Gyrokinetic Theory and Simulation of Experiments

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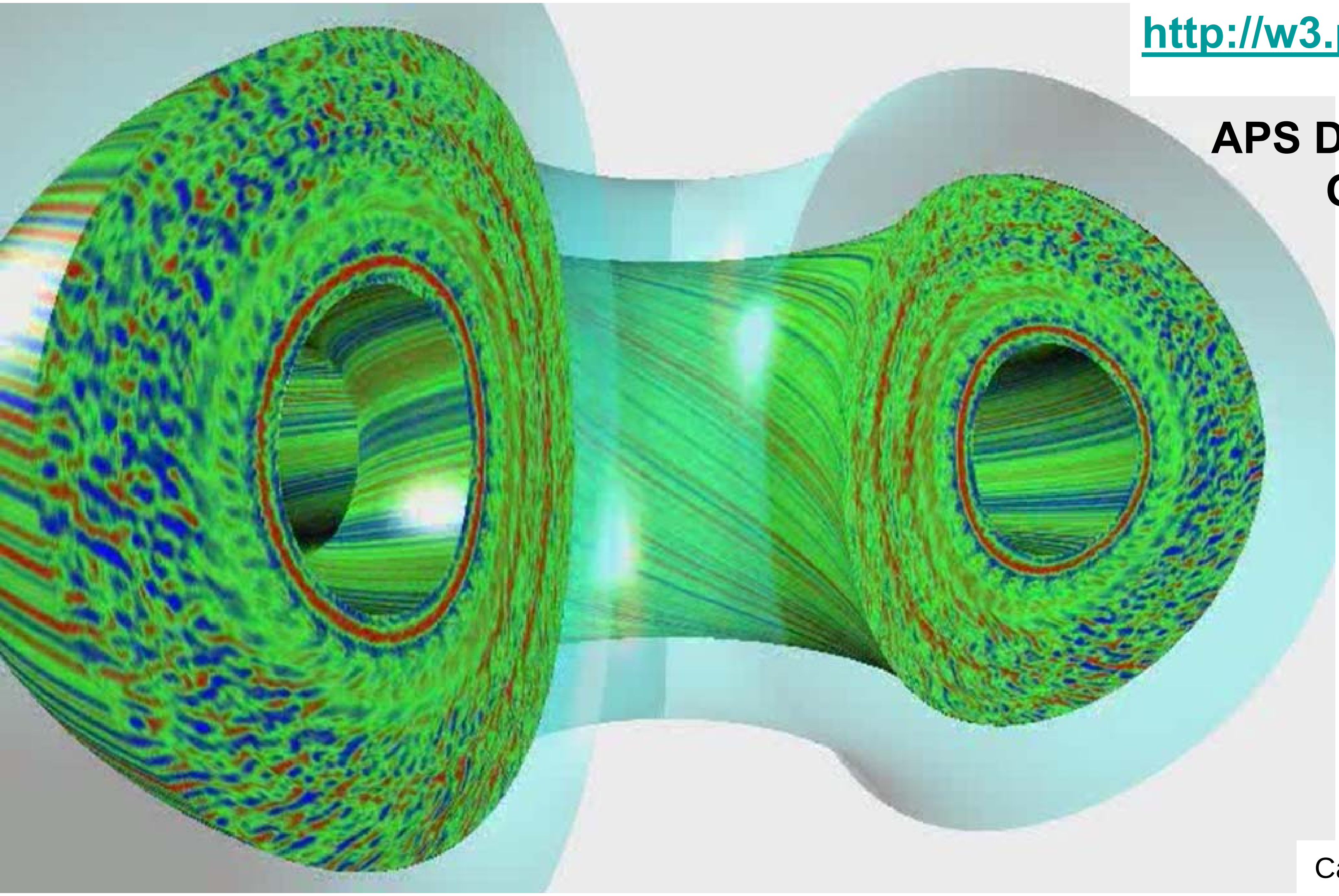
APS Div. Plasma Physics
Orlando, 11/12/2007



Candy, Waltz (General Atomics)

Gyrokinetic Theory & Simulation of Experiments

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1. Intuitive pictures of gyrokinetic turbulence, & how to reduce it
    - analogy with inverted pendulum / Rayleigh-Taylor instability
    - reducing turbulence with sheared flows, magnetic shear, plasma shaping → advanced tokamak & advanced stellarator designs

2. Development of & physics in gyrokinetic equations

3. Development of nonlinear 5-D simulations of gyrokinetic turbulence

4. Gyrokinetic simulations: physics studies & comparisons w/ expts.

5. Future challenges & opportunities
    - more detailed comparisons w/ expts incl. fluctuation diagnostics
    - directly couple turbulence simulations & long-time transport codes
    - Edge Turbulence, ELMs, transport barriers

Gyrokinetic Invited & Contributed Talks at this meeting:

- Invited talks borrowed from in this talk:

- **J E Kinsey, "First transport code simulations using the TGLF model", BI2.6, Monday, 12 noon**
- **Anne White, "Electron temperature fluctuations in the core of high-performance DIII-D plasmas", NI1.2, Wednesday, 10 AM**
- **G G Howes, "Turbulence in the solar wind: Theory, simulations and comparisons with observations", VI2.2 , Thursday 3:30 PM**

- More invited talks:

- David Mikkelsen, "A quantitative account of electron energy transport in an NSTX plasma", NI1.4, Wednesday, 11 AM
- Barrett Rogers, "Gyrokinetic simulations of plasma turbulence, transport and zonal flows in a closed field line geometry", Tuesday, 11 AM
- Z Lin, "Turbulent transport via wave-particle decorrelation in collisionless plasmas", CI1.3, Monday 3 PM
- J Lang, "Gyrokinetic delta f particle simulation of trapped electron mode driven turbulence", NI1.3, Wednesday 10:30 AM
- TS Hahm, "Turbulent equipartition theory of toroidal momentum pinch", YI1.6, Friday noon.

- Contributed Oral:

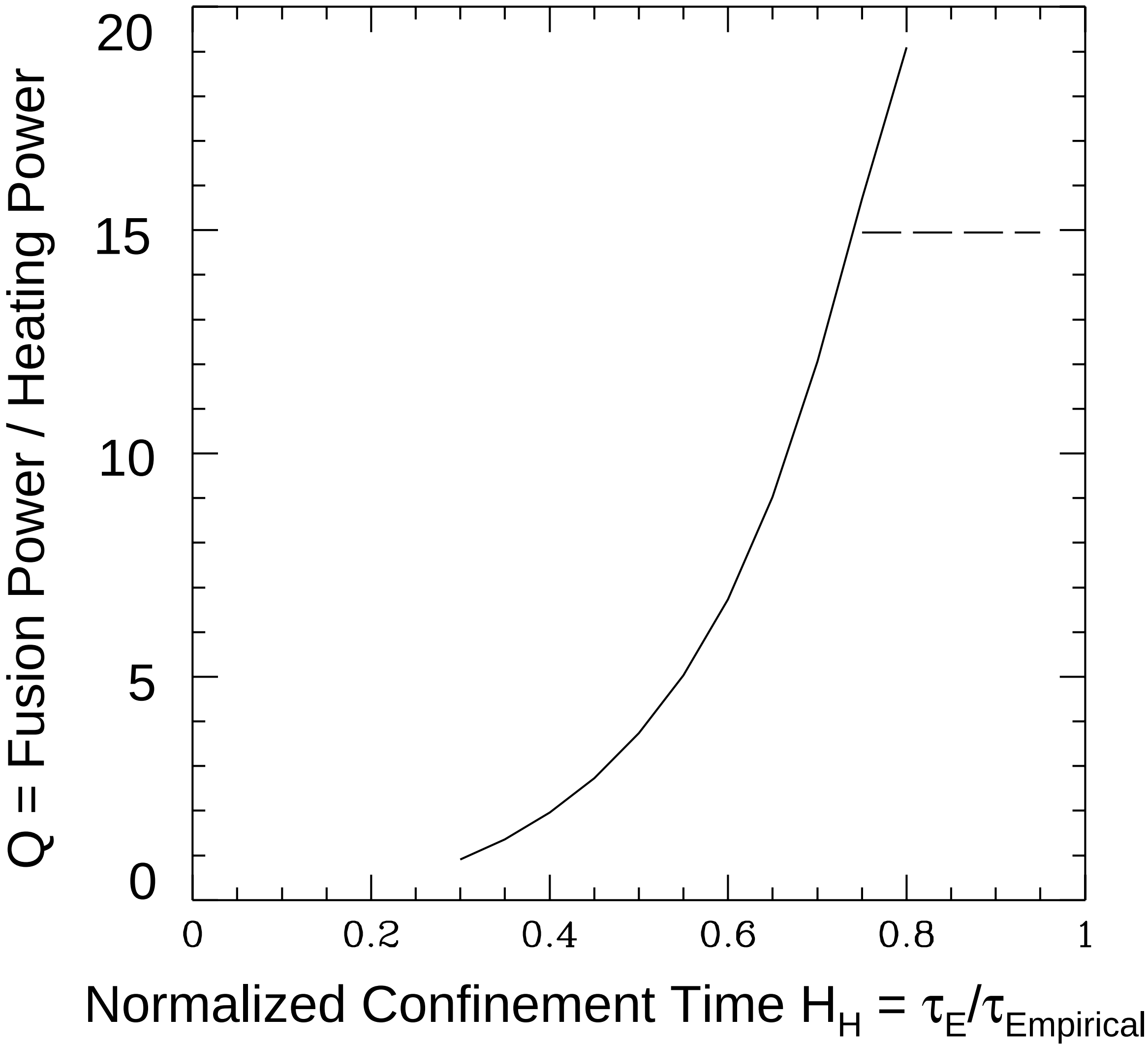
- Florian Merz, "Plasma microturbulence with dual drive", NO.00006, Wednesday 10:30 AM
- Ben McMillan, "Noise control in global gyrokinetic particle simulations", NO3.00010, Wednesday, 9:30 AM
- R V Budny, "Gyrokinetic simulations of electron density fluctuations and comparisons with measurement", NO3.00011, Wednesday 11:30 AM
- Yong Xiao, "Gyrokinetic simulation of trapped electron mode turbulence", NO3.00012, Wednesday 11:42 AM
- M Greenwald, "Particle transport and density peaking at low collisionality on Alcator C-Mod", PO3.00005, Wednesday, 2:48 PM
- Frank Jenko, "Decoupling of ion and electron heat transport via scale separation", TO3.00001, Thursday 9:30 AM
- Florian Merz, "Gyrokinetic turbulence simulations for stellarators", TO3.00002, Thursday 9:42 AM

Motivation & Summary

Fusion performance depends sensitively on confinement

Sensitive dependence on turbulent confinement causes some uncertainties, but also gives opportunities for significant improvements, if methods of reducing turbulence extrapolate to larger reactor scales.

$$\frac{dE}{dt} = P_{ext} + P_{fusion} - \frac{E}{\tau_E}$$



Caveats: best if MHD pressure limits also improve with improved confinement.
Other limits also: power load on divertor & wall, …

↓ turbulence & ↑ β could significantly improve fusion

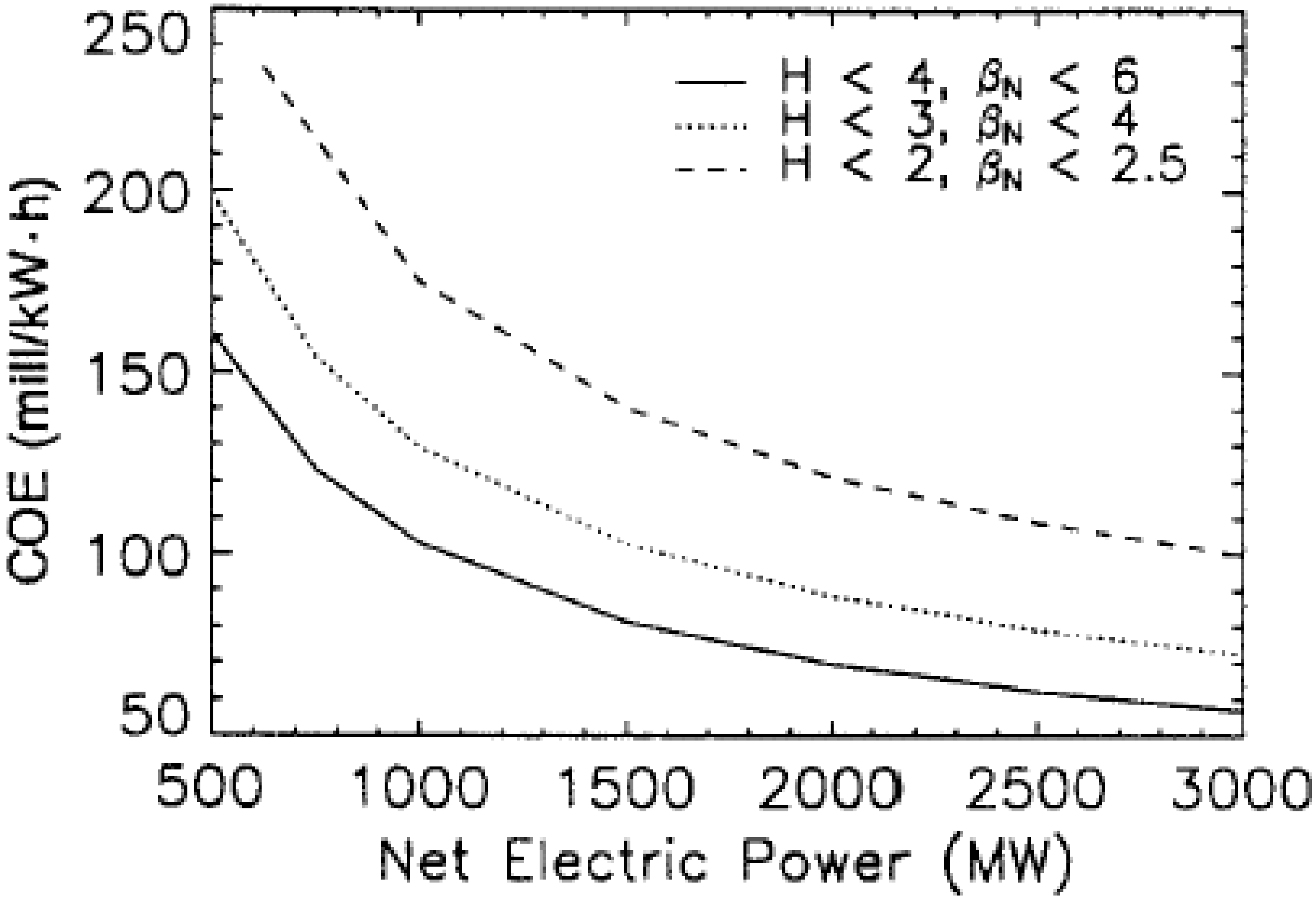


FIG. 4. Minimum COE steady state reactor parameters versus the net electric output. Cases are shown for three physics levels: (a) present day levels that would be sustainable in a non-transient manner in a conservatively designed system ($H \leq 2, \beta_N \leq 2.5$)*, (b) moderately improved physics* ($H \leq 3, \beta_N \leq 4$) *and (c) advanced physics* ($H \leq 4, \beta_N \leq 6$).

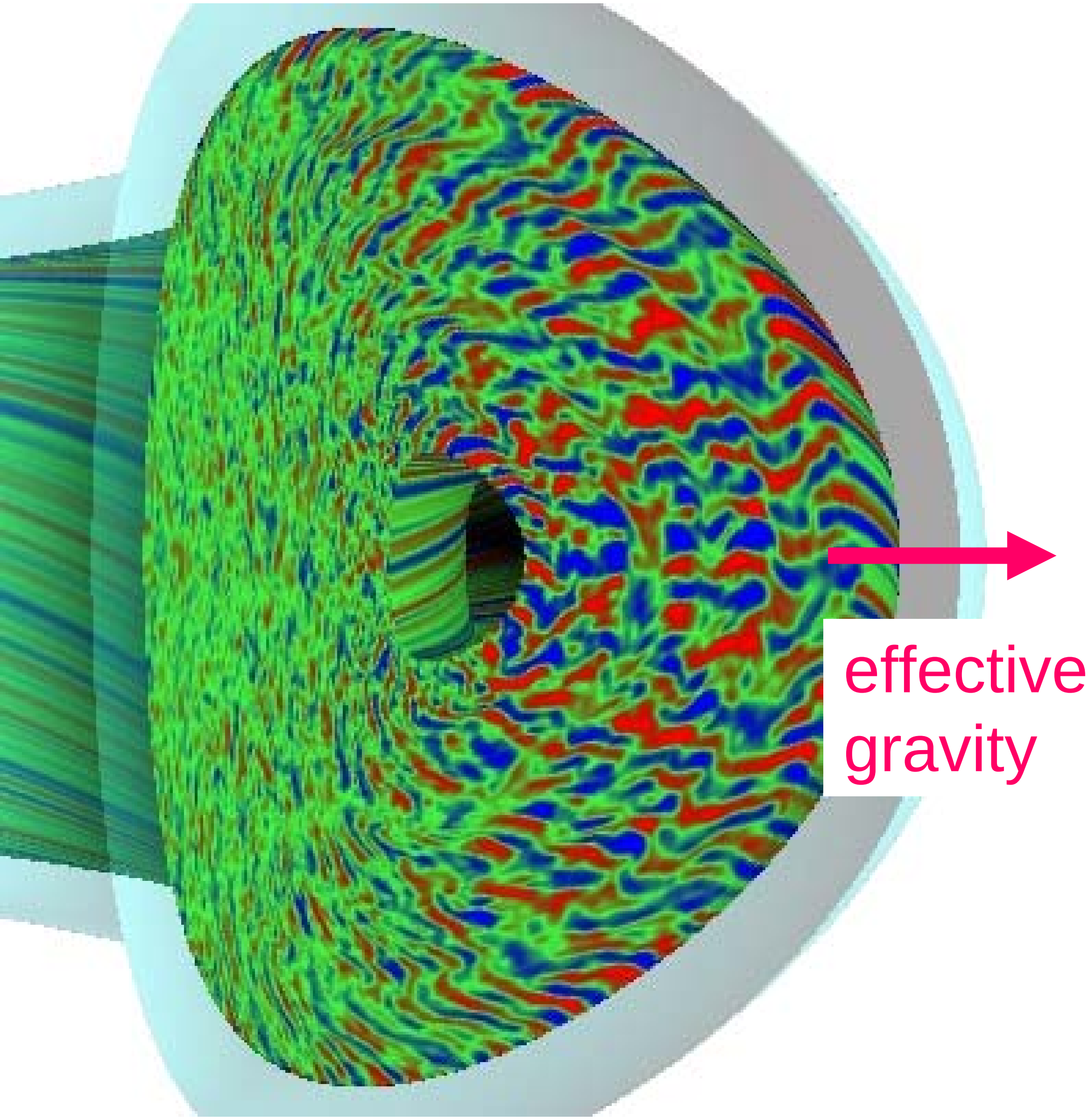
(Relative Cost of Electricity (COE) estimates in this study, see ARIES reactor studies for more detailed & lower costs estimates.)

Galambos, Perkins, Haney, & Mandrekas 1995 Nucl. Fus.

5-slide executive summary

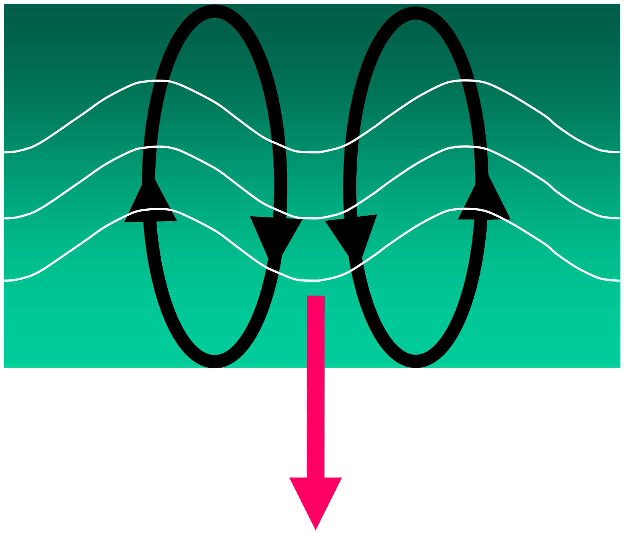
1. Intuitive pictures of gyrokinetic turbulence, & how to reduce it

- analogy w/ inverted pendulum / Rayleigh-Taylor instability
- reduce turbulence with sheared flows, magnetic shear, …



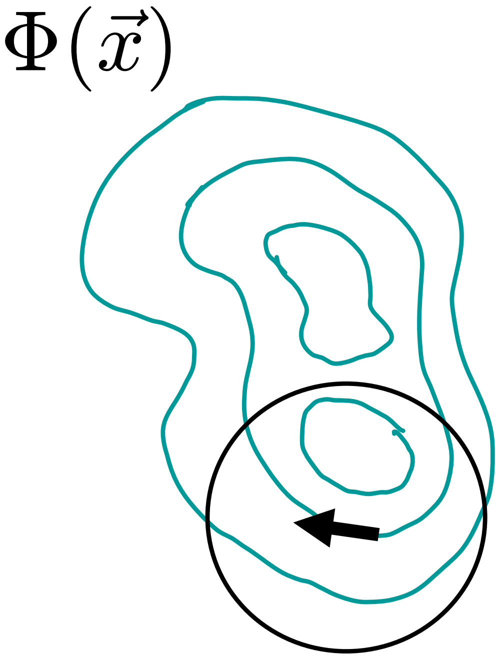
Inverted-density fluid

⇒Rayleigh-Taylor Instability



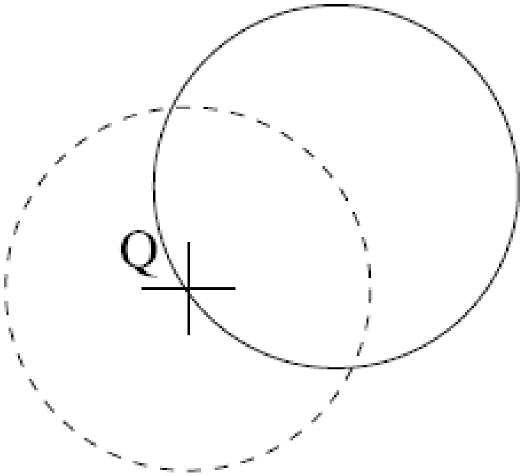
2. Development of & physics in gyrokinetic equations

if low frequencies $\omega << $ cyclotron frequency (Ω_c),
→ average over particle gyration, treat particles as rings of charge in spatially varying fields



$$E \times B \rightarrow -\nabla \underbrace{\langle \Phi \rangle} \times \vec{B}$$

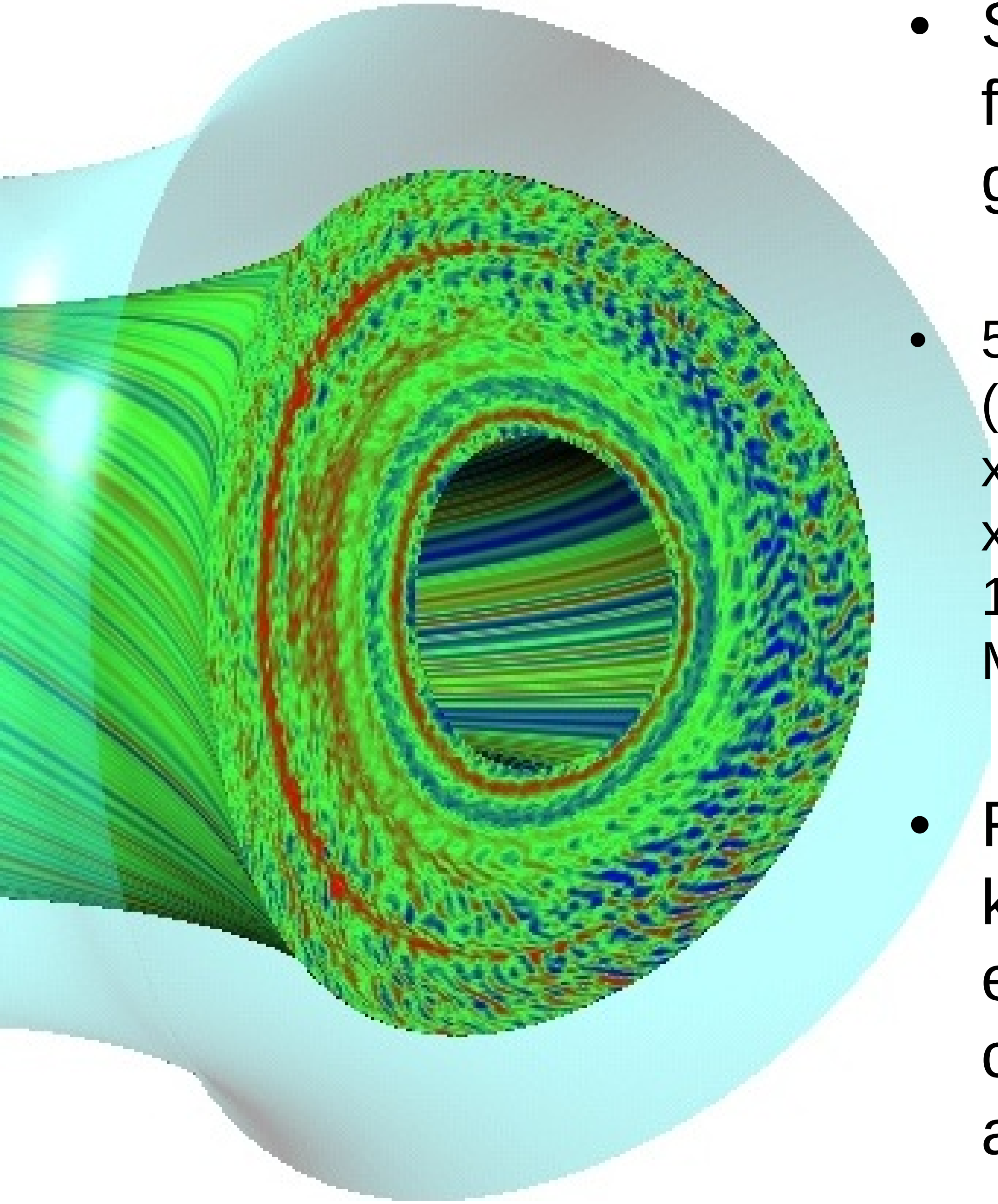
potential averaged around particle orbit, even if $k_{\perp}\rho_i$ large



When calculating charge at point Q, have to sum over all particles whose guiding centers are on the dashed line, & have to include small variation of particle density around gyro-orbit (→ polarization shielding)

Development of nonlinear gyrokinetics was a major breakthrough

3. Fairly Comprehensive 5-D Gyrokinetic Turbulence Codes Have Been Developed



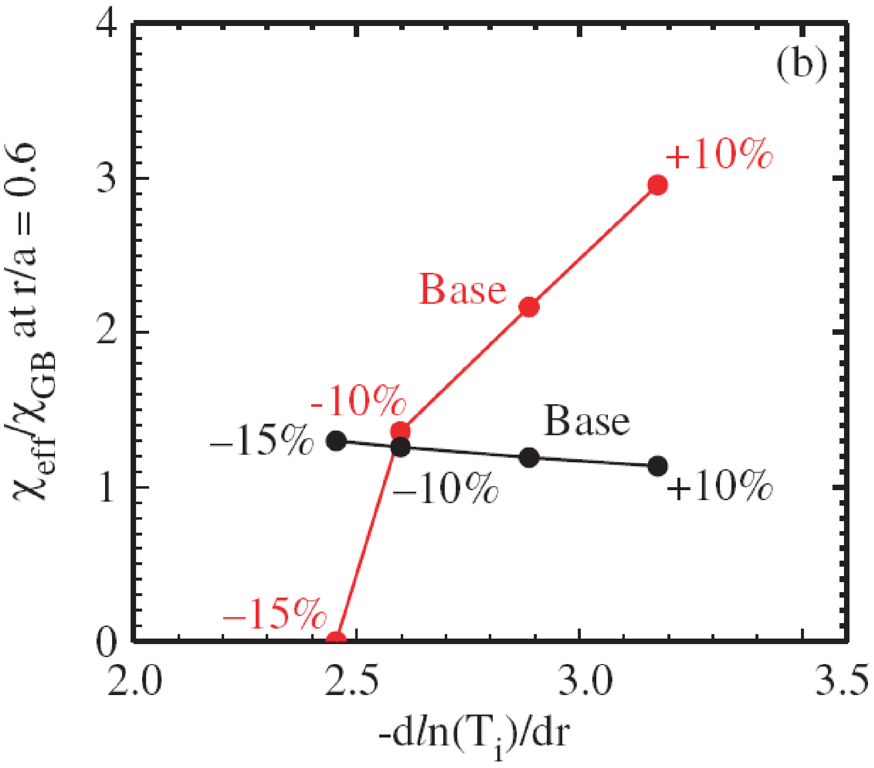
- Solve for the particle distribution function $f(r,\alpha,\theta,E,\mu,t)$ (avg. over gyration: 6D → 5D)

- 500 radii x 32 complex toroidal modes
  (96 binormal grid points)
  x 10 parallel points along half-orbits
  x 8 energies x 16 $v_{||}/v$
  12 hours on ORNL Cray X1E with 256 MSPs

- Realistic toroidal geometry, kinetic ions & electrons, finite-β electro-magnetic fluctuations, collisions. Sophisticated algorithms.

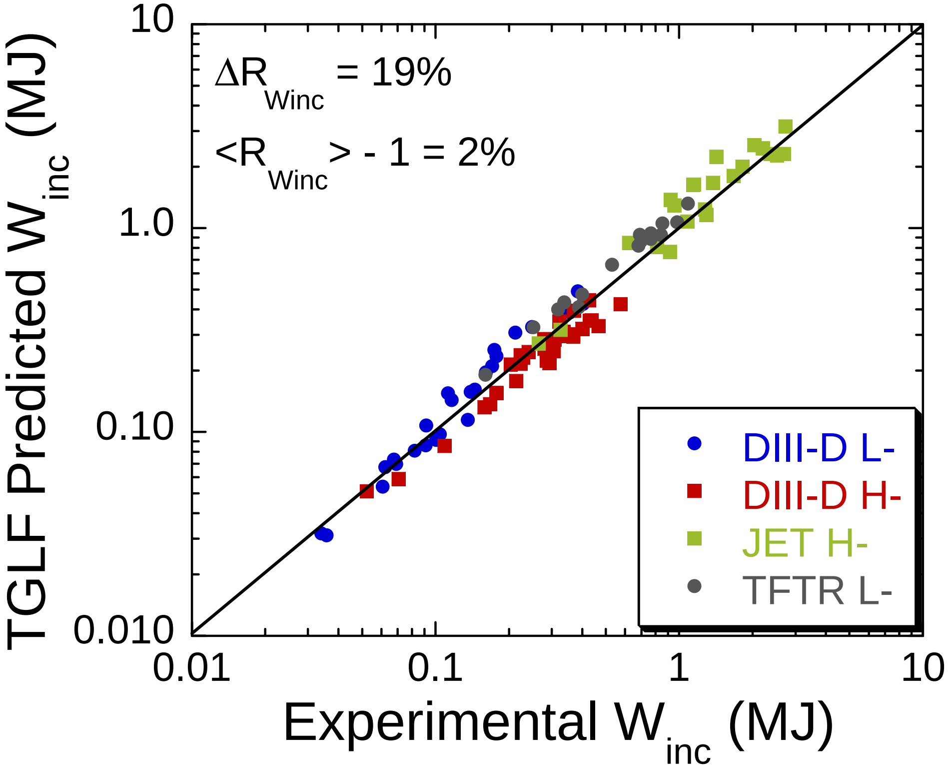
4. Gyrokinetic sims.: physics studies & comparisons w/ expts.

- Simulations often agree with core region of experiments within error bars on grad(T)



Candy & Waltz, PRL 2003, Waltz et al. 2005

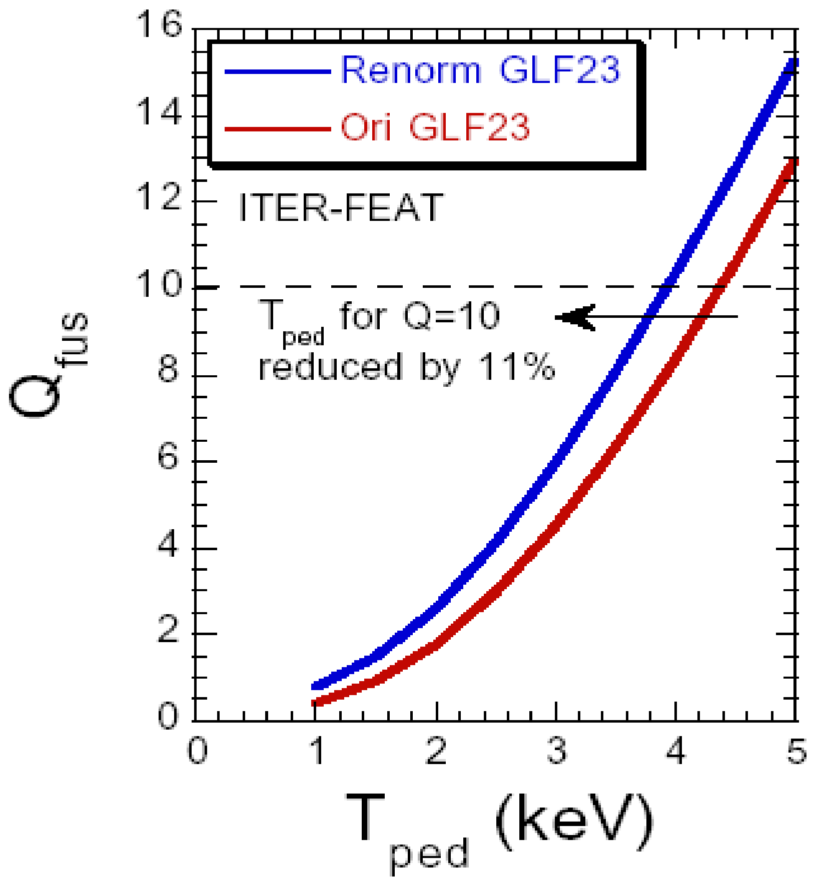
- TGLF transport model, based on gyrokinetic simulations, fits core of wide range of experiments



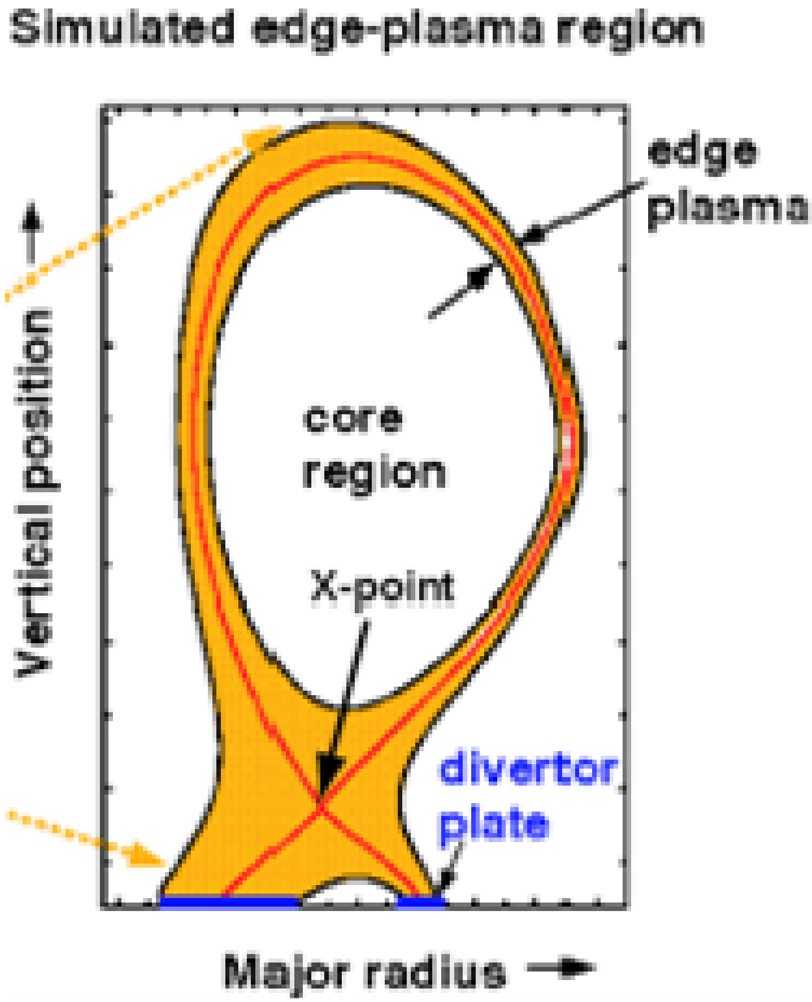
Kinsey BI2.6, Monday 12:00 Noon

5. Future challenges & opportunities:

- more detailed comparisons w/ expts incl. synthetic fluctuation diagnostics
- coupling turbulence simulations directly in long-time transport codes
- Edge Turbulence, very challenging but critical problem
  - Edge important: core depends on edge, ELMs, transport barriers
  - present core codes don't handle edge, need X-point separatrix, open & closed field lines, strong recycling, wide range of collisionality, …



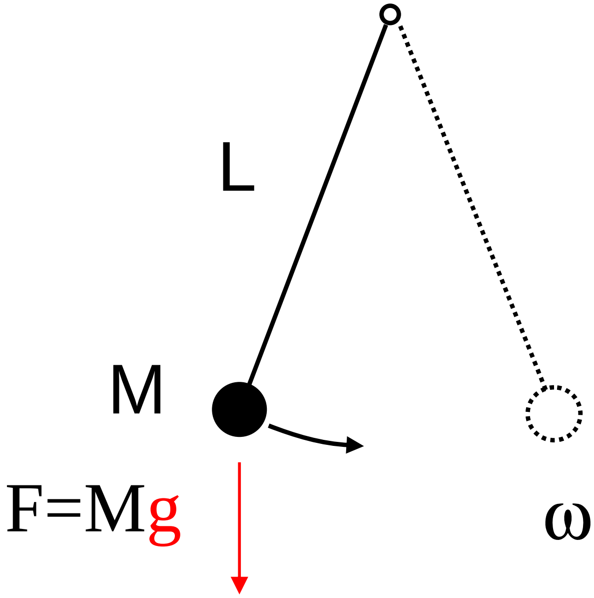
From Kinsey, Staebler, Waltz, Sherwood 2002.
Predictions for 2001 ITER-FEAT.



1. Intuitive pictures of gyrokinetic turbulence, & how to reduce it

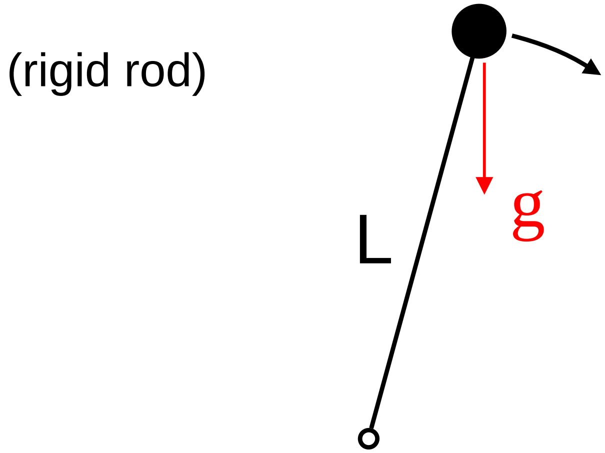
(many of these insights developed with gyrofluid simulations in 1990’s, but gyrokinetics needed for better accuracy.)

Stable Pendulum



$\omega=(g/L)^{1/2}$

Unstable Inverted Pendulum

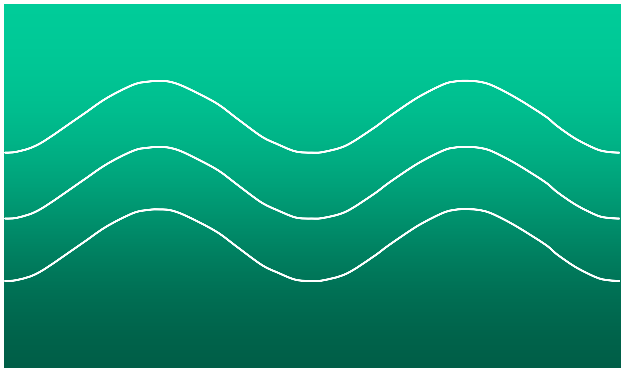


$\omega= (-g/|L|)^{1/2} = i(g/|L|)^{1/2} = i\gamma$

↑ Instability

Density-stratified Fluid

$\rho=\exp(-y/L)$

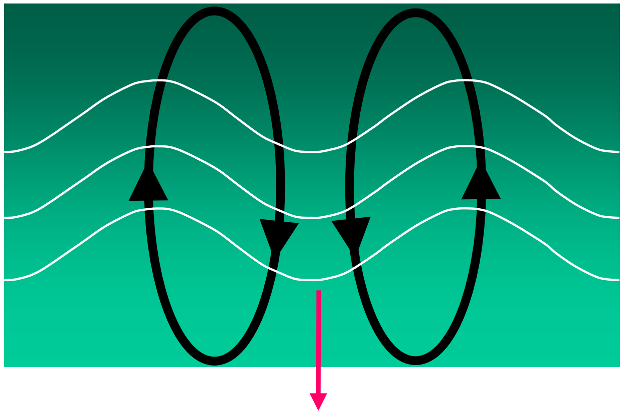


stable $\omega=(g/L)^{1/2}$

Inverted-density fluid

⇒Rayleigh-Taylor Instability

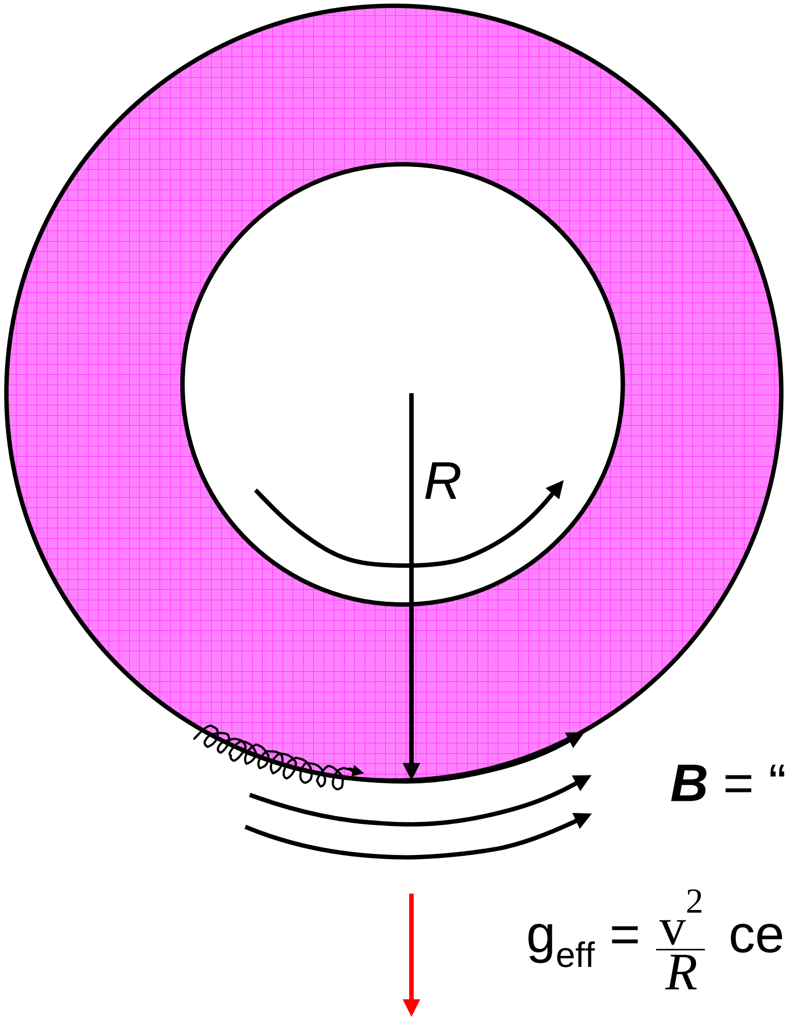
$\rho=\exp(y/L)$



Max growth rate $\gamma=(g/L)^{1/2}$

"Bad Curvature" instability in plasmas
≈ Inverted Pendulum / Rayleigh-Taylor Instability

Top view of toroidal plasma:



plasma = heavy fluid

B = "light fluid"

$g_{\text{eff}} = \frac{v^2}{R}$ centrifugal force

Growth rate:

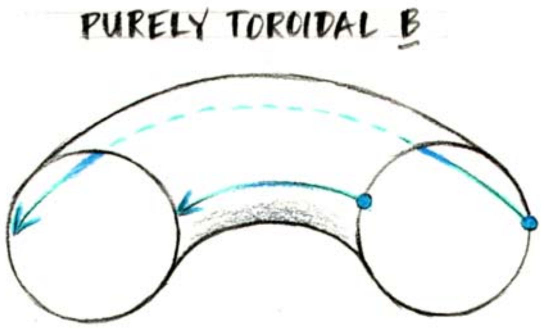
$$\gamma = \sqrt{\frac{g_{eff}}{L}} = \sqrt{\frac{v_t^2}{RL}} = \frac{v_t}{\sqrt{RL}}$$

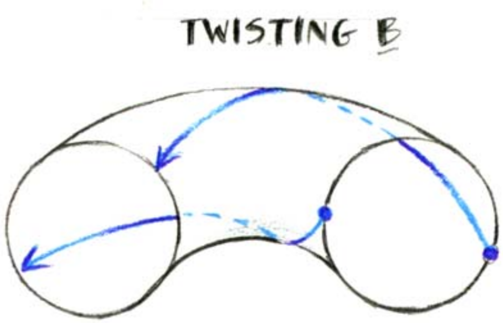
Similar instability mechanism
in MHD & drift/microinstabilities

$1/L = \nabla p/p$ in MHD,
$\propto$ combination of ∇n & ∇T
in microinstabilities.

The Secret for Stabilizing Bad-Curvature Instabilities

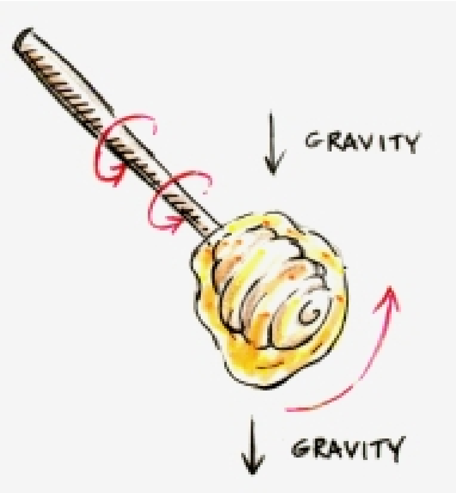
Twist in ***B*** carries plasma from bad curvature region to good curvature region:





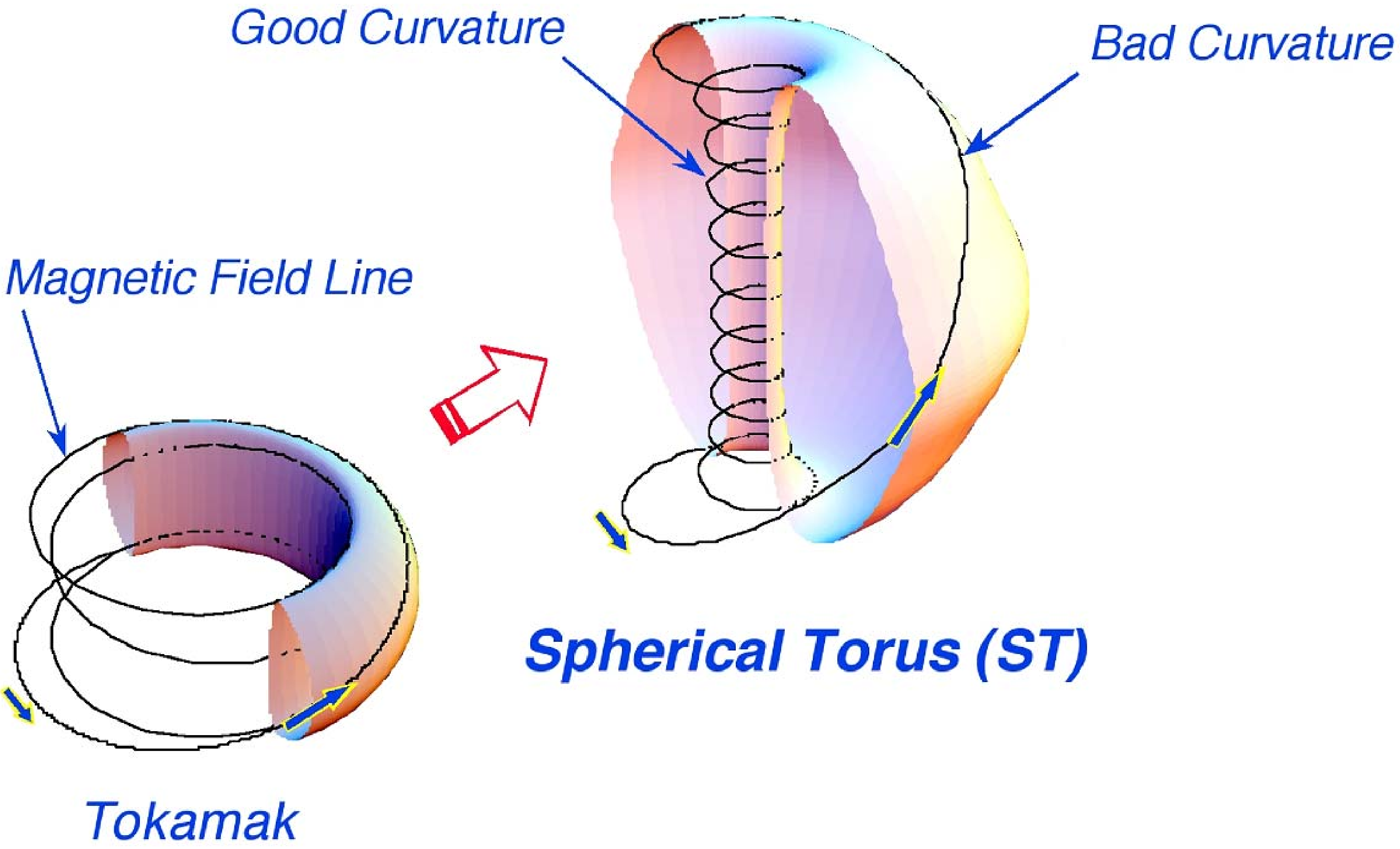
Unstable

Stable

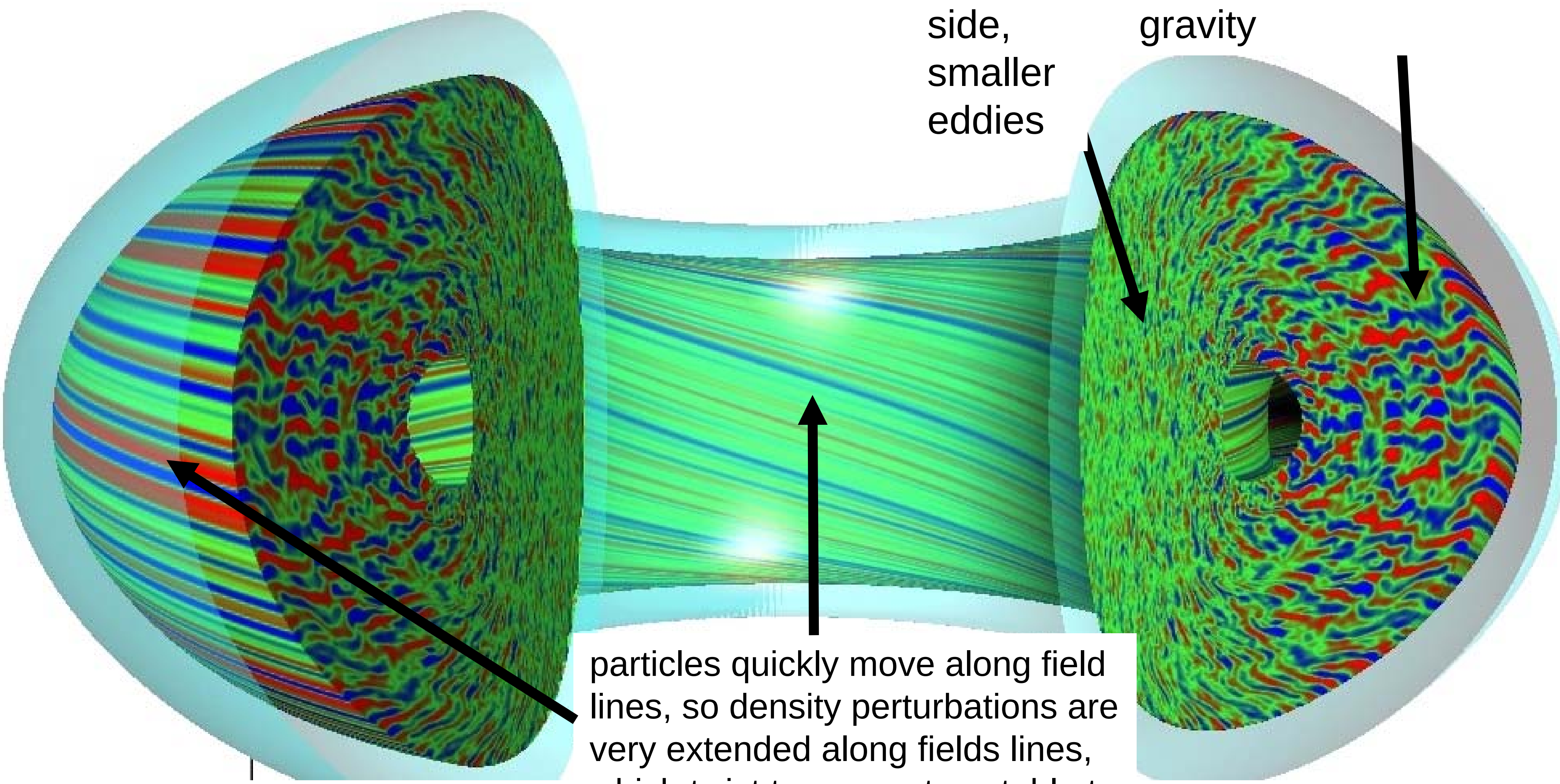


Similar to how twirling a honey dipper can prevent honey from dripping.

Spherical Torus has improved confinement and pressure limits (but less room in center for coils)



These physical mechanisms can be seen in gyrokinetic simulations and movies

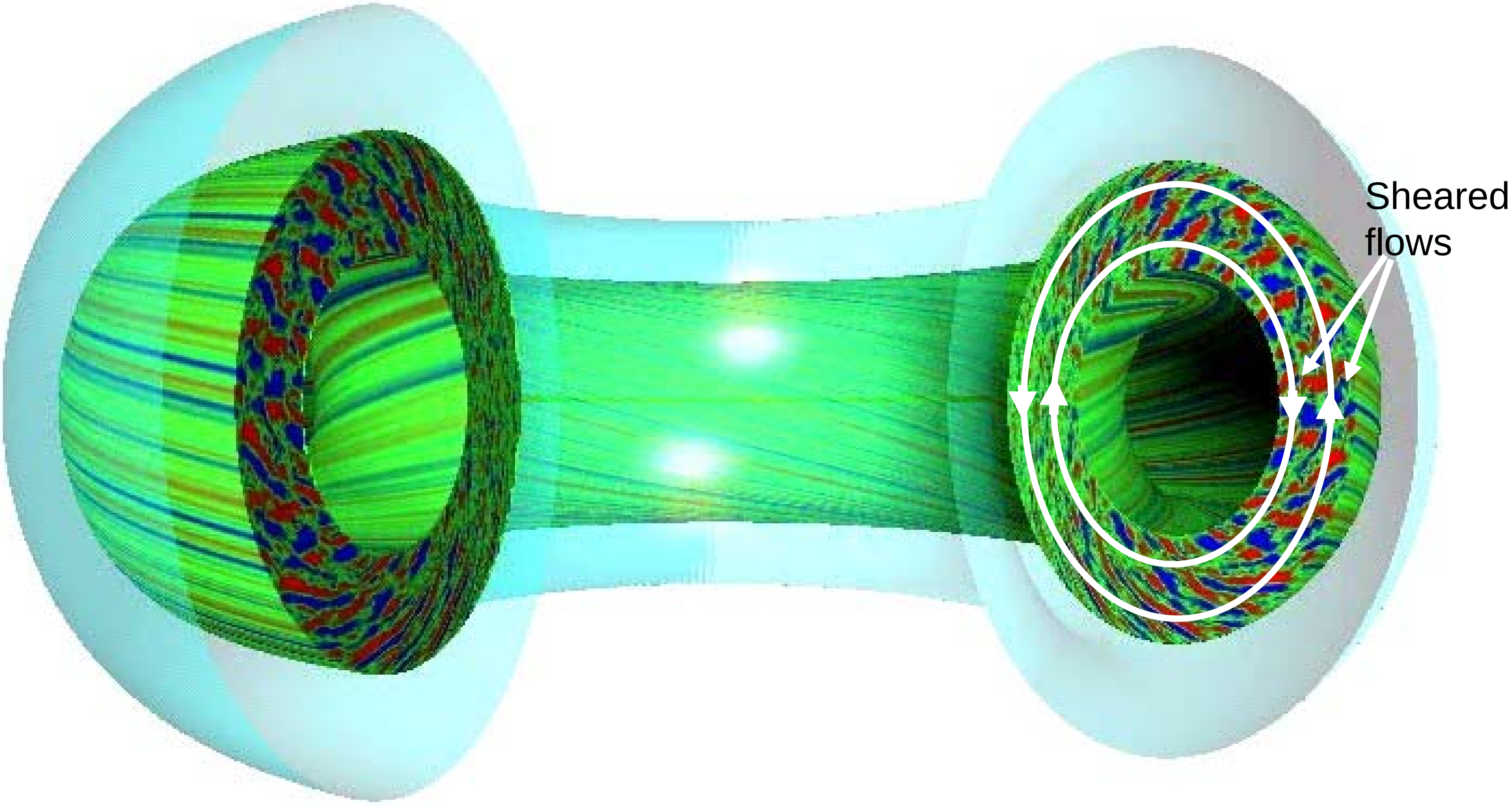


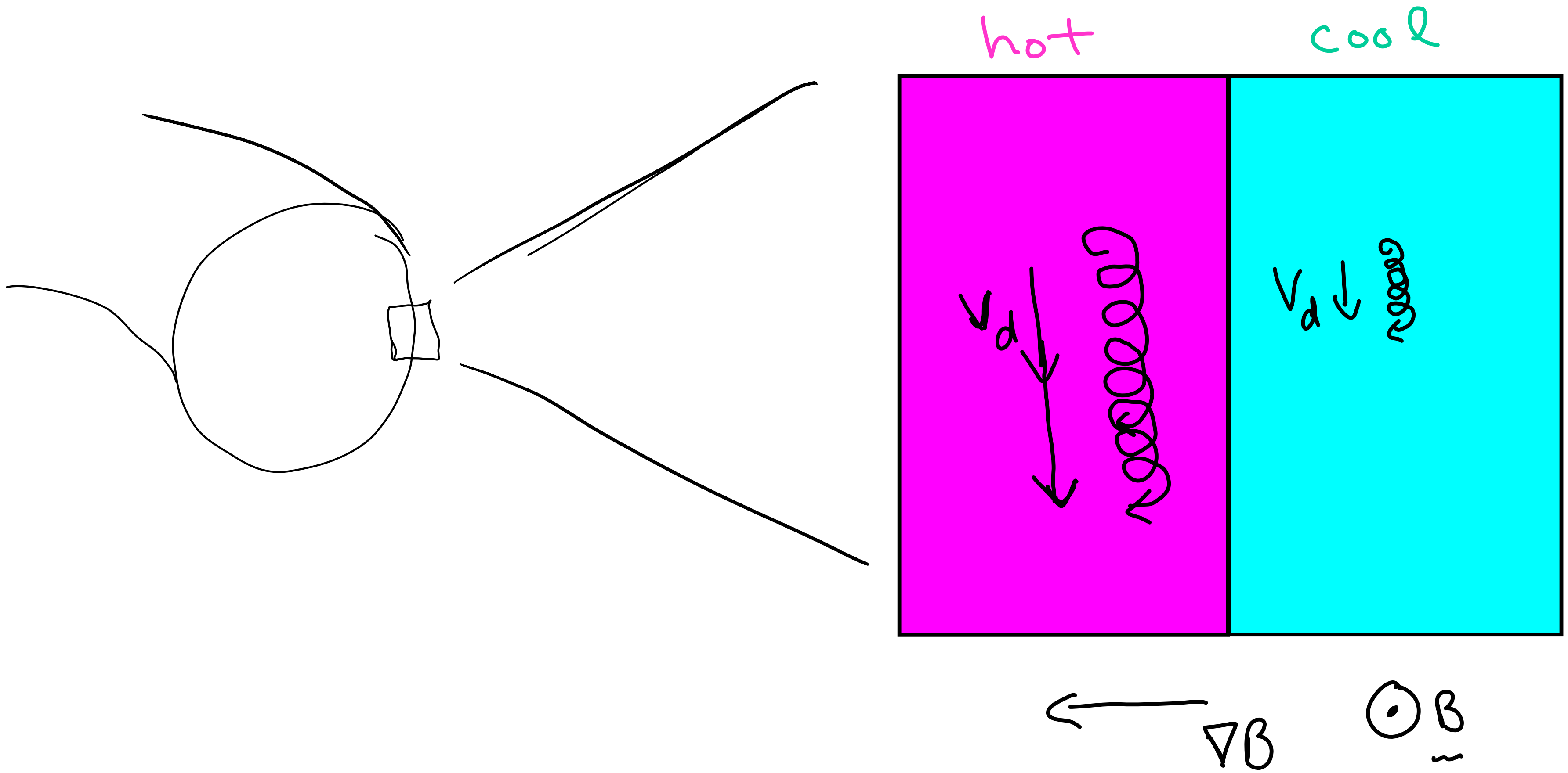
Stable side, smaller eddies

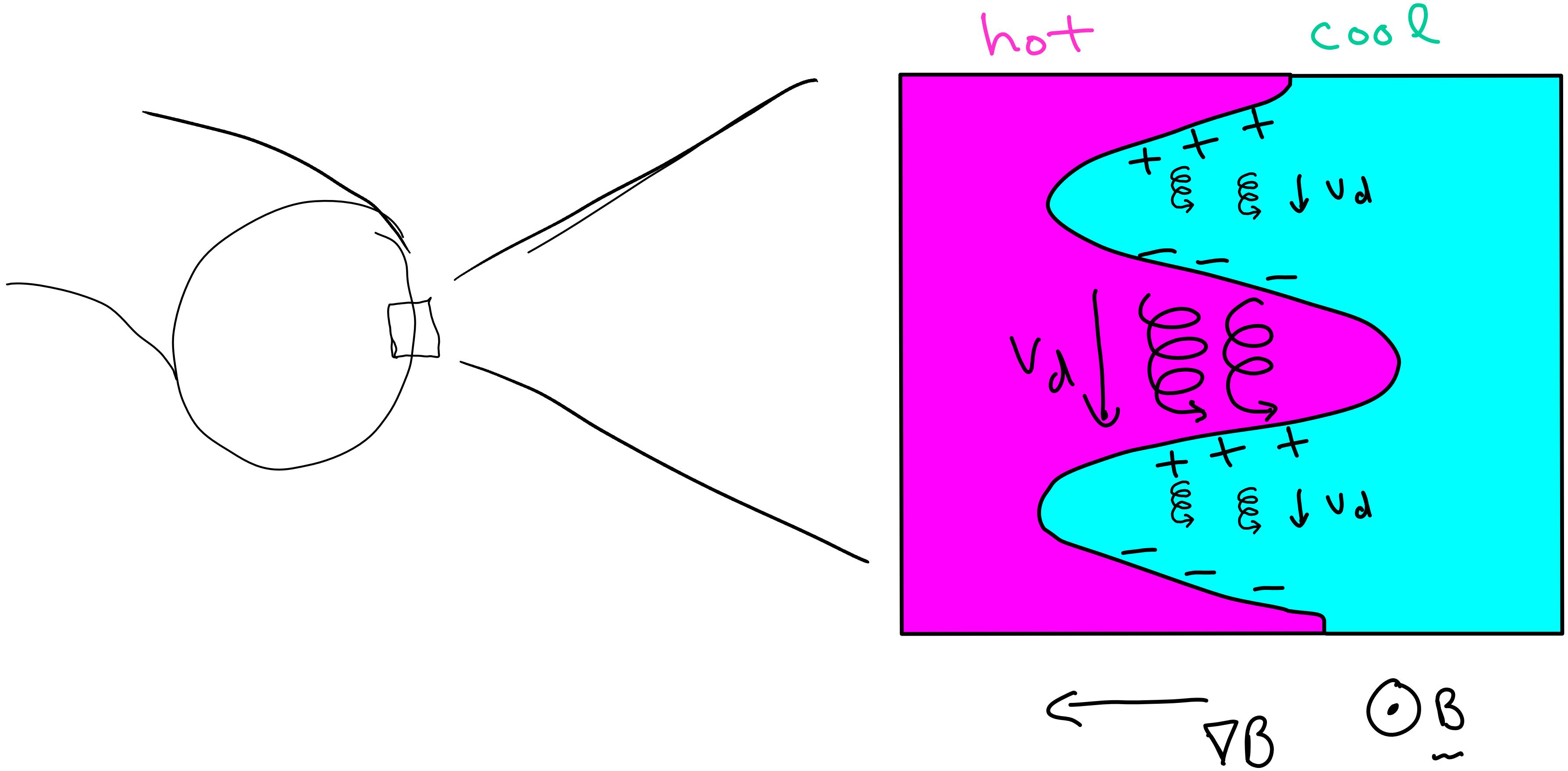
Unstable bad-curvature side, eddies point out, direction of effective gravity

particles quickly move along field lines, so density perturbations are very extended along fields lines, which twist to connect unstable to stable side

Movie http://fusion.gat.com/THEORY/images/3/35/D3d.n16.2x_0.6_fly.mpg from http://fusion.gat.com/theory/Gyromovies shows contour plots of density fluctuations in a cut-away view of a GYRO simulation (Candy & Waltz, GA). This movie illustrates the physical mechanisms described in the last few slides. It also illustrates the important effect of sheared flows in breaking up and limiting the turbulent eddies. Long-wavelength equilibrium sheared flows in this case are driven primarily by external toroidal beam injection. (The movie is made in the frame of reference rotating with the plasma in the middle of the simulation. Barber pole effect makes the dominantly-toroidal rotation appear poloidal..) Short-wavelength, turbulent-driven flows also play important role in nonlinear saturation.




hot
cool
V_d
V_d
∇B
$\odot \underset{\sim}{B}$


hot
cool
+ + +
v d
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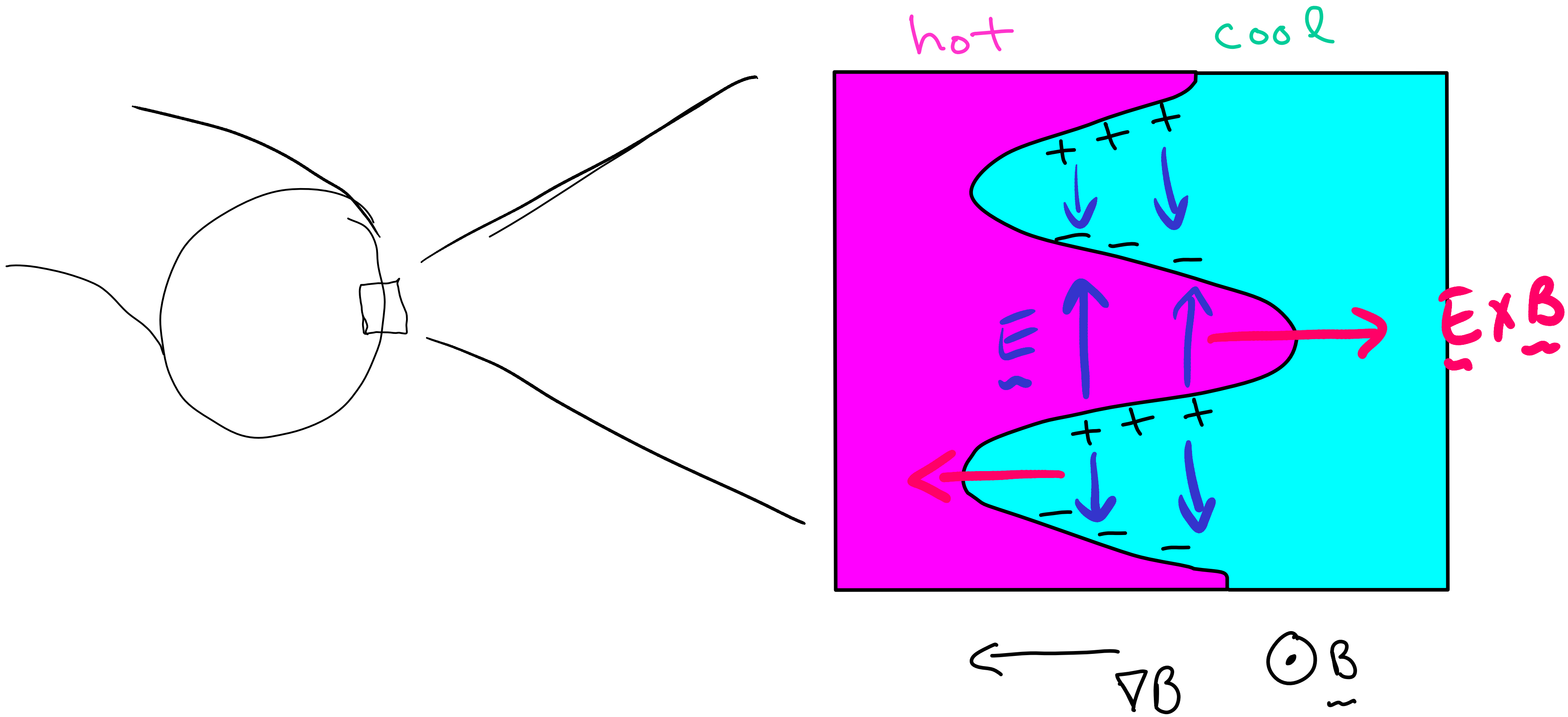
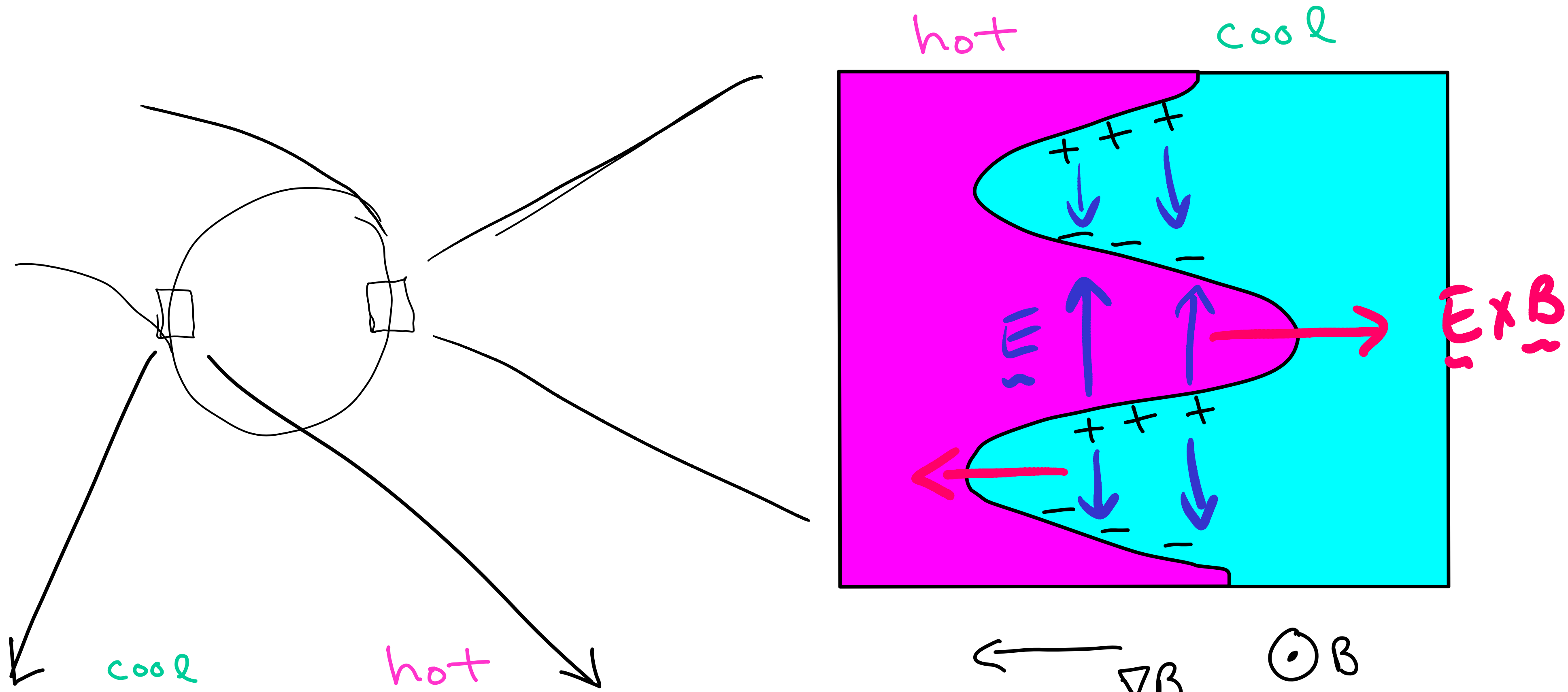
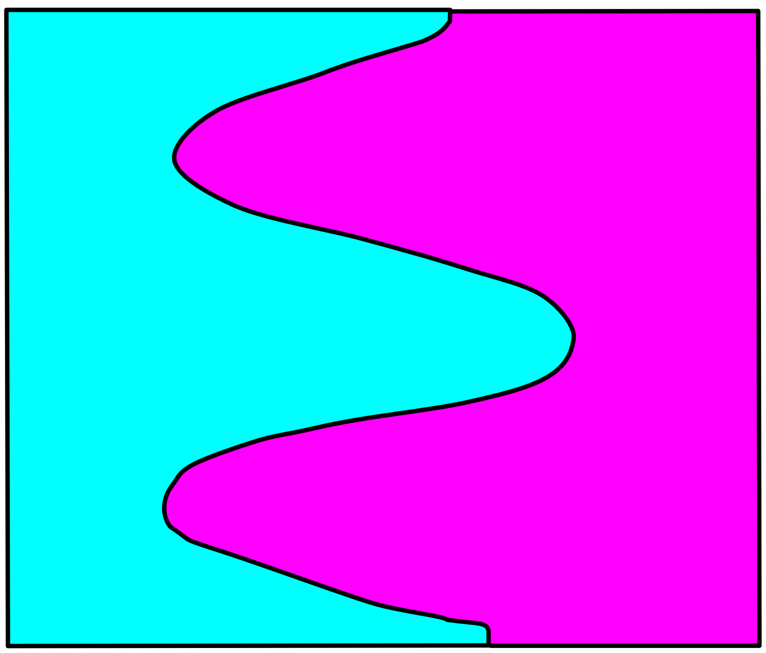
v d
+ + +
v d
– – –
∇B
⊙ B



Higher energy particles ∇B drift faster, creates charge separation & thus $\underset{\sim}{E}$ field, causes $E \times B$ flow that further accentuates perturbation. Positive feedback ⇒ instability.

Rosenbluth-Longmire picture





Can repeat this analysis on the good curvature side & find it is stable. (Leave as exercise.)

Rosenbluth-Longmire picture

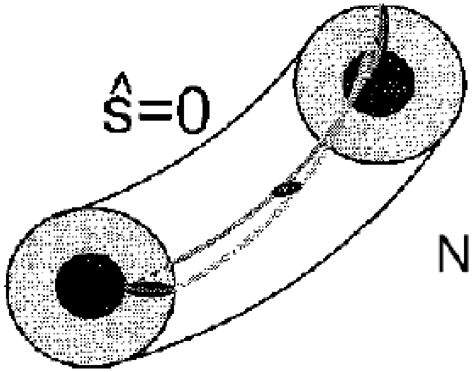
Simple picture of reducing turbulence by negative magnetic shear

Particles that produce an eddy tend to follow field lines.

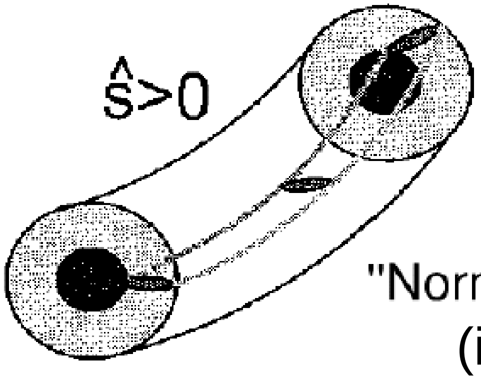
Reversed magnetic shear twists eddy in a short distance to point in the ``good curvature direction".

Locally reversed magnetic shear naturally produced by squeezing magnetic fields at high plasma pressure: ``Second stability" Advanced Tokamak or Spherical Torus.

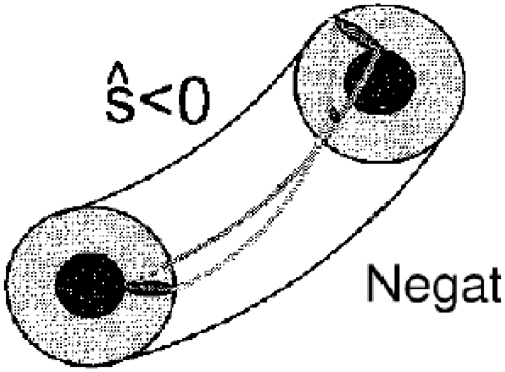
Shaping the plasma (elongation and triangularity) can also change local shear



No Magnetic Shear



"Normal" Magnetic Shear
(in std tokamaks)



Advanced Tokamaks

Negative Magnetic Shear
“Normal” in stellarators

Fig. from Antonsen, Drake, Guzdar et al. Phys. Plasmas 96
Kessel, Manickam, Rewoldt, Tang Phys. Rev. Lett. 94

2. Development of & physics in Gyrokinetic Eqs.

Development of gyrokinetic equations one of the triumphs of high-power theoretical plasma physics and asymptotic analysis

Interesting pre-history and history of gyrokinetics…

Key advance: Frieman & Chen show nonlinear version of gyrokinetics possible

Other advances: Hamiltonian/Lagrangian derivations, insure conservation properties, easier to go to higher order

Gyrokinetic Prehistory:

Chew-Goldberger-Low (1956) MHD-ordered Drift-Kinetic Eq.:

$$\epsilon \sim \frac{\text{frequency}}{\text{gyrofrequency}} \sim \frac{\omega}{\Omega_c} \sim \frac{\text{gyroradius}}{\text{gradient Length}} \sim \frac{\rho}{L} \ll 1$$

Later recognized MHD ordering demonstrates stability only for fast instabilities, with growth rates $\gamma \sim \epsilon\Omega_c$, and misses slow drift instabilities with $\gamma \sim \epsilon^2\Omega_c$:

$$\frac{\omega_*}{\Omega_c} \sim k_y \rho \frac{\rho}{L} \sim \epsilon^2$$

Extensions of CGL to higher $\mathcal{O}(\epsilon^2)$, but very complicated

Big Breakthrough: *Nonlinear* Gyrokinetics

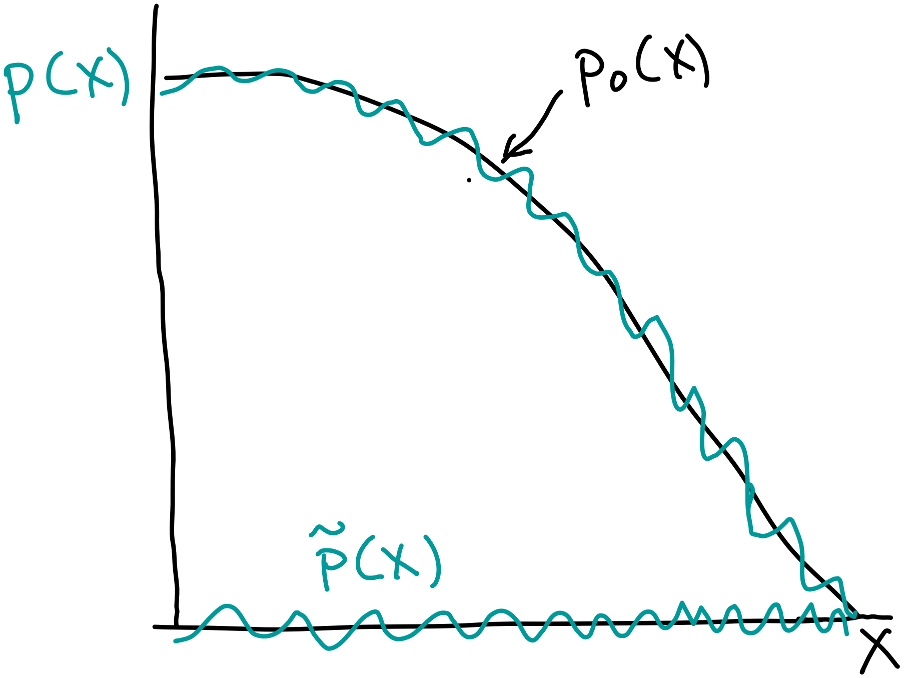
- Long, interesting history of linear gyrokinetics, 1960’s, 1970’s.
- E. A. Frieman & L. Chen 79-82, showed it is possible to gyro-average nonlinear terms and keep full FLR-effects for arbitrary $k_\perp\rho$, & get rigorous solution w/o closure problem
- (usually, averaging nonlinear terms → closure problems, such as fluid equation closures, statistical turbulence theories,...
  Perhaps understood by some, or in restrospect: J.B. Taylor ’67 demonstrated an adiabatic invariant still exists at arbitrary $k_\perp\rho$ …)
- GK ordering allows capture of drift/micro-instabilities & much of MHD at just order ε & not ε^2

$$\varepsilon \sim \frac{\omega}{\Omega} \sim \frac{\rho}{L} \sim \frac{\tilde{f}}{f_0} \sim \frac{e\Phi}{T_0} \quad \text{but} \quad k_\perp \rho \sim O(1)$$

$$\sim \frac{k_\parallel}{k_\perp}$$

Guided by expts., μwave scattering, physics insights

$$\epsilon \sim \frac{\omega}{\Omega} \sim \frac{\rho}{L} \sim \frac{\tilde{f}}{f_0} \sim \frac{e\Phi}{T_0} \sim \frac{k_\parallel}{k_\perp}$$ but $k_\perp \rho \sim O(1)$



Keep arbitrary $k_\perp \rho$,
FLR to all orders

Introduce two scales, L & $k_\perp$:

$$\nabla p = \underbrace{\nabla p_0} + \underbrace{\nabla \tilde{p}}$$

$$\frac{p_0}{L} \qquad \sim k_\perp \tilde{p}$$

even though

$$\tilde{f} << f_0$$

$$\nabla \tilde{f} \sim \nabla f_0$$

perturbations can locally flatten gradients, nonlinearities important

The electrostatic Gyrokinetic equation, in a Drift-Kinetic-like form for the full, gyro-averaged, guiding center density $\bar{f}(\vec{R}, v_\parallel, \mu, t)$:

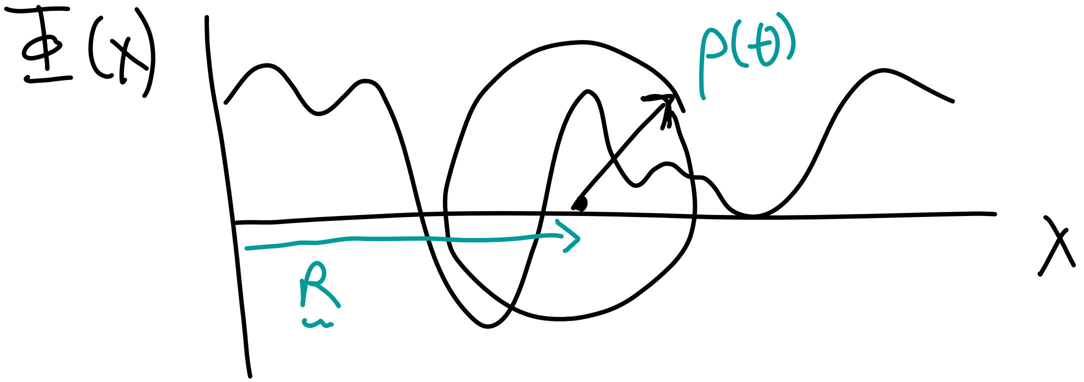
$$\boxed{\frac{\partial \bar{f}}{\partial t} + (v_\parallel \hat{\mathbf{b}} + \mathbf{v}_E + \mathbf{v}_d)\cdot\nabla\bar{f} + \left(\frac{q}{m}E_\parallel - \mu\nabla_\parallel B + v_\parallel(\hat{\mathbf{b}}\cdot\nabla\hat{\mathbf{b}})\cdot\mathbf{v}_E\right)\frac{\partial\bar{f}}{\partial v_\parallel} = 0}$$

$$\underset{\sim}{V}_E = -\frac{c}{B}\nabla\langle\Phi\rangle\times\hat{b} \qquad E_\parallel = -\hat{b}\cdot\nabla\langle\Phi\rangle \qquad \mu = \frac{1}{2}\frac{v_\perp^2}{B}$$

using gyroaveraged potential $\langle\Phi\rangle(\vec{R}) = \frac{1}{2\pi}\int d\theta\,\Phi(\underset{\sim}{R}+\rho(\theta))$

$$= \sum_{\underset{\sim}{k}} J_0(k_\perp\rho)\,\Phi_k\, e^{i\underset{\sim}{k}\cdot\underset{\sim}{R}}$$

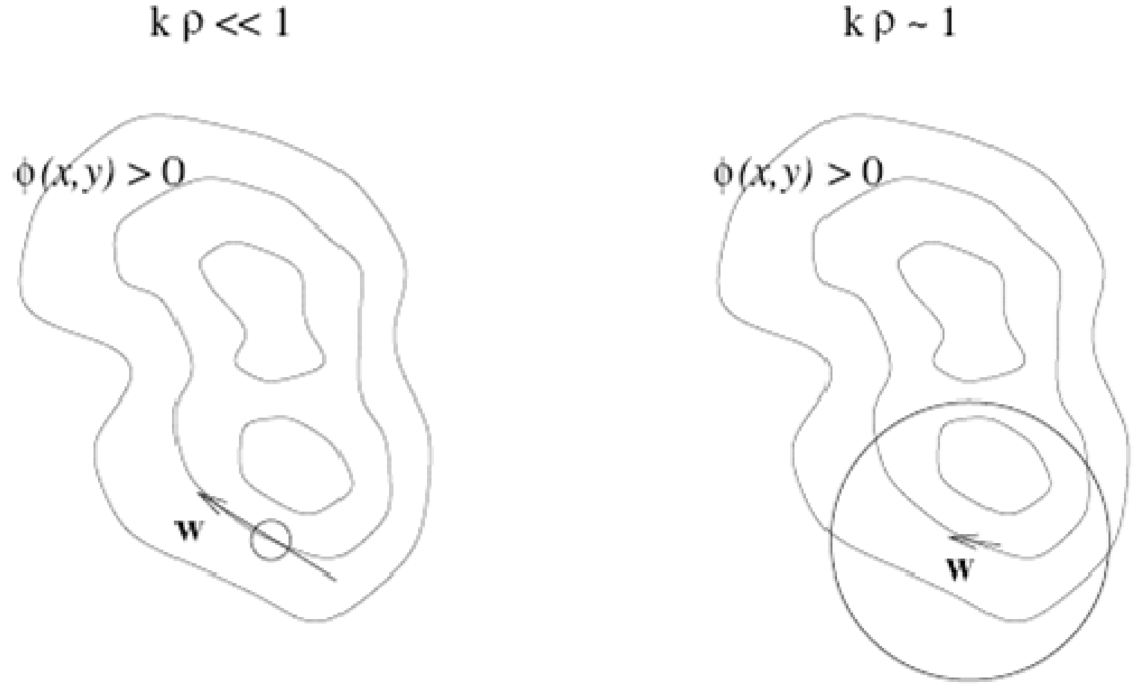
$\underset{\sim}{V}_d = \nabla B$ + curvature drift



The Meaning of Gyrokinetics

- low frequencies $\omega \ll \Omega_c = eB/mc$ for each species

treat particles as rings of charge in spatially varying fields



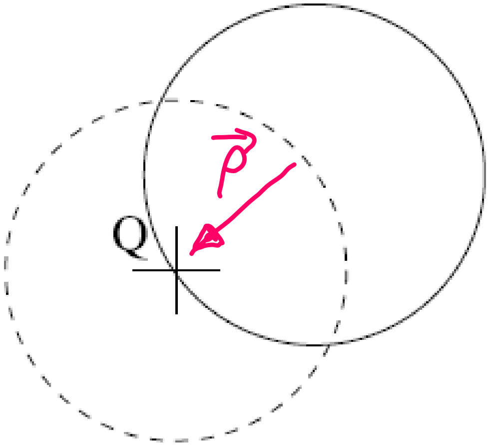
$k\rho \ll 1$ $k\rho \sim 1$

- reduced response: “gyroaveraging”
- reaction to fields, polarisation density: “gyroscreening”

(borrowed from B. D. Scott)

Density calculation for Poisson Eq. Includes Important Polarization shielding

$$n(\underset{\sim}{x}) = n_{gc} + n_{pol}$$



$$n_{gc}(\underset{\sim}{x}) = \int d^3v \; \bar{f}\left(\vec{X} - \vec{\rho}(\theta), v_{\parallel}, \mu\right)$$

$$n_{pol}(\underset{\sim}{x}) = n_0(\Gamma_0 - 1)\frac{e\Phi}{T_0} \approx -n_0 k_\perp^2 \rho_i^2 \frac{e\Phi}{T_0}$$

n_{pol} arises from gyro-phase dependent part:

$$\tilde{f}_\theta = -F_0 \frac{e}{T_0}\left[\Phi\left(\vec{R} + \vec{\rho}(\theta)\right) - \langle\Phi\rangle(\vec{R})\right]$$

Particles have an adiabatic response around their gyro-ring

alternative derivation: next order correction to adiabatic invariant mu

First Gyrokinetic PIC code

- Frieman & Chen had first derivation, but very complicated.

- Lee ’83 & ’87 derivations clearer, used Catto transformation to guiding center coordinates, & then asymptotic expansion. Made clearer the role of the polarization density (higher order polarization drift can be dropped from gyrokinetic equation, but resulting polarization density contributes to the gyrokinetic Poisson equation (because even small charge densities lead to large forces in plasmas)).

- Lee made clearer that GK polarization density eliminates small Debye scale and high frequency plasma oscillations, making simulations much more tractable. Demonstrates first GK PIC simulations (slab, electrostatic, 2-D on early 1980’s computers).

Modern Lagrangian/Hamiltonian Lie-Perturbation methods

Advantage: $df/dt = [H,f]$, make approximations to Hamiltonian/Lagrangian, but preserve important Hamiltonian properties: exact conservation of an energy H, phase-space, symplectic etc., easier to extend to full f instead of breaking up $f=f_0+f_1$, easier to extend to higher-order terms that may be important in some regimes (perhaps in edge turbulence where $f_1 << f_0$ assumption weak), etc.

Dubin, Krommes, Oberman, & Lee / borrowed from Littlejohn, Hamiltonian, slab, electrostatic

Hahm: Lagrangian approach better, extended to toroidal geometry & $dB_\perp$
Brizard: Lagrangian, extended to full $dB_\perp$ and $dB_{||}$, nonlinear properties
Dimits & Lodestro generalization of ordering
Sugama, others
Brizard-Hahm RMP 2007

Qin: To ensure total energy conservation exactly, use variational field theory for full system of particles & fields, with a particle Lagrangian & a field Lagrangian. Linear benchmarks with PEST MHD code, including kink mode. Higher-order extensions that may be useful near edge. Extensions to general frequency for RF resonant heating, etc.

3. Development of Nonlinear 5-D Simulations of Gyrokinetic Turbulence

The development of comprehensive gyrokinetic codes is one of the triumphs of computational/theoretical plasma physics (& of the modern explosion of computer power)

Main Comprehensive Gyrokinetic Codes

(Fully electromagnetic, being widely compared with expts.)

- GS2 (Dorland & Kotschenreuther) continuum, flux-tube
- GENE (Jenko, Garching) continuum, flux-tube
- GYRO (Candy & Waltz) continuum, global
- GEM (Parker and Chen) δF PIC, global

All of these codes include: toroidal geometry, general axisymmetric plasma shapes (GS2 & GENE do stellarators), multiple species, trapped and passing non-adiabatic electrons, electromagnetic fluctuations, collision operators, equilibrium scale ExB shear flow, GS2 & GENE use the $\rho_* \rightarrow 0$ limit local flux-tube (equivalent to thin annulus).

GYRO & GEM use global/non-local thick annulus, can include radial shear of ω_* profiles, turbulence spreading, etc. that can break gyro-Bohm scaling.

These gyrokinetic codes use a number of advanced algorithms.

Other Gyrokinetic Codes (1)

- Dimits PG3EQ: First toroidal gyrokinetic code in high-resolution flux tube limit including zonal flows (electrostatic w/ adiabatic electrons). Benchmark standard. Discovered Dimits Nonlinear Shift in the temperature gradient threshold, at low *q* have to go somewhat beyond the linear stability point for significant turbulence

- Global δF PIC codes (electrostatic, or fluid electron hybrid):

- Rick Sydora, LeBoeuf (UCLA, U. Alberta)
- S. E. Parker and Y. Chen (U.Col.)
- GTC (Z. Lin, U.C. I.)
- GTS (Weixing Wang PPPL), incl. plasma shaping, turbulence+neo
- M3D with gyrokinetic beam/thermal ions

- Edge/Global gyrokinetic codes (full f) in US:
  - TEMPEST (Xu, LLNL) continuum
  - ESL Edge Simulation Lab (Xu, R. Cohen, Colella) continuum full f
  - CPES Center for Plasma Edge Simulation (C.S. Chang) XGC edge PIC code

Other Gyrokinetic Codes (2)

- ORB5 (T-M Tran, A. Bottino, S. Jolliet, Lausanne, Garching) delta-f PIC, global, adiabatic electrons
- GYSELA (V. Grandgirard, X. Garbet, Y. Sarazin, …, Cadarache) semi-Lagrangian, global, adiabatic electrons
- FEFI, δ-FEFI (B.D. Scott, Garching) continuum, edge code, exploring algorithms
- ELMFIRE (J. Heikkinen, Finland) full F PIC, global (edge focus)
- G3D (Idomura, Japan) delta-f PIC, global
- G5D (Idomura, Japan) continuum, global (new, under development)
- GKV (Watanabe & Sugama, Japan) continuum, local

- gyrofluid codes based on moments of gyrokinetic equations, & fluid turbulence codes: give a lot of useful insight into turbulence, recent versions improve comparison with gyrokinetics (Bruce Scott, Beer-Hammett-Dorland-Waltz, Klauss Hallatschek, Rogers & Drake, Xu...)

Major Theoretical & Algorithmic Speedups

relative to simplest brute force, fully resolved, algorithm, for ITER $1/\rho_* = a/\rho \sim 700$

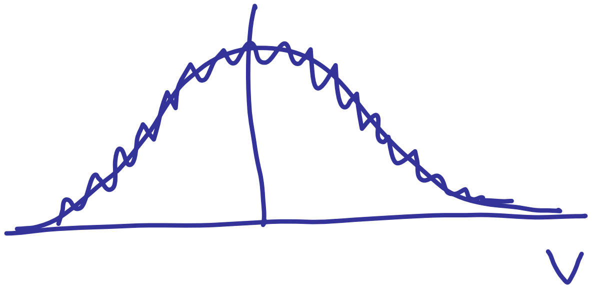
• Nonlinear gyrokinetic equation	
– ion polarization shielding eliminates plasma freq. $\omega_{pe}/\Omega_{ci} \sim m_i/m_e$	$\times 10^3$
– ion polarization eliminates ρ_e & Debye scales $(\rho_i/\rho_e)^3$	$\times 10^5$
– average over fast ion gyration, $\Omega_{ci} / \omega_* \sim 1/\rho_*$	$\times 10^3$
• Continuum or δf PIC, reduces noise, $(f_0/\delta f)^2 \sim 1/\rho_*^2$	$\times 10^6$
• Field-aligned coordinates (nonlinear extension of ballooning coord.)	
$\Delta_{\parallel} / (\Delta_{\perp} q R / a) \sim a / (q R \rho_*)$	x70
• Flux-tube / Toroidal annulus wedge, ↓ simulation volume	
– $k_\theta \rho_i = 0, 0.05, 0.1, \ldots, 1.0$ $n = 0, 15, 30, \ldots, 300$ (i.e., 1/15 of toroidal direction)	x15
– $L_r \sim a/5 \sim 140\ \rho \sim 10$ correlation lengths	x5
• High-order / spectral algorithms in 5-D, $2^5 \times 2$	x64
• Implicit electrons	x5-50
• Total combined speedup of all algorithms	$\times 10^{23}$
• Massively parallel computers (Moore's law 1982-2007)	$\times 10^5$

Major Theory/Algorithm Advances

- δf PIC, reduces PIC noise

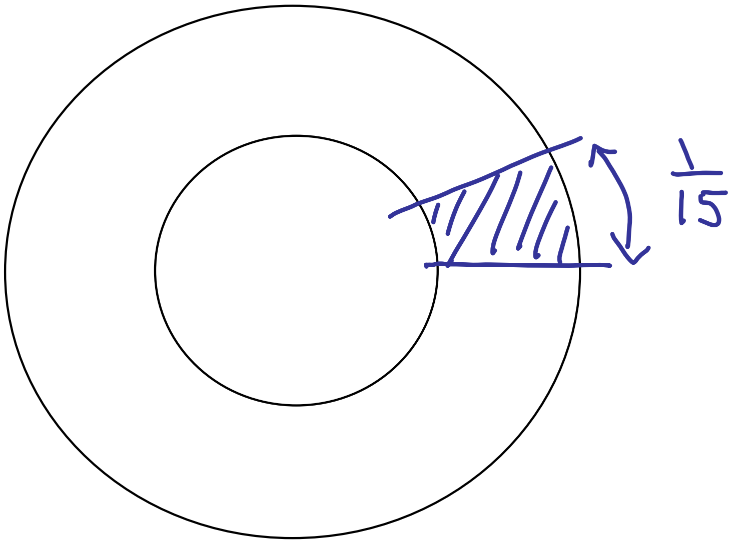
  (Kotschenruether, Dimits, Parker, Hu & Krommes, Aydemir)

  $f(x,v,t)$ = smooth $f_0(v)$ + weighted particles δf

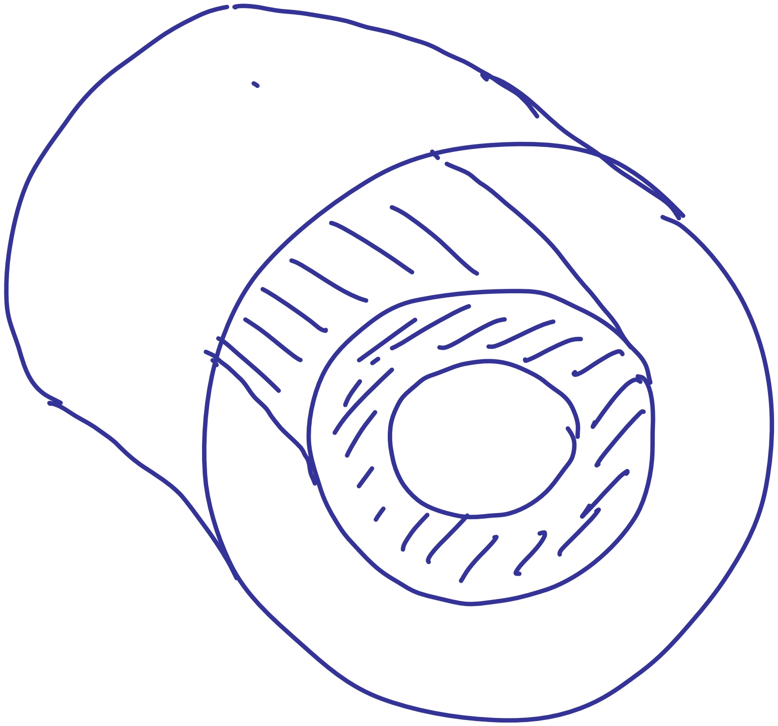


- Field-aligned coordinates, local flux-tube / toroidal annulus wedge (Cowley, Beer, Hammett, Dimits) (similar to shearing box simulations of accretion turbulence in astro)
- General toroidal annulus wedge (Waltz, Candy, Rosenbluth):

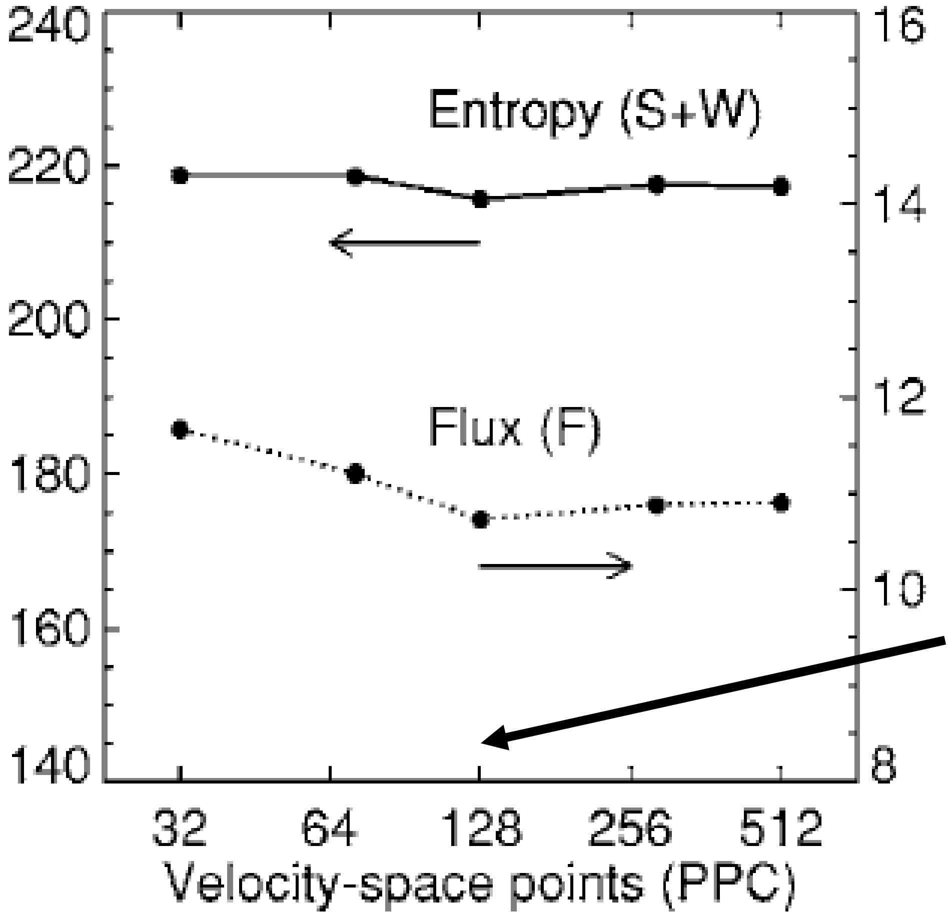
Top view:



Side view:



Good convergence in velocity space



suppressed 0

Std: 8 energies X 16 pitch angles

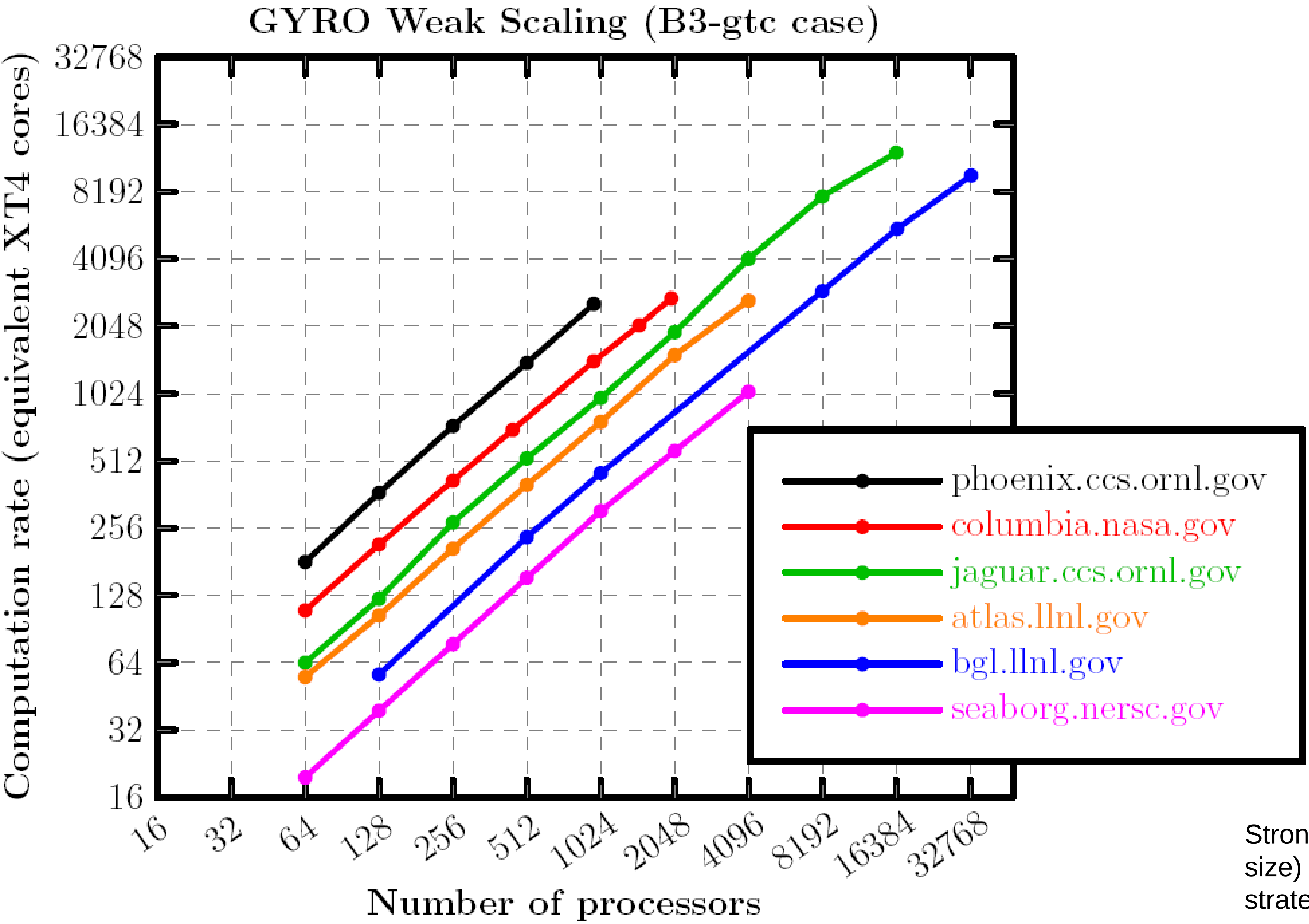
FIG. 5. Velocity-space scan, starting from low resolution (32 points) and working up to very high resolution (512 points), with the baseline case at 128 points. The upper curve (solid line) is the intensity $S+W$, and the lower curve (dotted line) is the ion heat flux. The results suggest that the baseline velocity-space resolution is more than adequate, and no significant fine structure in velocity-space develops.

Gauss-Legendre integration w/ 8 energies exact for Maxwellian X 8th order polynomial, super-exponential convergence.

(for comparison typical PIC code uses 10’s of particles/cell)

Candy & Waltz, Phys. Plasmas 13, 032310 (2006)

5-D continuum codes show excellent scaling to large # of processors

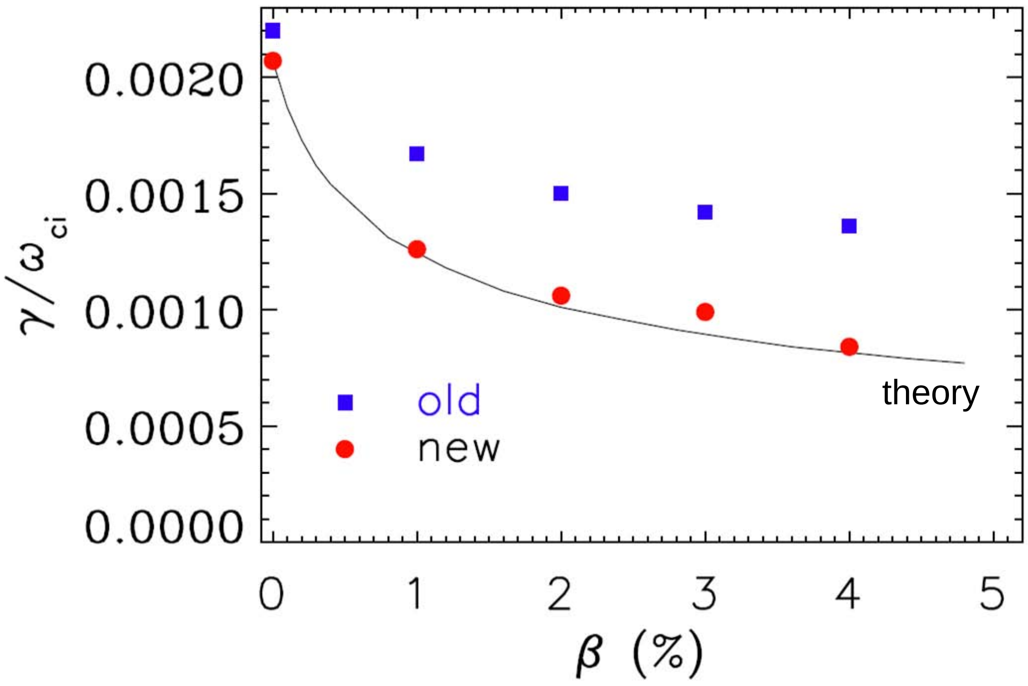


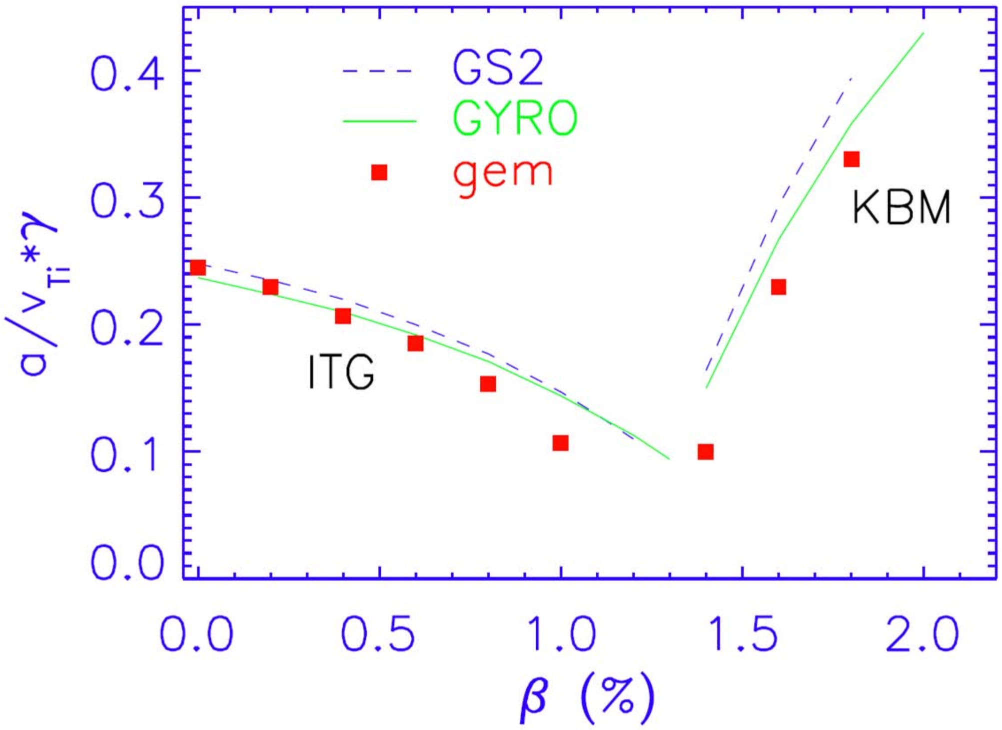
Strong scaling (fixed problem size) for GS2 & GYRO demonstrated over a factor of 1000 processors.

Various US Supercomputers including the ORNL Cray XT4 (jaguar), X1E (phoenix) and LLNL IBM BlueGene/L (bgl).

Candy, Dimits, Fahey, Mikkelsen

Y. Chen & S. Parker breakthrough: working GK PIC algorithm for magnetic fluctuations





Long standing “high-β” problem in GK PIC codes, first observed by Cummings (’94). Chen & Parker JCP 03 fixed problem by careful treatment of two large terms (arising from canonical momentum transformation) that nearly cancel

Benchmark 3 independent PIC & continuum codes

Global code approaches local flux-tube limit as $\rho_* \rightarrow 0$

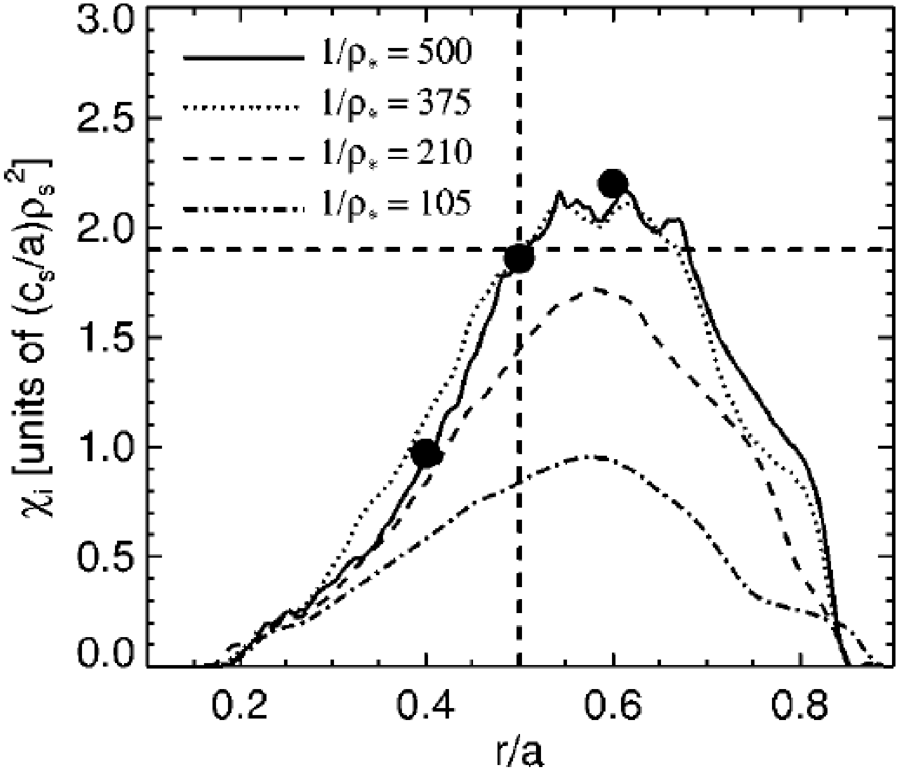


FIG. 3. Curves show radial profile of χ_i averaged over the time i
$400 \leqslant (c_s/a)t \leqslant 900$, as computed by GYRO. Solid dots show resu
three separate GS2 flux-tube simulations.

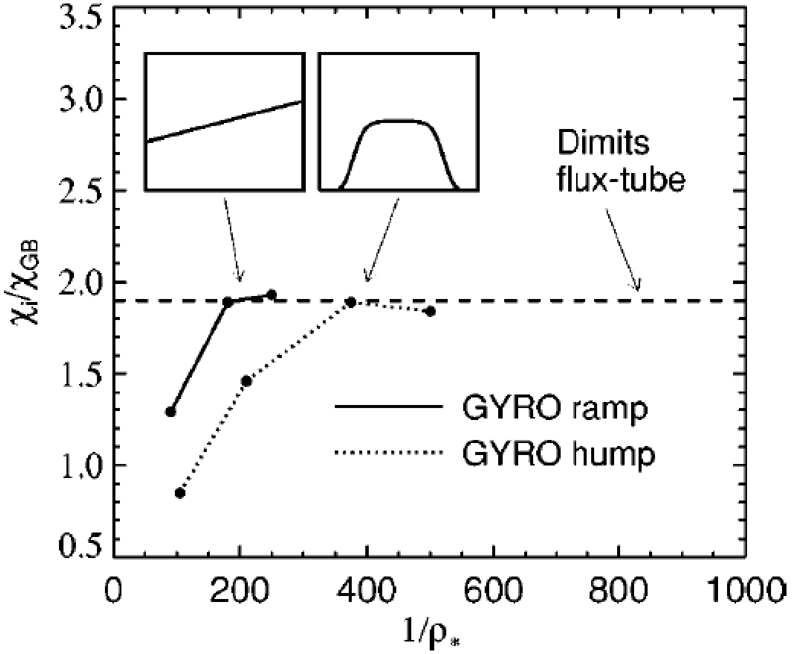
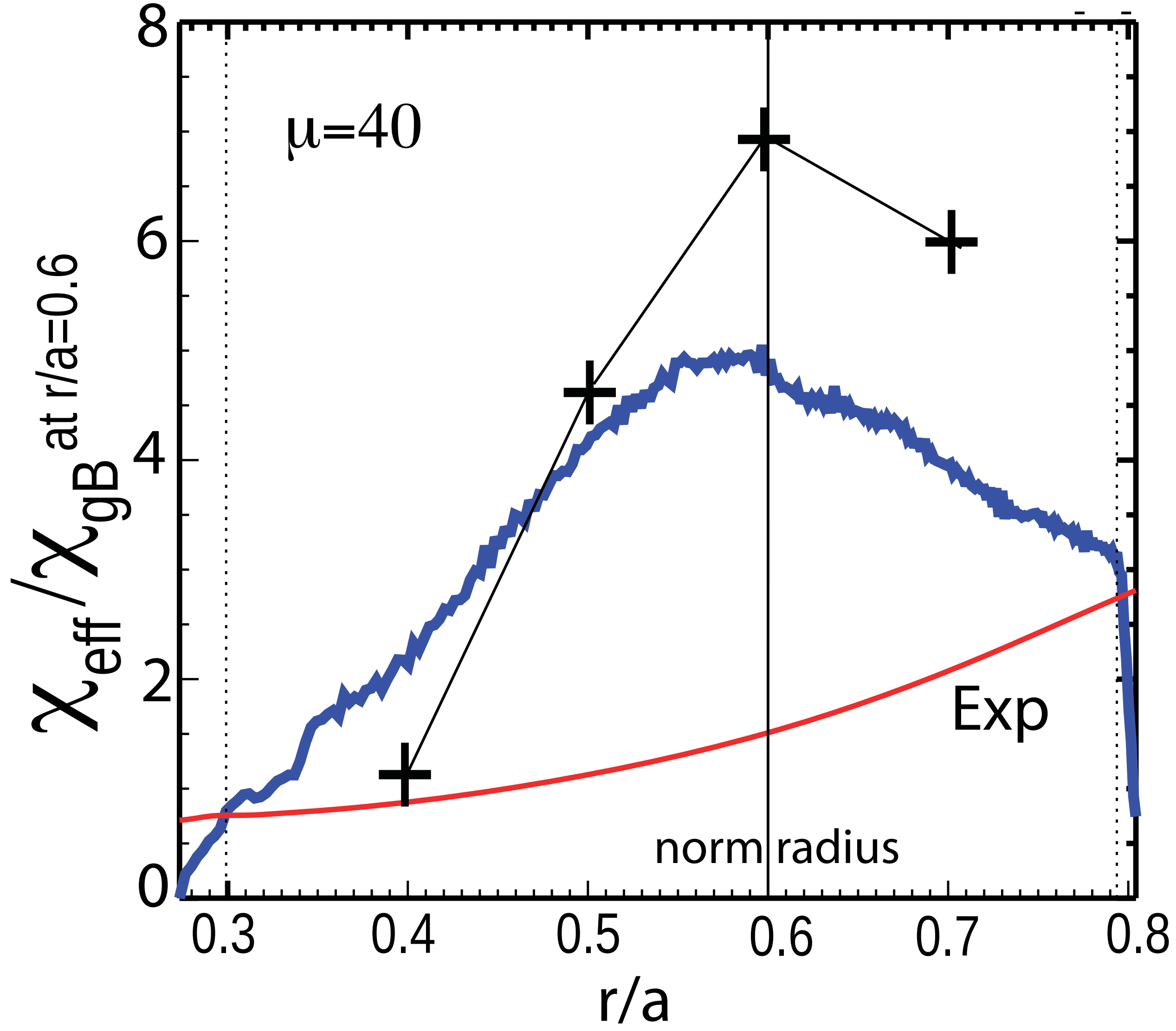


FIG. 4. Comparison of the GYRO data for the GTC *hump* profile (dotted curve) defined in Eq. (1), with a GYRO simulation of the *ramp* profile (solid curve) defined in Eq. (2). The *hump* results are the same as shown in Fig. 1. Note that the more realistic *ramp* profile exhibits gyro-Bohm scaling at substantially smaller system size.

Candy et al. PoP 04

Moderate amount of turbulence spreading occurs in some cases



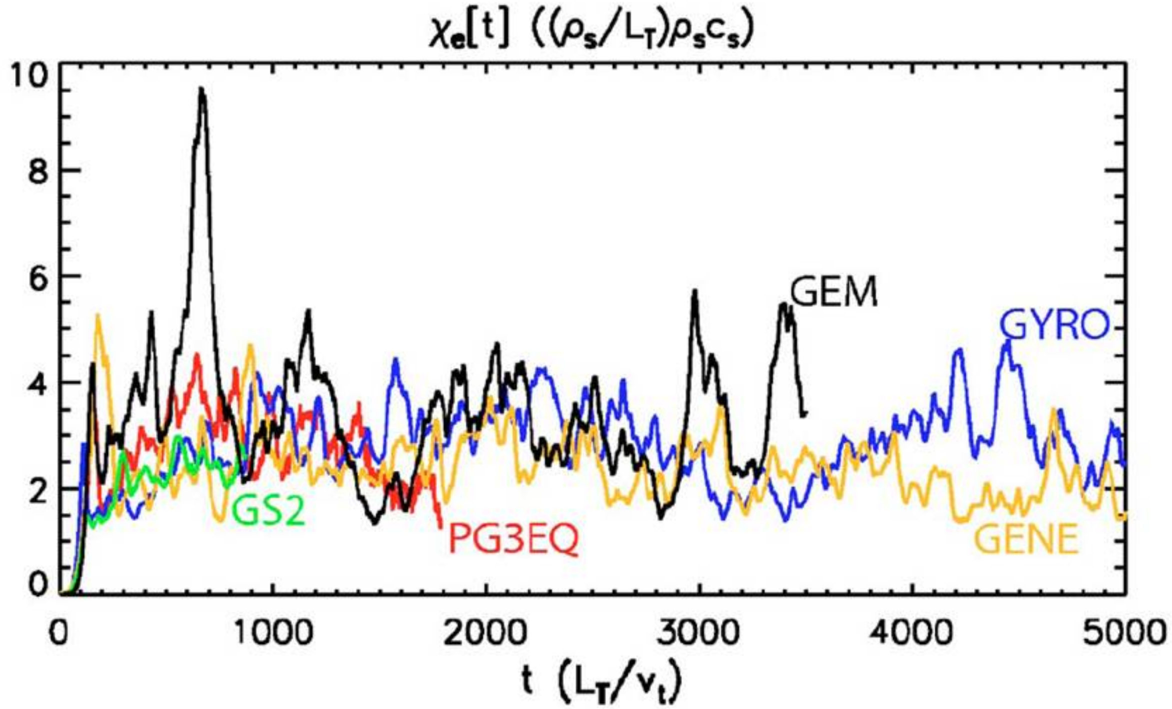
amount of spreading:
- ~ 0.1 a
- ~ 2-5 radial correlation lengths
- ~ 20-50 ρ_i

see also Hahm et al. 2004, Lin et al. 2002, Garbet et al., Newman, Xu

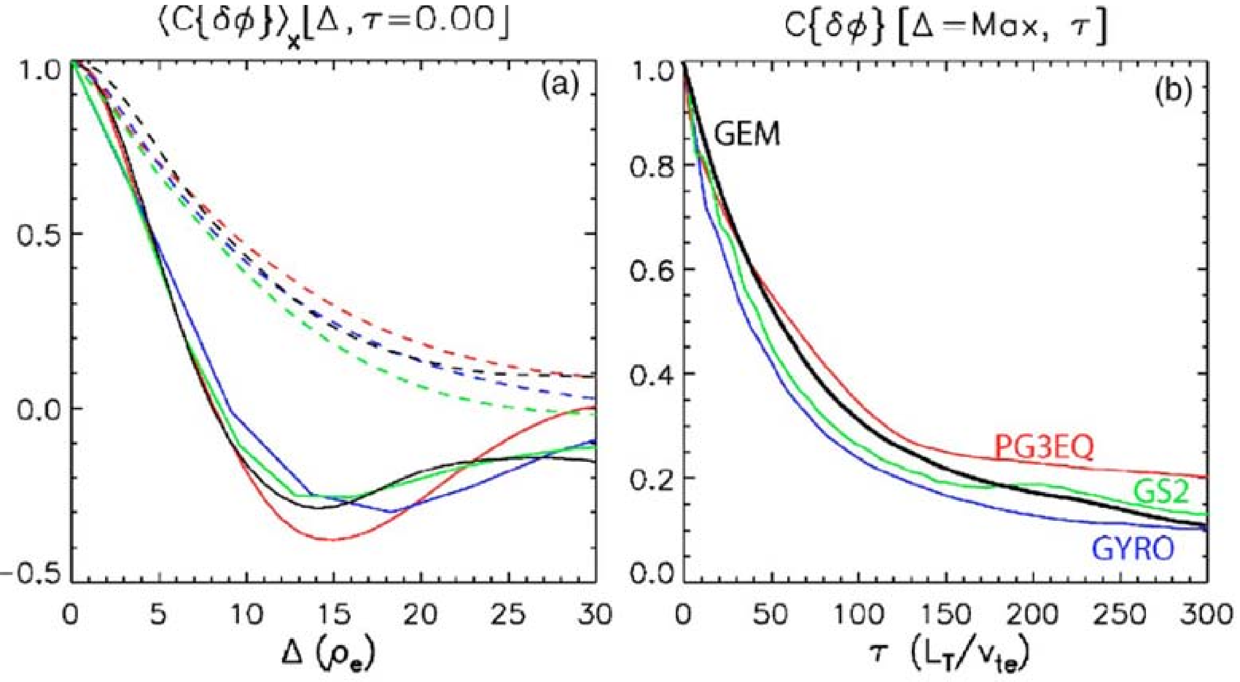
DIII-D L-mode

Waltz, Candy, Petty 2006 PoP 13, 072304

Successful Benchmarks of Independent Gyrokinetic Codes



Good agreement in χ (+/- 10% on long time average $t > 1000$ a/c_s) between 3 continuum and 2 PIC codes



Correlation functions agree well

Nevins et al. 2007

Continuum & PIC Gyrokinetic codes

(Eulerian & Lagrangian/Monte-Carlo)

Very useful to have both types of codes: very different numerical properties,
Very useful independent cross-check on each other

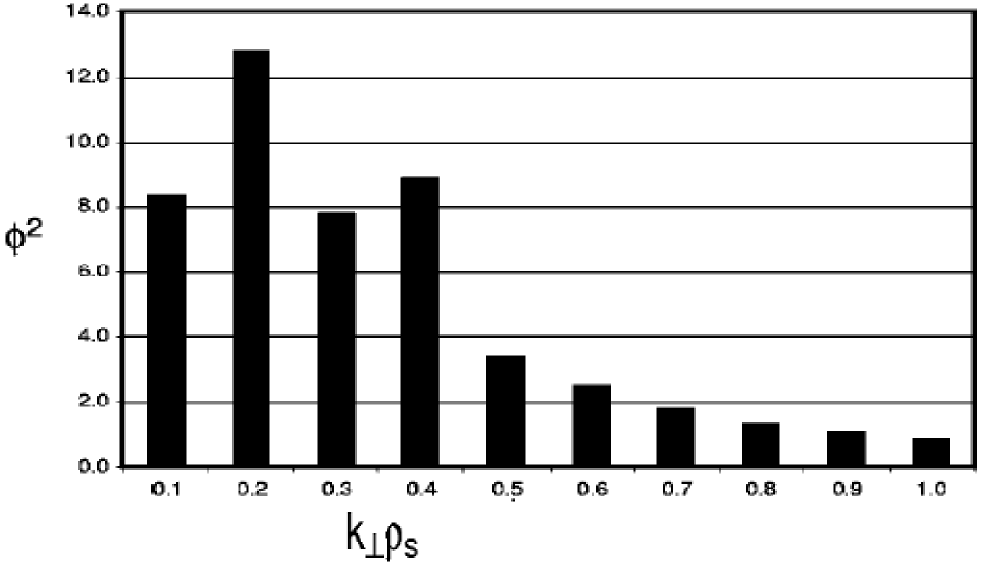
“Continuum” codes use a range of advanced algorithms: pseudo-spectral,
high-order upwind, conservative Arakawa methods, finite-element, etc.,
not just simple grid.

4. Gyrokinetic Simulations: Physics studies & comparisons with expts

2002: early detailed comparisons of gyrokinetic simulation and DIII-D experiment

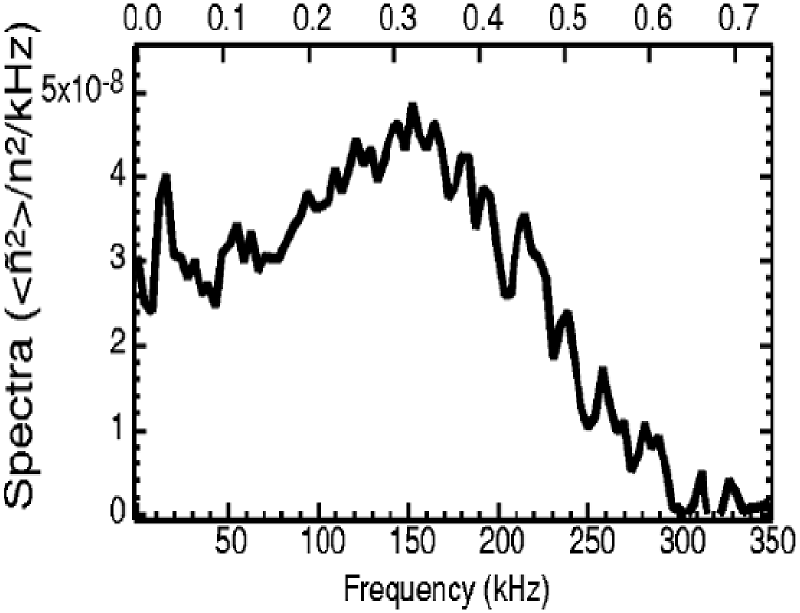
GS2 gyrokinetic simulation of fluctuation spectrum

(Ross & Dorland, PoP 2002)



Qualitatively similar to shape of measured spectrum

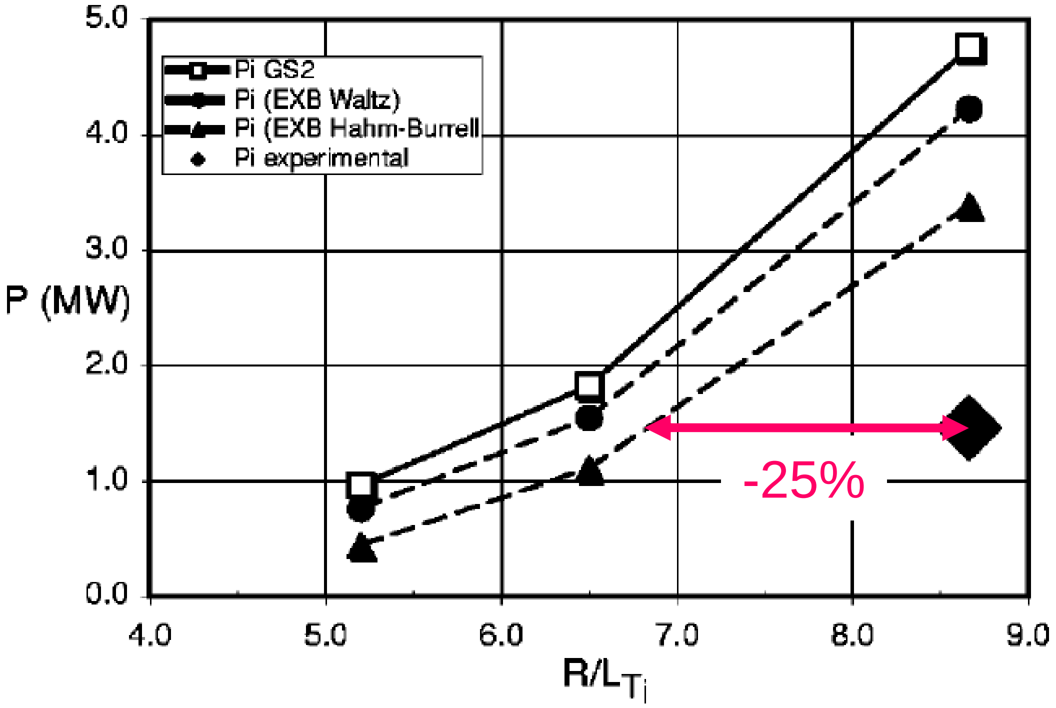
(Ross, Bravenec, Dorland, et al. 2002) (BES measurements by McKee, Fonck, et al.)

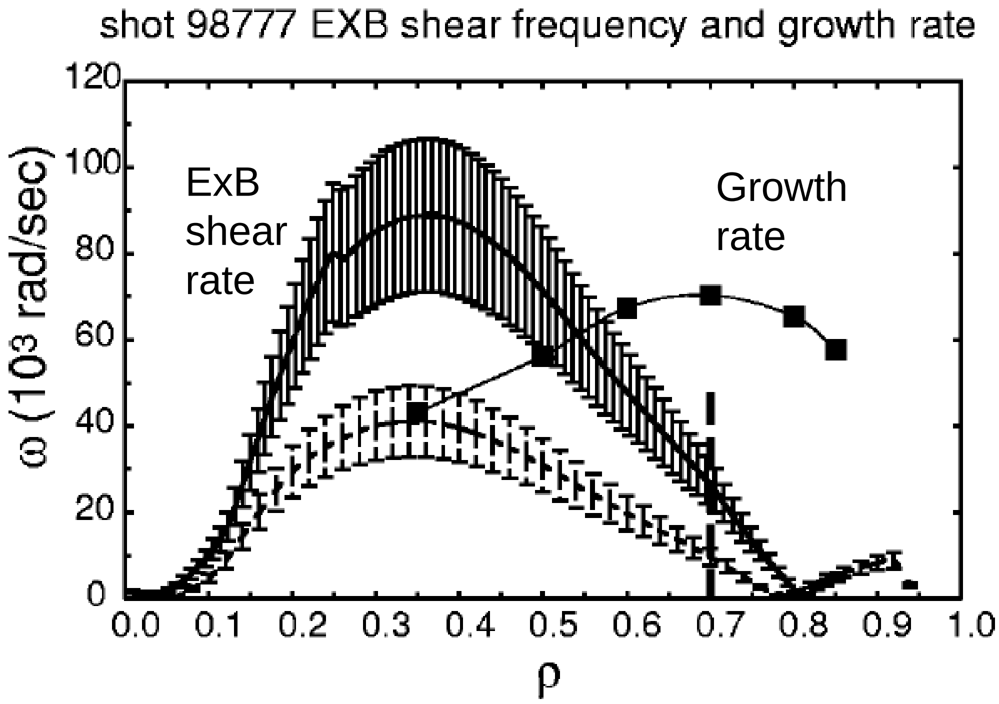


Absolute amplitude of simulation fluctuations too large, but see next slide.

Instrumental viewing volume contributes to roll off at high k, working on synthetic diagnostics to better account for instrumental effects

Ross, Dorland 2002: detailed comparisons of gyrokinetic simulation and DIII-D L-mode measurements



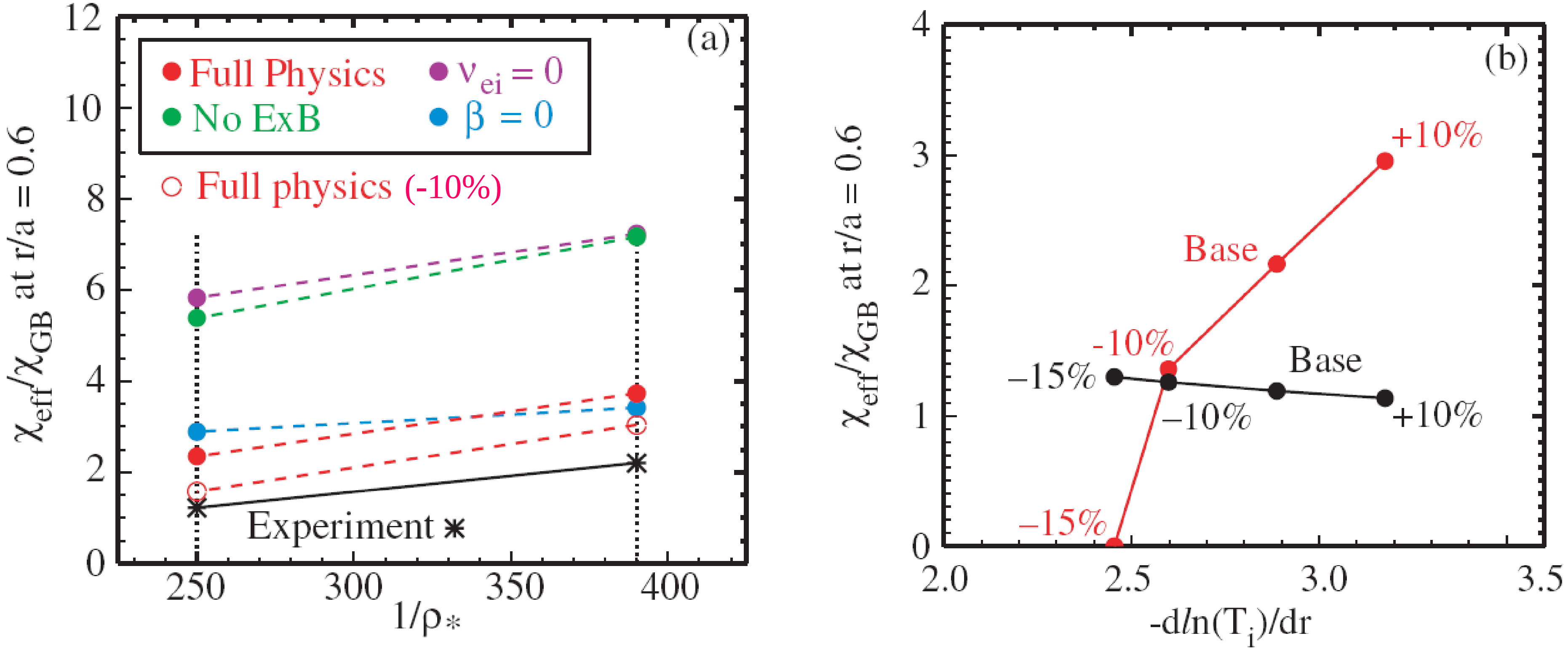


Need to ↑ ∇T here

Need to ↓ ∇T here

- Simulation gives heat flux 2x experiment at r/a=0.7
- *25%* ↓ ∇T outside error bars if applied everywhere, but may be within error bars since need to ↑ ∇T for r/a < 0.55

Comparison of GYRO Code & Experiment



Gyrokinetic turbulence codes now including enough physics (realistic geometry, sheared flows, magnetic fluctuations, trapped electrons, fully electromagnetic fluctuations) to explain observed trends in thermal conductivity, in many regimes.

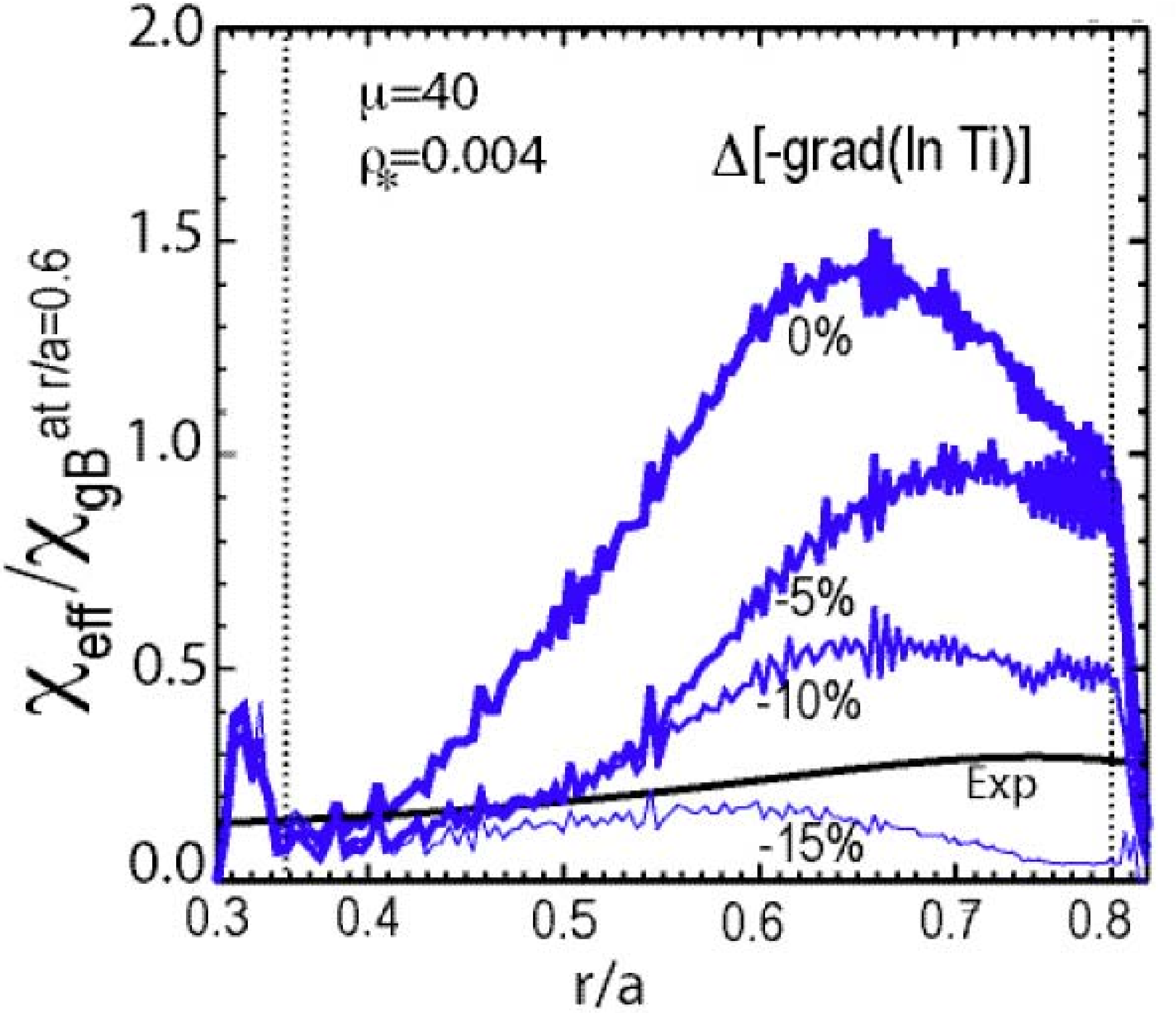
Big improvement over 15 years ago, when there were x10 – x100 disagreements between various analytic estimates of turbulence & expts.

Now within experimental error on temperature gradient. Importance of critical gradient effects emphasized in 1995 gyrofluid-based IFS-PPPL transport model.

Caveats: Remaining challenges: quantitative predictions of internal transport barriers, test wider range of parameters, & more complicated edge turbulence.

Candy & Waltz, PRL03, Waltz Nucl. Fus. 05

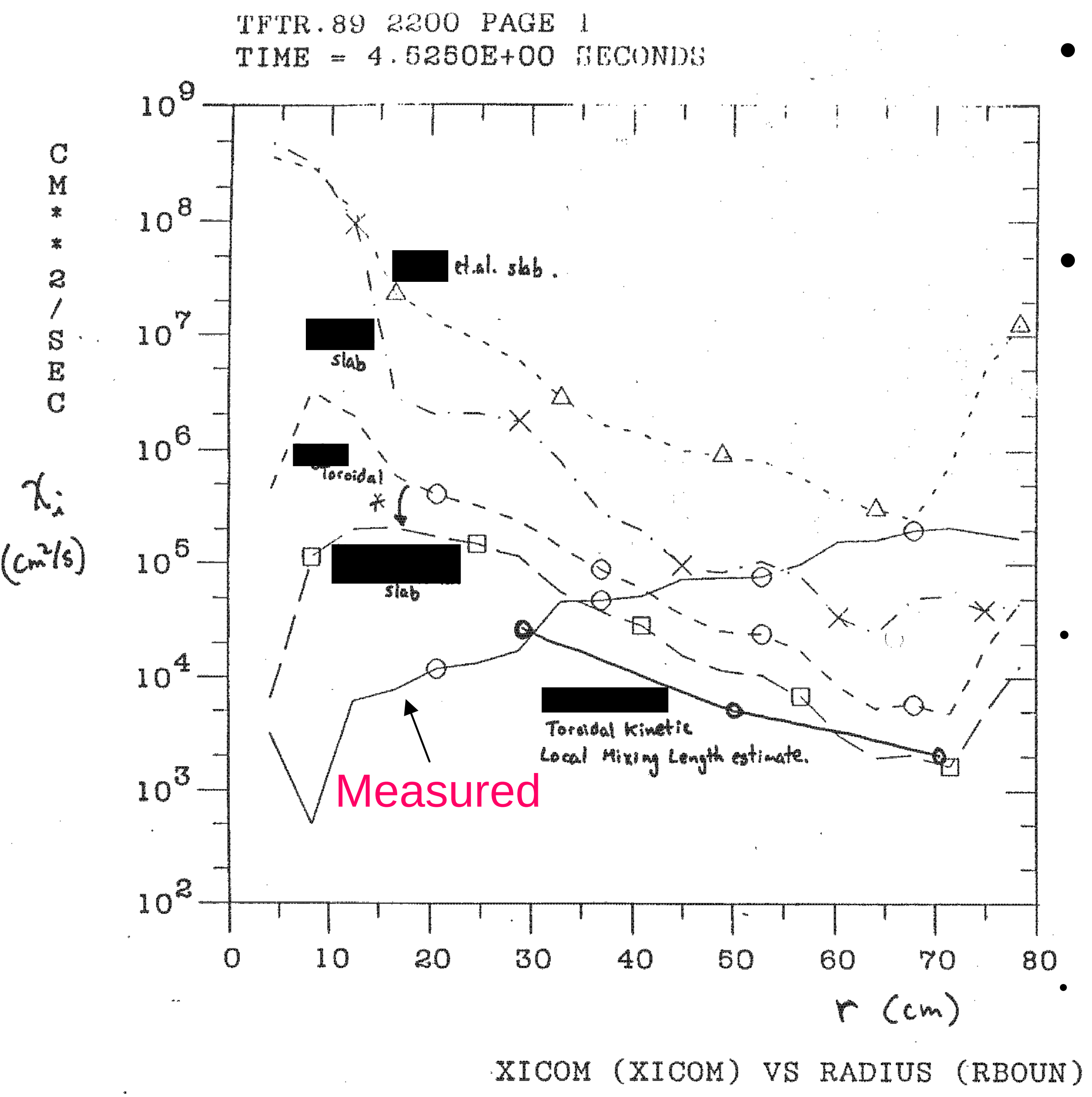
ITG often within 5% of threshold in core



DIII-D H-mode

Waltz, Candy, Petty 2006 PoP 13, 072304

1980's analytic turbulence theories had large disagreements (x10-1000) with experiments



- Very smart people, but very hard problem

- Recent gyrokinetic simulations (and models based on them) now compare much better with experiments. We've made a lot of progress.

- This plot made in 1990. We and many theories didn't appreciate at that time the importance of getting thresholds for marginal stability accurate, … Much discussion about marginal stability in LM & SS, but pellet experiments apparently drive $\eta_i > \eta_i^{crit}$ (slab theory) without changing transport. Proposed at the time: may not have been beyond marginal stability for toroidal modes (Rewoldt & Tang, 1990, Horton et al. 1992)

- see also S.D. Scott et al., Phys. Fluids B 1990

R/L_{Te} dependence for ‘large’ density gradients

$R/L_n > 2.5$

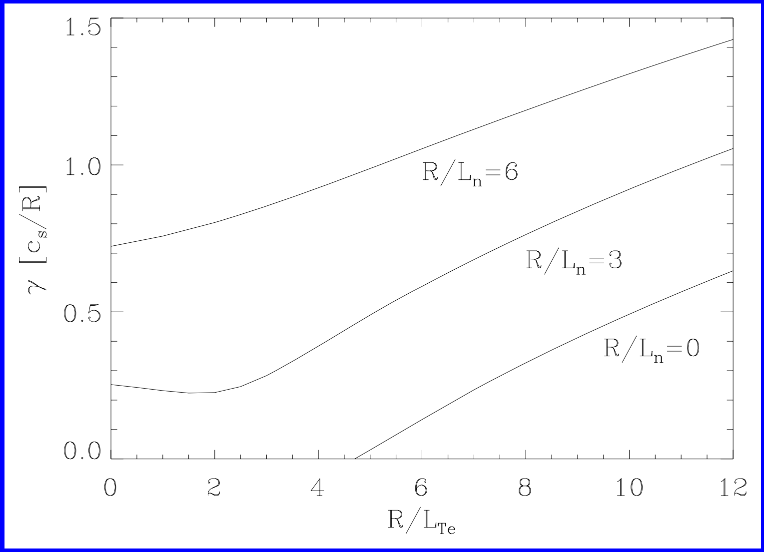
Conventional (quasi-)linear models:

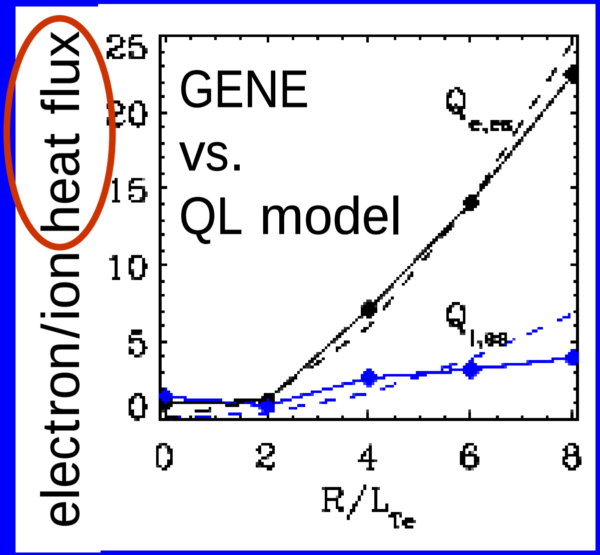
no critical gradient (density gradient drive)

Nonlinear simulations and new quasilinear model:

effective critical gradient

electron heat flux has offset-linear scaling





$$\chi_e = \chi_s \, q^{\nu} \left(\frac{R/L_{T_e} - \kappa_C}{R/L_{T_e}} \right) \frac{\rho_s^2 c_s}{R}$$

$\chi_s \gtrsim 100$

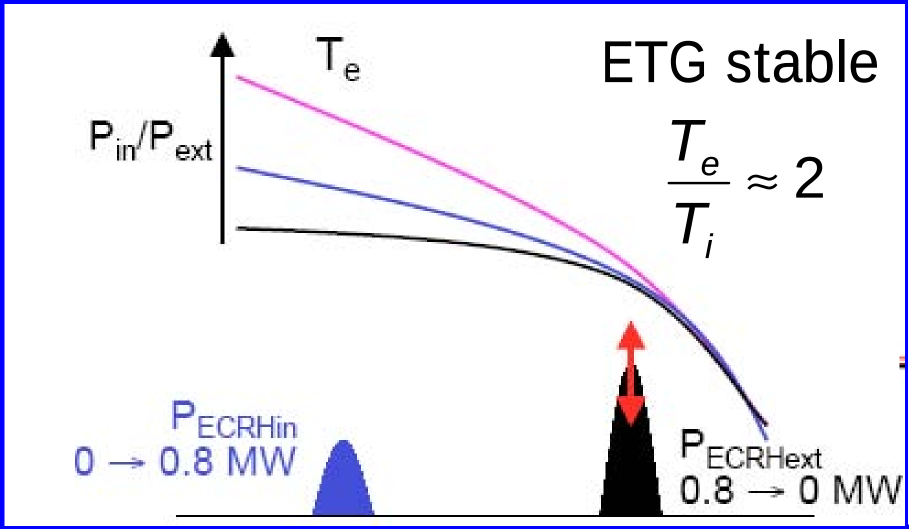
- similar as in adiabatic ITG case
- implies Te profile stiffness
- coupling of particle and electron heat flux

Jenko, Angioni, et al. IAEA06

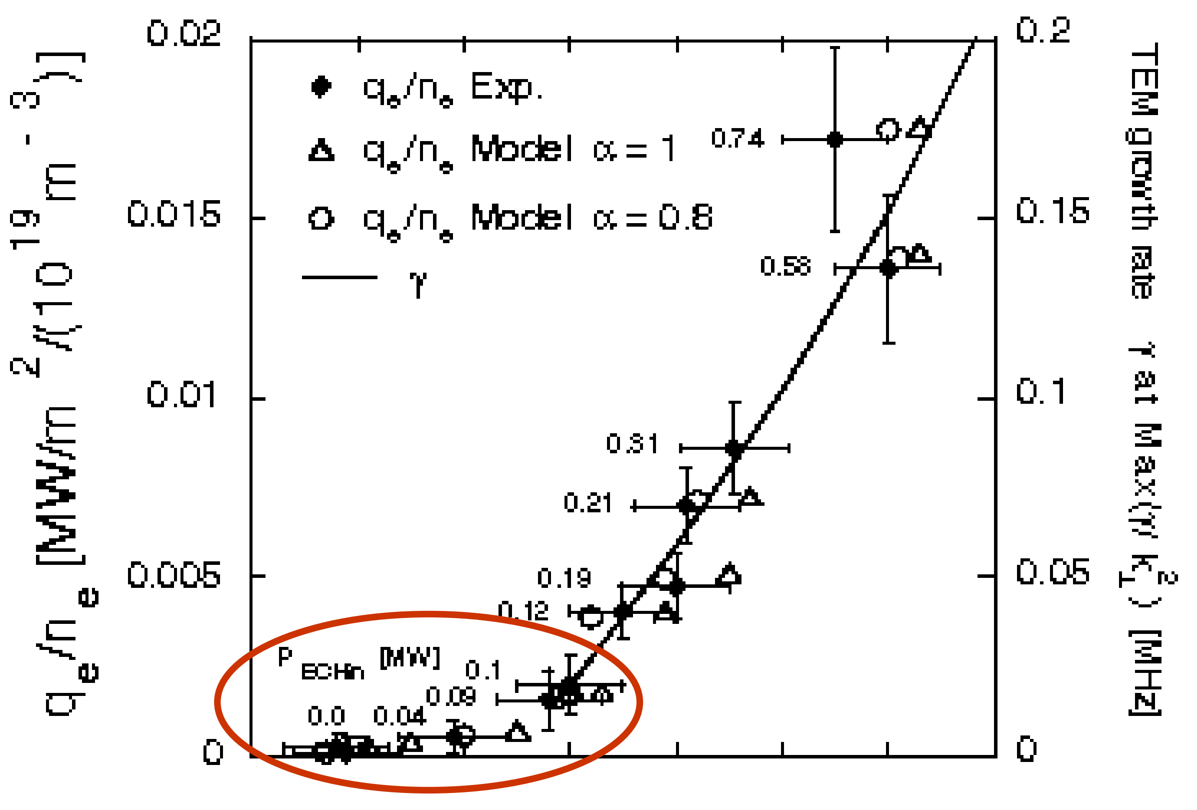
http://www-pub.iaea.org/MTCD/Meetings/FEC2006Presentations/Presentations/20-oct-06/talk_EX8-5Ra.ppt

Existence of a threshold in R/L_{Te}

- AUG L-mode plasmas
  [0.8 MW ECRH, little OH)
- gradual reduction of central ECRH, balanced by increase of off-axis heating



[F. Ryter et al., PRL 2005]



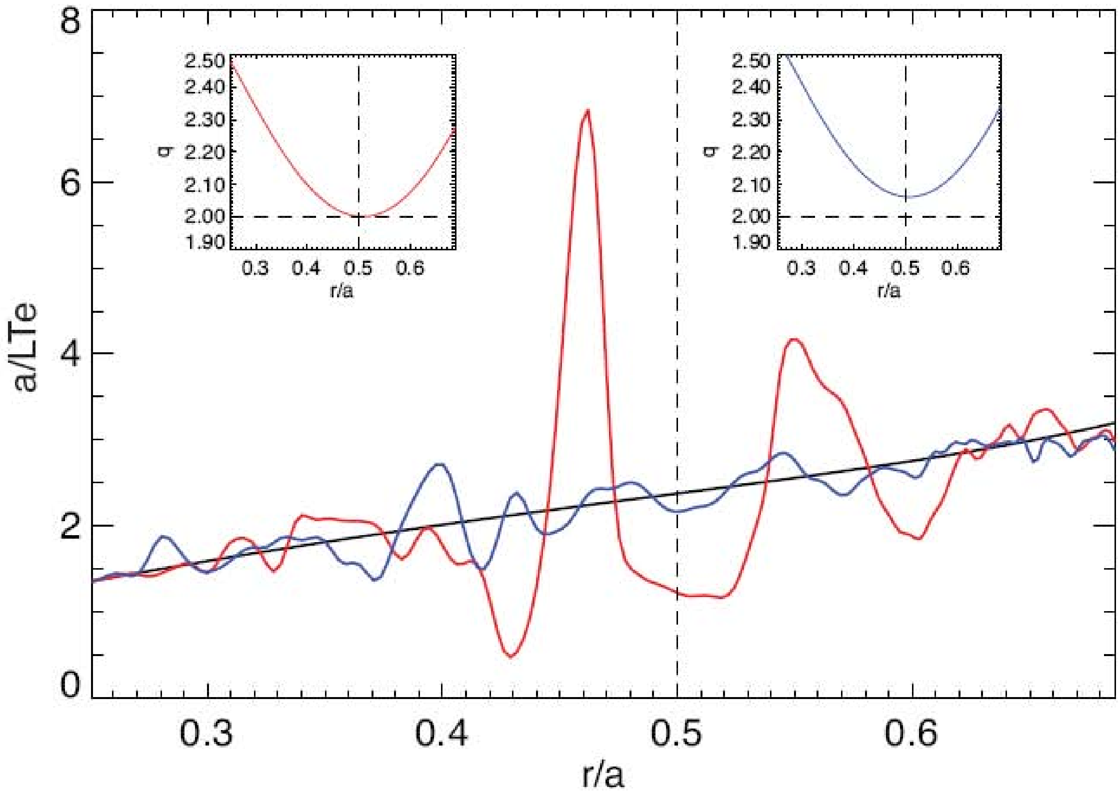
Threshold behavior is observed directly; power balance and transient transport consistent with both linear gyrokinetics and CG model.

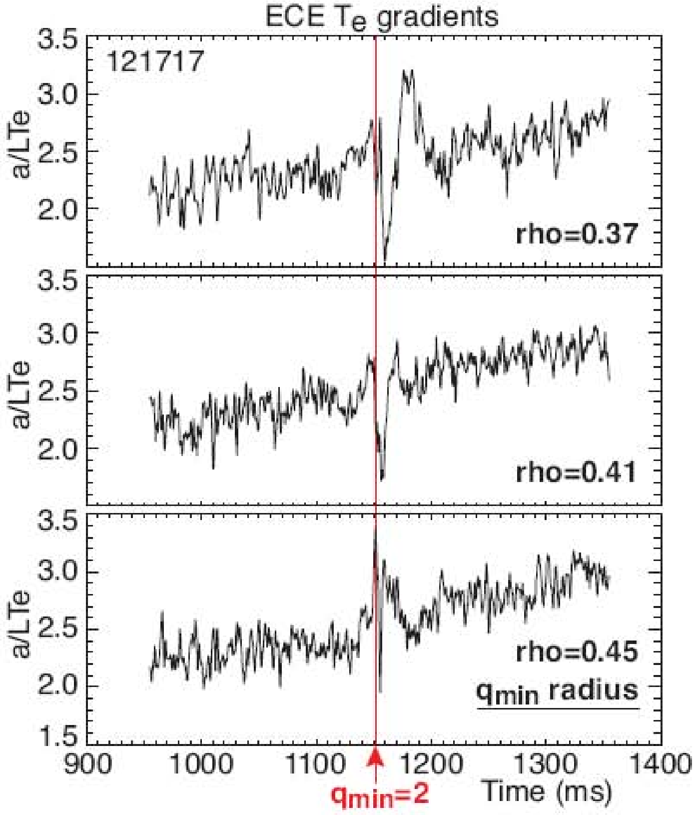
Jenko, Angioni, et al. IAEA06

http://www-pub.iaea.org/MTCD/Meetings/FEC2006Presentations/Presentations/20-oct-06/talk_EX8-5Ra.ppt

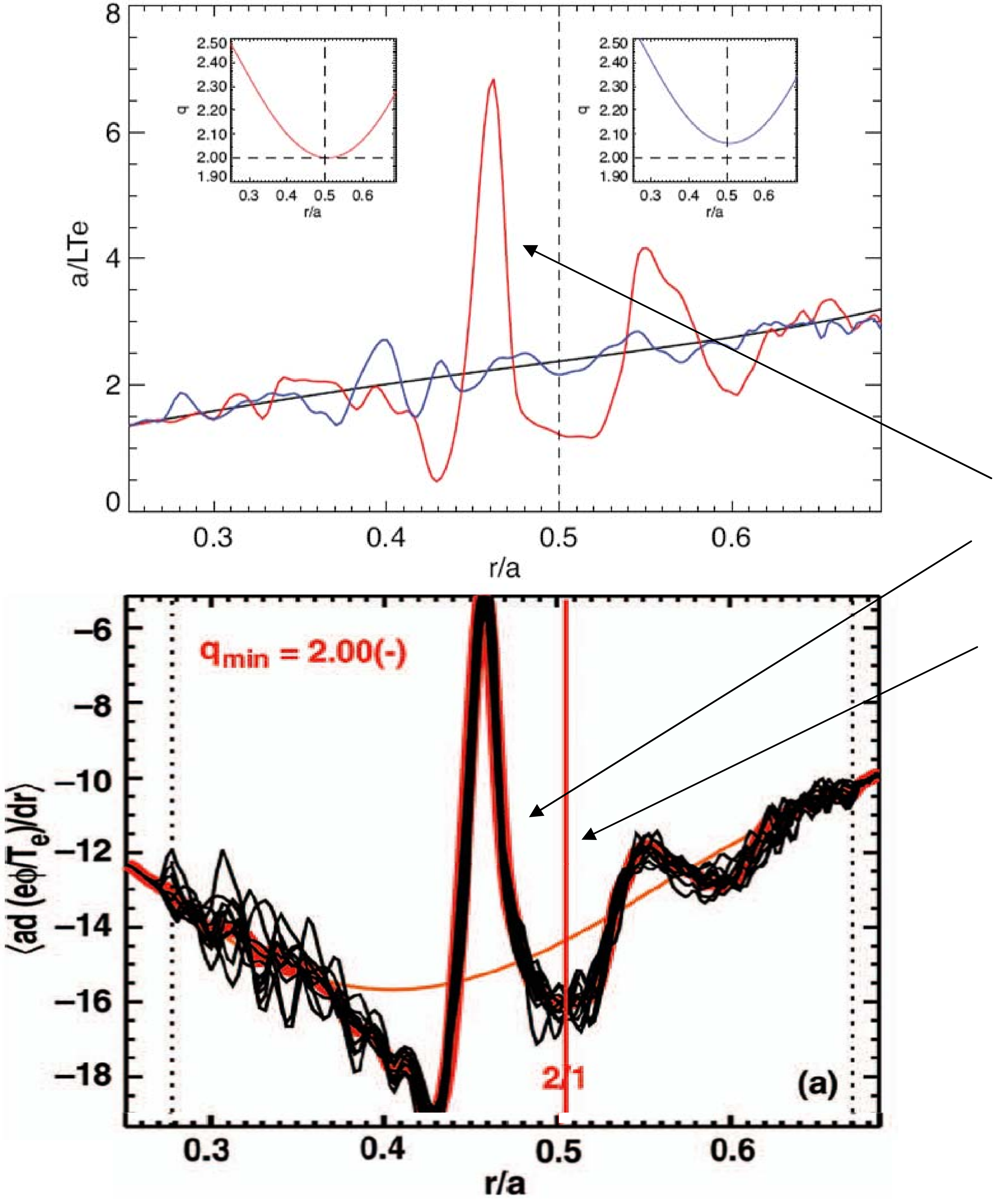
GYRO discovers electron temperature corrugation

- **Once kinetic electrons were added to GYRO electron temperature corrugations (n=0 modes) were observed to form at low order rational surfaces.**
  - These agree well with DIII-D measurements.





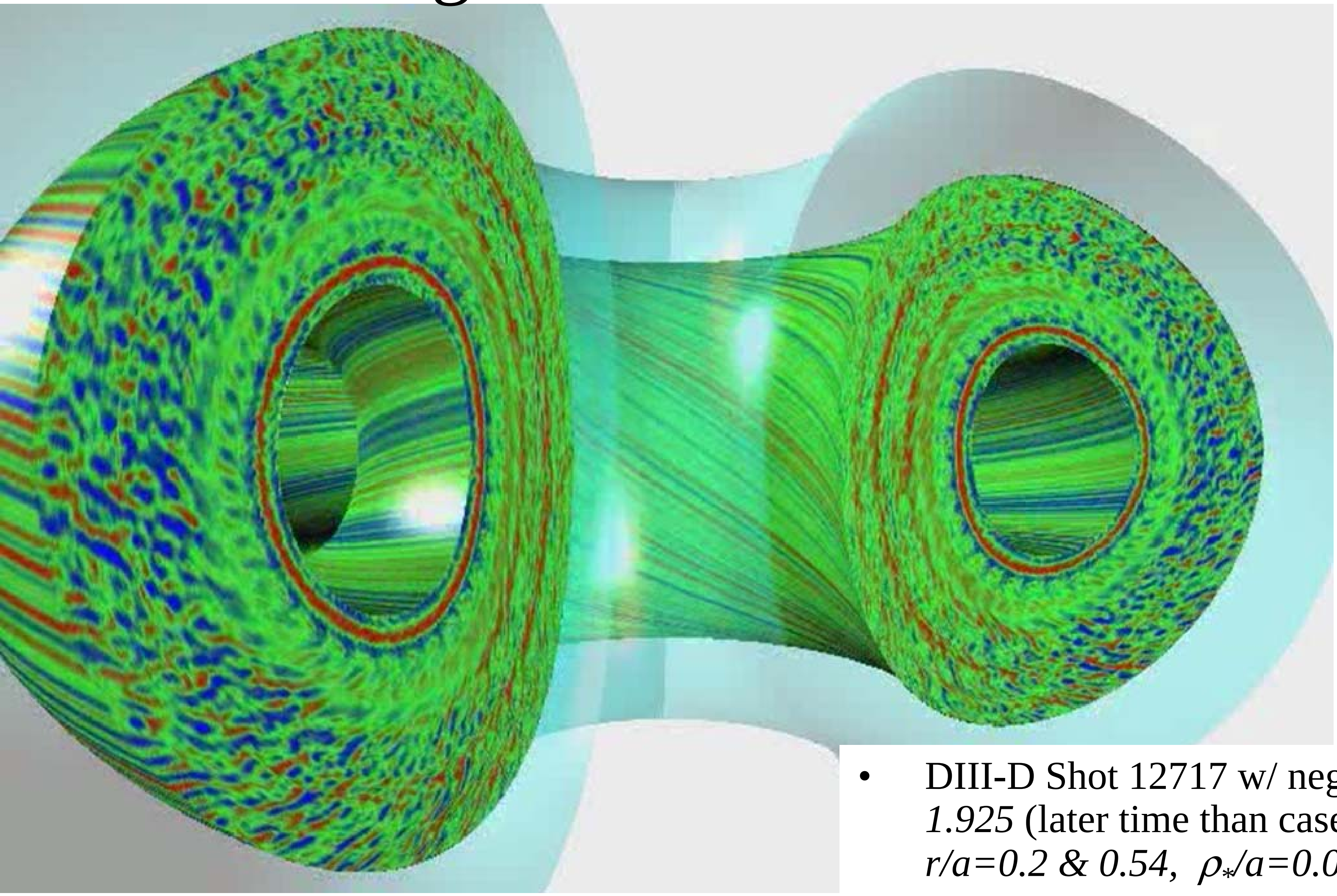
Waltz, Austin, Burrell, Candy, PoP 2006



Temperature corrugation and
large sheared zonal flow
occur near
low-order rational surface

Waltz, Austin, Burrell, Candy, PoP 2006

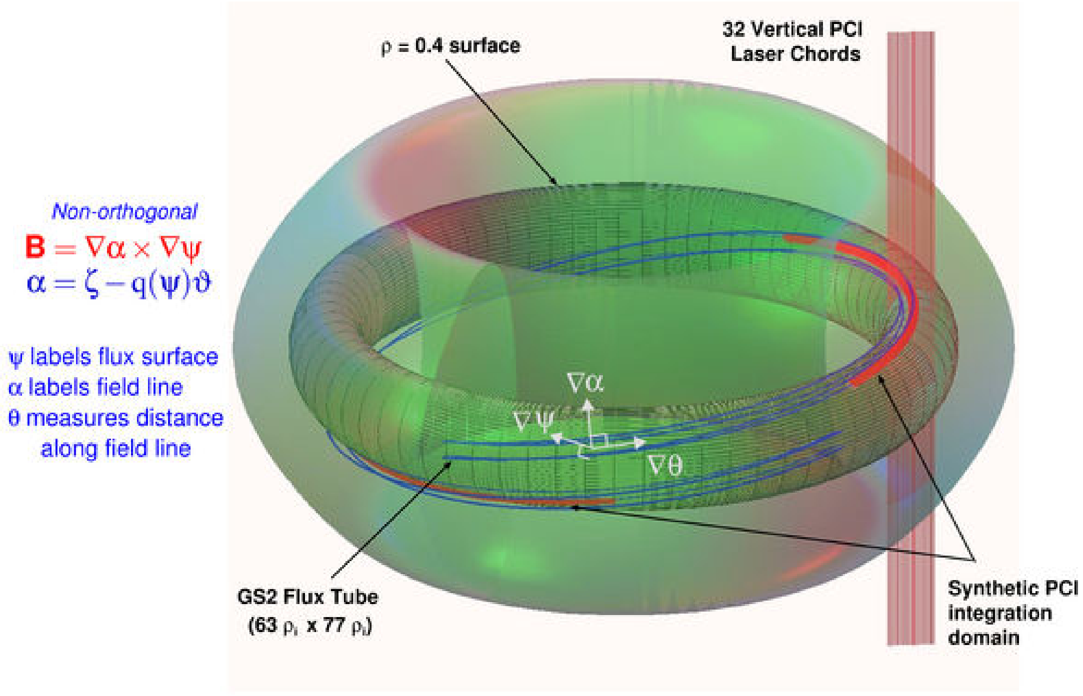
Full-physics GYRO simulation of Negative Central Shear DIII-D case



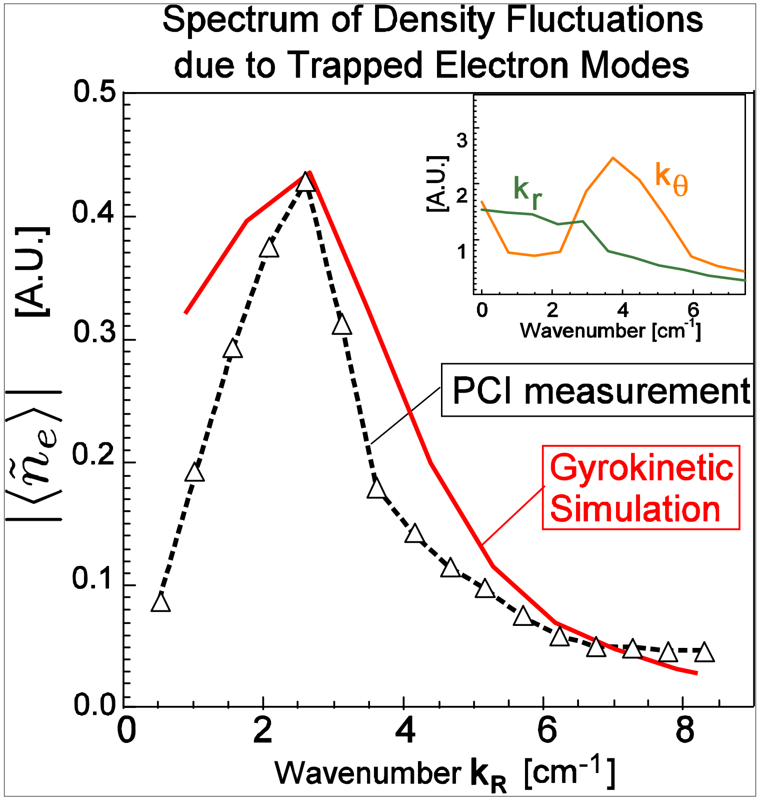
Movie of density fluctuations from GYRO simulation
http://fusion.gat.com/THEORY/images/0/0f/N32o6d0.8.mpg
from http://fusion.gat.com/theory/Gyromovies

Waltz, Austin, Burrell, Candy, PoP 2006

- DIII-D Shot 12717 w/ negative central shear, $q_{min} = 1.925$ (later time than cases in paper), $q=2$ @ $r/a=0.2$ & 0.54, $\rho_*/a=0.003$
- experimental *grad(T)* used, but reduced ExB shearing rate by 20% to get finite turbulence
- 500 radii x 32 complex toroidal modes (96 binormal grid points) x 10 parallel points along half-orbits x 8 energies x 16 $v_{||}/v$, 12 hours on ORNL Cray X1E with 256 MSPs



- Transform $k_R = (\nabla R \cdot \nabla\psi/|\nabla\psi|)k_\psi + (\nabla R \cdot \nabla\alpha/|\nabla\alpha|)k_\alpha$
- Instrument function: Gaussian beam, finite aperture, reference beam k_R~0

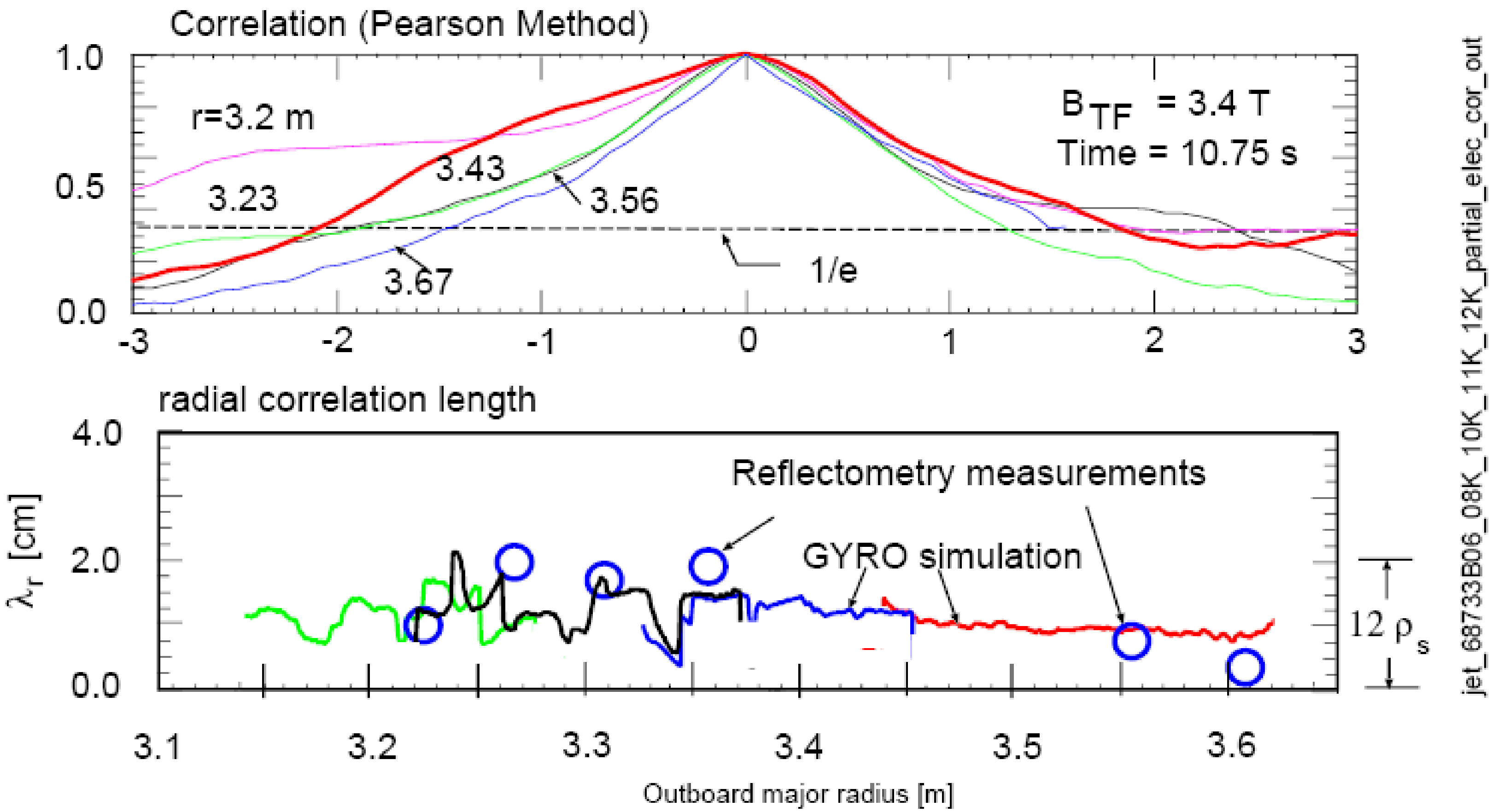


Synthetic phase contrast imaging diagnostic in GS2 [Ernst, IAEA06].

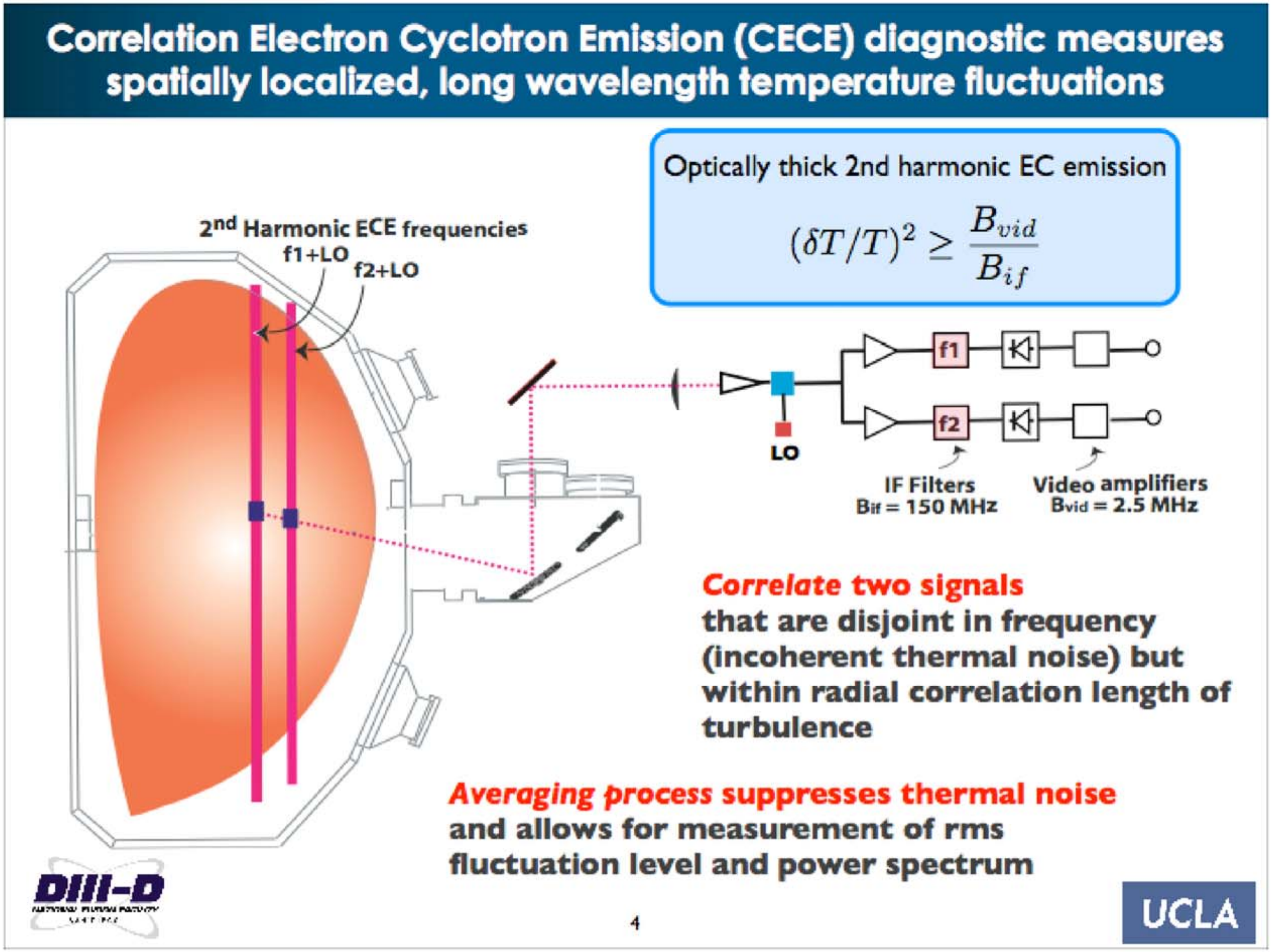
Direct observation of TEM turbulence: Nonlinear GS2 simulations, with synthetic diagnostic, reproduce wavelength spectrum from phase contrast imaging in Alcator C-Mod ITB [Ernst-IAEA06].

Radial correlation consistent with reflectometry

- Correlation of $\tilde{n}_e(r_1, t)$ and $\tilde{n}_e(r_2, t)$
- λ_r defined by Δr where correlation decreases below 1/e
- Magnetic axes at 2.97m and outboard separatrix at 3.85m



Budny, Kramer, Mazzucatto, JET, et al., NO3.11, Wed. 11:30 am



Correlation Electron Cyclotron Emission (CECE) diagnostic measures spatially localized, long wavelength temperature fluctuations

Optically thick 2nd harmonic EC emission

$$(\delta T/T)^2 \geq \frac{B_{vid}}{B_{if}}$$

***Correlate* two signals**
that are disjoint in frequency (incoherent thermal noise) but within radial correlation length of turbulence

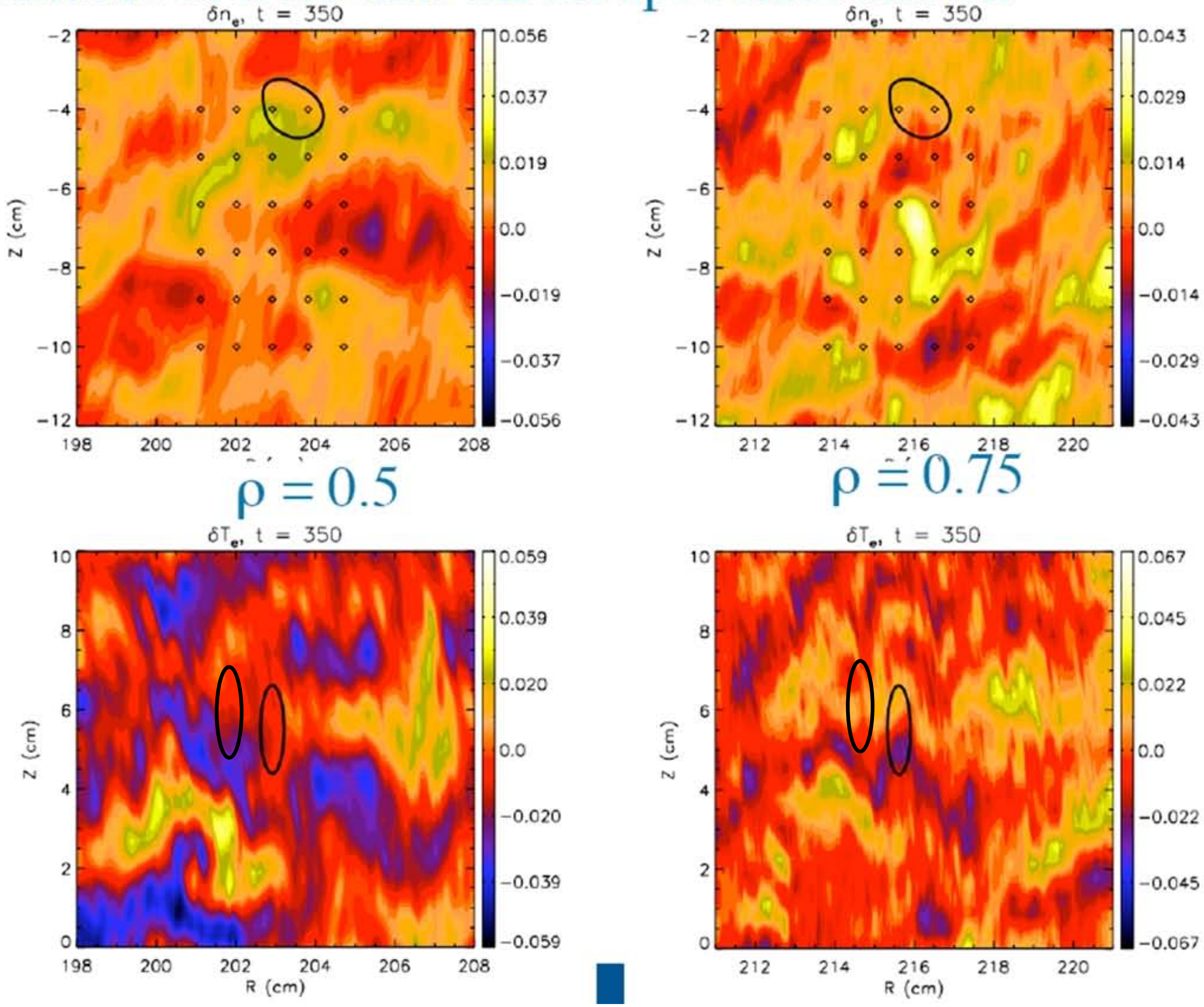
***Averaging process* suppresses thermal noise**
and allows for measurement of rms fluctuation level and power spectrum

Anne White’s invited talk NI1.2, Wed. 9:30 AM

ρ = 0.5 vs ρ = 0.75 spot sizes

50% contours of BES and CECE spot sizes shown

δn_e



$\rho = 0.5$

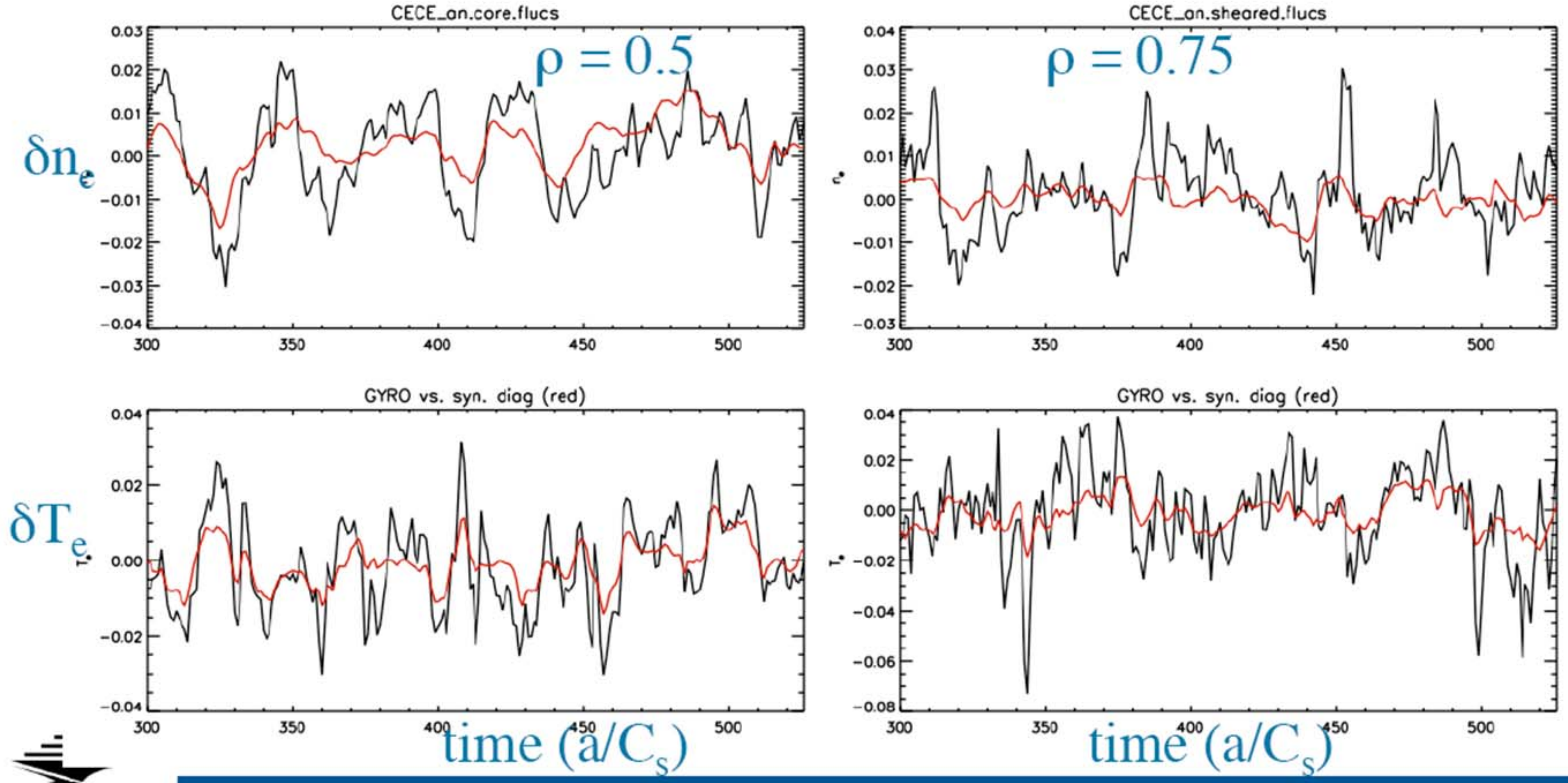
$\rho = 0.75$

δT_e

Holland, White, GYRO synthetic diagnostics (UCSD, UCLA, General Atomics)

Clear loss of sensitivity to high frequency components

- **Black is GYRO, red is synthetic BES/CECE**

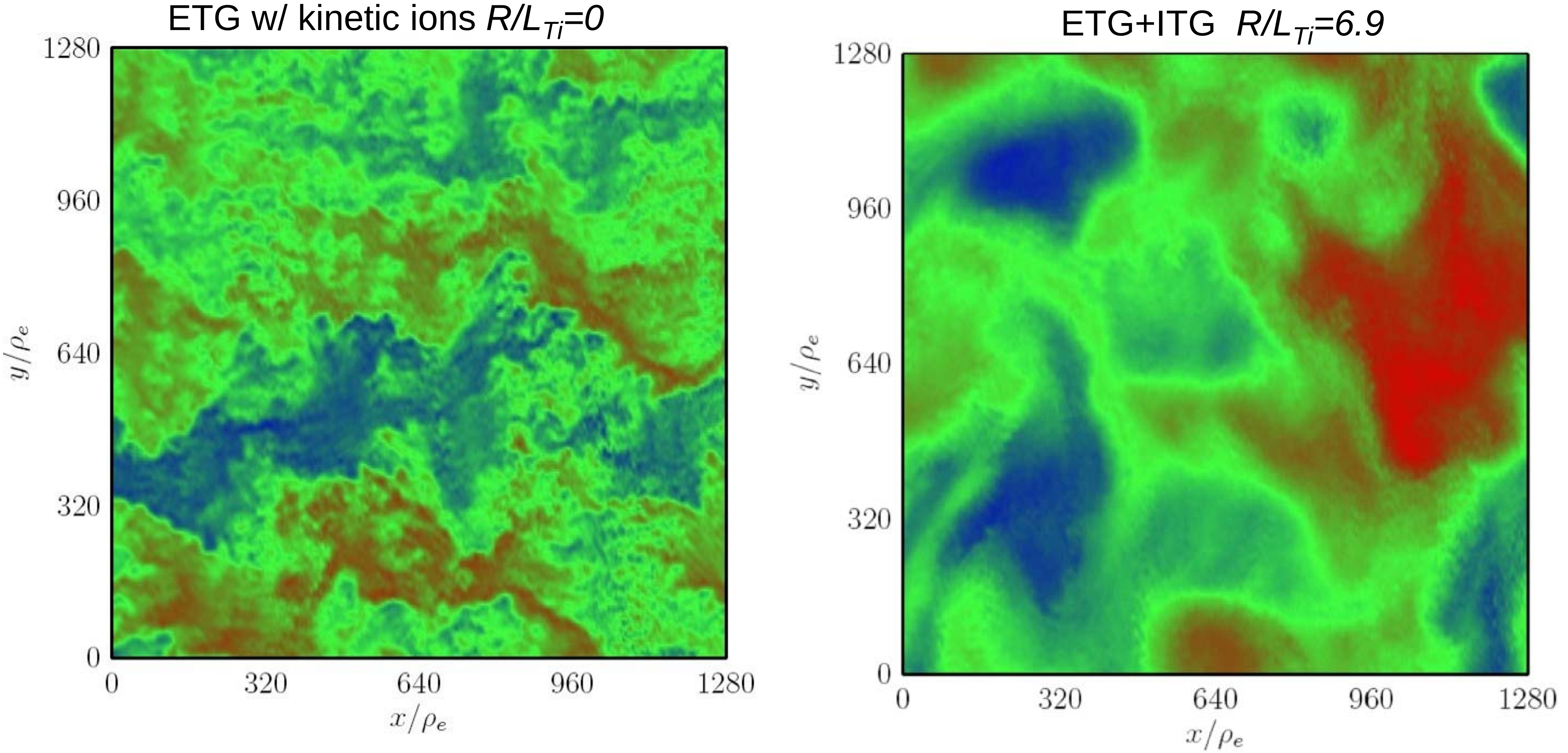


UCSD

GENERAL ATOMICS

Largest GYRO simulations used to study interaction of ITG & ETG Turbulence

- 1280 ρ_e x 1280 ρ_e x 20 parallel pts/orbit x 8 energies x 16 $v_{||}/v$
- electrons + kinetic ions, $m_i/m_e = 20^2 - 30^2$
- 5 days on DOE/ORNL Cray X1E w/ 720 Multi-Streaming Processors



ETG w/ kinetic ions R/L_{Ti}=0

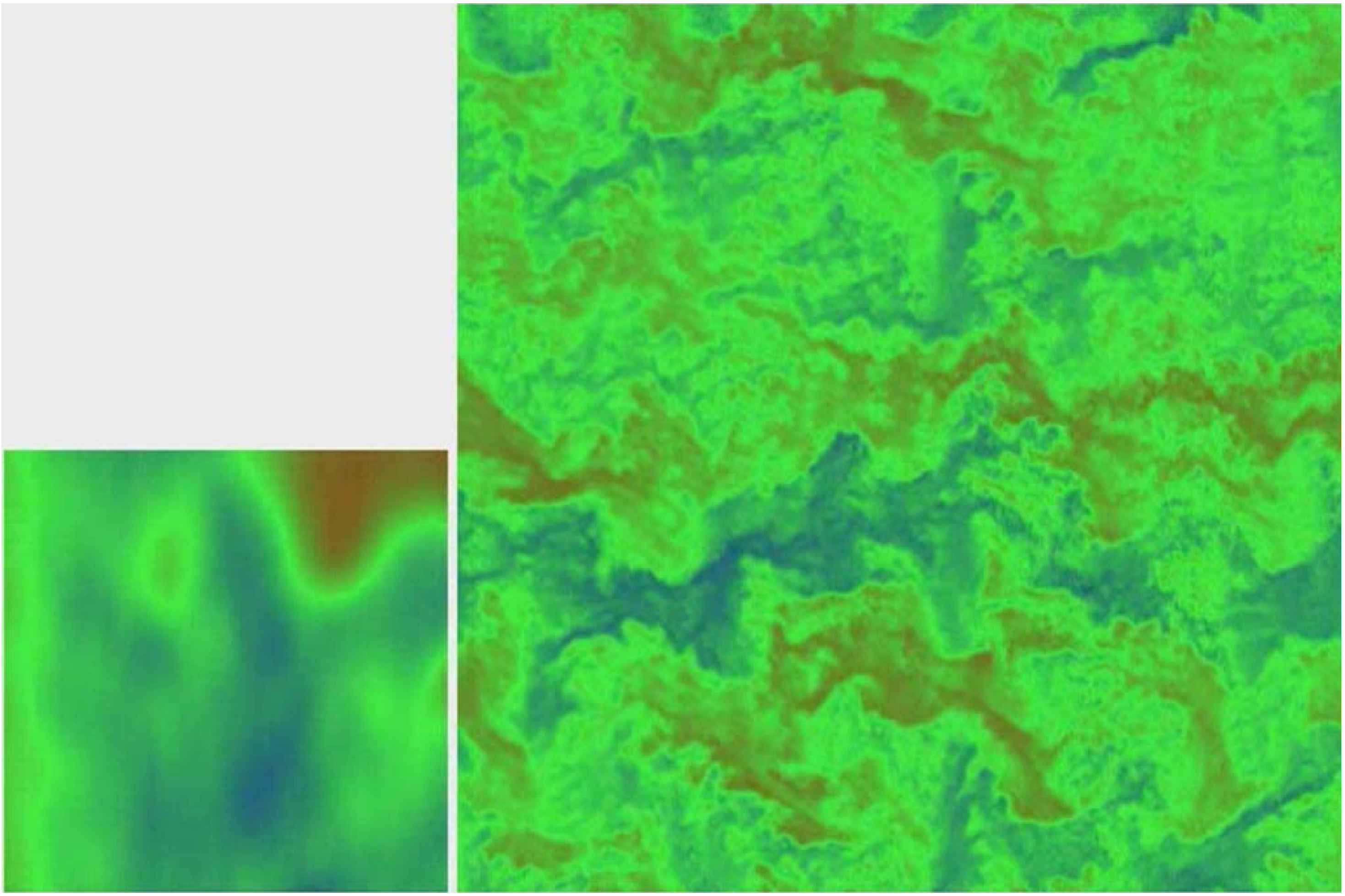
ETG+ITG R/L_{Ti}=6.9

Candy, Waltz, et al. JPSC 2007

ETG + kinetic ion GYRO simulation movie

- large box on right: full simulation domain, 1280 ρ_e x 1280 ρ_e = 64 ρ_i x 64 ρ_i
- small box on lower left: zoom in on a 64 ρ_e x 64 ρ_e patch

http://fusion.gat.com/THEORY/images/1/1f/ETG-ki.mpg from http://fusion.gat.com/theory/Gyromovies



Candy & Waltz

ETG fluctuations ($k_\perp \rho_i > 1$) may account for significant fraction of transport in some plasmas

Simple scaling from ITG to ETG:

$$\chi_{itg} \sim C_{itg}\, \rho_i^2\, v_{ti}/L$$
$$\chi_{etg} \sim C_{itg}\, \rho_e^2\, v_{te}/L \sim \chi_{itg}/60$$

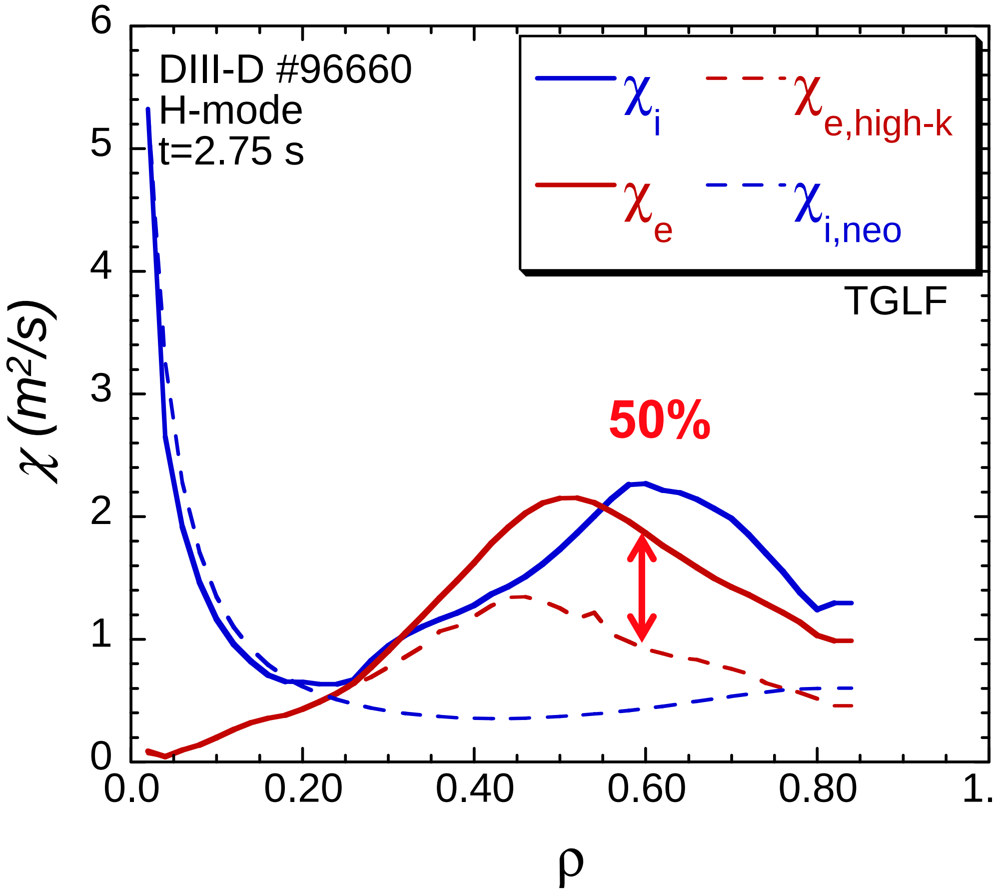
But Dorland & Jenko (2000) showed ETG turbulence larger because:

perpendicular adiabatic ions for ETG gives more shielding of zonal electric fields than does parallel adiabatic electrons for ITG.

Candy showed ETG will be reduced by kinetic ions, more so if strong ITG turbulence

ITG can be weak near marginal stability w/ ExB shear. TGLF transport model shows ETG / high-k TEM may still be important in some cases.

DIII-D H-mode

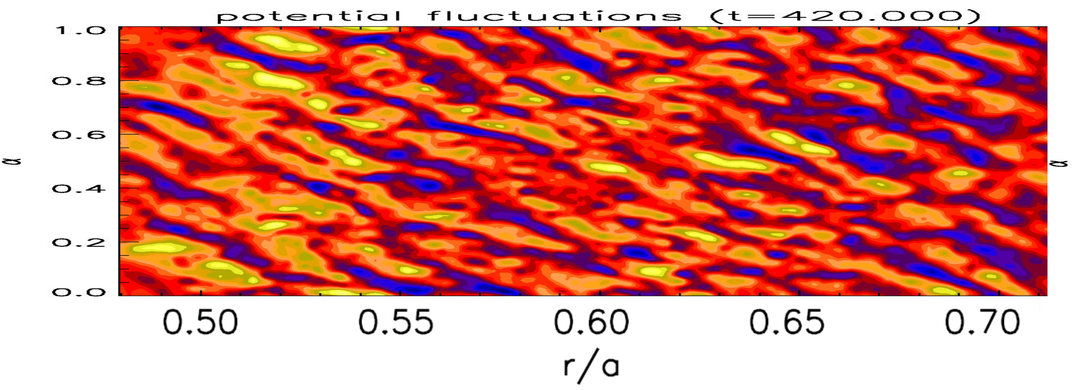


J. Kinsey BI2.6, Monday 12:00 Noon

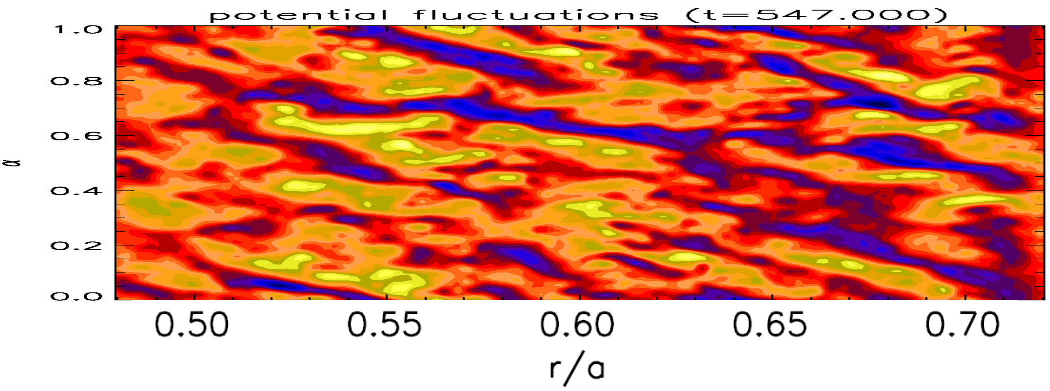
ExB shear can affect even ETG

GYRO ETG-ki sim. turbulent e flux ~ 1 MW (NSTX expt ~ 2 MW)
ExB shearing rate varied from 2X to 1/4 experimental rate.
Eddies grow longer (and wider) as shearing rate is reduced.

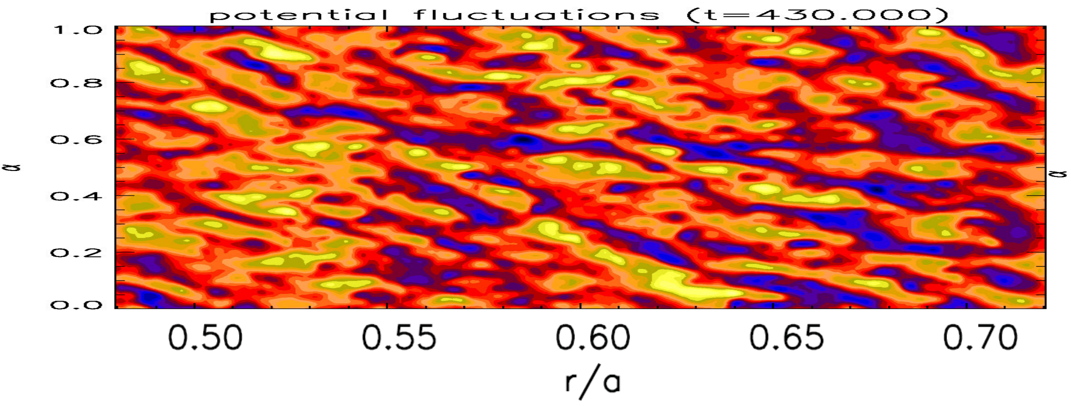
2X



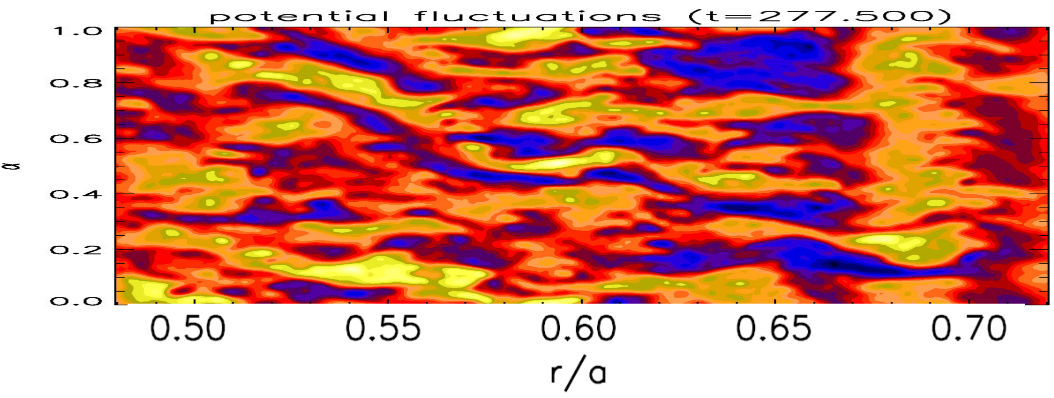
1/2X



experimental ExB rate



1/4X



Radial domain ~400 ρ_e.

Mikkelsen, NSTX

- database of 400+ GYRO simulations available

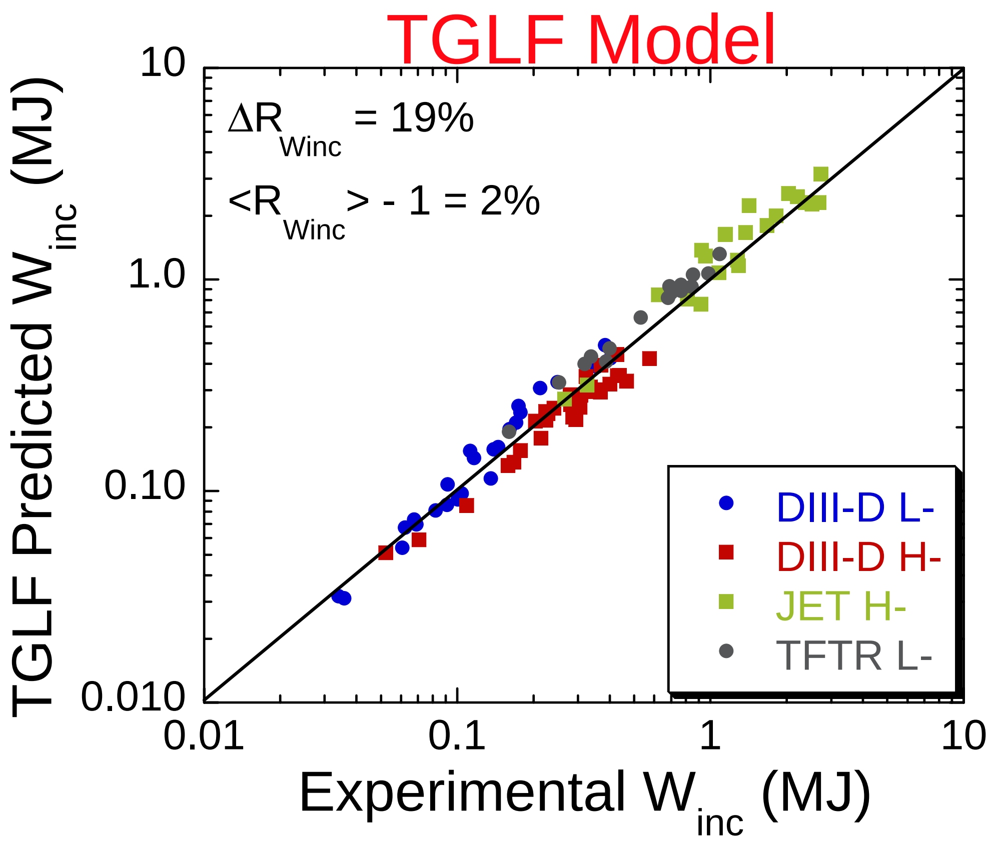
- can be used to test & fit theories

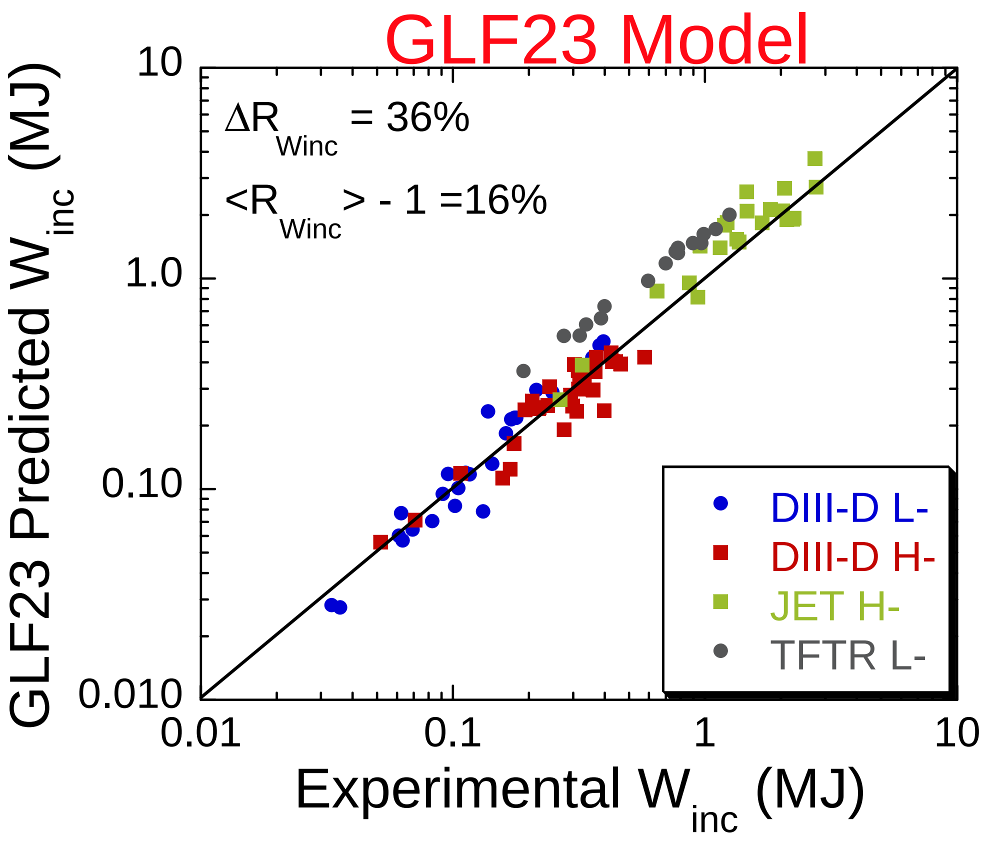
- Used to develop improved transport model, TGLF, which fits experiments better than GLF23 over a wider range of parameters

TGLF exhibits lower average global errors than GLF23 for a large L- and H-mode profile database of 96 discharges

- **Database: 25 DIII-D L-,33 DIII-D H-, 22 JET H-, 16 TFTR L-mode discharges**
- **Avg RMS errors in W_{inc} is 19% for TGLF, 36% for GLF23**
- **Avg RMS error in W_{tot} is ΔR_{Wtot}=10% for TGLF, 20% for GLF23**

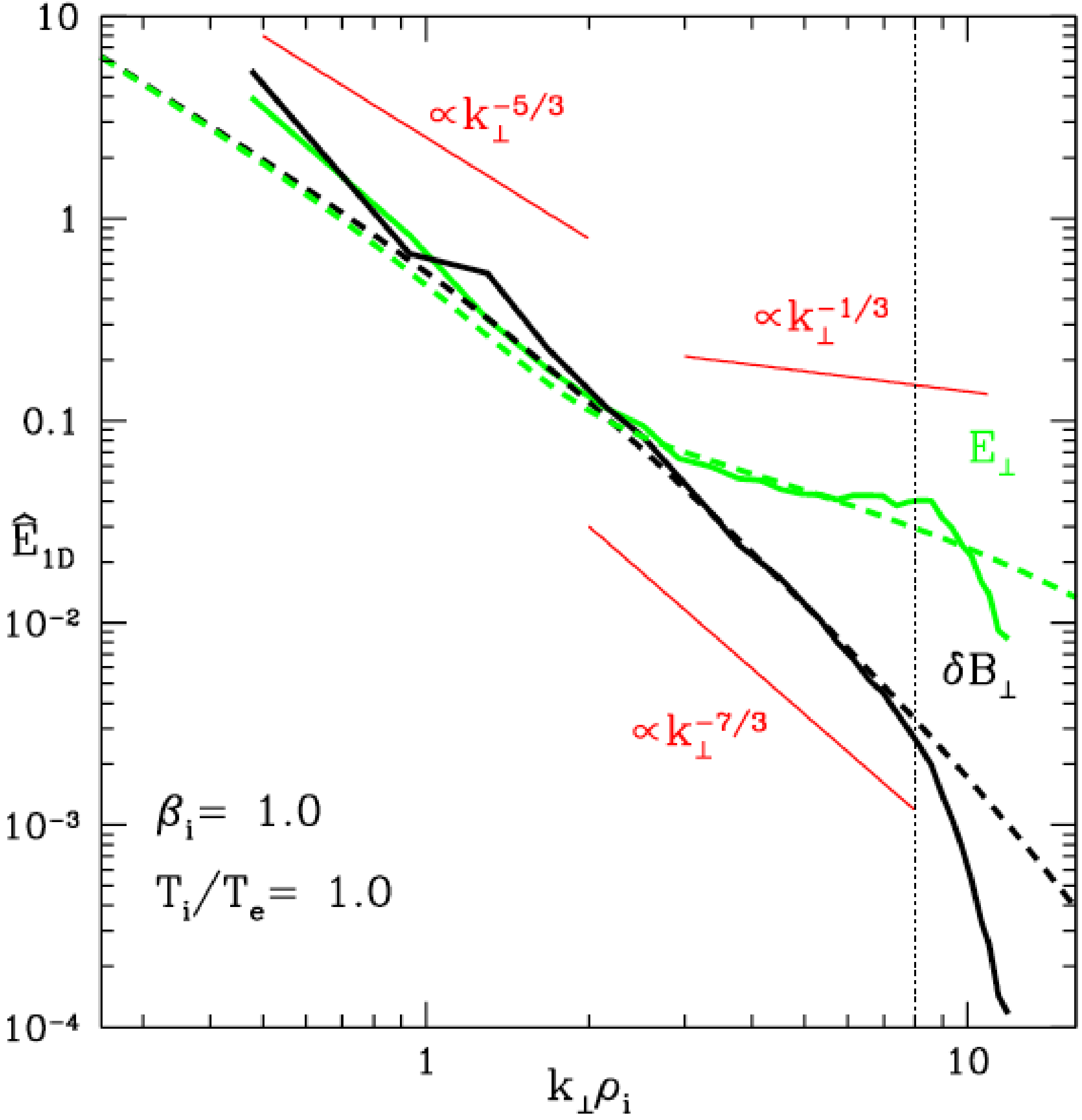
Wider range of conditions than earlier GLF23 comparisons





J. Kinsey BI2.6, Monday 12:00 Noon

Gyrokinetic codes applied to astro/solar physics:
Transition from MHD to Kinetic Alfven Wave Cascade



AstroGK/GS2 simulation

- Consistent with theory:
  MHD: -5/3 spectra in E & B
  KAW: -1/3 E spectrum and
  -7/3 B spectrum

- See talk by Greg Howes for more:
  Session VI2: Thursday 3:30pm
  Turbulence in the Solar Wind: Theory, Simulations, and Comparisons with Observations

$$(n_x, n_y, n_z, n_\xi, n_E, n_s) = (64, 64, 128, 64, 8, 2)$$

More than 500,000,000 computational meshpoints

Howes, Dorland, Quataert, Cowley, et al.

5. Future challenges & opportunities

- more detailed comparisons w/ expts incl. synthetic fluctuation diagnostics

- move from flux prediction to profile prediction mode:
  - makes experimental comparisons easier, more direct
  - a step to coupling short-time turbulence simulations & long-time transport codes
  - transport code coupling: study ITB formation, heat/cold pulse perturbative studies

- Many multiscale problems here, incl. Neoclassical Tearing Mode interaction with turbulence

- Most important & difficult problem: **Edge Turbulence**, ELMs, H-mode transport barrier

Areas of possible improvements for core gyrokinetic codes

- Dominant terms that can break gyro-Bohm scaling have been included (shear in profiles, turbulence spreading)

- However, there are some small ρ_* terms and small $k_{||}/k_{\perp}$ terms that have been dropped for convenience. Could be put in.

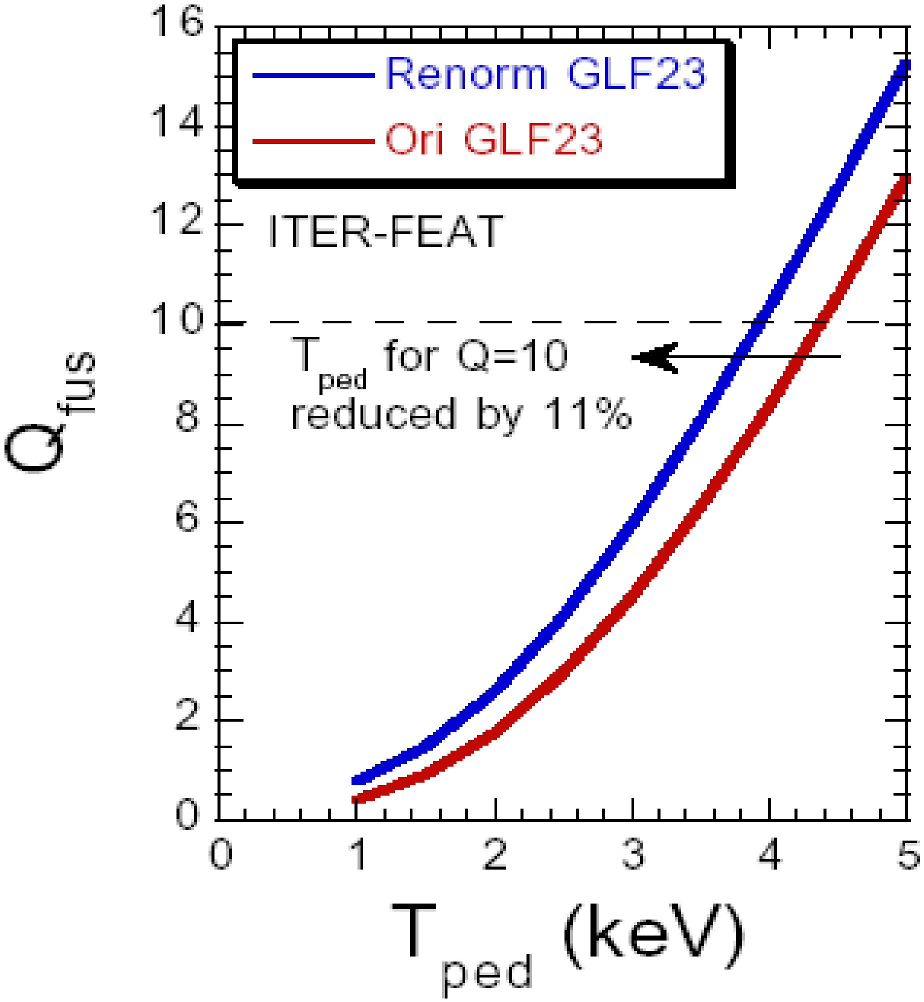
- Collision operators simplified to various degrees, improve

- Can’t handle separatrix, not efficient for high collisionality regimes (like edge), and so need new edge gyrokinetic codes…

Fusion performance depends sensitively on Edge

Sensitive dependence on turbulent confinement causes some uncertainties, but also gives opportunities for significant improvements, if methods of reducing turbulence extrapolate to larger reactor scales.

$$\frac{dE}{dt} = P - \frac{E}{\tau_E}$$

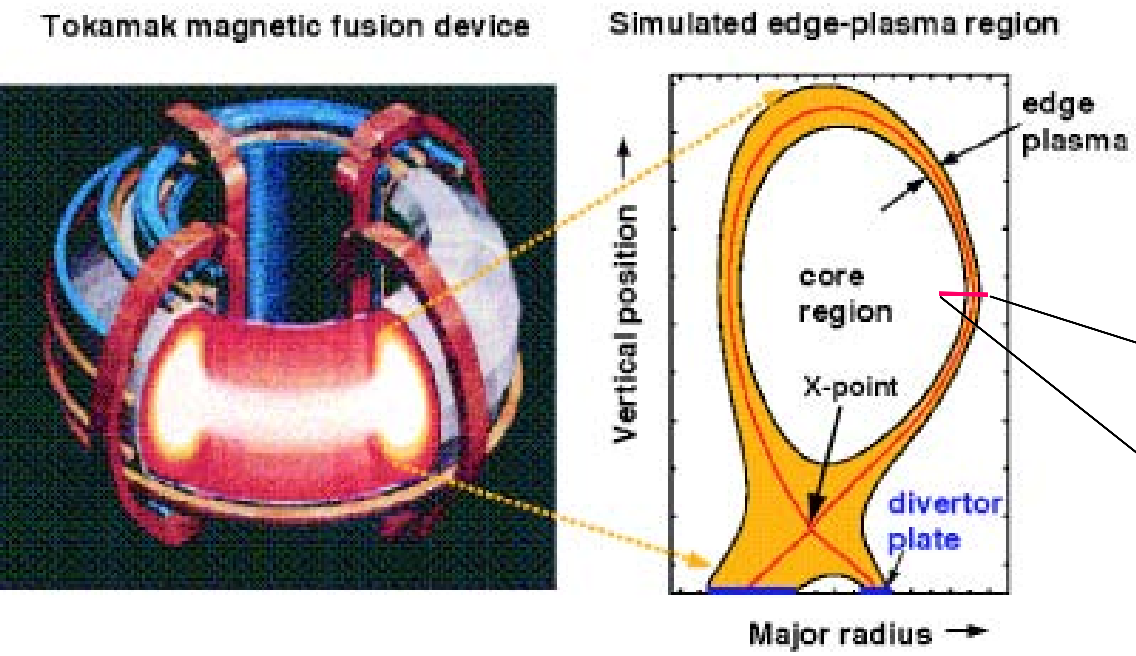


From Kinsey, Staebler, Waltz, Sherwood 2002.
Predictions for 2001 ITER-FEAT.

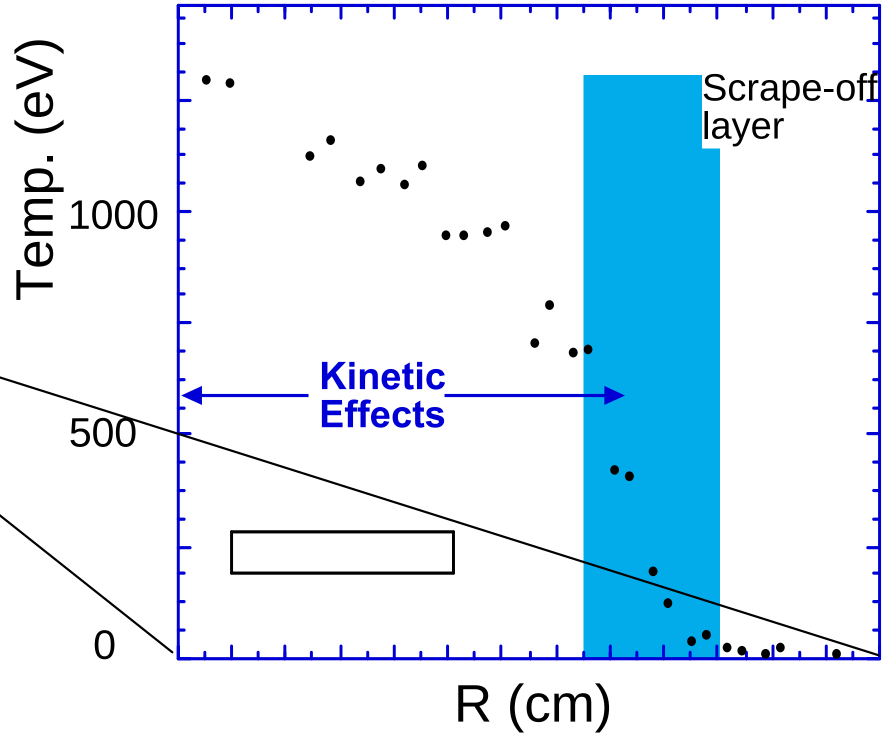
Lithium may help ↑ edge T

Caveats: best if MHD pressure limits also improve with improved confinement.
Other limits also: power load on divertor & wall, …

Edge boundary layer very important & uncertain



Schematic views of divertor tokamak and edge-plasma region (magnetic separatrix is the red line and the black boundaries indicate the shape of magnetic flux surfaces)



Edge pedestal temperature profile near the edge of an H-mode discharge in the DIII-D tokamak. [Porter*2000*]. Pedestal is shaded region.

- Marginal stability: Tcore ∝ Tped

- Periodic instabilites in edge region can dump outer ~5-10% of plasma onto divertor plates. Might be manageable, or divertor erodes, melts?

Beginning Work: Edge Gyrokinetic Turbulence

- Crucial: Need large H-mode pedestal & small ELMs, fusion Q depends on T_{ped}

- Complicated:
  - Character of edge turbulence different: not ITG/TEM but nonlinear / drift resistive ballooning, strong non-adiabatic electrons, significant magnetic fluctuation…
  - Open & closed field lines, X-point, H-mode forms near separatrix
  - Strong sources & sinks, neutral recycling, radiation, particle fuelling, Debye sheath boundary conditions
  - Large variation in density and & temperature over scale of simulation
  - need algorithms that can handle high and low collisionality regimes
  - Not a large separation of equilibrium and fluctuation scales, need accurate conservative full-F code

- (edge fluid work: B.D. Scott, Rogers & Drake, Hallatschek, Xu)

- Two initial gyrokinetic efforts:
  - ESL/TEMPEST, continuum approach (R. Cohen, LLNL, LBNL, GA, PPPL, …)
  - CPES, PIC approach (C.S. Chang, NYU, Colorado, PPPL, …)

Other Unfinished Gyrokinetic Work:

- Sometimes GLF23/TGLF transport models predict too little transport near the magnetic axis. Something missing? Turbulence spreading? Microtearing modes? ETG?

- While gyrokinetic simulations & transport models often predict temperature profiles within ~10% experimental uncertainties, there are some outliers which need further study. This relatively good accuracy is in part a consequence of stiff transport with critical gradients, which makes the prediction of temperature profiles less sensitive to uncertainties in turbulence saturation levels, but which can also make it more difficult to quantify when other transport mechanisms (like ETG, microtearing, gyro-Bohm breaking effects, or turbulence spreading) might be playing some role.

- Turbulent transport in ST's: Long-wavelength ITG/TEM stabilized . Microtearing modes?, ETG?, ?, nonlinear saturation?

- Transport barrier formation, transition threshholds, etc.

- Interaction of low-n MHD (NTM) and high-n turbulence…

- Gyrokinetics in stellarators, alternate concepts like RFPs, …

Gyrokinetic Theory & Simulation of Experiments

1. Intuitive pictures of gyrokinetic turbulence, & how to reduce it
    - analogy with inverted pendulum / Rayleigh-Taylor instability

2. Development of & physics in gyrokinetic equations

3. Development of nonlinear 5-D simulations of gyrokinetic turbulence

4. Gyrokinetic simulations: physics studies & comparisons w/ expts.

5. Future challenges & opportunities

    - more detailed comparisons w/ expts incl. fluctuation diagnostics
    - coupling fast-time turbulence simulations & long-time transport codes
    - Edge Turbulence, ELMs, transport barriers

Selected Gyrokinetic References

- This talk available at w3.pppl.gov/~hammett/talks
- 3 GYRO movies shown (d3d.n16.2x_06_fly, n32o6d0.8, & ETG-ki) from http://fusion.gat.com/theory/Gyromovies
- Web sites for 4 main gyrokinetic codes discussed here (incl. refs., documentation):
  - GYRO (Waltz & Candy, GA): fusion.gat.com/theory/Gyro
  - GS2 (Dorland & Kotschenreuther, U. Maryland/Texas): gs2.sourceforge.net
  - GENE (Jenko, Garching): www.ipp.mpg.de/~fsj
  - GEM (Parker & Chen, U. Colorado): cips.colorado.edu/simulation/gem.htm

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- S.E. Parker and Y. Chen (U. Colorado)
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- T.S. Hahm, A. Brizard, W.W. Lee, W. Tang, J. Krommes, T. Stoltzfus-Dueck
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- DOE Scientific Discovery Through Advanced Computing (SciDAC)
  - Center for the Study of Plasma Microturbulence
  - Edge Simulation Laboratory
  - Earlier DOE SciDAC & Computational Grand Challenge projects, including Plasma Microturbulence Project & Numerical Tokamak Project
- DOE National Energy Research Supercomputing Center (NERSC)
- Many others…