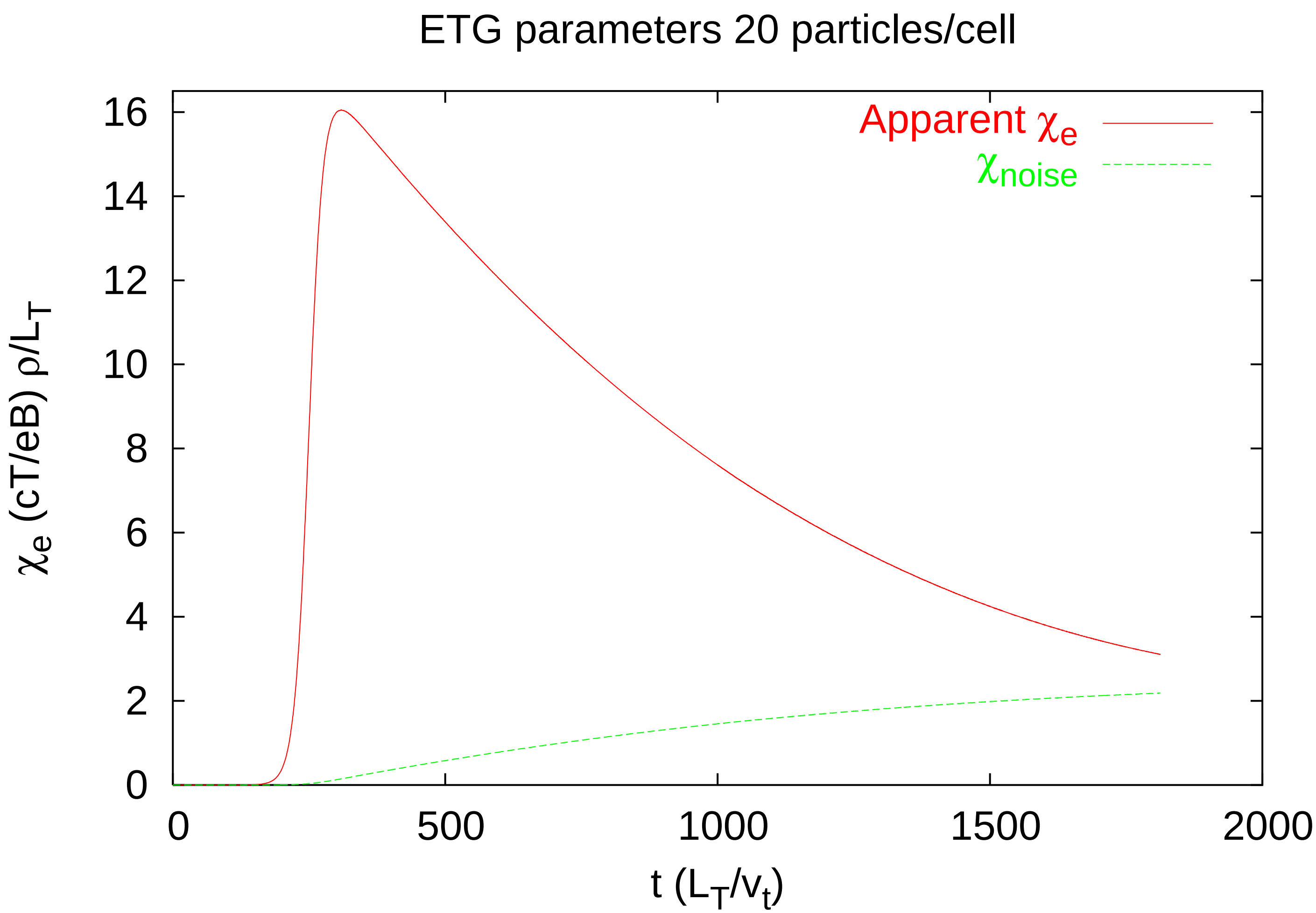
ETG is reduced by noise while ITG is enhanced by noise because the very long thin eddies of ETG are more sensitive to noise diffusion.

Initial large peak in χ probably Ignored as a transient overshoot

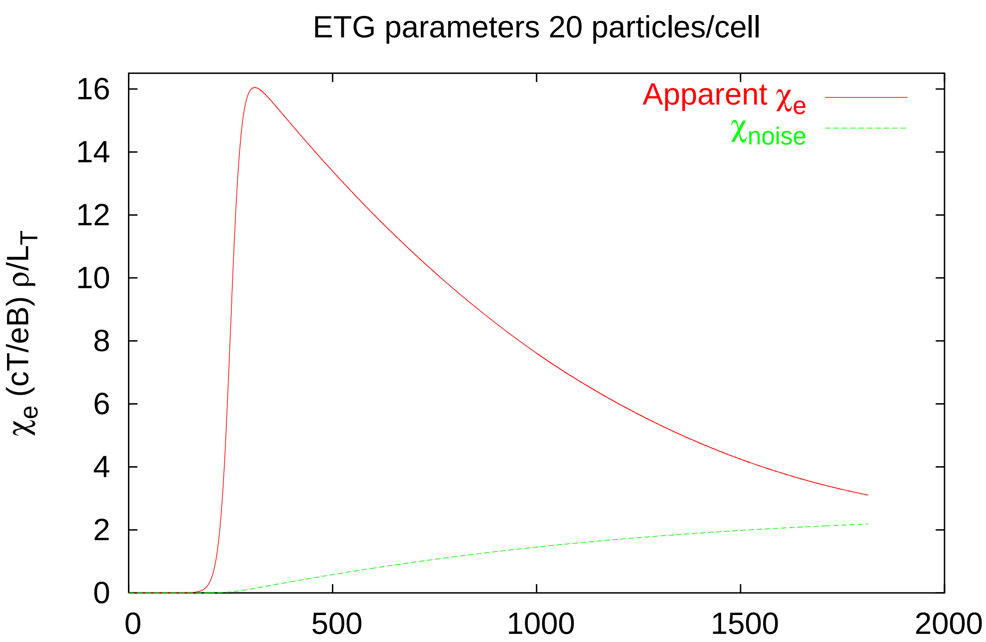
In the absence of noise, assumed that ETG χ would have risen to 17 for these parameters.
Noise suppresses to χ ~ 3, explains low values in Z. Lin simulations.



Model predicts significant suppression of ETG turbulent χ due to noise, for 5, 10, and even 20 particles per cell.

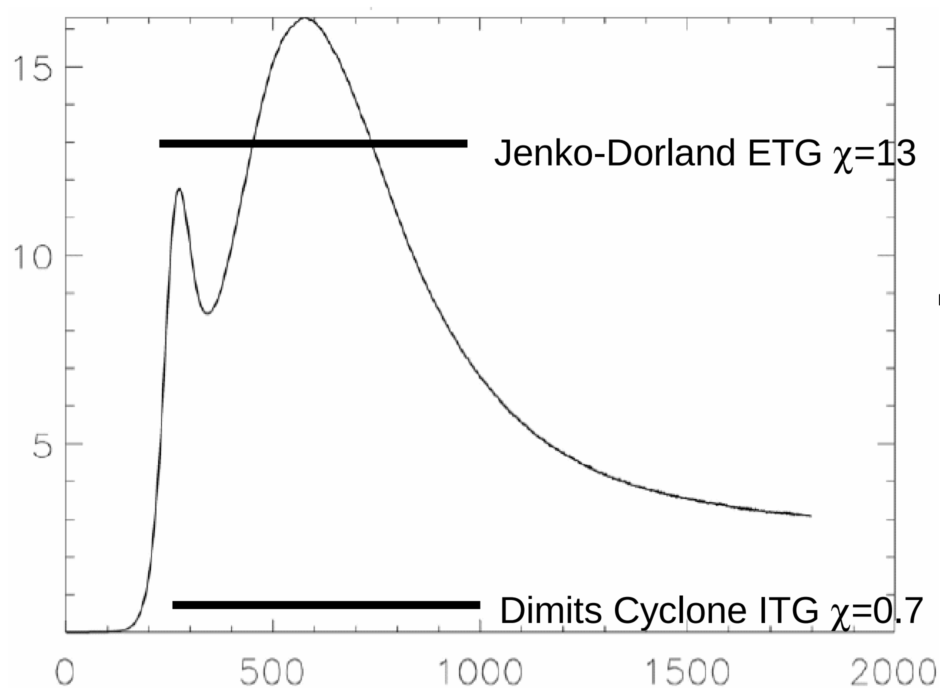


Increasing # of particles just lengthens the duration of initial "transient", but eventually gives same apparently converged χ~3. Initial overshoot (missing here) in real turbulence would drive noise harder, cause χ to drop to apparent "convergence" more quickly. Convergence issues obscured by usual long-time variability in real turbulent fluxes, and long initial period of high χ perhaps dismissed as time needed for zonal flows to eventually grow up.



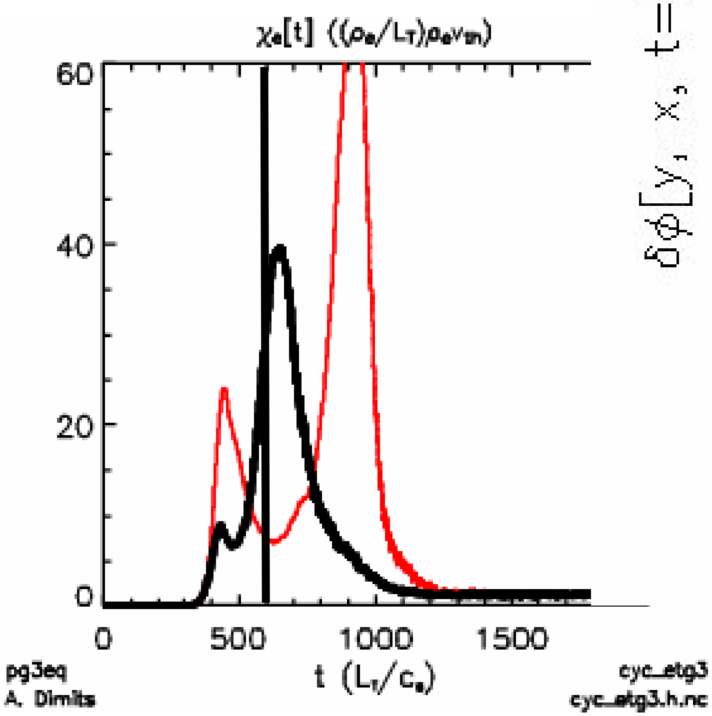
Toy model qualitatively similar to Z. Lin's $\chi_{etg}(t)$. Predicts significant suppression of ETG turbulence for typical GTC parameters, even for 20 particles/cell.

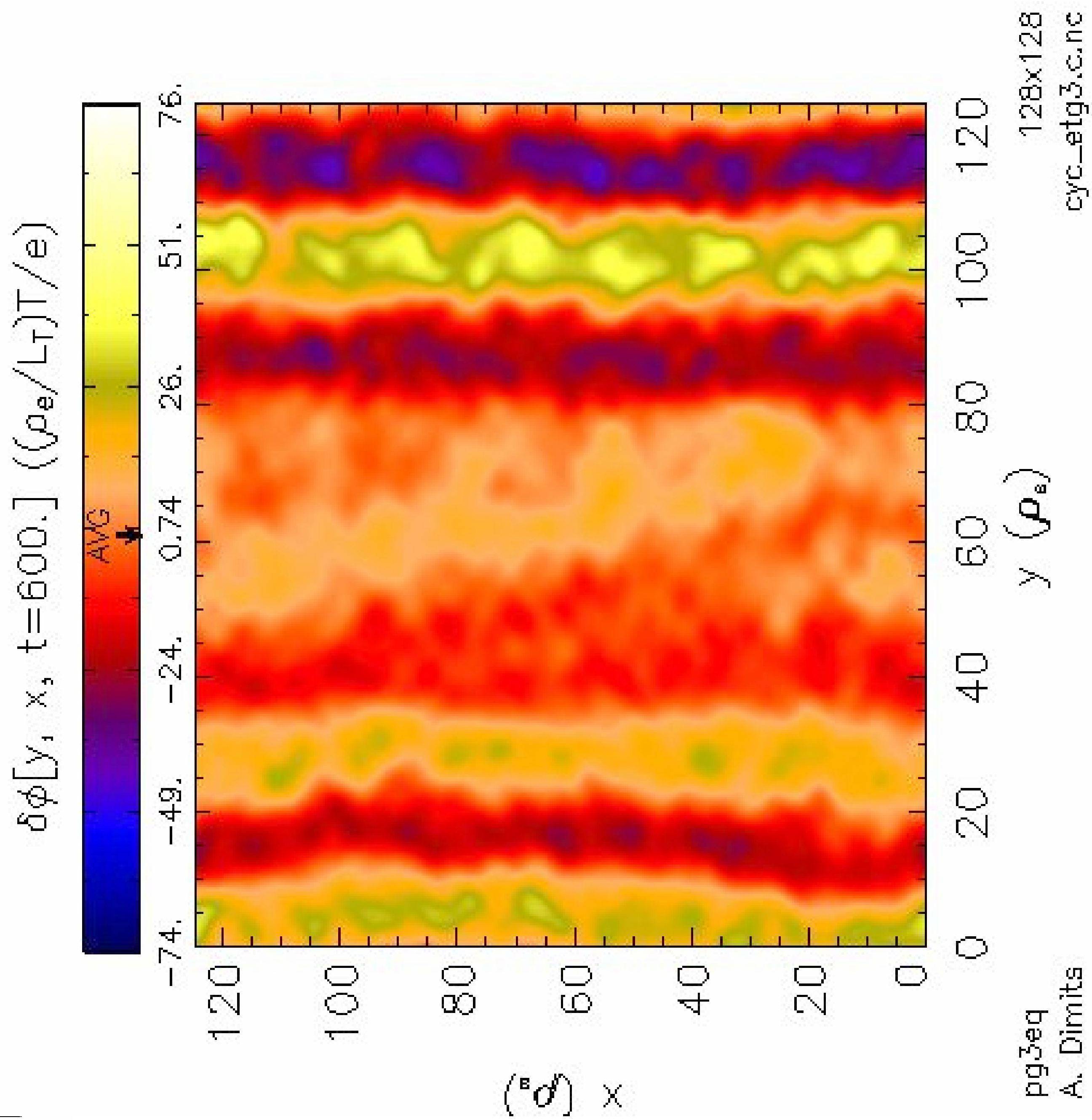
Z. Lin's reported value of $\chi_{etg} \sim 3$ is expected level for isotropic noise diffusion that gives $k_\perp^2\ \chi_{etg} \sim \gamma_{lin}$

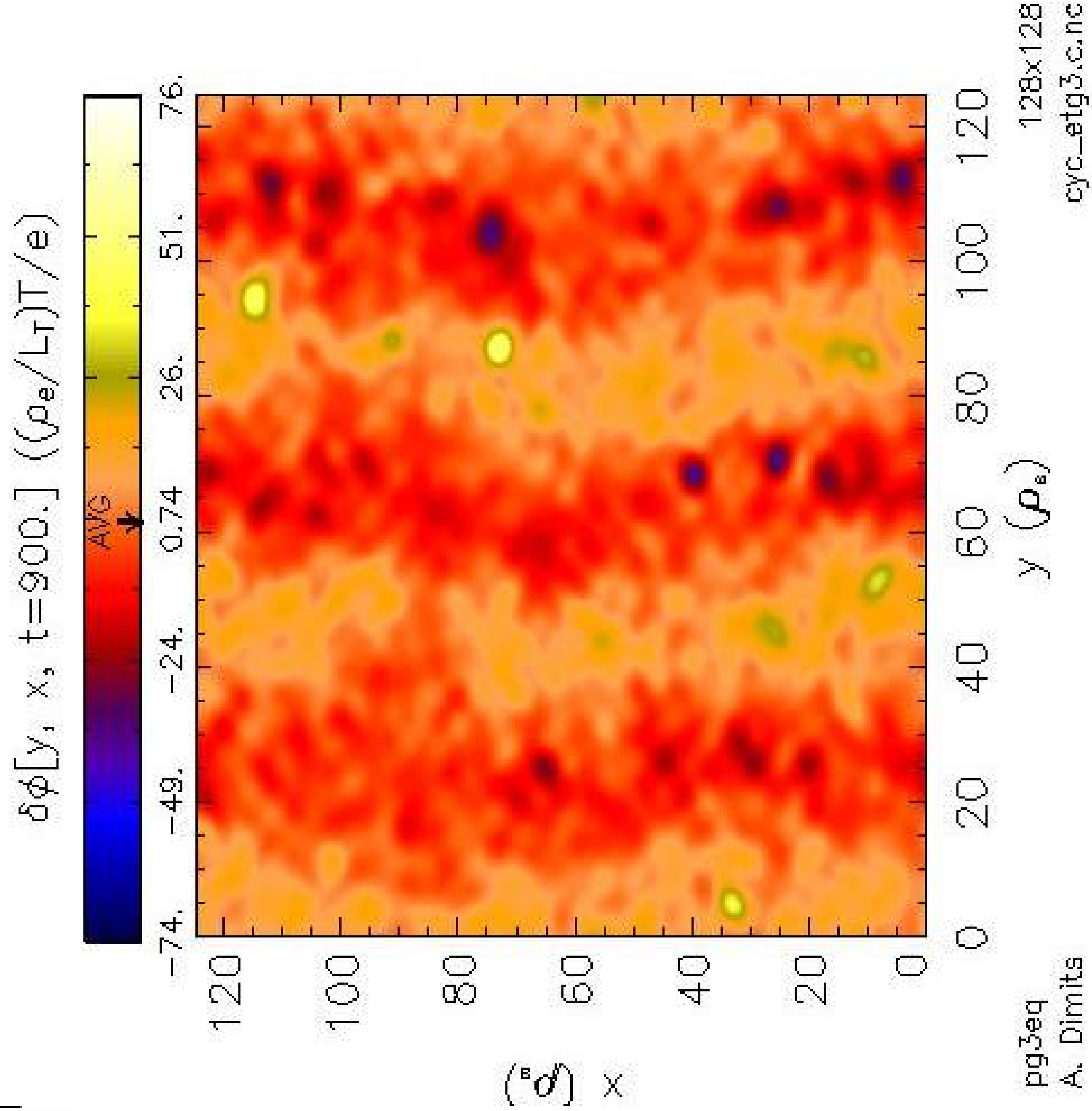


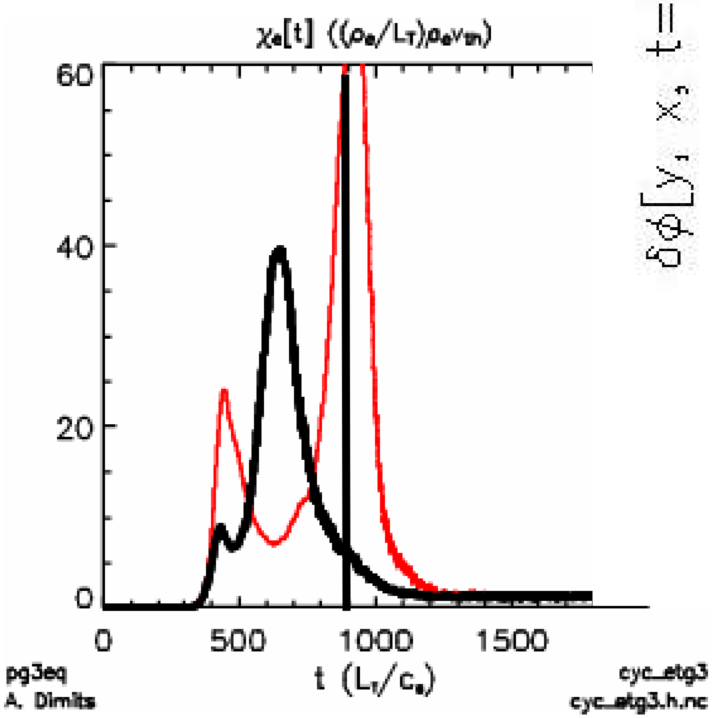
From Z.Lin's IAEA 2004 slides (at URL below). Believed to be $\chi_{etg}(t)$. Initial large values of χ_{etg} comparable to Jenko-Dorland $\chi_{etg} \sim 13$, but quickly generates large weights & thus noise that suppresses turbulence.

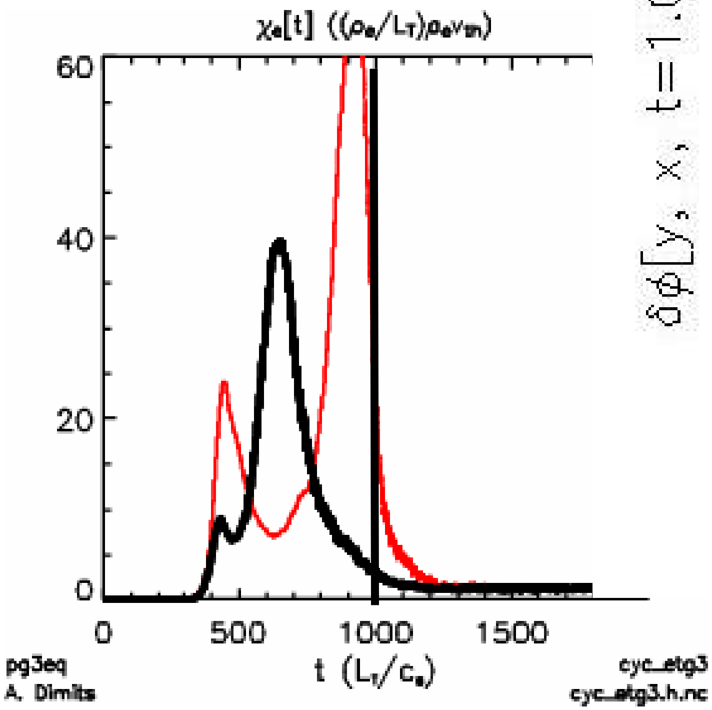
http://www.cfn.ist.utl.pt/20IAEAConf/presentations/T5/2T/5_H_8_4/Talk_TH_8_4.pdf


χe[t] ((ρe/LT)ρeVte)
60
40
20
0
0
500
1000
1500
t (LT/cs)
pg3eq
A. Dimits
cyc_etg3
cyc_etg3.h.nc

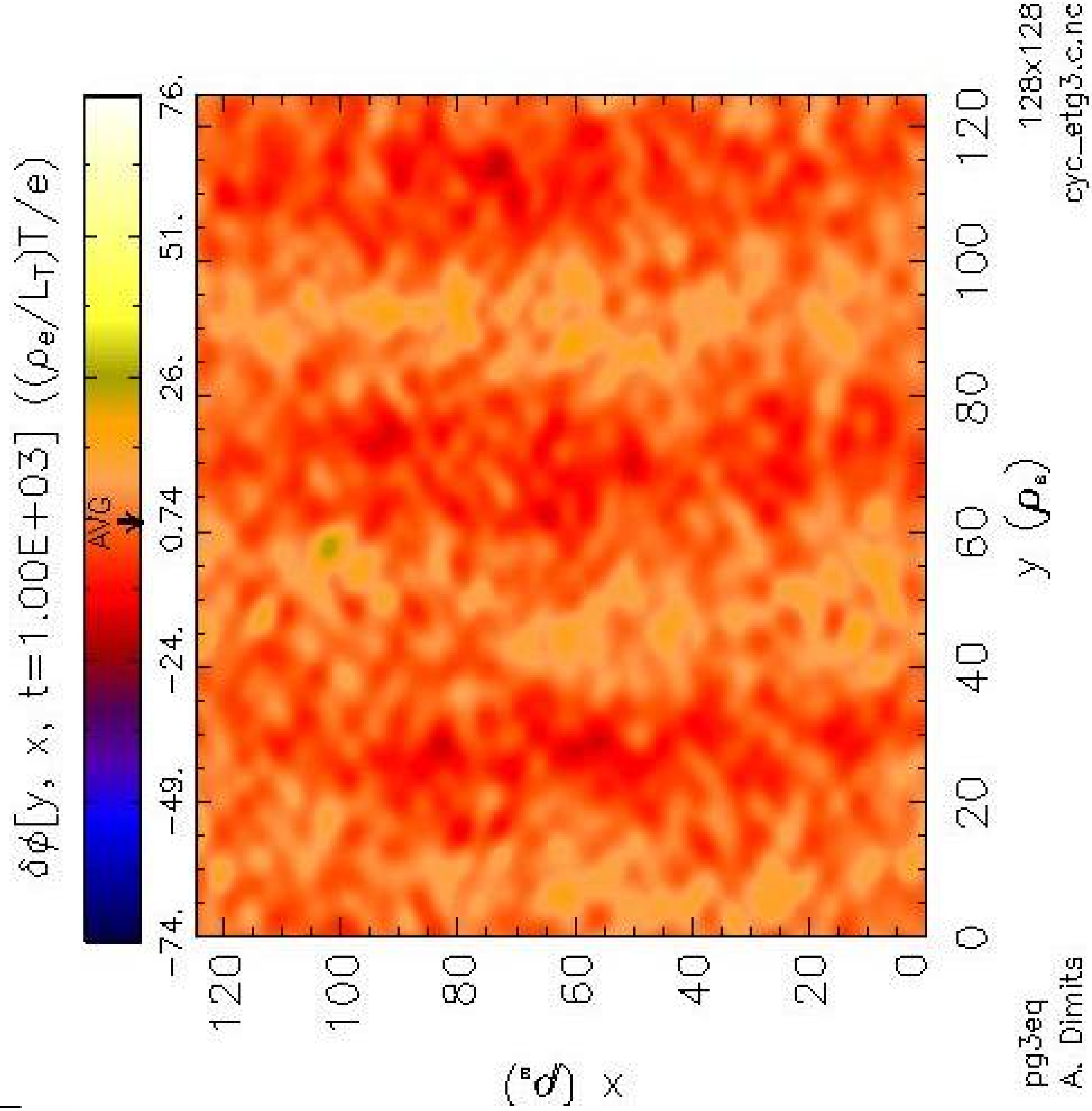

δφ[y, x, t=600.] ((ρe/LT)T/e)
AVG
76.
51.
26.
0.74
−24.
−49.
−74.
y (ρs)
x (ρs)
128x128
cyc_etg3.c.nc
pg3eq
A. Dimits


δφ[y, x, t=900.] ((ρe/LT)T/e)
AVG
76.
51.
26.
0.74
−24.
−49.
−74.
y (ρs)
x (ρs)
128x128
cyc_etg3.c.nc
pg3eq
A. Dimits

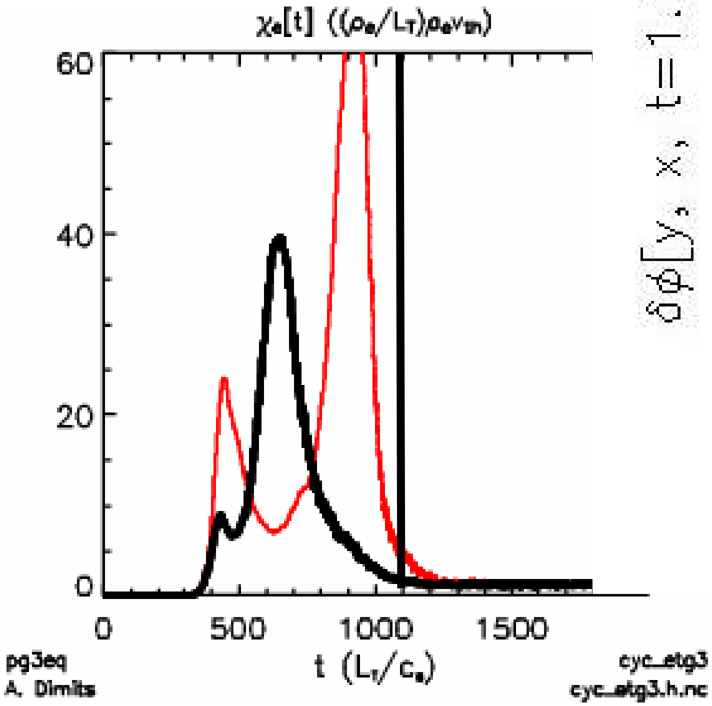

χe[t] ((ρe/LT)ρevth)
60
40
20
0
500
1000
1500
t (LT/cs)
pg3eq
A. Dimits
cyc_etg3
cyc_etg3.h.nc

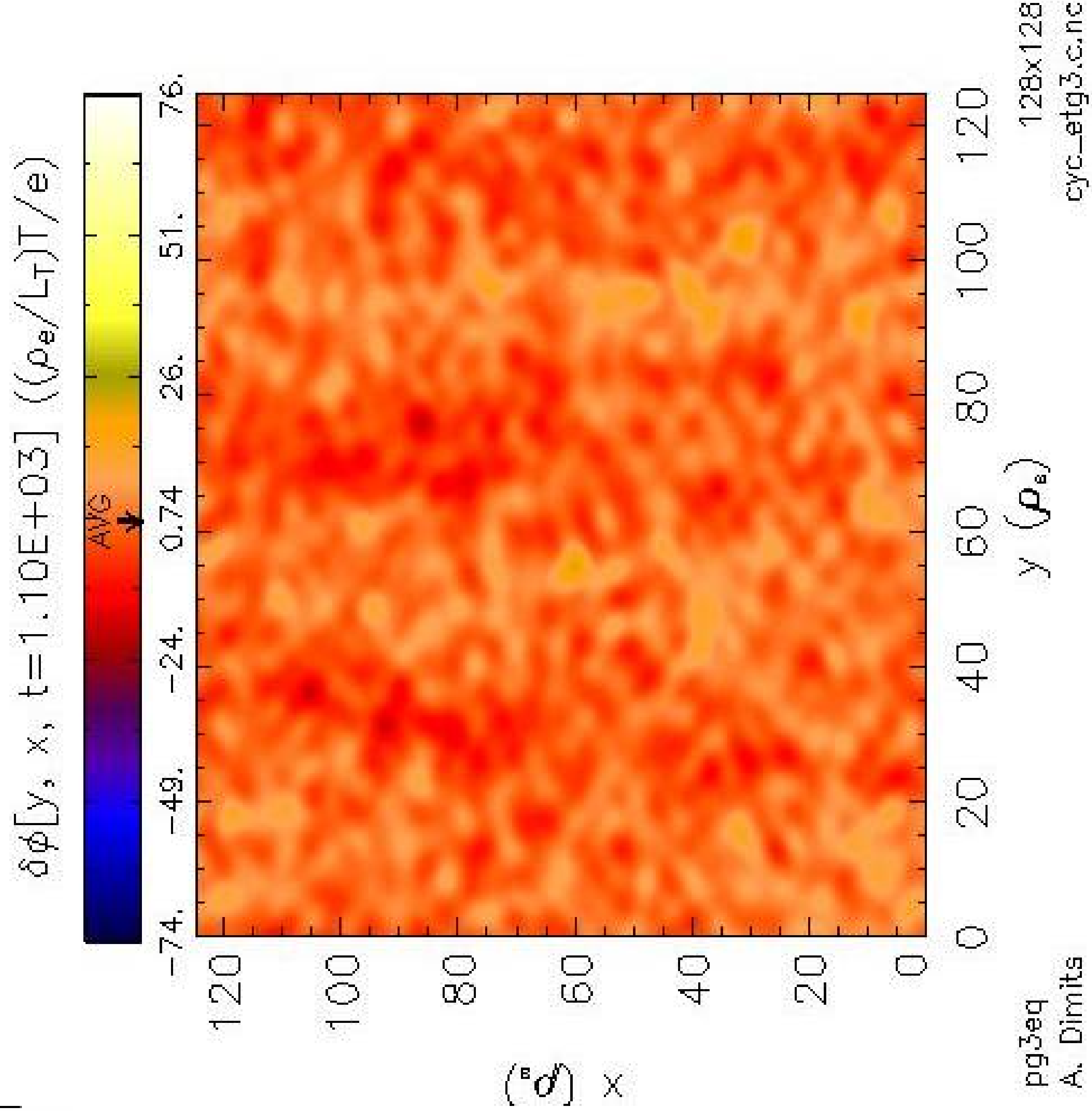


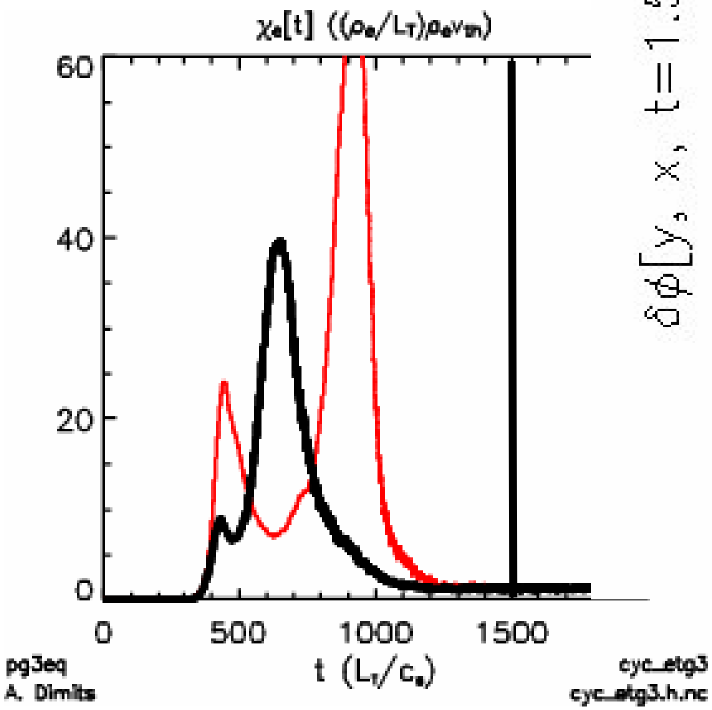
χe[t] ((ρe/LT)ρevth)
60
40
20
0
0
500
1000
1500
t (LT/cs)
pg3eq
A. Dimits
cyc_etg3
cyc_etg3.h.nc

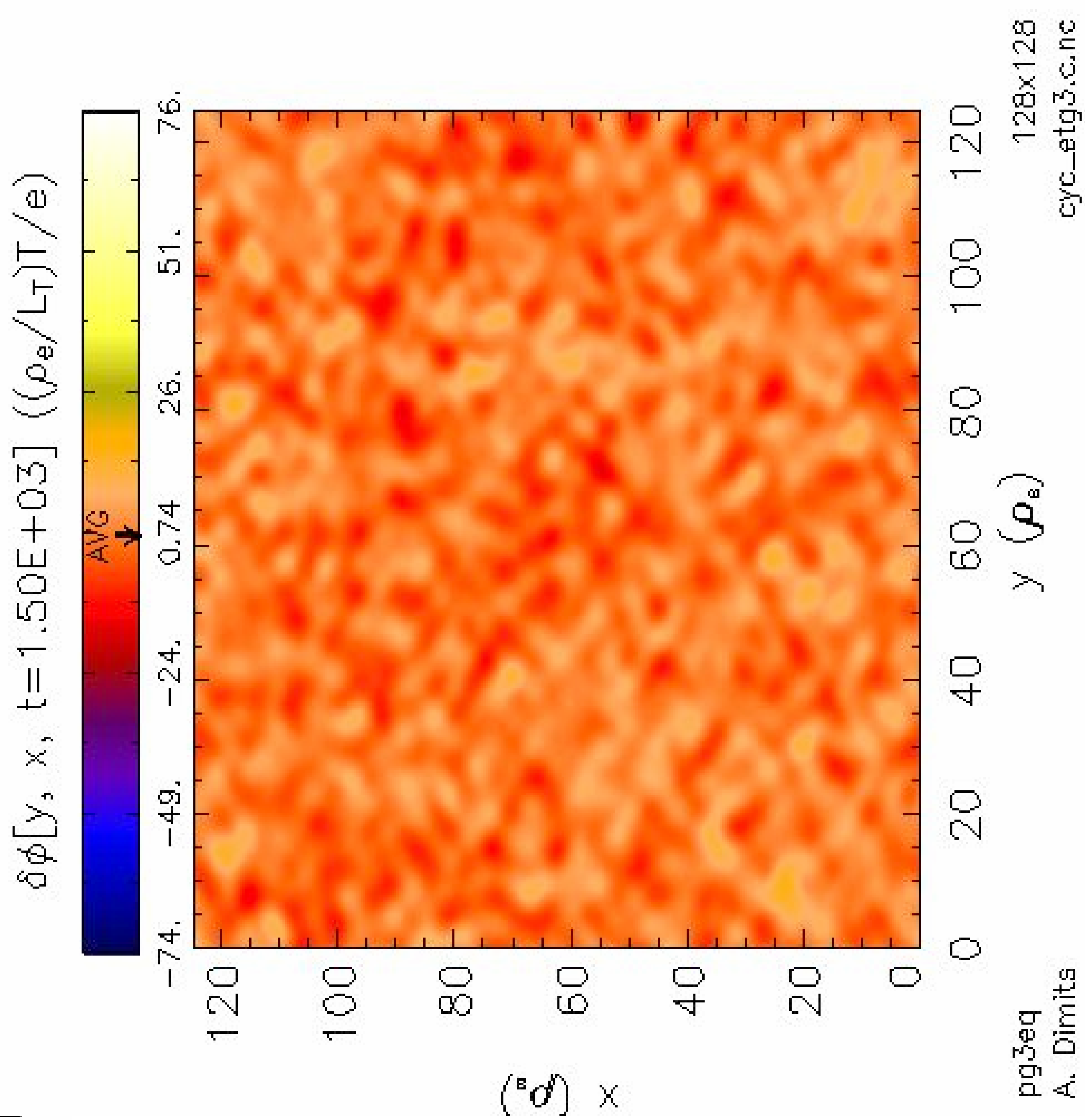


δφ[y, x, t=1.00E+03] ((ρe/LT)T/e)
AVG
76.
51.
26.
0.74
−24.
−49.
−74.
y (ρs)
x (ρs)
128x128
cyc_etg3.c.nc
pg3eq
A. Dimits

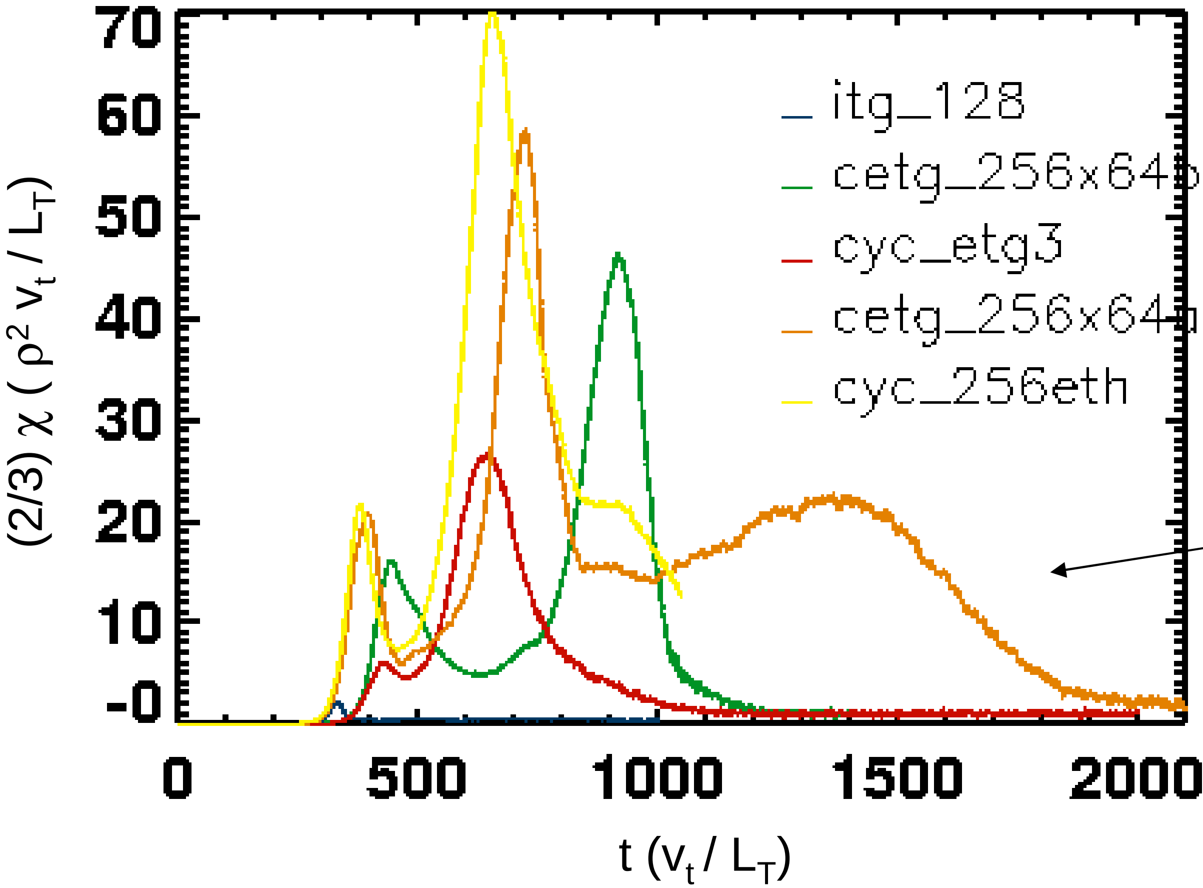

$\chi_e[t]$ $((\rho_e/L_T)\rho_e v_{te})$
0
20
40
60
0
500
1000
1500
t (L_T/c_s)
pg3eq
A. Dimits
cyc_etg3
cyc_etg3.h.nc


$\delta\phi$[y, x, t=1.10E+03] $((\rho_e/L_T)T/e)$
76.
51.
26.
0.74
−24.
−49.
−74.
AVG
y (ρ_s)
x (ρ_s)
128x128
cyc_etg3.c.nc
pg3eq
A. Dimits


χe[t] ((ρe/LT)ρeVth)
60
40
20
0
0
500
1000
1500
t (LT/cs)
pg3eq
A. Dimits
cyc_etg3
cyc_etg3.h.nc


δφ[y, x, t=1.50E+03] ((ρe/LT)T/e)
AVG
76.
51.
26.
0.74
-24.
-49.
-74.
y (ρs)
x (ρs)
128x128
cyc_etg3.c.nc
pg3eq
A. Dimits

Dimits doing scans to study convergence, trapping effects



256x 64x32 r/R=0

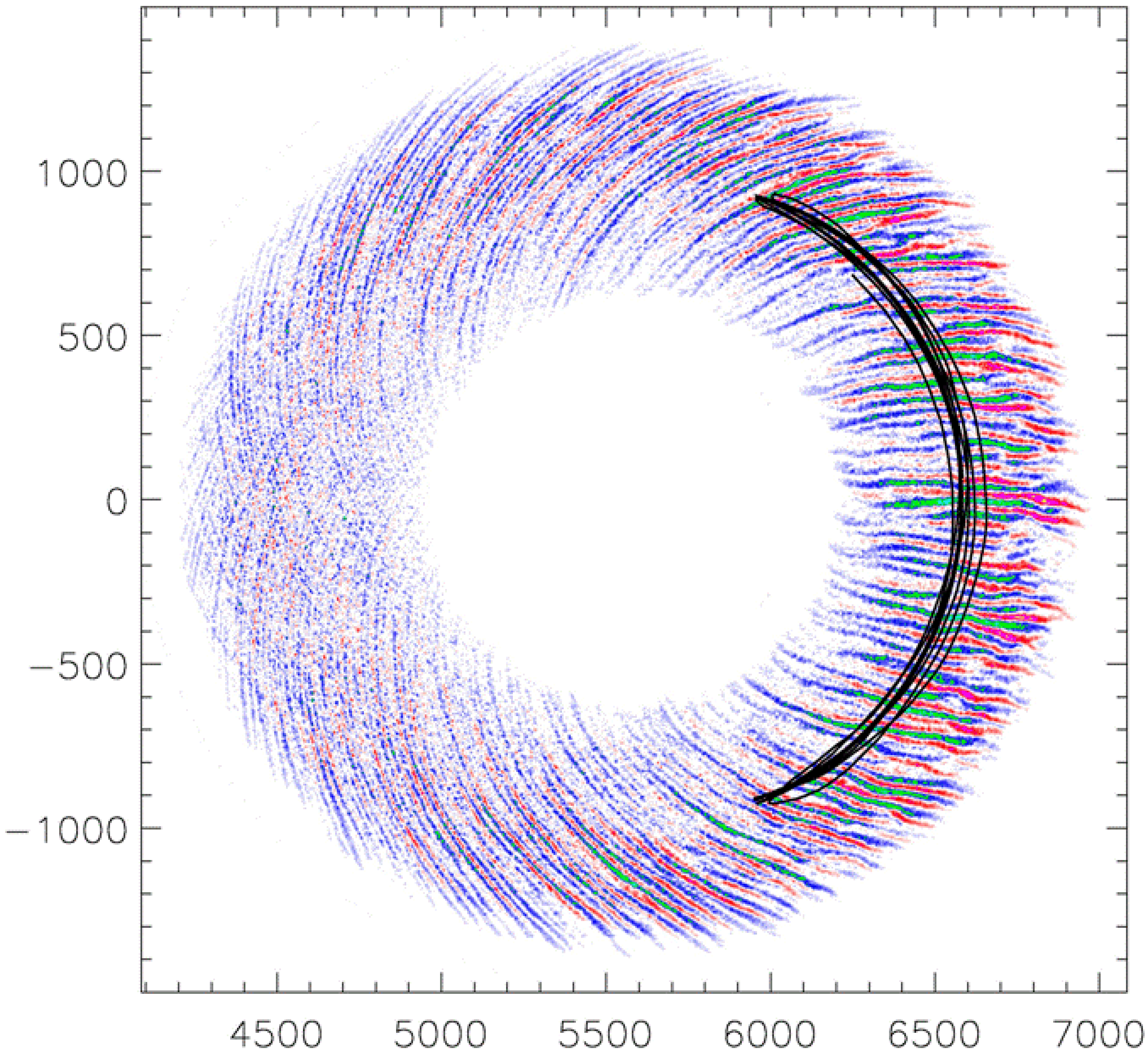
128x128x64 r/R=0

256x 64x32 r/R=0.18

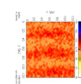
256x256x32 r/R=0.18

With trapped particles, spectrum shifts to lower k_θ, takes longer for noise to grow to larger level for $k_\perp^2 D_{noise} > \gamma_{lin}$

16 particles/cell

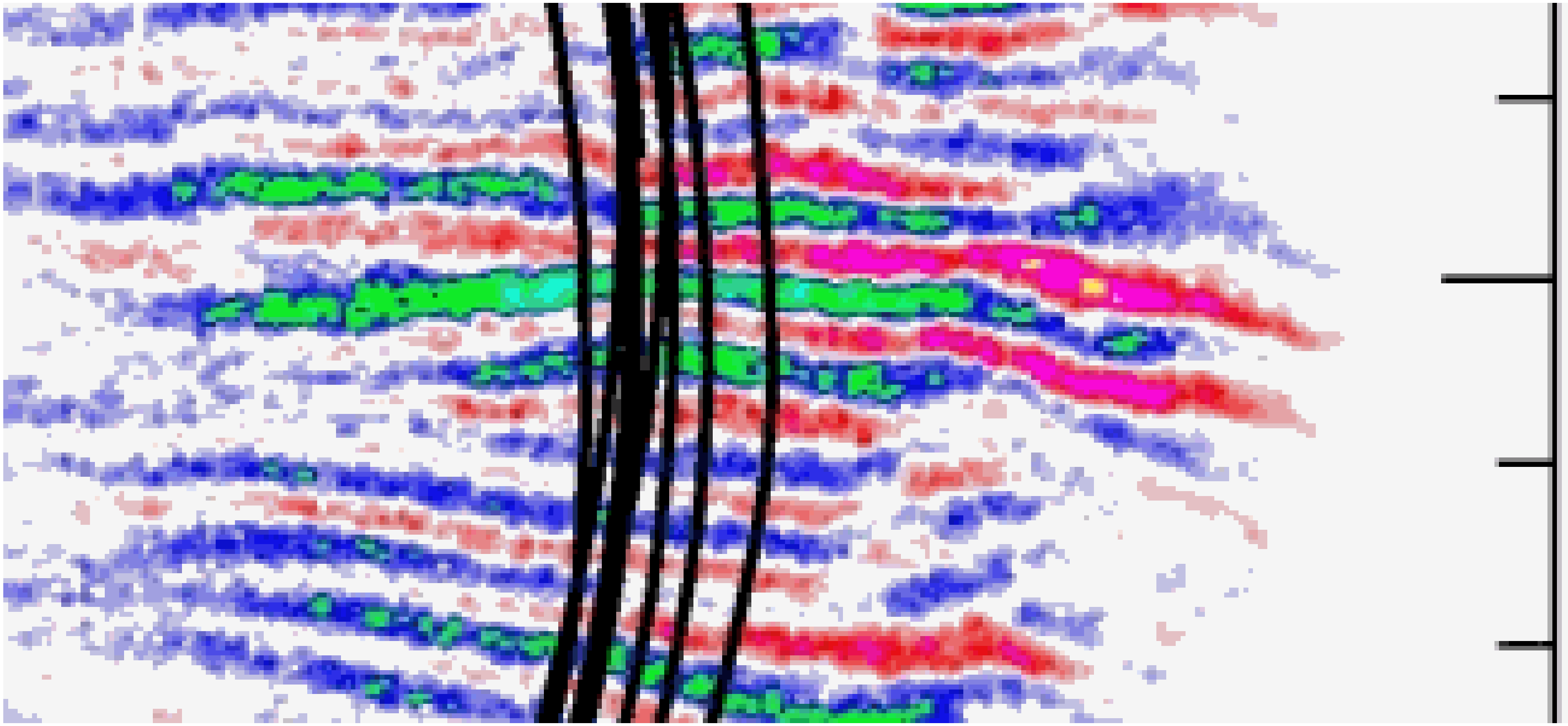


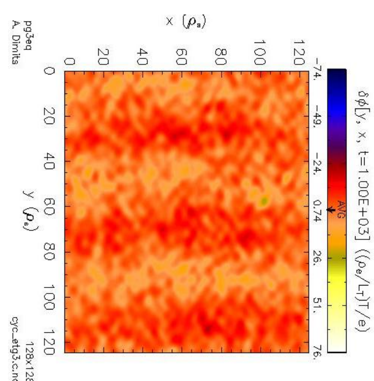
Dimits contour plots at
Same scale as Z. Lin's



Dimits contour plot at t=1000, when χ_e ~ 4 x final χ_{noise}. This is when noise effects are strong enough to reduce χ_e to ~1/2 of Jenko-Dorland, but noise hasn't completely taken over yet.

But if one shrinks the contour plot to the scale used in Z. Lin's plots, then the eye (and the finite resolution of the computer screen) will average out the noise to make it less apparent.





If we blow up Z. Lin's contour plot, then we can see the noise at small scales more easily. It looks roughly comparable to Dimits' contour plot at t=1000 (when $\chi_e \sim 4$ x final $\chi_{noise} \sim 1/2\ \chi_{Jenko\text{-}Dorland}$, and the simulation is transitioning to the noise-dominated state, but noise hasn't completely taken over yet.).

Eyeball comparisons depend on choice of color table, smoothing in graphics, etc. Need more quantitative measures of noise.

(Sometimes the above plot is rendered at low quality when printed or converted to a pdf file. A high resolution original image is at: http://w3.pppl.gov/theory/Image2.gif and is displayed at http://w3.pppl.gov/theory/GPSC.html

Discussion of results

•Large initial transients in $\chi_{etg}(t)$ seen by Dimits & by Z. Lin are larger than or comparable to Jenko & Dorland χ_{etg}. But this high χ_{etg} quickly drives weights so large that $k_\perp^2 D_{noise} \sim \gamma_{lin}$ and the turbulence is suppressed or significantly reduced.

• Scanning from 5-20 particles/cell appears to be converged (but isn't) because it just changes time it takes for weights to build up to give $k_\perp^2 D_{noise} \sim \gamma_{lin}$. This can be misunderstood as an ignorable initial transient.

• ETG eddies are radially very extended but still short scale in poloidal direction, so it only takes a little bit of $D_{noise} \ll$ Jenko-Dorland χ_{etg} to suppress or significantly reduce the turbulence. Radially extended eddies of ETG are more sensitive to particle noise than ITG is, requires many more particles to converge.

• Transition from turbulence-dominated state to noise-dominated state expected to slower in Z. Lin's simulations then in Dimits initial simulations (which have large overshoots due to their smaller size).

Caveats

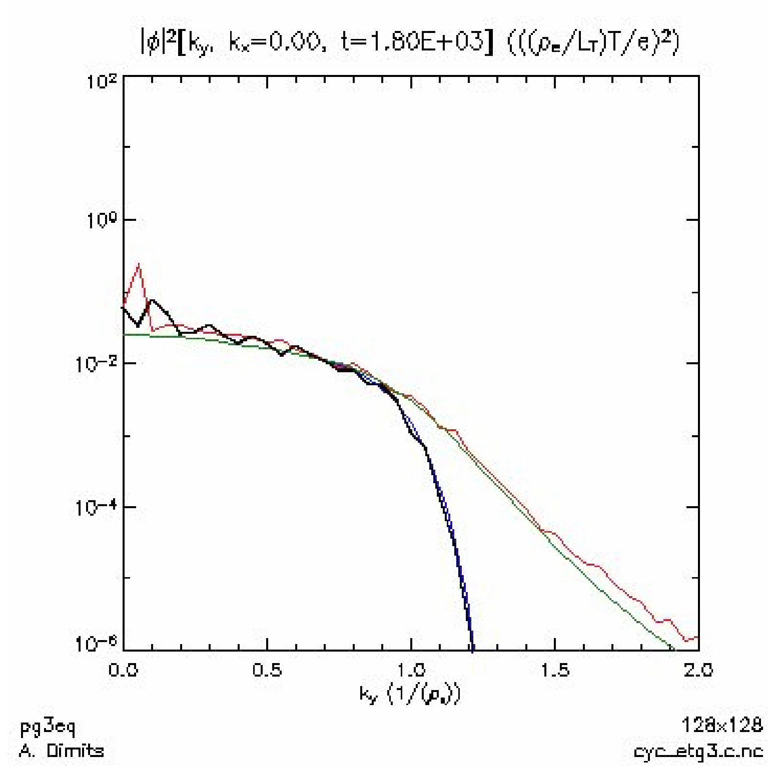
- Used guestimates of filtering parameters for Z. Lin's ETG simulations, based on Lin's previous ITG spectra.
- Would be best to compare with Lin's plots of $\chi(t)$ for different numbers of particles, or do a particle number scan in another gyrokinetic code.
- Crude model of turbulent saturation. Misses interactions with secondary instabilities that can give initial overshoot and increase noise level.
- Neglected differences in zonal components of noise due to differences in ITG/ETG zonal flow dynamics.
- Fluctuation-dissipation theorem used uniform plasma dielectric in unsheared slab geometry, should be modified in a turbulent medium etc.?
- Long-time scale variability often seen in $\chi(t)$ makes detection of trends harder
- Should calculate energy weighted thermal diffusion more consistently?

Work in Progress

- Recently extended calculation of noise spectrum $<|\Phi_k|^2>(\omega)$ to include renormalized treatment of nonlinearities on dielectric shielding, and on shielded test-particle trajectories.
- Presently working to include this improved $<|\Phi_k|^2>(\omega)$ in the calculation of the test-particle diffusion.
- Various simple Pade approximations used for Z function in correlation times and FLR factors. More precise calculations tested to see if they improve the transition from the turbulence-dominated to noise-dominated states.
- Improved toy model being developed (motivated vaguely by form of EDQNM/RMC). Slight improvements in transition from turbulene to noise.
- Various other improvements in calculation in progress.

Conclusions

- Rigorous calculation of noise spectrum based on Test Particle Superposition Principle (extending Krommes 93 calculation to include numerical filtering factors) agrees very well (no free parameters) with observed spectrum at late times in Dimits' gyrokinetic ETG simulations, confirming that noise quickly grows to dominate those ETG results.



- ETG eddies are radially very extended but still short scale in poloidal direction & so are sensitive to even a small amount of diffusion in the poloidal direction. Calculations of D_{noise} for Z. Lin's recent ETG simulations (using Lee-Tang entropy balance to give the rms weights) indicates $k_{\perp}^2 D_{noise} \sim \gamma_{lin}$. Suggests that Z. Lin's ETG results are dominated by noise that is suppressing or significantly reducing the apparent turbulence.

- ETG simulations require many more particles for convergence than ITG.

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