Particle Noise-Induced Diffusion & Its Effect on ETG Simulations

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Acknowledgments: S. Cowley, B. Cohen, W. Dorland, F. Jenko, J. Krommes, M. Kotschenreuther

Outline

- Simple estimate of the spectrum of potential fluctuations due to a discrete number of gyrokinetic particles, shielded by an adiabatic species. Within a factor of 2 of the more detailed calculation based on an extension of Krommes' 1993 using the classic fluctuation-dissipation theorem or test-particle superposition principle.

- Agrees very well with Dimit's gyrokinetic PIC ETG simulations (with no free parameters!)

- Renormalized calculation of noise-induced ExB diffusion, D_{noise}, for a test-particle in this spectrum of random potential fluctuations.

- Theory determines D_{noise} if given rms particle weights.
Can monitor $k_\perp^2 D_{noise}$ to insure it is $<< \gamma_{lin}$ for a reliable simulation.

- Toy model for interaction of noise and turbulence, useful for predicting results if one doesn't have access to weights. Also explains how noise can increase transport in some cases (ITG) and decrease it in other cases (ETG).

Theoretical Noise Spectrum Agrees Well with Dimits PIC ETG Simulation at Late Times

- Discrete particle noise in PIC codes is quantifiable — well studied in past:
  - Langdon '79 – Birdsall&Langdon '83, Krommes '93

⇒ Useful code verification tool. We can develop objective criteria to determining when discrete particle noise is a problem

$S_\phi(k)$ from simulation:

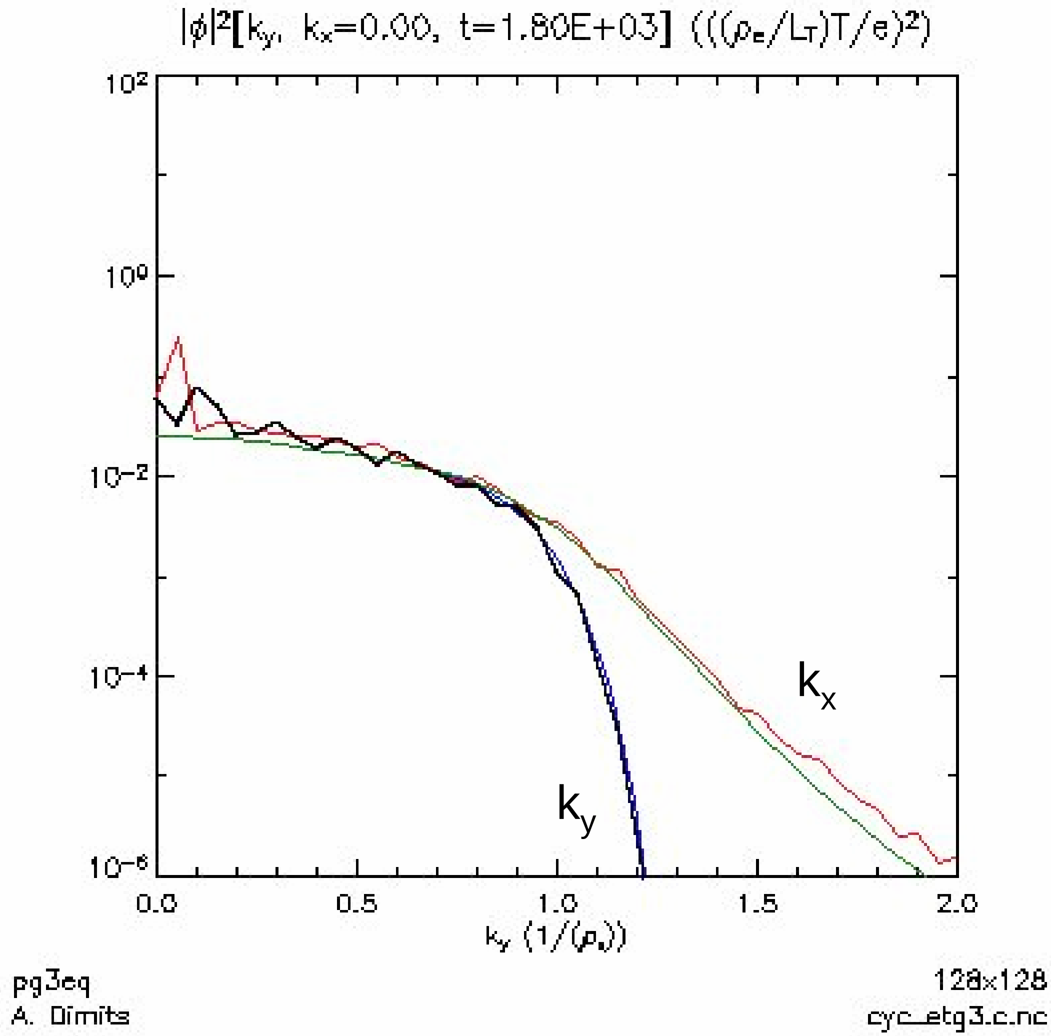
$S_\phi(k_x=0,\ k_y\quad)$ —black

$S_\phi(k_x\quad,\ k_y=0)$ —red/brown

$S_\phi(k)$ from noise:

$S_{noise}(k_x=0,\ k_y\quad)$ —blue

$S_{noise}(k_x\quad,\ k_y=0)$ —green



Jenko & Dorland found ETG turbulence >> ITG turbulence (in Gyro-Bohm units)

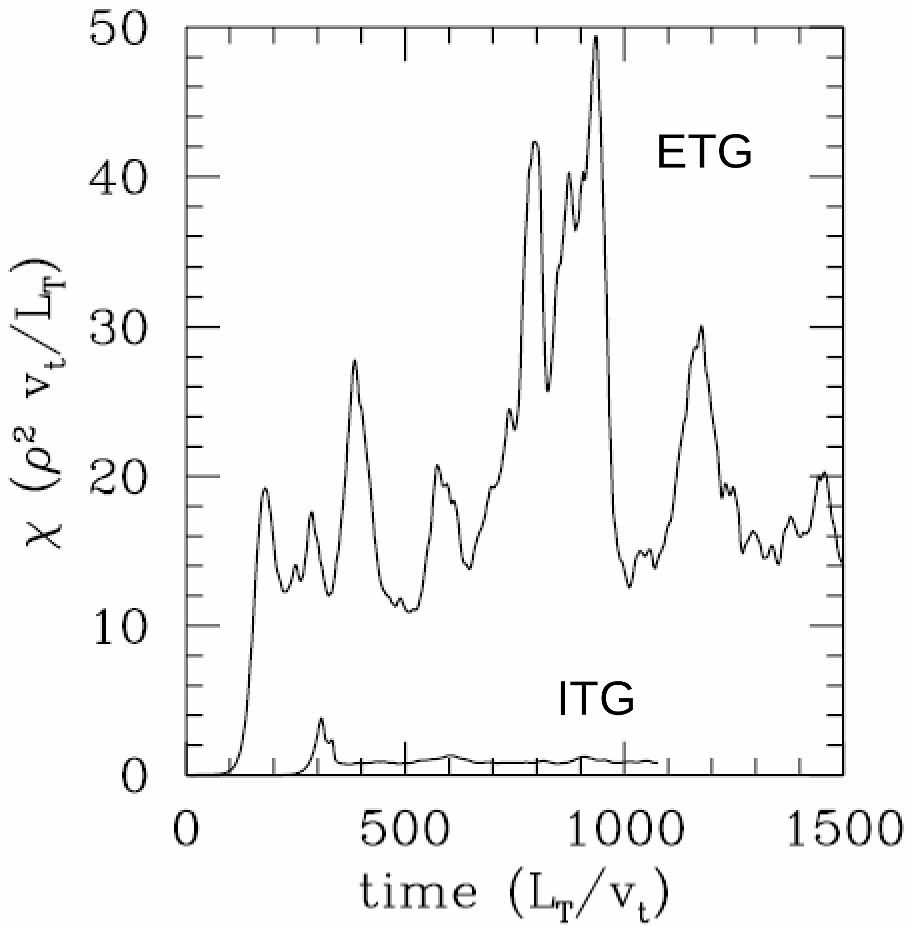


FIG. 1. χ_e^{ETG} (upper curve) and χ_i^{ITG} (lower curve) for similar parameters.

(Dorland & Jenko 2000, see also Jenko & Dorland 2002: with larger box, Lx=512 ρ, report χ_e = 13)

ETG eddies are radially extended streamers

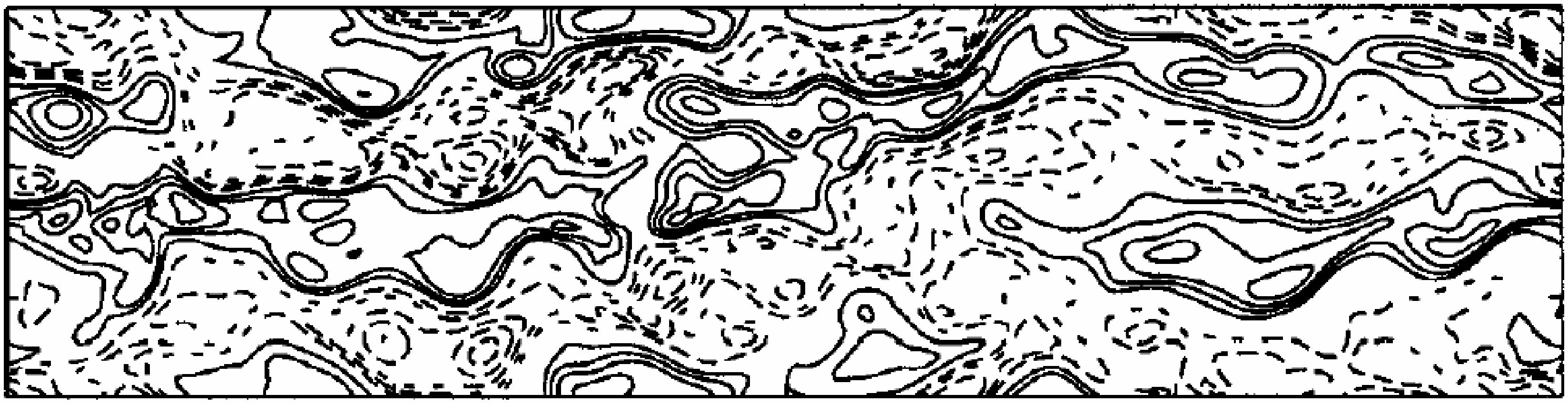


FIG. 2. Characteristic ϕ contours in the outboard x-y plane. This snapshot was taken at the end of the ETG run shown in Fig. 1. The figure is $256\rho_e \times 64\rho_e$.

High ETG transport relative to ITG transport theoretically understood as due to difference in adiabatic response for ions vs. electrons ==> reduces ETG zonal flows ==> ETG streamers get to higher velocity and are more elongated. (Rogers & Dorland, Jenko & Dorland 2000, 2002, etc.)

(Jenko & Dorland 2000)

Key ITG/ETG Difference: different adiabatic response to zonal flows

ITG turbulence, adiabatic electron response:

$$n_e = n_i$$

$$n_{e0}\frac{e}{T}\left(\Phi - \left\langle\Phi\right\rangle\right) = \int d^3v\, J_0 f_i - n_{i0}\left(1 - \Gamma_0(k_\perp \rho_i)\right)\frac{e}{T}\Phi$$

Flux-surface averaged potential, electrons adiabatic because $k_{||}=v_{te} >> \omega$ don't respond to zonal flows ($k_{||}$=0, pure E_r).

ETG turbulence, adiabatic ion response:

$$n_i = n_e$$

$$n_{i0}\frac{e}{T}\Phi = \int d^3v\, J_0 f_e + n_{ei0}\left(1 - \Gamma_0(k_\perp \rho e)\right)\frac{e}{T}\Phi$$

Ions adiabiatic because $k_\perp \rho_i >> 1$. Ions CAN shield zonal flows.
↓ zonal flows --> streamers elongate --> transport ↑

Detailed secondary/tertiary instability analysis includes this, explains ITG/ETG saturation level differences, scalings (Rogers, Dorland, Jenko papers)

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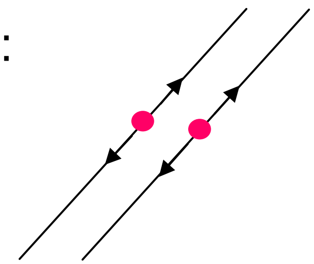
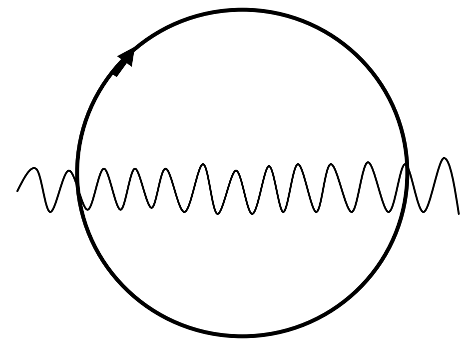


electrons ***don't*** respond to zonal flows ($k_{||}$=0, pure E_r).
since electrons are adiabatic because $k_{||} v_{te} >> \omega$

ETG turbulence, adiabatic ion response:



Ions do shield zonal flows for ETG
Since ions are adiabiatic because $k_{\perp}\rho_i >> 1$.
↓ zonal flows --> streamers elongate --> transport ↑

Detailed secondary/tertiary instability analysis includes this, explains ITG/ETG saturation level differences, scalings (Rogers, Dorland, Jenko papers)

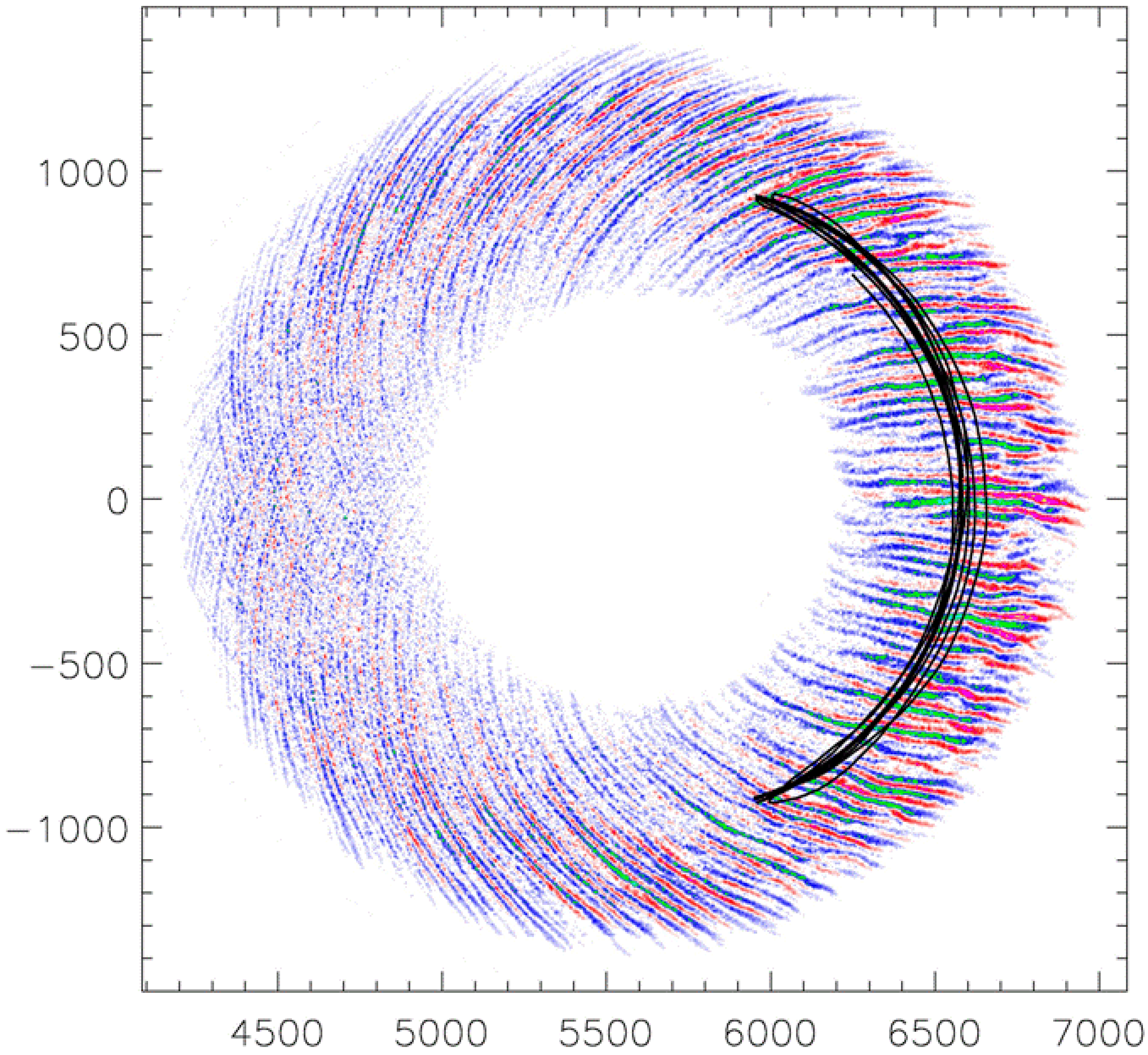
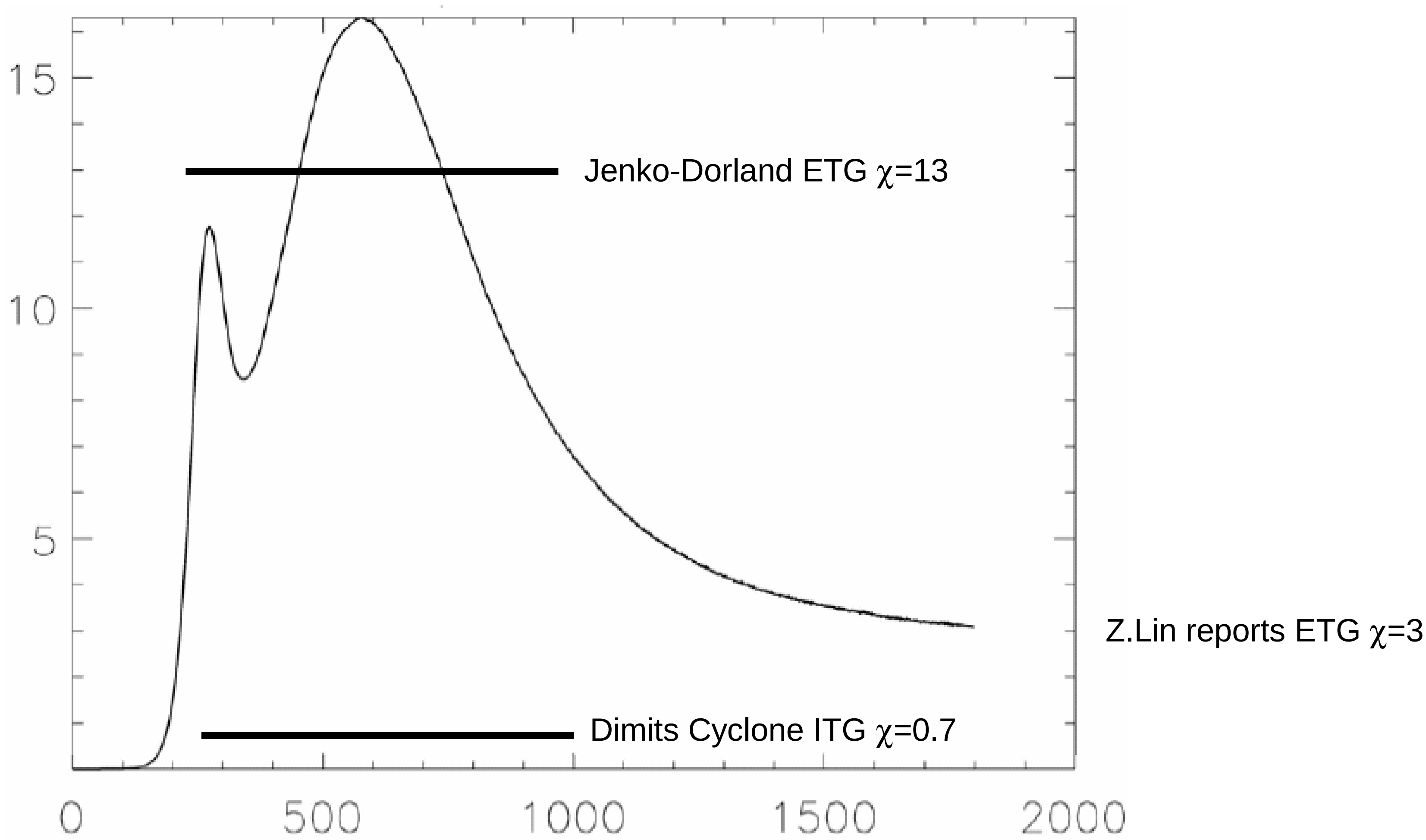


Fig. of etg streamers
from Z. Lin global PIC
simulations IAEA 2004



From Z.Lin's IAEA 2004 slides (at URL below). Believed to be $\chi_{etg}(t)$. Initial large values of χ_{etg} comparable to Jenko-Dorland 2002 χ_{etg} ~ 13. Ignoring initial "transient", reported result is χ_{etg} ~ 3. Scanned 5 to 20 particles/cell.

http://www.cfn.ist.utl.pt/20IAEAConf/presentations/T5/2T/5_H_8_4/Talk_TH_8_4.pdf

Standard approach to discrete particle noise

Particle discreteness ==> Fluctuations ==> Collision operator

Klimontovich Eq. ==> Vlasov Eq. + Collisions C(f)

diffusion in velocity
from $\langle \delta E^2 \rangle$ fluctuations
$\propto 1/(n\lambda_D^3)$

Gyrokinetic Klimontovich Eq. ==> Gyrokinetic Eq. + $C_{GK}(f)$

diffusion in g.c. position
from $\langle (\delta E \times B)^2 \rangle$ fluctuations
$\propto 1/(nV_{smooth})$

Various standard approaches to calculating C(f): binary collision operator cut off at Debye shielding scale, BBGKY hierarchy, etc.

Krommes' Calculation of Noise Spectrum

Krommes' 1993 calculation of the gyrokinetic noise spectrum uses the classic fluctuation-dissipation theorem, and shows equivalent results from the test-particle superposition principle (shielded test particles can be treated as independent). (see also W.W. Lee 1987, classic paper by A.B. Langdon 1979)

Krommes' calculation used shielding by linear dielectric from gyrokinetic equation in a slab, uniform plasma. Hu & Krommes 94 extended to δf.

We have extended Krommes' test-particle superposition calculation to:

- Treat one species as adiabatic instead of with particles.

- Include factors for finite-size particle shape S (accounts for interpolation of particle charge to grid, and forces from grid to particles) & S_{filt} factor for explicit filtering of Φ. Important for quantitative comparisons.

- Use a renormalized dielectric, including a $k_\perp^2 D_{NL}$ term on the non-adiabatic part of the shielding cloud, and including random walks in the test particle trajectories instead of assuming straight-line trajectories. Affects frequency spectrum of fluctuations, but not the frequency-integrated k spectrum.

Applying thermal noise to non-equilibrum systems

Can't directly apply Fluctuation-Dissipation Theorem or Test-Particle Superposition Principle to a linearly unstable plasma, because they use the dielectric response to calculate particle correlations and shielding, and rely on all poles being in lower half ω plane. In an unstable plasma, the linear dielectric leads to amplification of noise, not shielding.

Standard approach is indirect: use FDT or TPSP to calculate spectrum of fluctuations and the resulting collision operator C(f) in thermal equilibrium, then use that C(f) to study collisional effects on non-equlibrium problems (e.g., effects of collisions on trapped particle modes or reconnection). Good approximation if sufficient separation of time/space scales.

Our calculation of $|\Phi_{noise}(k,\omega)|^2$ spectrum and D_{noise} follow similar approach. D_{noise} dominated by high $k_\perp$ & high $k_{||}$, so a self-consistent regime exists where

$$\nu_{noise} \sim |k_{||}|\, v_t + k_\perp^2\, D_{noise} >> \gamma,\ \omega_{*}\ \Gamma_0$$

Alternatively, as suggested in Hu & Krommes '94, etc., one could use a renormalized nonlinear dielectric to model nonlinearly saturated turbulent system. (All perturbations decay in nonlinearly saturated system. Future work…)

Simple Estimate of Noise: Randomly Positioned Particles

Fourier conventions:

$$\Phi(\vec{x}) = \frac{1}{V}\sum_{\vec{k}} e^{i\vec{k}\cdot\vec{x}}\tilde{\Phi}_{\vec{k}}$$

$$\tilde{\Phi}_{\vec{k}} = \int_V d^3x\, e^{-i\vec{k}\cdot\vec{x}}\,\Phi(\vec{x})$$

Adiabatic species + polarization density = “bare” guiding center contribution

Gyrokinetic Poisson Eq:
(W.W. Lee, Phys. Fluids ’83)

$$n\frac{e\Phi}{T} + n(1-\Gamma_0)\frac{e\Phi}{T} = S_{filt}S\int d^3\mathrm{v}\, J_0\,\delta f$$

$$= S_{filt}S\sum_i w_i J_{0i}\delta(\vec{x}-\vec{x}_i)$$

$$\left|\frac{e\tilde{\Phi}_k}{T}\right|^2 = \frac{S_{filt}^2 S^2}{n^2(2-\Gamma_0)^2}\sum_i\sum_j w_i w_j J_{0i}J_{0j}e^{-i\vec{k}\cdot\left(\vec{x}_i-\vec{x}_j\right)}$$

$$\left\langle\left|\frac{e\tilde{\Phi}_k}{T}\right|^2\right\rangle_N = \frac{S_{filt}^2 S^2}{n^2(2-\Gamma_0)^2}\sum_i w_i^2 J_{0i}^2$$

neglecting
particle
correlations

Quantifying Particle Discreteness (2)

(a partially correlated fluctuation spectrum)

- More detailed calculation following Krommes93 gyrokinetic test-particle superposition calculation, including dielectric shielding in kinetic response, numerical filtering/interpolation factors, resonance broadening renormalization:

$$\left\langle \left| \frac{e\phi_k}{T} \right|^2 \right\rangle_H = \frac{V^2 \langle w_i^2 \rangle S_{filter}^2(k) S^2(k) \Gamma_0(k_\perp^2 \rho_{th}^2)}{N_p \left[2 - \Gamma_0(k_\perp^2 \rho_{th}^2)\right]\left[2 - \left(1 - S_{filter} S^2 d_\parallel(k)\right)\Gamma_0(k_\perp^2 \rho_{th}^2)\right]} \xrightarrow[k \to 0]{} \frac{V^2 \langle w_i^2 \rangle}{2N_p}$$

- The fully uncorrelated spectrum (for comparison)

$$\left\langle \left| \frac{e\phi_k}{T} \right|^2 \right\rangle_N = \frac{V^2 \langle w_i^2 \rangle S_{filter}^2(k) S^2(k) \Gamma_0(k_\perp^2 \rho_{th}^2)}{N_p \left[2 - \Gamma_0(k_\perp^2 \rho_{th}^2)\right]^2} \xrightarrow[k \to 0]{} \frac{V^2 \langle w_i^2 \rangle}{N_p}$$

Test Particle Superposition with Renormalized Trajectories

Test Particle Superposition Principle: Trajectories of “dressed” test particles can be treated as statistically independent. Dominant correlations included by a shielding cloud (calculated using the plasma dielectric response) that follows each moving test particle.

Intuitive approach, usually agrees with the rigorous Fluctuation-Dissipation Theorem in thermal equilibium (Krommes’93, Rostoker, …).

Include resonance-broadening type of renormalization of test particle trajectory:

$$\left\langle \exp(-i\mathbf{k}\cdot\left(\mathbf{x}_i(t)-\mathbf{x}_i(t_2)\right)) \right\rangle = \exp(-ik_\parallel \mathrm{v}_{\parallel,i}(t-t_2) - D_{t.p.}k_\perp^2 |t-t_2|)$$

Renormalization important for consistently handling both

- weak noise limit, where $D_{noise} \propto \Phi^2_{noise}$, and
- strong noise limit, where $D_{noise} \propto \Phi_{noise}$.

Renormalized Dielectric Shielding

Nonlinear gyrokinetic Eq. (uniform slab, electrostatic):

$$\frac{\partial \delta f}{\partial t} + \mathrm{v}_{\|}\hat{b}\cdot\nabla\delta f + \frac{c}{B}\hat{b}\times\nabla J_0\Phi\cdot\nabla\delta f = -\mathrm{v}_{\|}\left(\hat{b}\cdot\nabla J_0\frac{q\Phi}{T}\right)F_{Max,0}$$

If ExB velocity is small-scale random fluctuations, treat as random walk diffusion:

$$\frac{\partial \delta f}{\partial t} + \mathrm{v}_{\|}\hat{b}\cdot\nabla\delta f - D_{NL}\nabla_{\perp}^{2}\delta f = -\mathrm{v}_{\|}\left(\hat{b}\cdot\nabla J_0\frac{q\Phi}{T}\right)F_{Max,0}$$

Better renormalization (Catto 78): nonlinearity affects only non-adiabatic part of δf:

$$\frac{\partial \delta f}{\partial t} + \mathrm{v}_{\|}\hat{b}\cdot\nabla\delta f - D_{NL}\nabla_{\perp}^{2}\left(\delta f + F_{Max.0}J_0\frac{q\Phi}{T}\right) = -\mathrm{v}_{\|}\left(\hat{b}\cdot\nabla J_0\frac{q\Phi}{T}\right)F_{Max,0}$$

Insures no nonlinear damping of a thermal equilibrium solution (the adiabatic solution) (Catto78, Krommes81, Krommes02). Combined with using the same D_{NL} for the shielding cloud as the $D_{t.p.}$ for the test-particle trajectory, preserves the form of the Fluctuation-Dissipation Theorem.

Detailed Calculation of Noise-Spectrum Incl. Self-Shielding

Potential induced by shielded test particle density ρ_{ext}:

$$\Phi = \frac{4\pi q \rho_{ext}}{k^2 \varepsilon(\vec{k},\omega)}$$

$$\varepsilon(\vec{k},\omega) = \frac{k_D^2}{k^2}\left[\frac{T}{T_a}\left(1-\delta_{k_\parallel}\right) + 1 - \Gamma_0 + S_{filt} d_\parallel S^2 \left\langle J_0^2 \right\rangle \left(1 + \zeta Z(\zeta + i\zeta_D)\right)\right]$$

Gyrokinetic dielectric shielding including simple renormalized D_{NL} model of nonlinear effects on shielding cloud and test-particle random walk trajectory, $\zeta_D = k_\perp^2 D_{NL}/(|k_{\|}|\ v_t\ 2^{1/2})$. Integrating $<|\Phi_k|^2>(\omega)$ over all ω gives a result independent of D_{NL} (but D_{nl} important for getting frequency spectrum right to calculate test-particle diffusion). Resulting k spectrum:

$$\left\langle \left| \frac{e\tilde{\Phi}_{noise,k}}{T} \right|^2 \right\rangle = \frac{V^2 \left\langle w^2 \right\rangle}{N} \frac{S_{filt}^2 S^2 \left\langle J_0^2 \right\rangle}{\left[\frac{T}{T_a}\left(1-\delta_{k_\parallel}\right) + 1 - \Gamma_0 + \underbrace{S_{filt} d_\parallel S^2 \left\langle J_0^2 \right\rangle}\right]\left[\frac{T}{T_a}\left(1-\delta_{k_\parallel}\right) + 1 - \Gamma_0\right]}$$

Only difference from simple random-particle spectrum. $< \Phi_k^2>$ only 50% lower at low $k_\perp$ (adiabatic electrons already got half of shielding), equal at high $k_\perp$ (ion shielding vanishes)

Why Particle Weights Grow in Time

$$\frac{\partial f}{\partial t} + \left(\mathrm{v}_{\parallel} + \mathrm{v}_{ExB}\right) \cdot \nabla f + \frac{q}{m} E_{\parallel} \frac{\partial f}{\partial \mathrm{v}_{\parallel}} = 0$$

$$\frac{Df}{Dt} = 0$$

Clever δf algorithm to reduce noise: f = smooth f_0 + particles δf

$$\frac{D}{Dt}\delta f = -\frac{D}{Dt} f_0 \approx -\mathrm{v}_{ExB} \cdot \nabla f_0$$

$$\delta f \approx \left(x - x_0\right)\frac{df_0}{dx}$$

f = constant along a particle's trajectory. But as particle moves to position where local f_0 is different than the f where the particle started, the weight grows to represent the difference.

$$\frac{dw_{rms}^2}{dt} = \frac{d}{dt}\frac{\left\langle \left(\delta f\right)^2 \right\rangle}{f^2} \approx \frac{2\chi_{tot}}{L_T^2}$$

entropy balance in steady state
W.W. Lee & W. Tang 88

Note: In the following plots comparing the noise formula on the last page with $<\Phi^2(k_x,k_y)>$ spectra from Dimits gyrokinetic PIC simulations, there are no free parameters.

Detail: Nevin's GKV diagnostic is the $k_\perp$ spectrum at z=0, so have to sum the expression for the noise Φ_k^2 spectrum over all k_z:

$$\left\langle \left| \frac{e\tilde{\Phi}_{k_x,k_y}(z=0)}{T} \right|^2 \right\rangle = \frac{1}{L_z^2} \sum_{k_z} \left\langle \left| \frac{e\tilde{\Phi}_k}{T} \right|^2 \right\rangle$$

Simulation Verification (1)
The Transverse (to *B*) Fluctuation Spectrum

Requires:

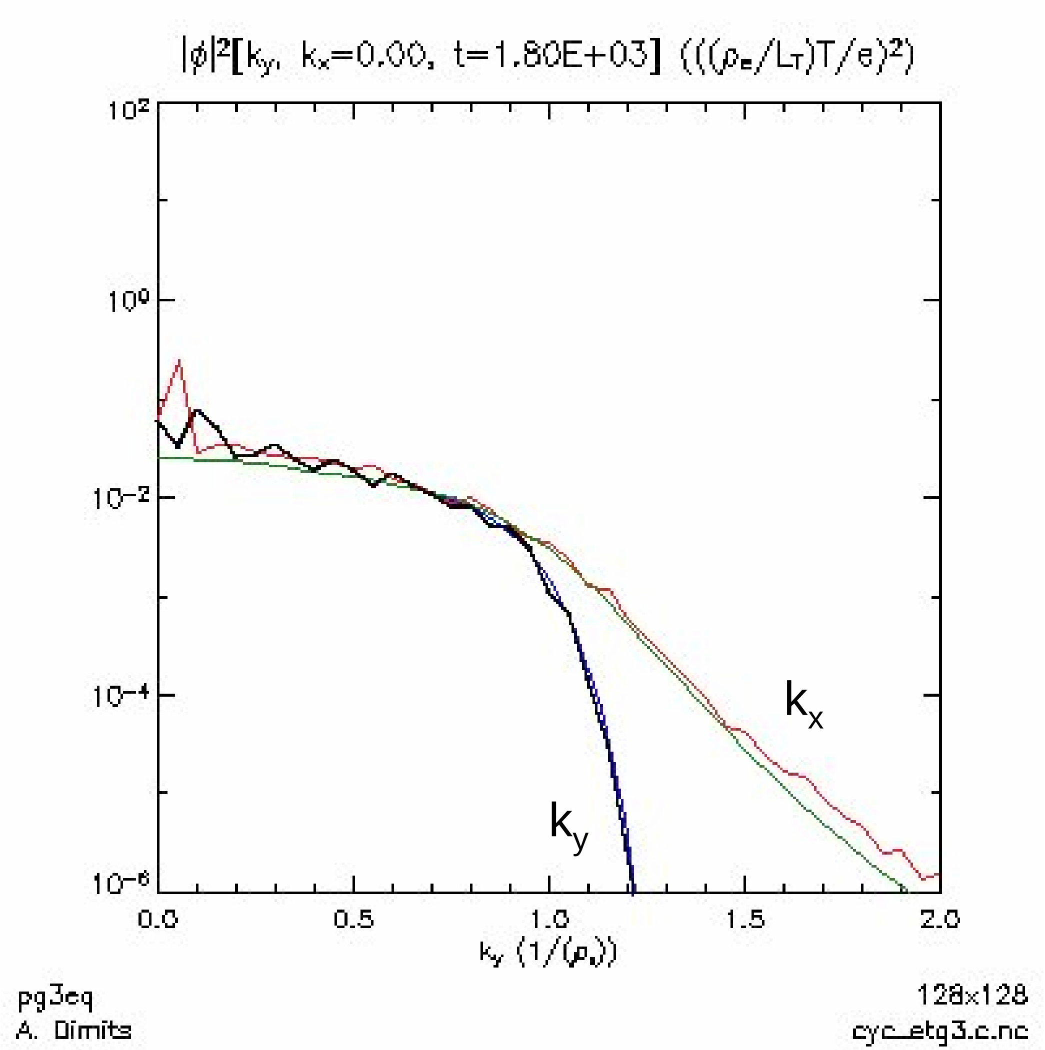
- From Simulation,
  - Fluctuation data in plane $\perp$ to ***B***
  - The time-series $\langle w^2 \rangle(t)$
  - Numerical details about the field-solve
- A mixed representation, $\langle |\phi|^2 \rangle(k_x,k_y,z)$

$$\left\langle \left| \frac{e\phi_{k_x,k_y}(z)}{T} \right|^2 \right\rangle = \frac{1}{L_z^2} \sum_{k_z} \left\langle \left| \frac{e\phi_{k_x,k_y,k_z}}{T} \right|^2 \right\rangle =$$

$$\approx \frac{\langle w^2 \rangle}{n_p (L_x L_y \Delta z)} \left\{ \frac{\Delta z}{2\pi} \int_{-\pi/\Delta z}^{\pi/\Delta z} \frac{S_{filter}^2 \Gamma_0}{[2-\Gamma_0][2-(1-S_{filter} d_{\|})\Gamma_0]} dk_z \right\}$$

$\Rightarrow$ Predicted noise spectrum fits the data.
No Free Parameters!

$\Rightarrow$ This simulation has a noise problem.



Discrete Particle Noise can sometimes be an issue for Particle-in-cell simulations

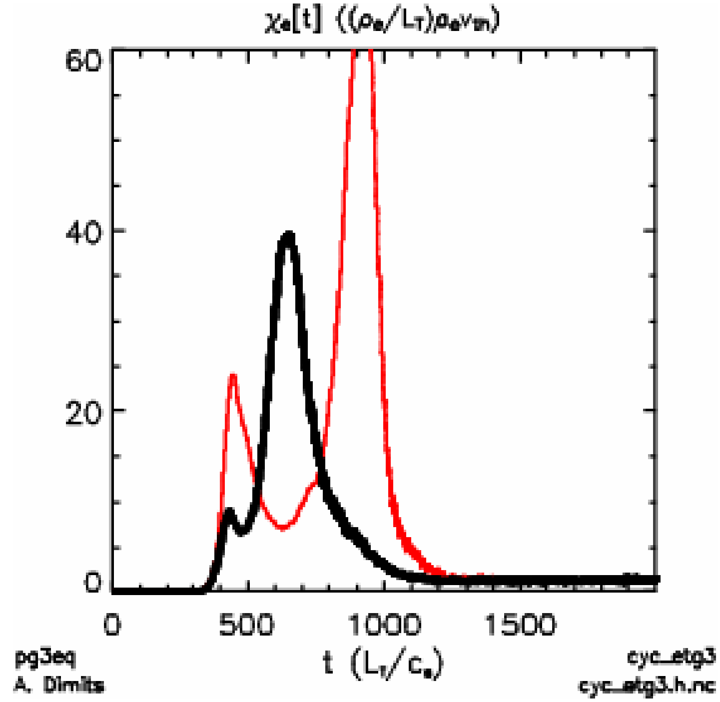
- Red curve: 16 particles/cell, 256 x 64 x 32
- Black curve: 8 particles/cell 128 x 128 x 64
- The mean square particle weight $<w^2>(t)$ measures discrete particle noise

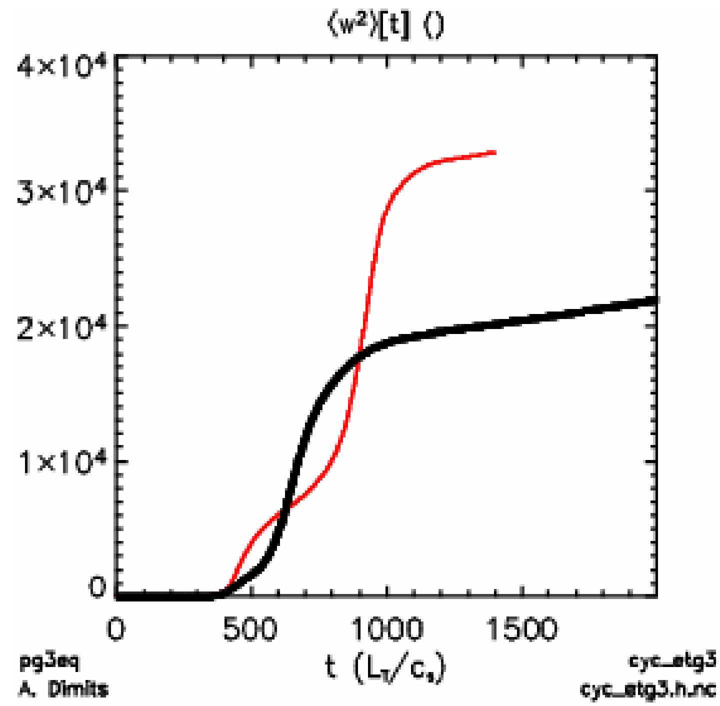
- Entropy Theorem of Lee & Tang:

$$\frac{dw_{rms}^2}{dt} \approx \frac{2\chi_e}{L_T^2}$$

- χ_e is larger (in GK-units, $\rho^2 v_{th}/L_T$) Discrete particle noise a greater issue for ETG that ITG?

- Discrete particle noise looks important for $t > 700\ L_T/v_{th}$ in PG3EQ simulations





Total & Noise part of ϕ spectrum $S_\phi(k)$ at t=

$S_\phi(k)$ from simulation:

$S_\phi(k_x\!=\!0,\ k_y\quad)$ —black

$S_\phi(k_x\quad,\ k_y\!=\!0)$ —red/brown

$S_\phi(k)$ from noise:

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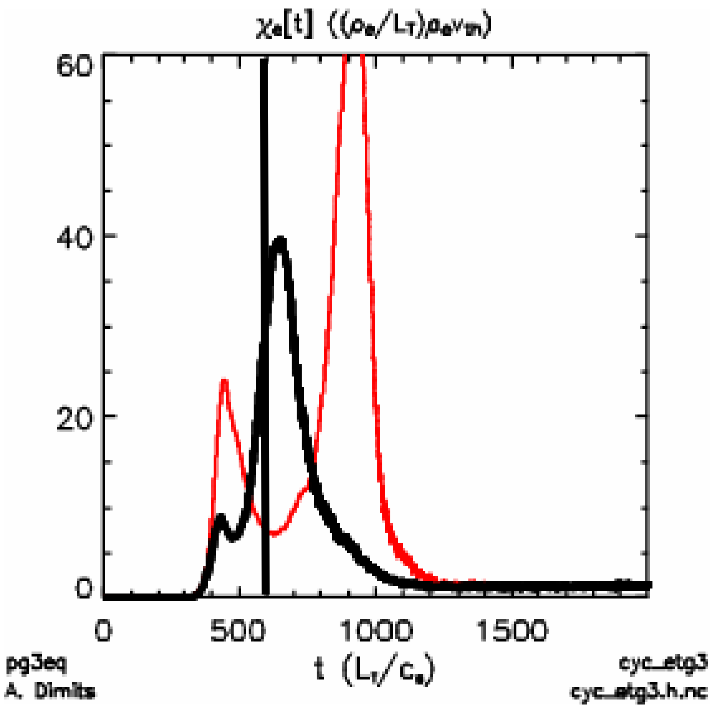

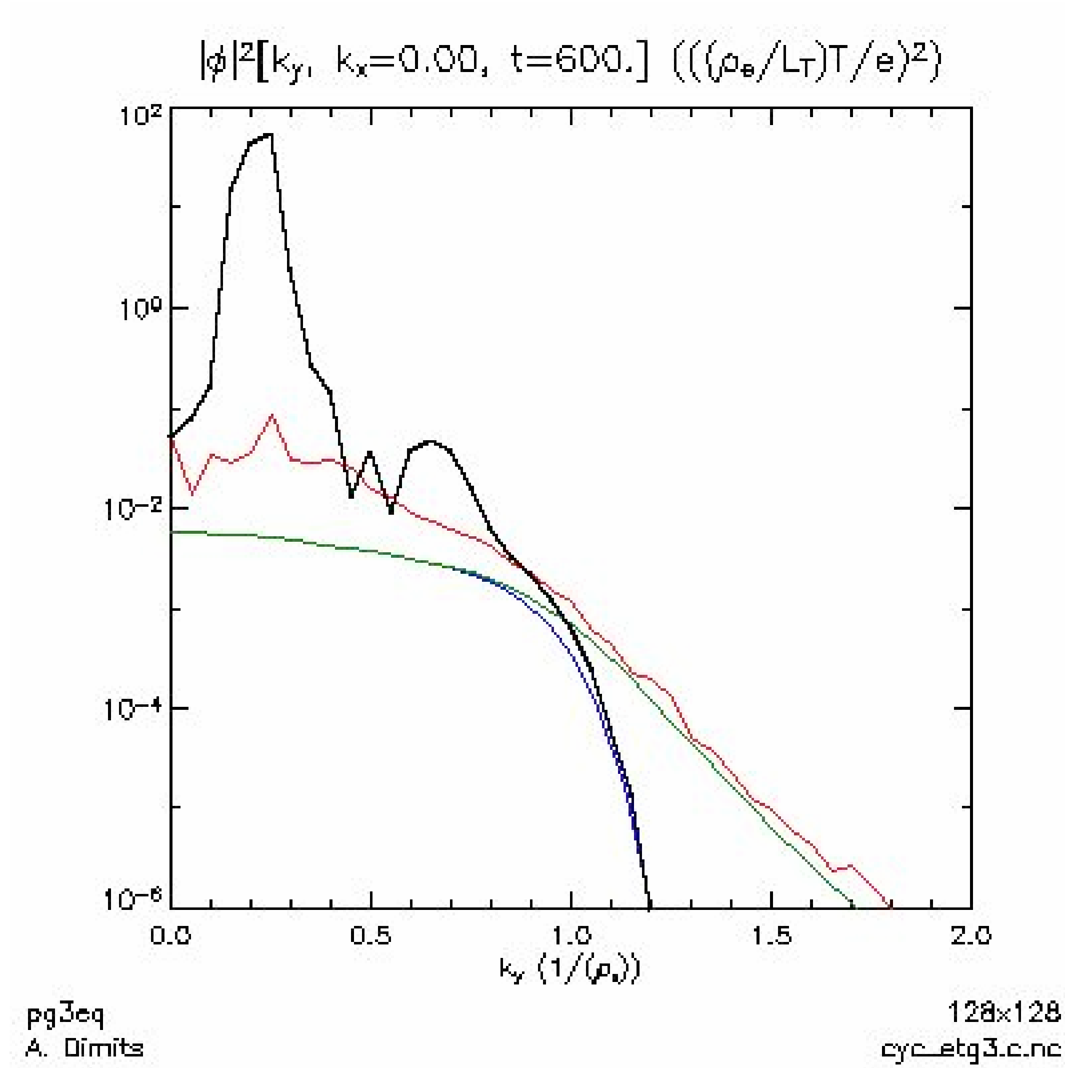

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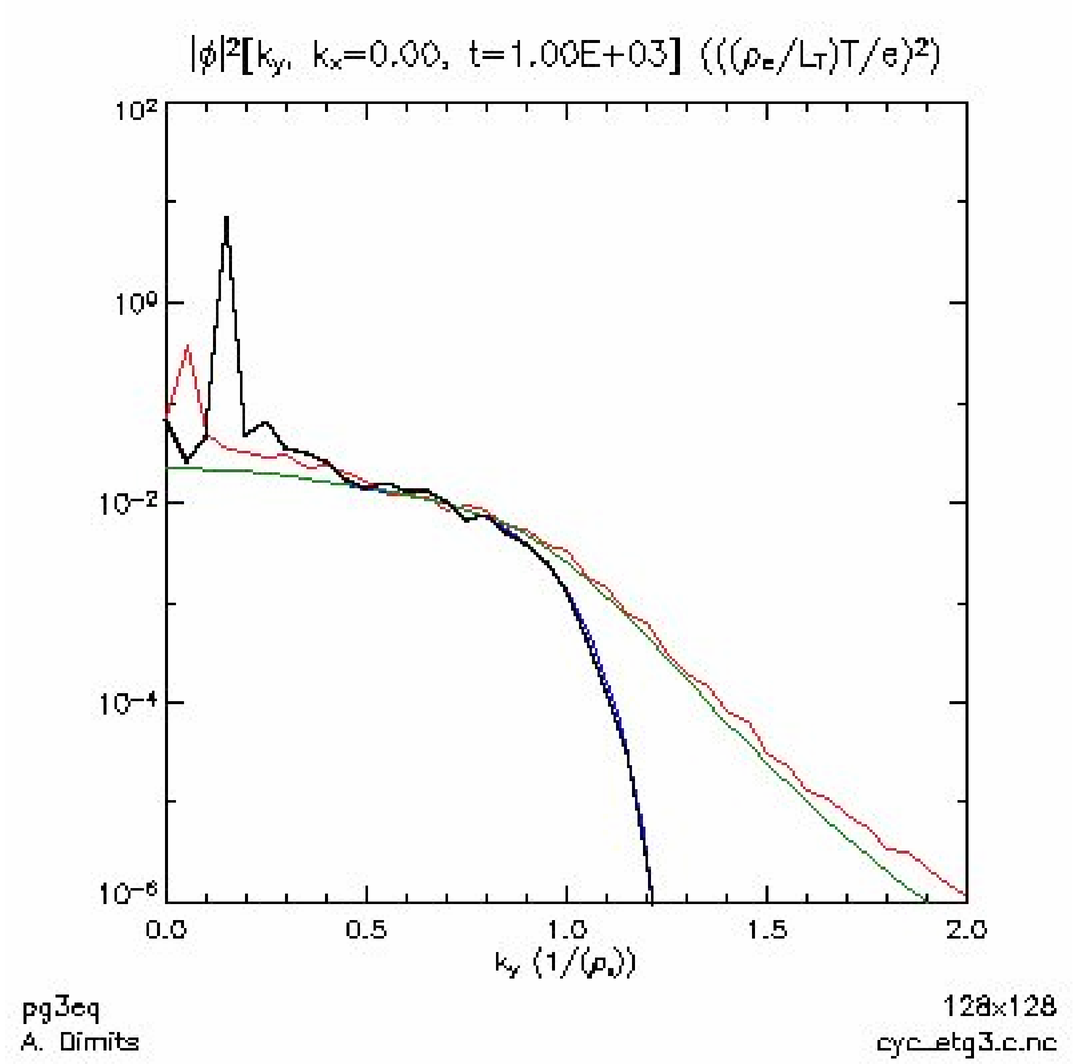

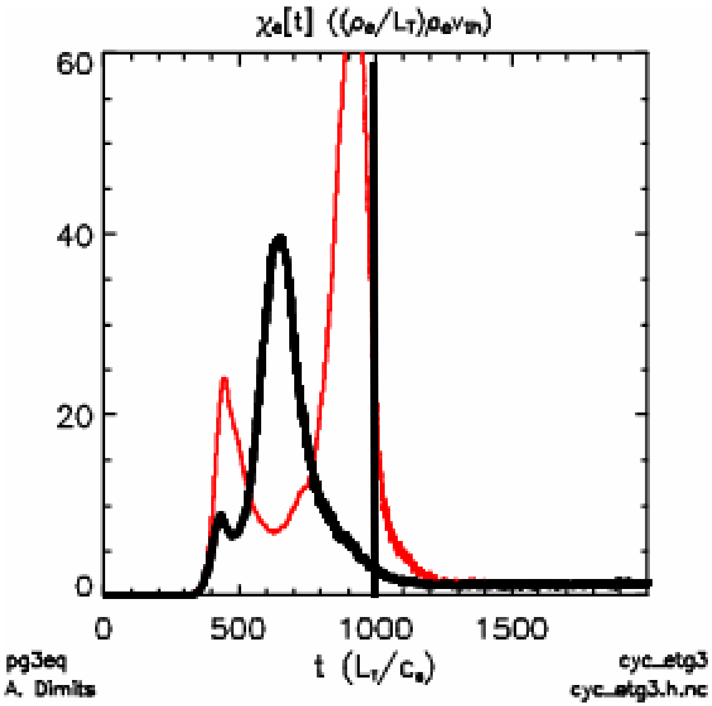

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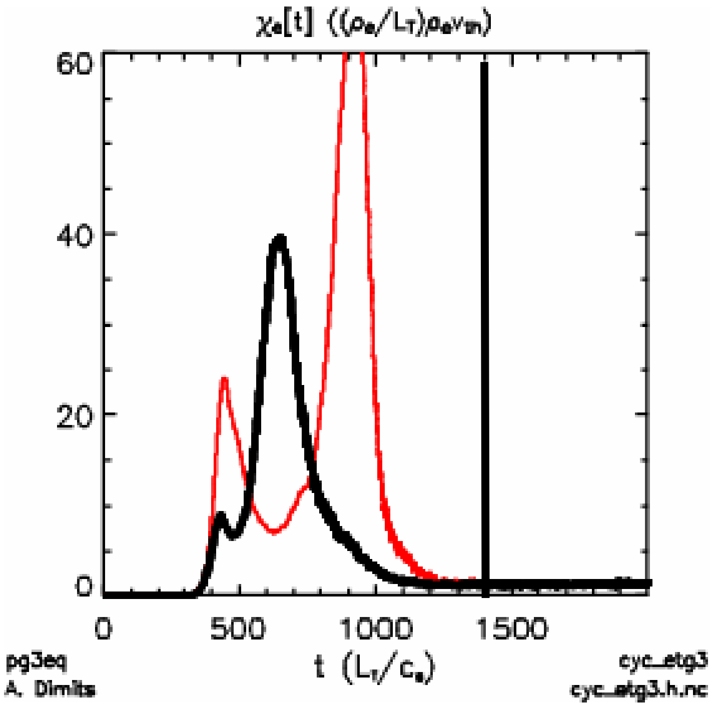

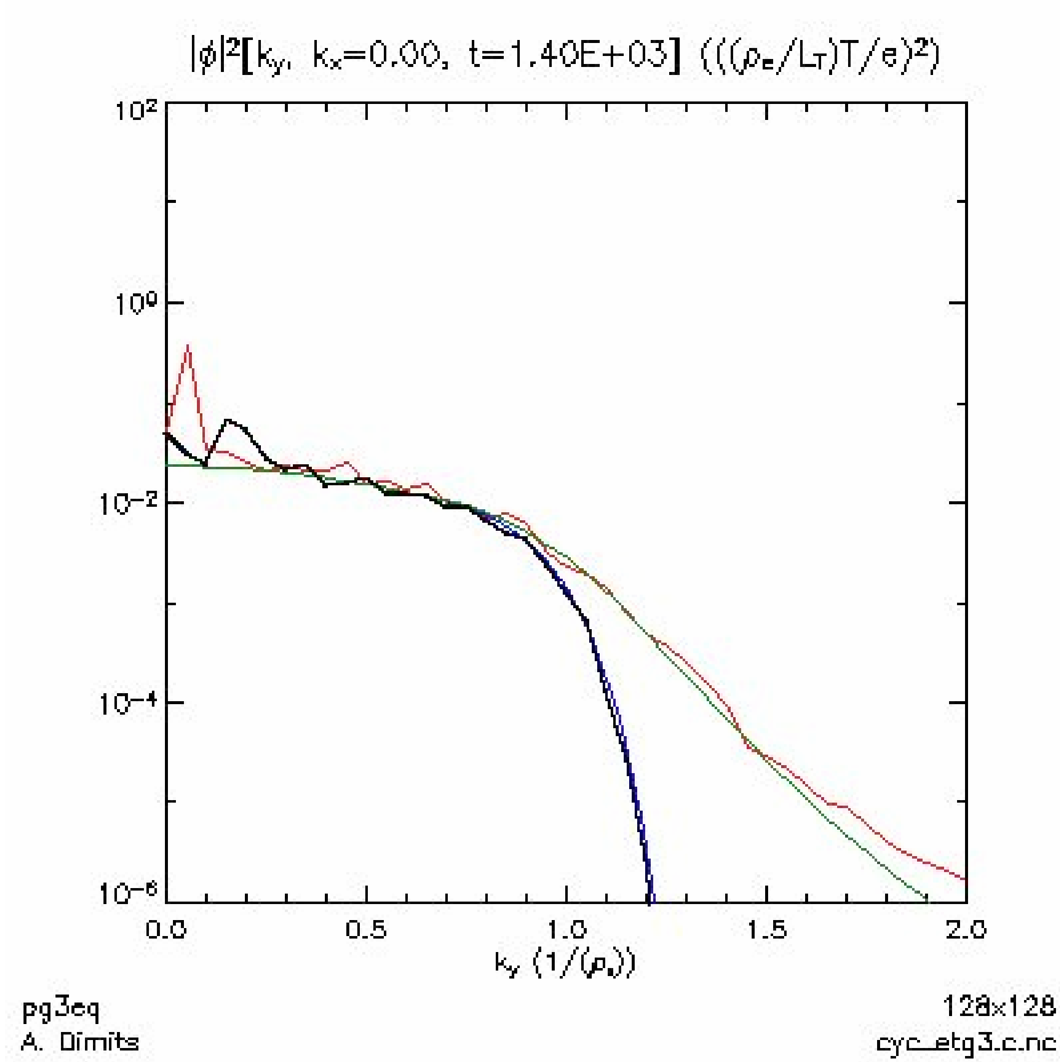

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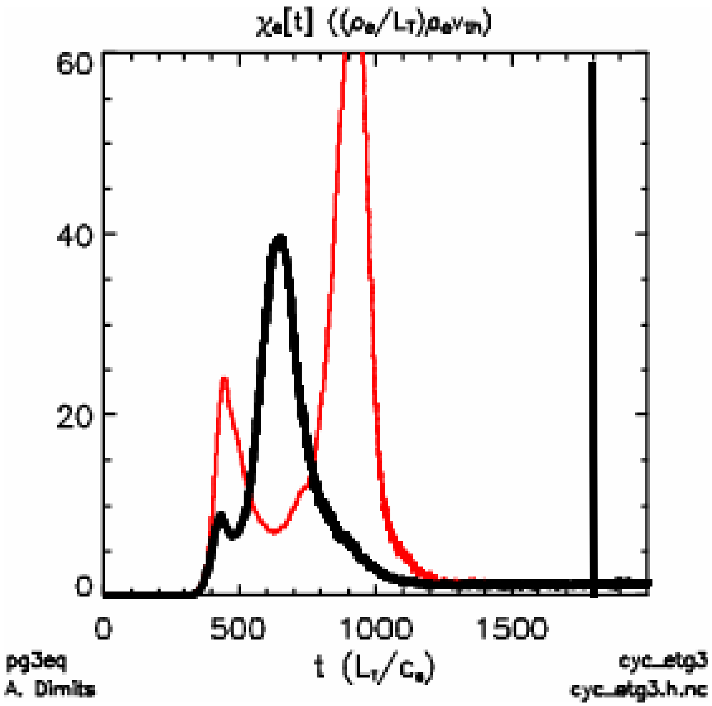
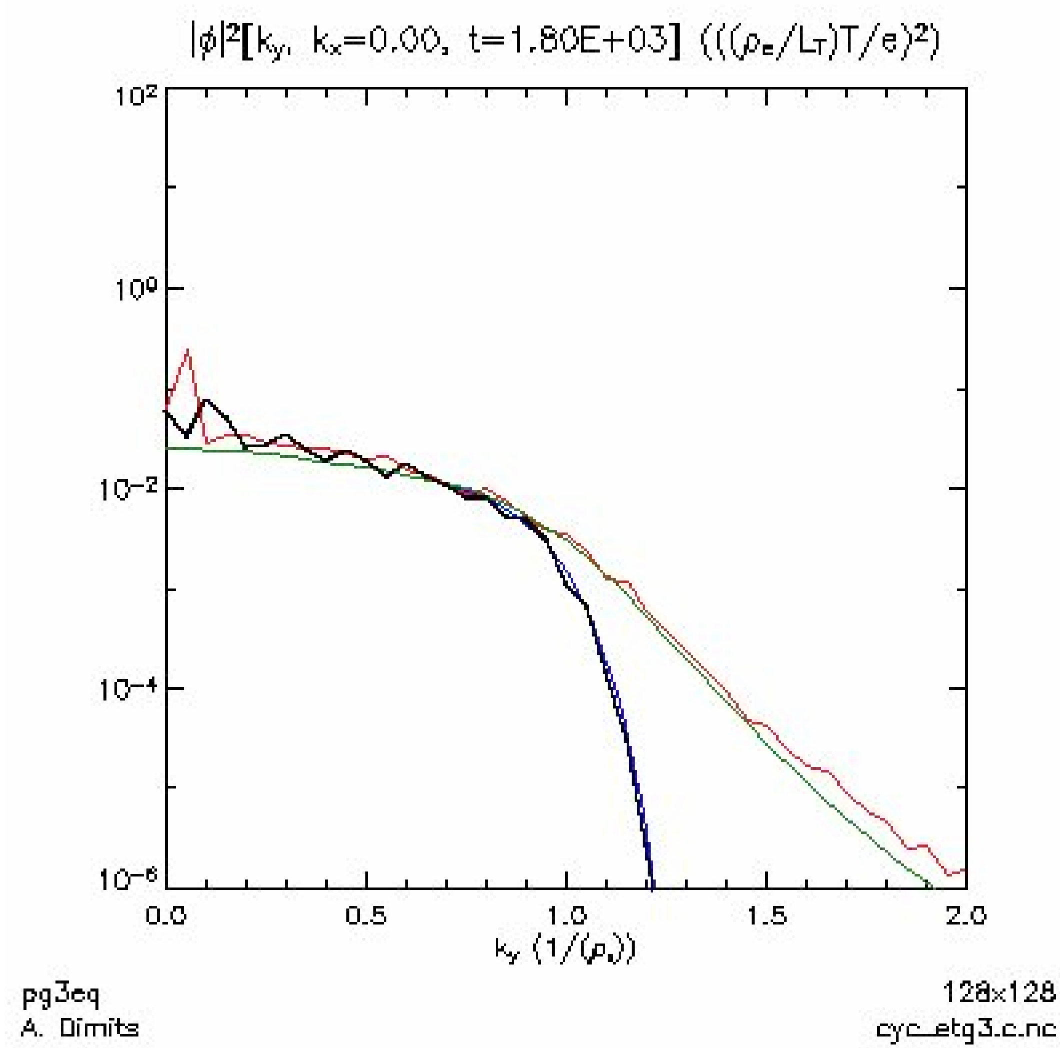




$1/(D_{noise}\, k_\perp^2) \sim 1/((2/3)\; 2\; 0.05^2) \sim 300$

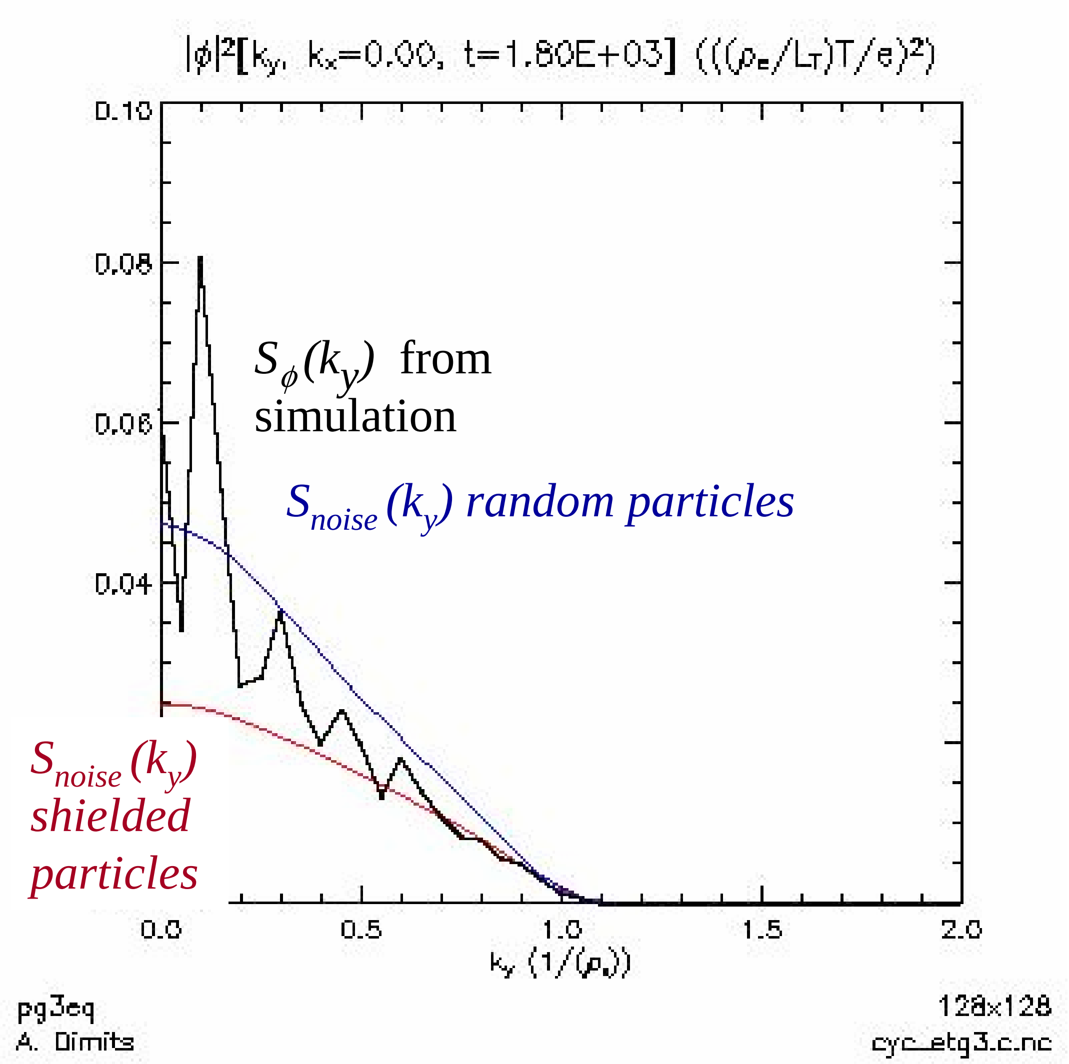
Noise calculation including shielding more accurate.

Simple noise calculation assuming randomly located particles is at most a factor of 2 higher than noise from test-particle superposition principle, including shielding cloud of other particles.

The two noise calculations approach each other for $k_y \rho_e >> 1$, where FLR makes shielding ineffective.

Simple noise from random particles slightly overpredicts observed spectrum.

Noise calculated including shielding from linear gyrokinetic dielectric fits observations better at $k_y \rho_e > 0.5$, provides lower bound on observation.



Simulation Verification (2)
The Fluctuation Intensity

A less computationally intensive diagnostic

$$\left\langle \left| \frac{e\phi}{T} \right|^2 \right\rangle = \frac{1}{V^2} \sum_k \left\langle \left| \frac{e\phi_k}{T} \right|^2 \right\rangle = \frac{\langle w^2 \rangle}{n_p V_{shield}}$$

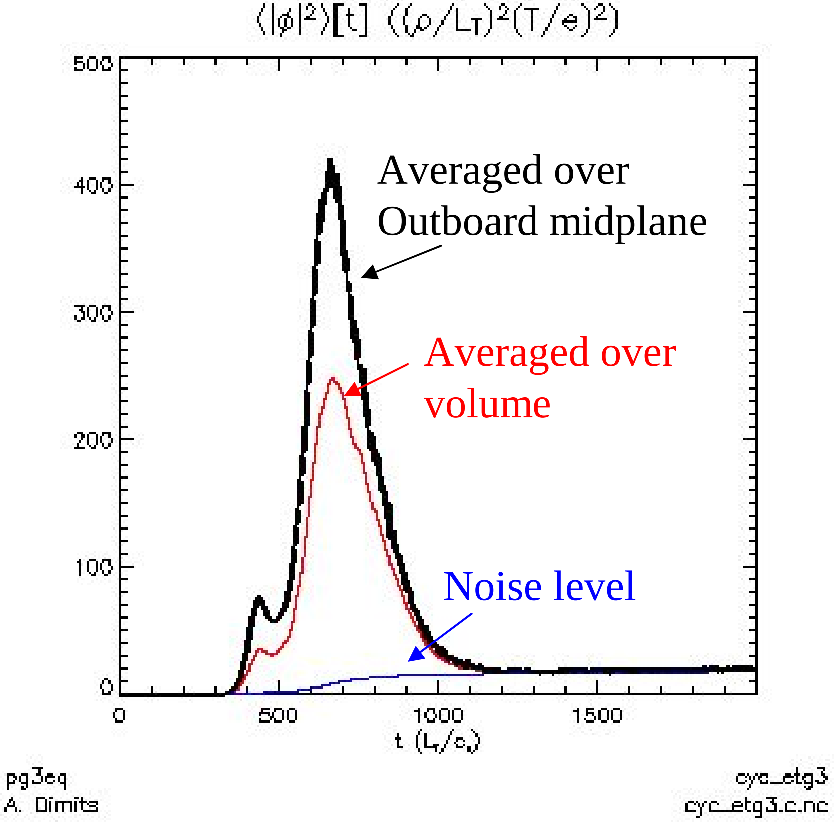
where

$$V_{shield}^{(H)} \equiv \left\{ \frac{1}{(2\pi)^3} \int d^3k \frac{S_{filter}^2 \Gamma_0(k_\perp^2 \rho_{th}^2)}{[2-\Gamma_0][2-(1-S_{filter} d_\parallel)\Gamma_0]} \right\}^{-1}$$

$$V_{shield}^{(N)} \equiv \left\{ \frac{1}{(2\pi)^3} \int d^3k \frac{S_{filter}^2 \Gamma_0(k_\perp^2 \rho_{th}^2)}{[2-\Gamma_0(k_\perp^2 \rho_{th}^2)]^2} \right\}^{-1}$$

Typically $V_{shield} \approx 30\, \Delta x \Delta y \Delta z$

Cyclone base-case-like ETG
Fluctuation Intensity



Simulation Verification (3)
The Fluctuation Energy Density

Fluctuation energy density may be a more relevant diagnostic:

- Has direct physical significance (energy associated with ExB motion)
- Closely related to transport coefficient $D \approx \langle V_{ExB}{}^2 \rangle \tau_{corr}$

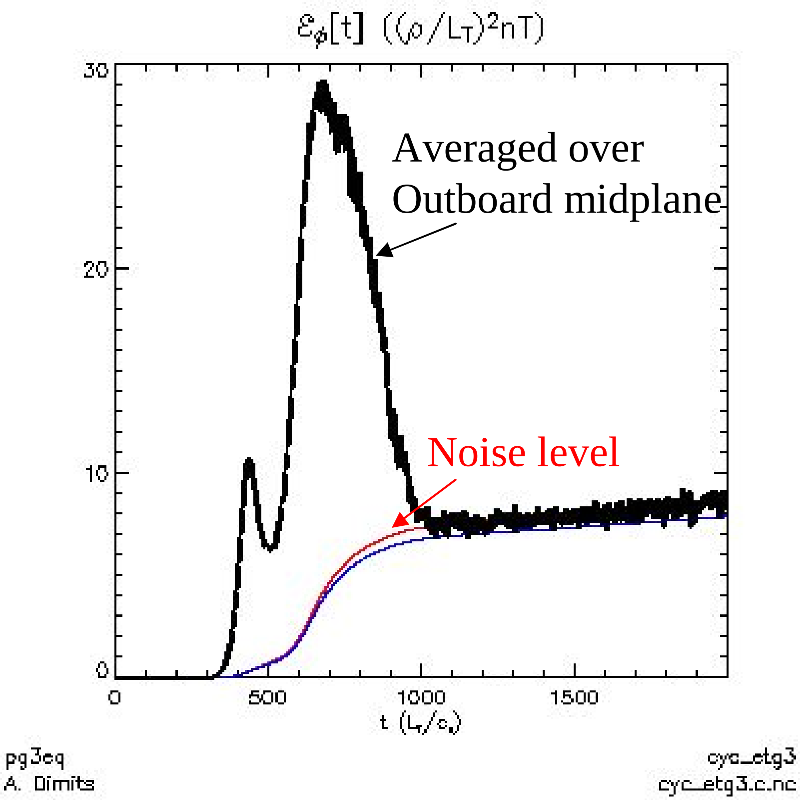
$$-\frac{\omega_p^2}{\Omega_c^2}\left\langle \frac{\phi \nabla_\perp^2 \phi}{4\pi} \right\rangle = nT \frac{\left\langle w^2 \right\rangle}{n_p V_{shield}} \left\langle K_\perp^2 \rho^2 \right\rangle_{noise}$$

where

$$\left\langle K_\perp^2 \rho^2 \right\rangle_{noise}^{(H)} \equiv \left\{ \frac{V_{shield}}{(2\pi)^3} \int d^3k \frac{K_\perp^2(k)\rho^2 S_{filter}^2(k)\Gamma_0(k_\perp^2 \rho_{th}^2)}{\left[2-\Gamma_0\right]\left[2-\left(1-S_{filter} d_\parallel\right)\Gamma_0\right]} \right\}$$

$$\left\langle K_\perp^2 \rho^2 \right\rangle_{noise}^{(N)} \equiv \left\{ \frac{V_{shield}}{(2\pi)^3} \int d^3k \frac{K_\perp^2(k)\rho^2 S_{filter}^2(k)\Gamma_0(k_\perp^2 \rho_{th}^2)}{\left[2-\Gamma_0(k_\perp^2 \rho_{th}^2)\right]^2} \right\}$$

Cyclone base-case-like ETG Fluctuation Energy



Renormalization of Noise-induced test-particle diffusion

$$D_{noise} = \int_{-\infty}^{t} dt' \left\langle \mathrm{v}(x(t),t)\,\mathrm{v}(x(t'),t') \right\rangle$$

$$= \left\langle \mathrm{v}_{ExB}^2 \right\rangle \tau_c \quad \propto \frac{1}{V^2} \sum_{\vec{k}} k_y^2 J_0^2 \,|\, \phi_{noise,k} \,|^2 \, \frac{1}{|\, k_\parallel \,|\, \mathrm{v}_t}$$

(in simple limits)

$$\rightarrow \frac{1}{V^2} \sum_{\vec{k}} k_y^2 J_0^2 \,|\, \phi_{noise,k} \,|^2 \, \frac{1}{|\, k_\parallel \,|\, \mathrm{v}_t + k_\perp^2 D_{noise} + \nu_{turb}}$$

$$\propto \frac{L_z}{V^2} \sum_{k_x, k_y} k_y^2 J_0^2 \,|\, \phi_{noise,k} \,|^2 \log\left(\frac{|\, k_{\parallel,\max} \,|\, \mathrm{v}_t + k_\perp^2 D_{noise} + \nu_{turb}}{k_\perp^2 D_{noise} + \nu_{turb}} \right)$$

Test-particle diffusion coefficient has a logarithmic divergence in the correlation time if integration is over straight-line trajectories. Use standard trick of treating trajectories as stochastic random walks consistent with diffusion. (Singularity also resolved now that I've included renormalized nonlinearities in calculation of noise spectrum.) Include also model of effect of larger-scale turbulence on smaller scales as turbulent shearing rate ν_{turb} (but results insensitive to this).

Noise-induced test-particle diffusion

Integrate over properly weighted (ω,k) spectrum of noise fluctuations to find test-particle diffusion coefficient. Used a renormalized propagator to resolve a logarithmic divergence in the correlation time. (Have also included est. of turbulent shearing decorrelation, etc.)

$$D_{noise} \propto \left\langle \left(\frac{e\Phi_{noise}}{T} \right)^2 \right\rangle \frac{a_{\|}}{L_T} \frac{b_{\max}^2}{\log(1+2b_{\max})} \log\left(1 + c_0 \frac{(1+b_{\max})}{(2+b_{\max})} \frac{k_{\|\max} \mathrm{v}_{ti}}{D_{noise} k_{\perp\max}^2} \right)$$

Note: in large D_{noise} limit, this gives $D_{noise} \sim \Phi_{noise} \sim w_{rms}$, not $\sim w_{rms}^2$. Essential to get linear growth of D_{noise} as seen in Dimits' particle # scan.

$$\frac{dw_{rms}^2}{dt} \propto (D_{turb} + D_{noise}) \sim w_{rms}$$

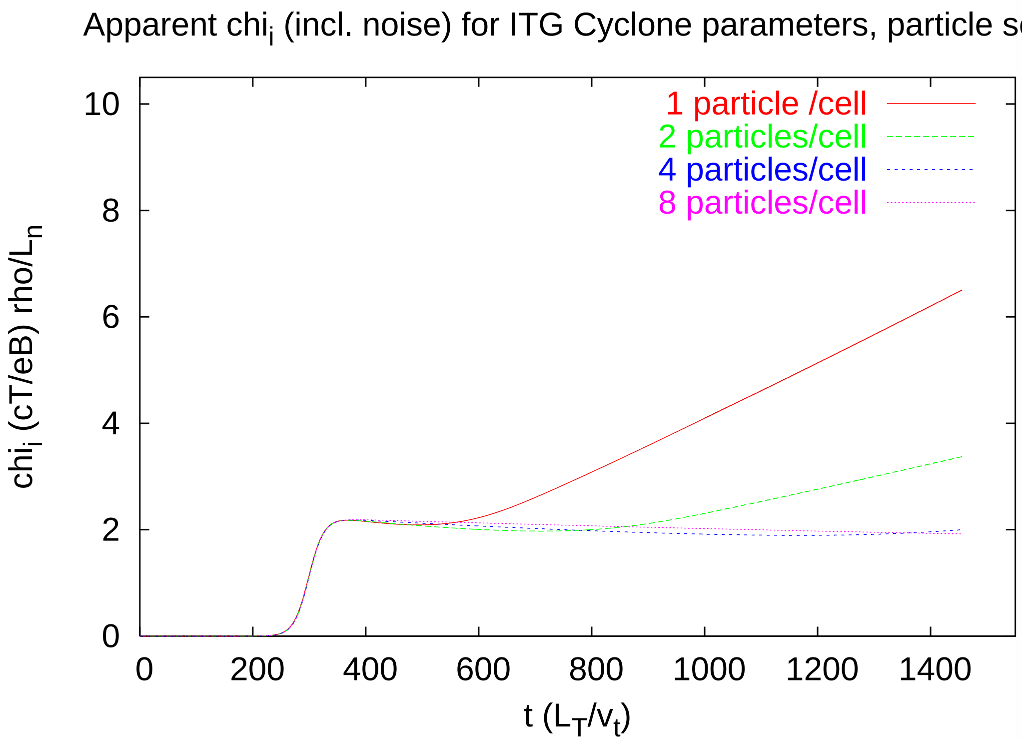
$$\frac{dw_{rms}}{dt} \propto const.$$

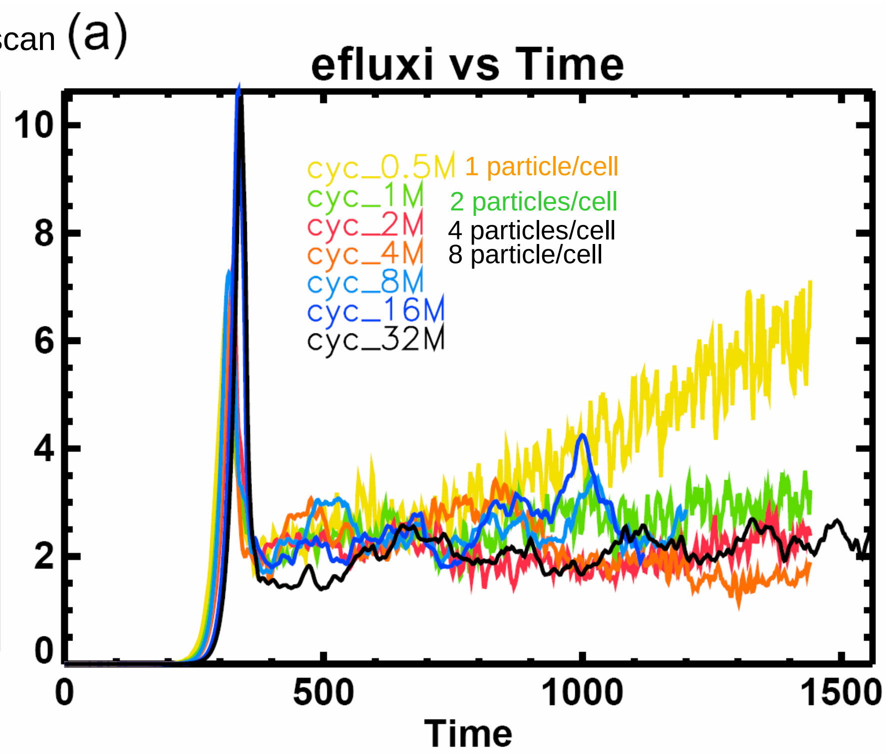
Noise-induced test-particle diffusion

$$D_{noise} \propto \left\langle \left(\frac{e\Phi_{noise}}{T} \right)^2 \right\rangle \frac{a_\parallel}{L_T} \frac{b_{\max}^2}{\log(1+2b_{\max})} \log\left(1 + c_0 \frac{1+b_{\max}}{2+b_{\max}} \frac{k_{\parallel\max} \mathrm{v}_{ti}}{D_{noise} k_{\perp\max}^2} \right)$$

But it is also important to keep the logarithmic corrections so that in the low D_{noise} limit, $D_{noise} \sim \Phi_{noise}^2 \sim w_{rms}^2$ which gives a much smaller D_{noise} (otherwise noise would dominate from near the beginning of the simulation).

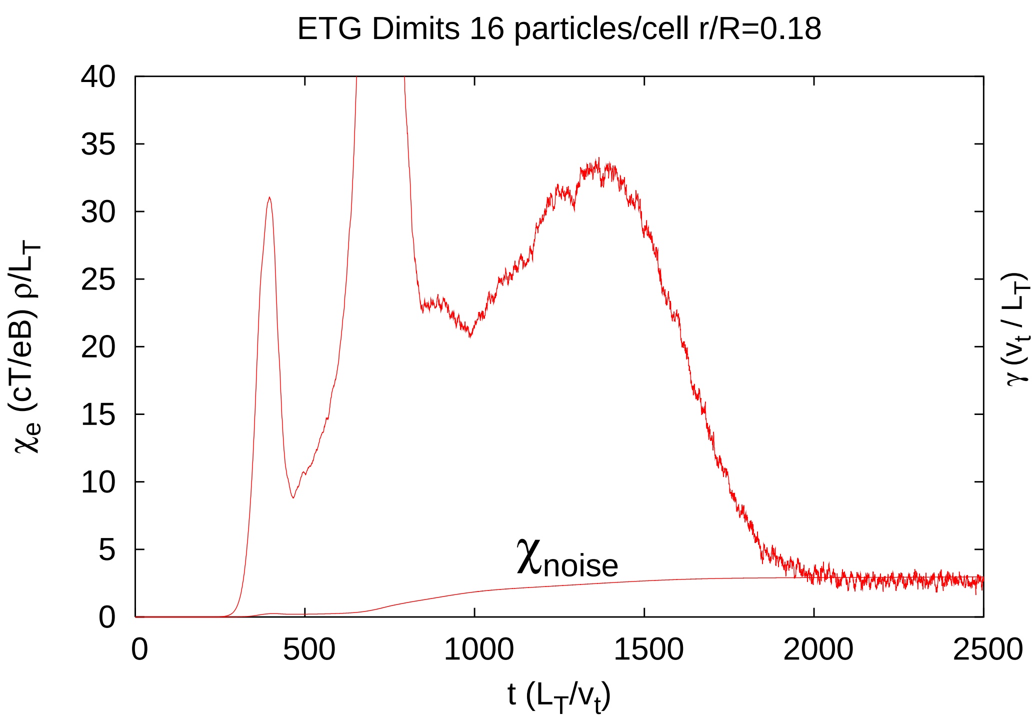
Model of noise-induced diffusion agrees well with particle scan in ITG simulation (Dimits 98/2000)



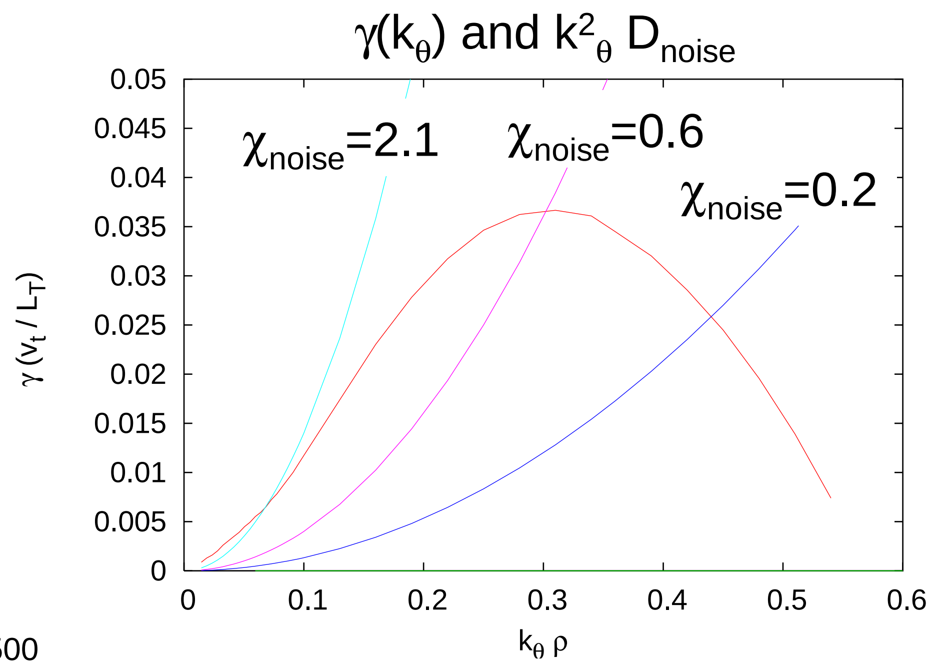


One adjustable order-unity coefficient in model for D_{noise} (approximations used In integrals for renormalized decorrelation time), fit to match Dimits' 1 particle/cell case

Dimits GK ETG simulations demonstrate χ_e falls to χ_{noise} by end of run, when $D_{noise}\, k_\perp^{\,2} > \gamma$

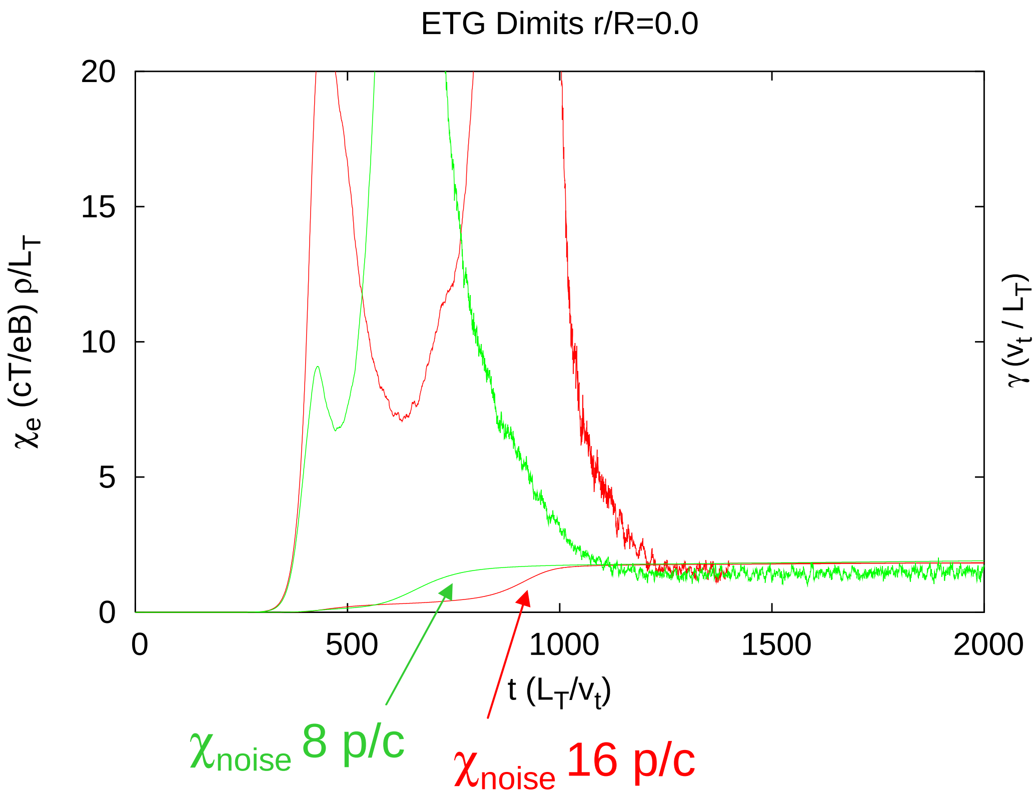


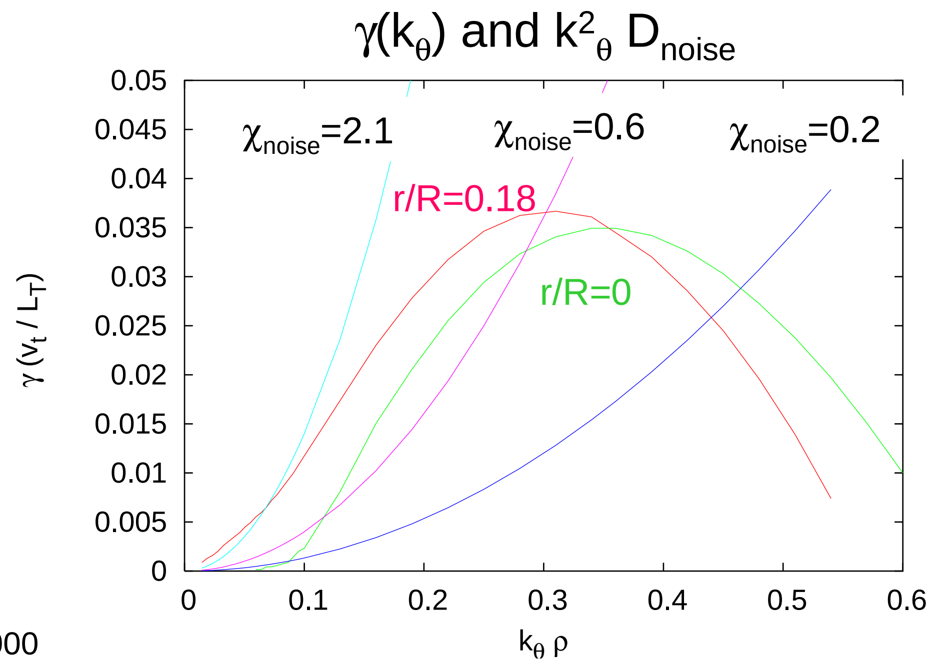
χ_{noise}=0.2, 0.6, 2.1



Growth rate for Cyclone base case with s-alpha geometry.

Noise follows expected trends as particle number varied & trapping turned off



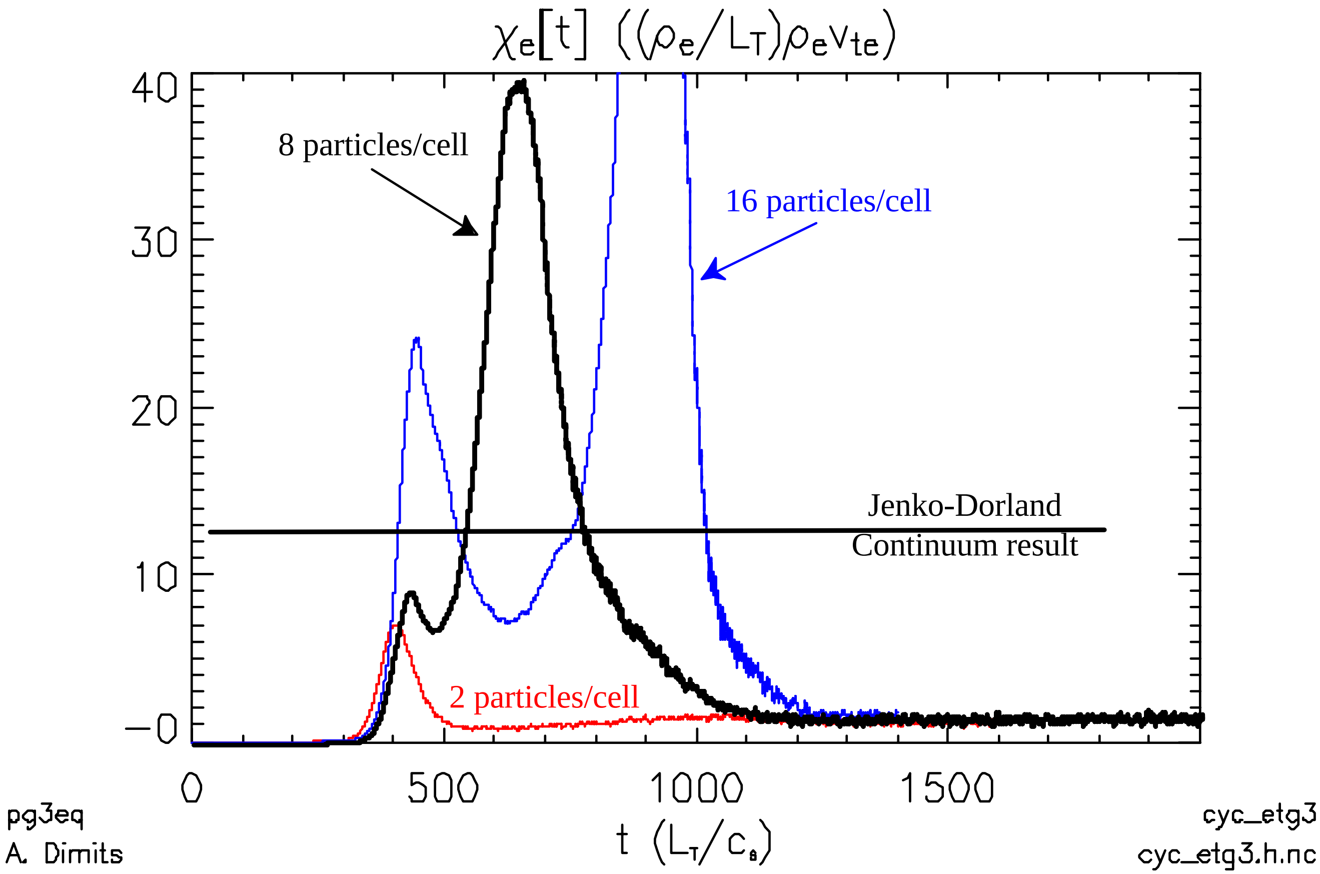


Turning off particle trapping (*r/R*=0) significantly reduces γ at low *k*

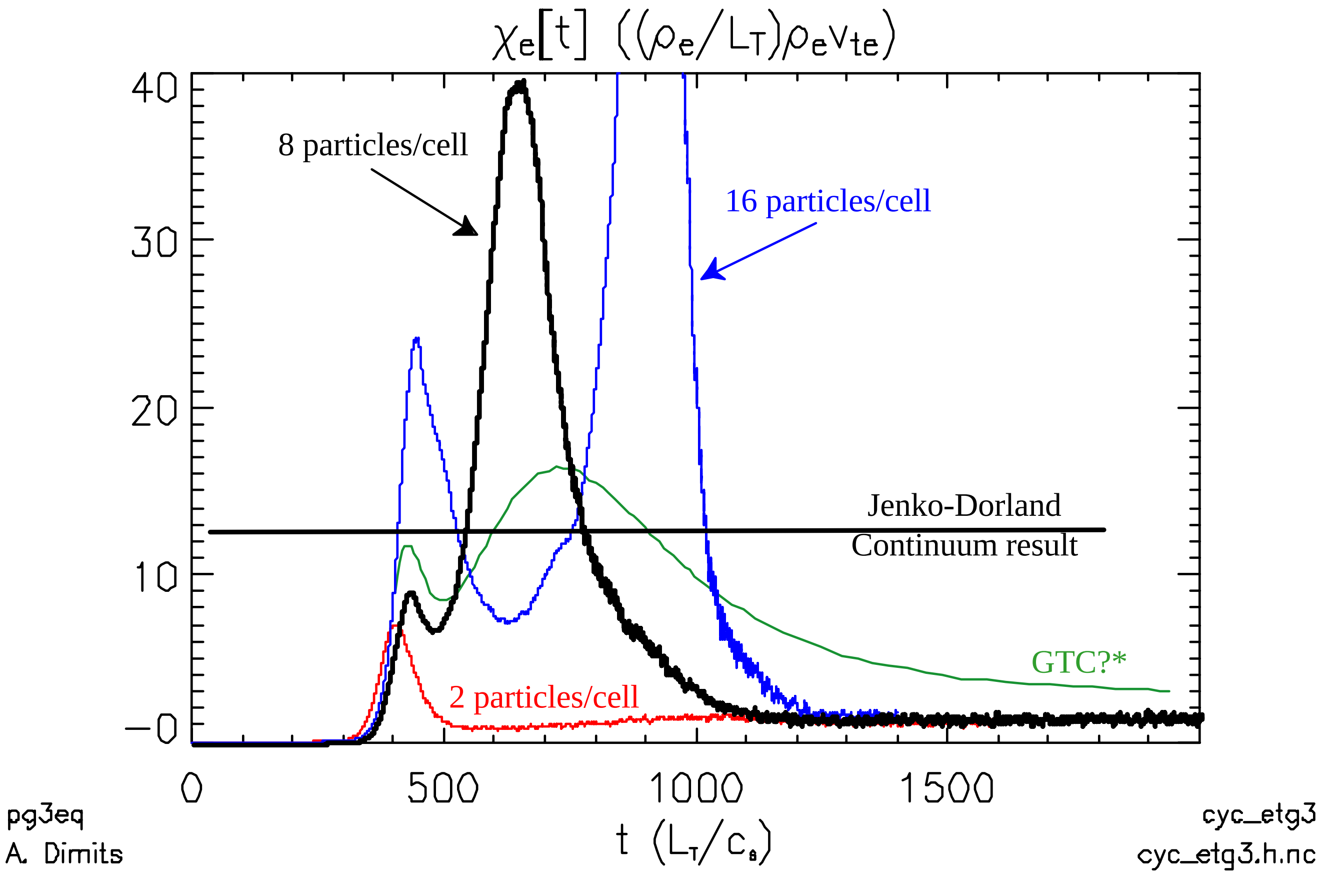
$$\chi_{noise} \propto \frac{w^2}{N} \propto \frac{\int^t dt'\chi(t)}{N}$$

Increasing # particles just leads to longer initial period of high χ_{tot}, so final χ_{noise} is not sensitive to N.

Discrete Particle Noise Suppresses Transport In Cyclone-base-case like ETG Simulations

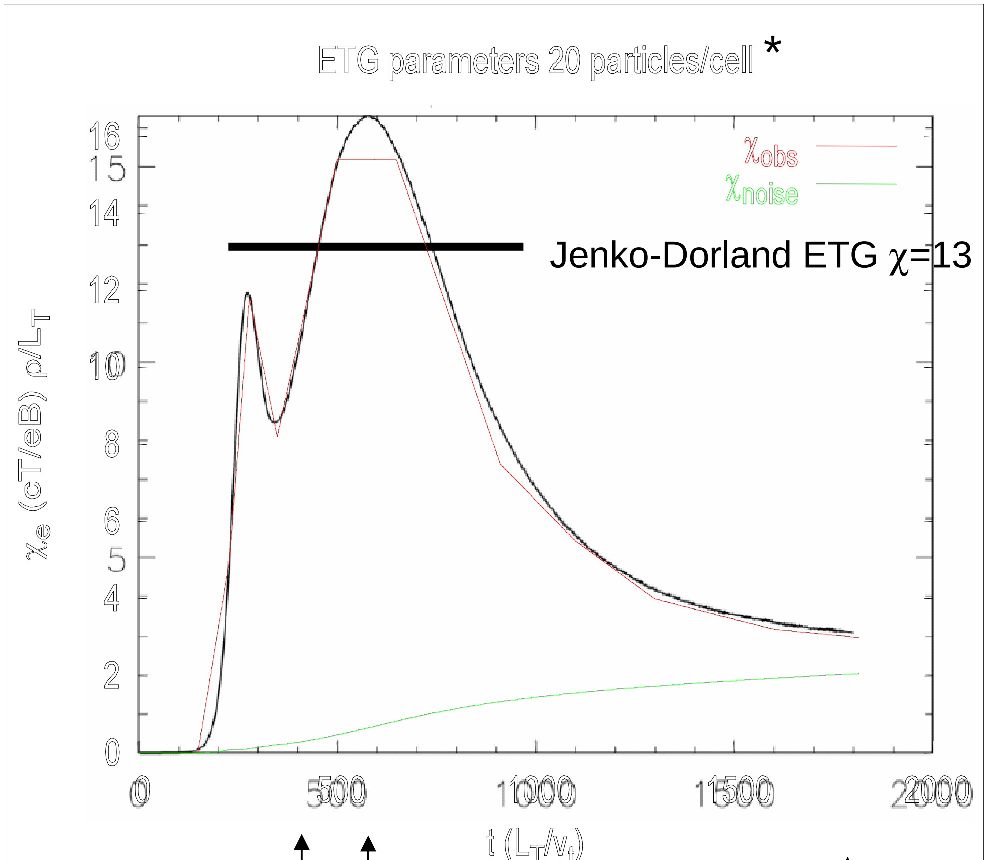


Discrete Particle Noise Suppresses Transport In Cyclone-base-case like ETG Simulations

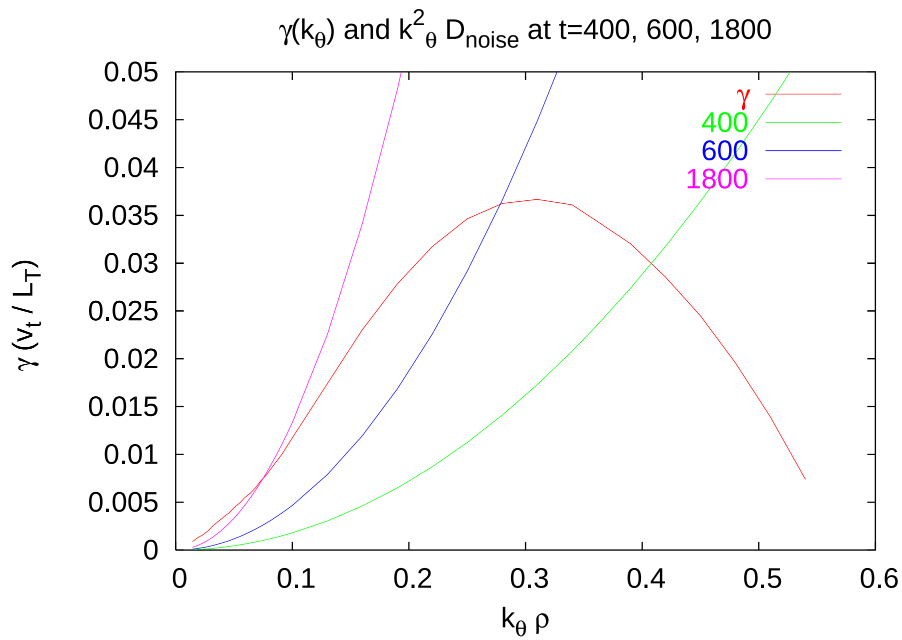


*GTC curve from Slide #13 of Z. Lin's IAEA presentation, which can be found at:
http://www.cfn.ist.utl.pt/20IAEAConf/presentations/T5/2T/5_H_8_4/Talk_TH_8_4.pdf

$D_{noise}\, k_\perp^2$ >~ γ by end of Z. Lin ETG simulation $\rightarrow$Turbulence drive reduced (except at very low k_θ)



χ_{noise}=0.2, 0.6, 2.1

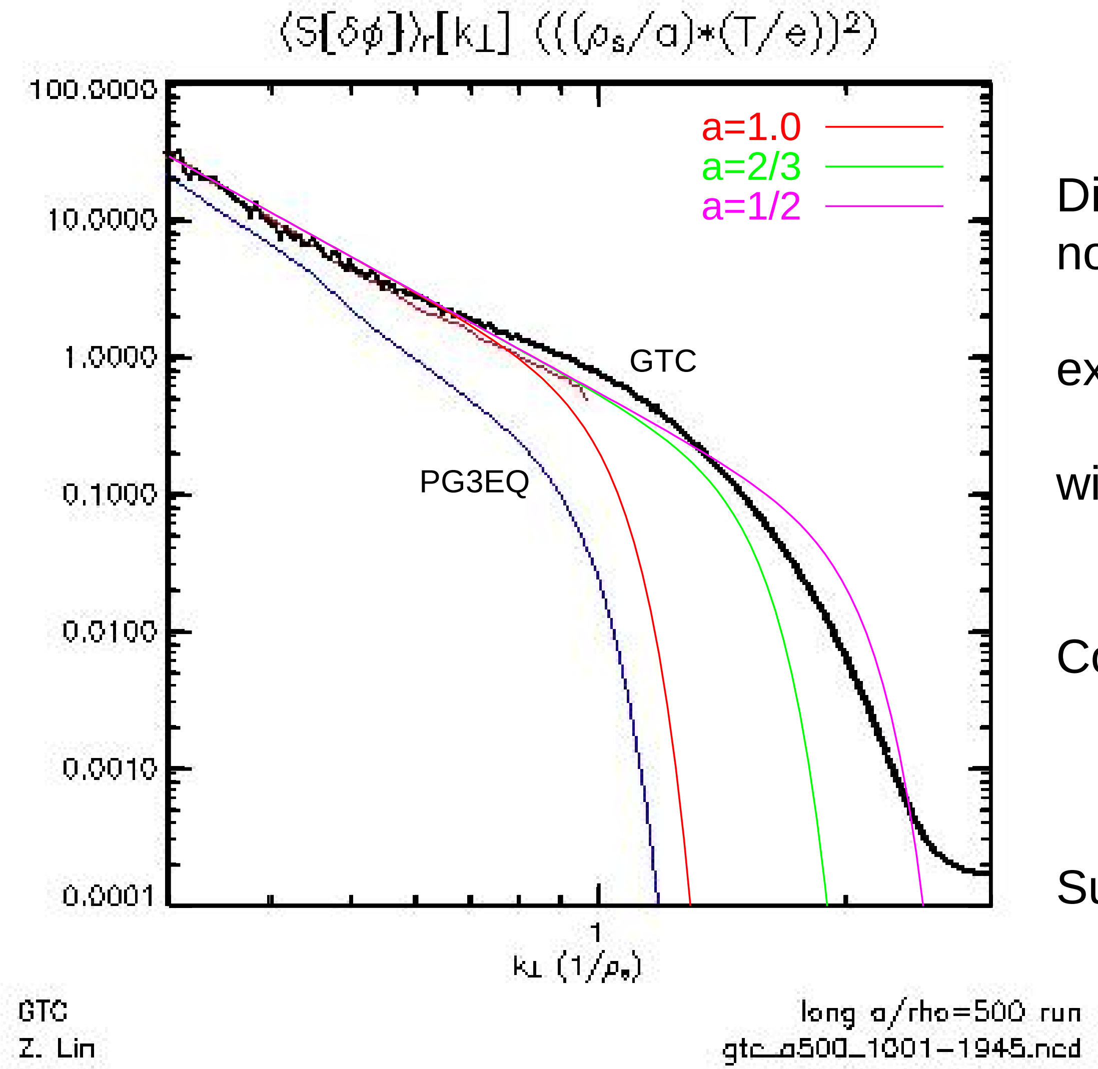


Growth rate for Cyclone base case with s-alpha geometry.
$\gamma \uparrow$ ~20% with real circles.

* Note added Oct. 22, 2005: see errata at http://w3.pppl.gov/~hammett/talks/2005/Sherwood-errata.html

Believed to be χ_{etg}(t). From Z.Lin's IAEA 2004 slides
http://www.cfn.ist.utl.pt/20IAEAConf/presentations/T5/2T/5_H_8_4/Talk_TH_8_4.pdf

Estimate of Particle Filtering Parameters



Dimits PG3EQ uses spectral / non-local filter

$\exp(-(ak_y\Delta y)^8)/(1+(ak_x\Delta x)^8)/(1+ak_z\Delta z)^8$

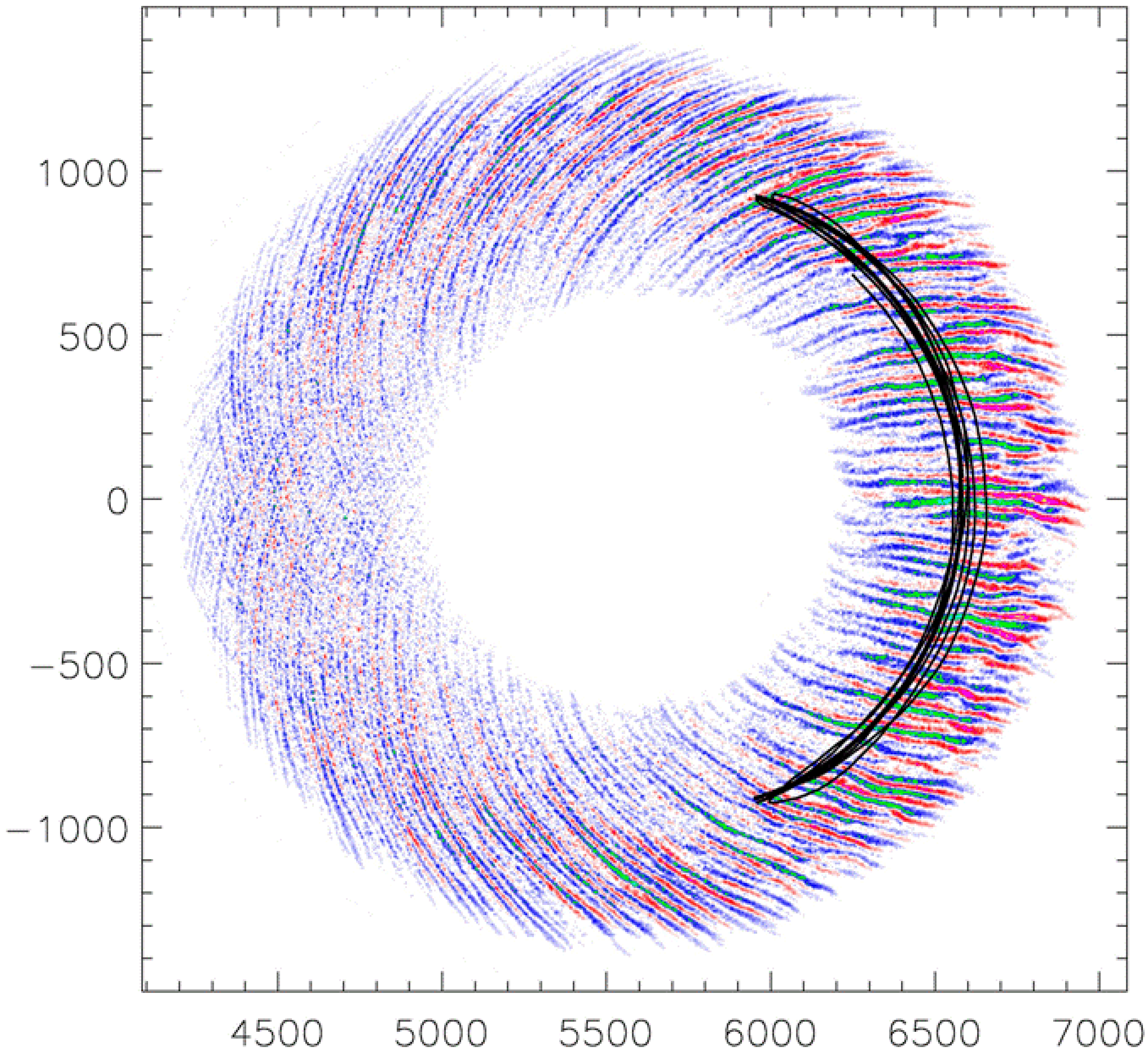
with a=1

Comparison of GTC spectra with

$S \sim (1/k^{3.3}) \exp(-(ak_y\Delta y)^8)$

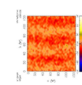
Suggests filter width a~2/3 roughly.

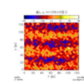
Data for ITG spectra from Z. Lin & A. Dimits provided to W. Nevins for a paper showing similarities between codes for ITG turbulence. Small differences at low k not significant, depend on time averaging.



Dimits contour plots at Same scale as Z. Lin's

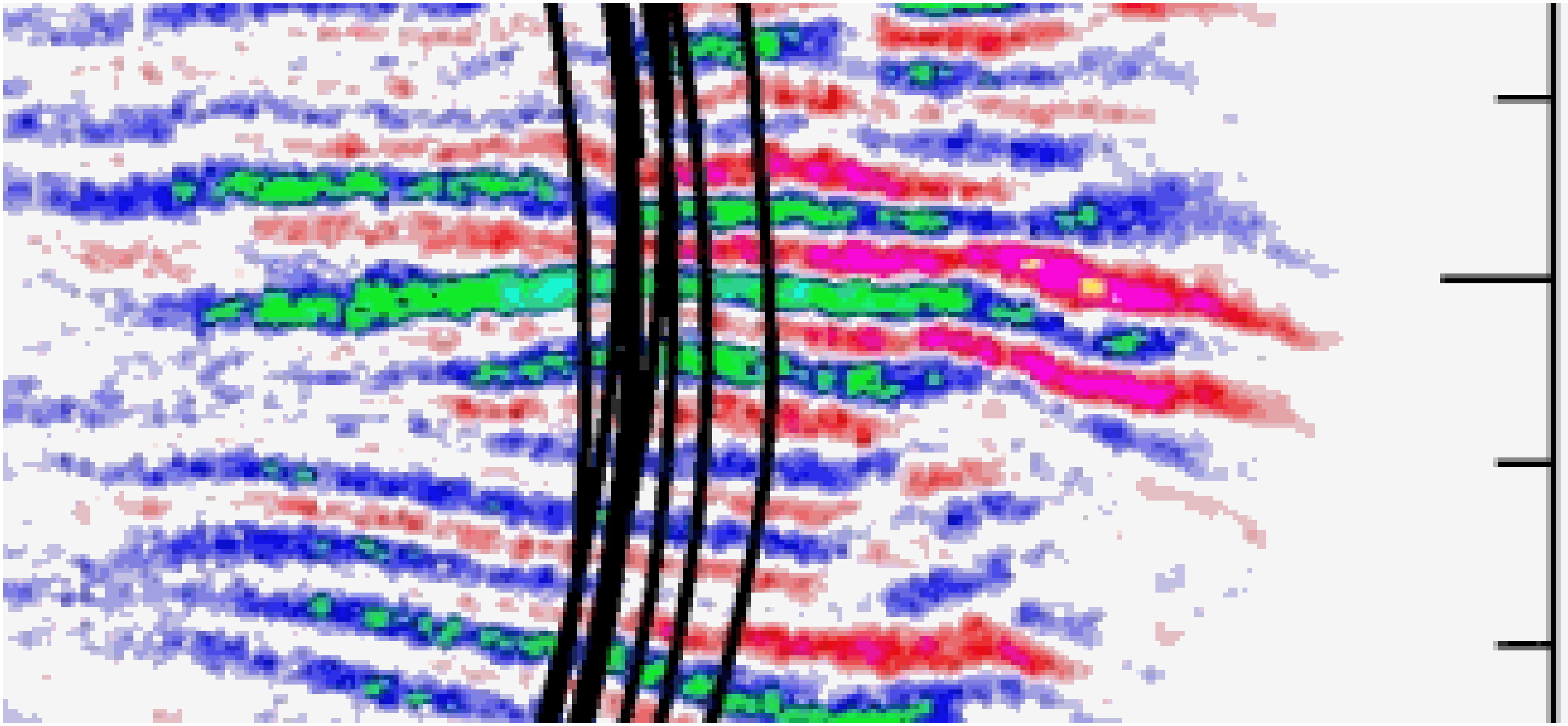
Dimits contour plot with Re-scaled color bar

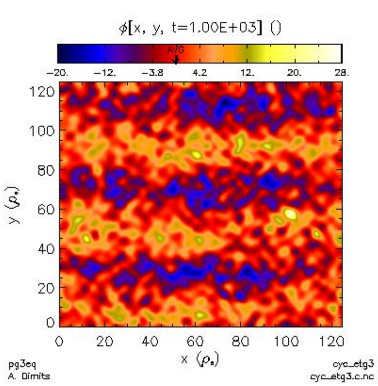


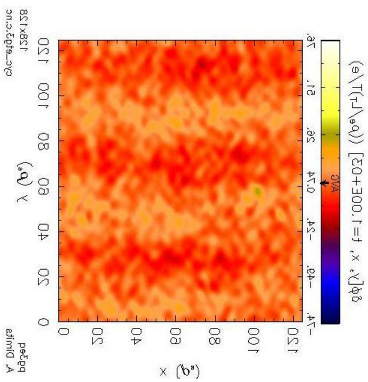


Dimits contour plot at t=1000, when χ_e ~2 x final χ_{noise}. This is when noise effects are strong enough to reduce χ_e to ~1/4th of Jenko-Dorland result, but ETG mode is still apparent.

But if one shrinks the contour plot to the scale used in Z. Lin's plots, then the eye (and the finite resolution of the computer screen) will average out the noise to make it less apparent.







If we blow up Z. Lin's contour plot, then we can see the noise at small scales more easily. It looks roughly comparable to Dimits' contour plot at t=1000 (when χ_e ~ 2 x final χ_{noise} ~ 1/4th $\chi_{Jenko\text{-}Dorland}$.

Eyeball comparisons depend on choice of color table, smoothing in graphics, etc. as illustrated by two versions of Dimits' contour plot to left which differ only in the color table employed. Hence, we need more quantitative measures of noise than the "eyeball test".

Discussion of results

•Large initial transients in χ_{etg}(t) seen by Dimits & by Z. Lin are larger than or comparable to Jenko & Dorland χ_{etg}. But this high χ_{etg} quickly drives weights so large that $k_\perp^2 D_{noise} \sim \gamma_{lin}$ and the turbulence is suppressed or significantly reduced.

• Scanning from 5-20 particles/cell appears to be converged (but isn't) because it just changes time it takes for weights to build up to give $k_\perp^2 D_{noise} \sim \gamma_{lin}$. This can be misunderstood as an ignorable initial transient.

• ETG eddies are radially very extended but still short scale in poloidal direction, so only takes a little bit of D_{noise} << Jenko-Dorland χ_{etg} to suppress or significantly reduce the turbulence. Radially extended ETG eddies more sensitive to noise than ITG is, requires many more particles to converge.

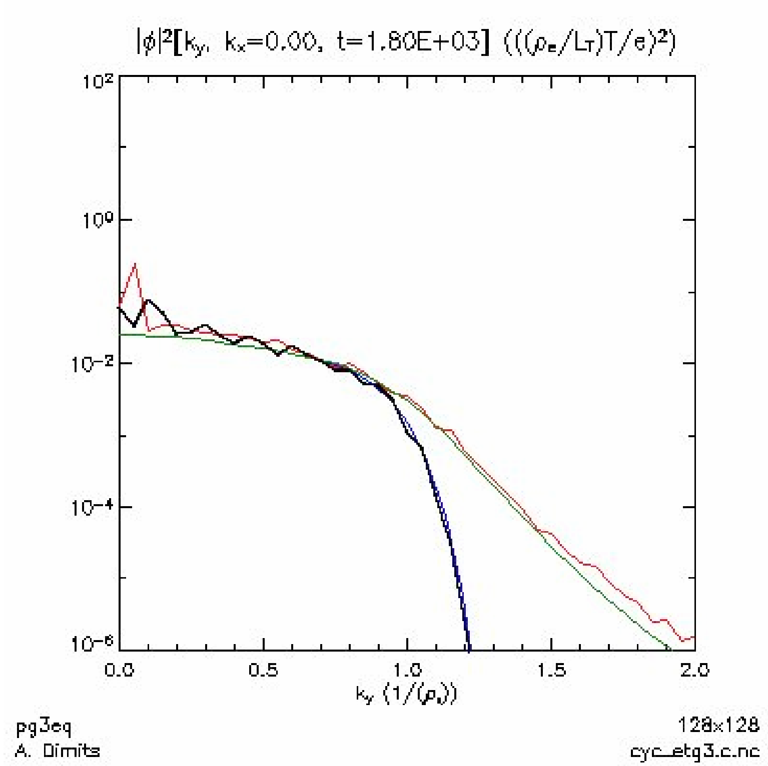
$$\boxed{D_{total} = \frac{\gamma - k_\perp^2 D_{noise}}{k_r^2} + D_{noise}}$$

• Transition from turbulence-dominated state to noise-dominated state expected to slower in Z. Lin's simulations then in Dimits initial simulations (which have large overshoots due to their smaller size).

Caveats

- Used guestimates of filtering parameters for Z. Lin's ETG simulations, based on Lin's previous ITG spectra.
- Would be best to compare with Lin's plots of $\chi(t)$ for different numbers of particles, or do a particle number scan in another gyrokinetic code.
- Neglected differences in zonal components of noise due to differences in ITG/ETG zonal flow dynamics.
- Fluctuation-dissipation theorem used uniform plasma dielectric in unsheared slab geometry. Probably good approximation but would be interesting to try a renormalized model of turbulent dielectric.
- Long-time scale variability often seen in $\chi(t)$ makes detection of trends harder
- Should calculate energy weighted thermal diffusion more consistently?

Conclusions



- Detailed calculation of noise spectrum based on Test Particle Superposition Principle (extending Krommes 93 calculation to include numerical filtering factors) agrees very well (no free parameters) with observed spectrum at late times in Dimits' gyrokinetic ETG simulations (chosen with parameters similar to Z. Lin's simulations), confirming that noise grows to dominate those ETG results.

- ETG eddies are radially very extended but still short scale in poloidal direction & so are sensitive to even a small amount of diffusion in the poloidal direction. Calculations of D_{noise} for Z. Lin's recent ETG simulations (using Lee-Tang entropy balance to give the rms weights) indicates $k_\perp^2 D_{noise} \sim \gamma_{lin}$. Suggests that Z. Lin's ETG results are dominated by noise that is suppressing or significantly reducing the apparent turbulence.

- ETG simulations require many more particles for convergence than ITG.

References

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Note: these slides are as presented in a talk and a poster at the 2005 International Sherwood Fusion Theory Conference, with just a few typos fixed (sign error in adiabatic response in slide #16, missing S^2 in denominator of slide #14 but not #17, vertically flipping the lower contrast pg3eq contour plots in slides #39&40), add an explanation on slide #34. These changes were made April 24. On Oct. 22, 2005 added a pointer to an errata note on slide # 37 and here:

See errata note at http://w3.pppl.gov/~hammett/talks/2005/Sherwood-errata.html