Introduction to Plasma Physics

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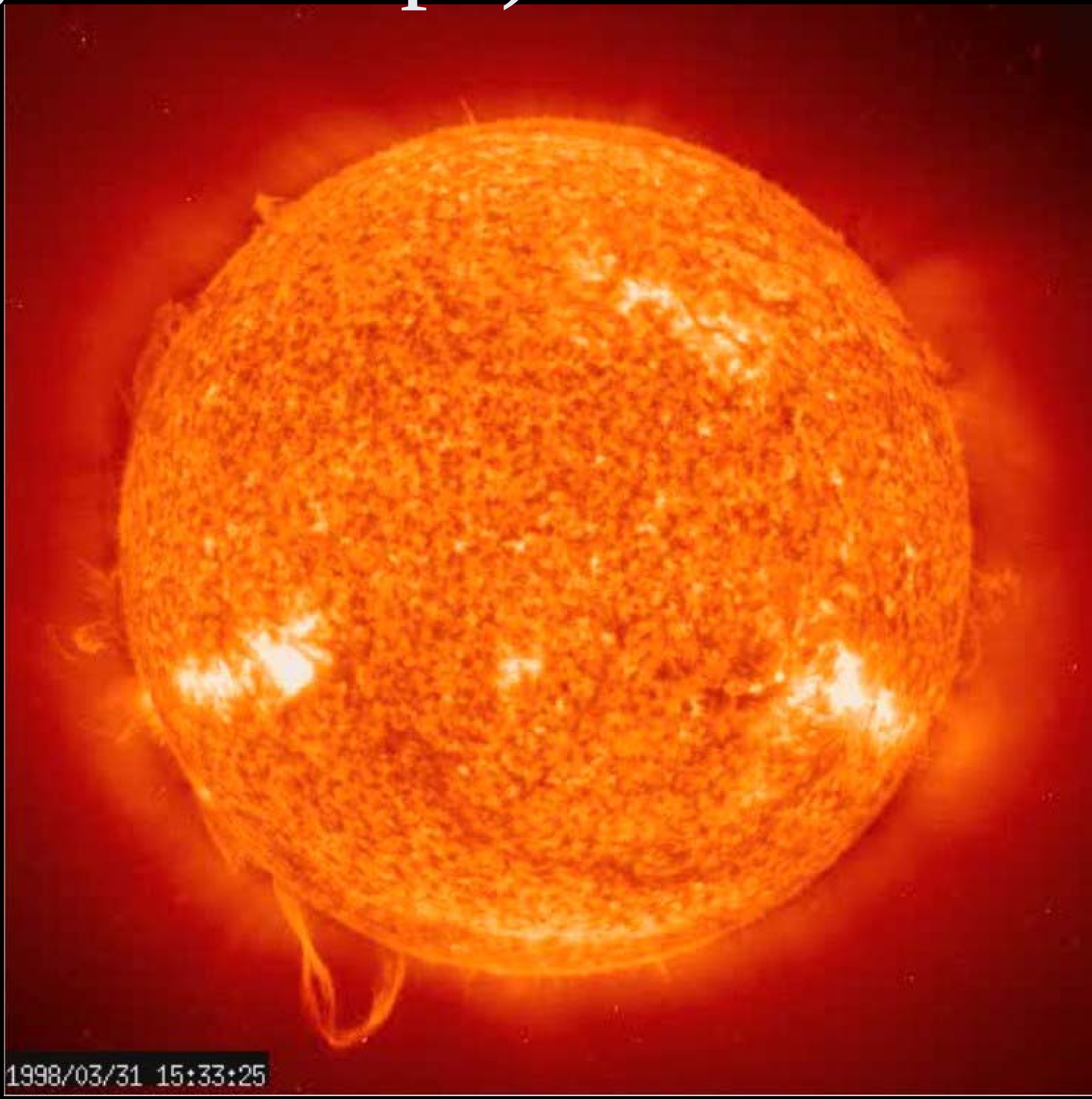
acknowledgements: Many slides borrowed from Prof. Fisch, Prof. Goldston, others

Introduction to Plasma Physics

- Visual gallery of wide variety of plasmas & applications: space & astrophysics, plasma etching, …
- Overview of status of magnetic fusion energy research
- Fundamentals of plasmas:
  - 4th state of matter
  - weak coupling between pairs of particles, but
  - strong collective interactions:
    - Debye shielding
    - electron plasma oscillations
  - hierarchy of length scales, expressed in terms of fundamental plasma parameter

Λ = # of particles in a Debye sphere

Erupting Prominences (plasma in magnetic loops)

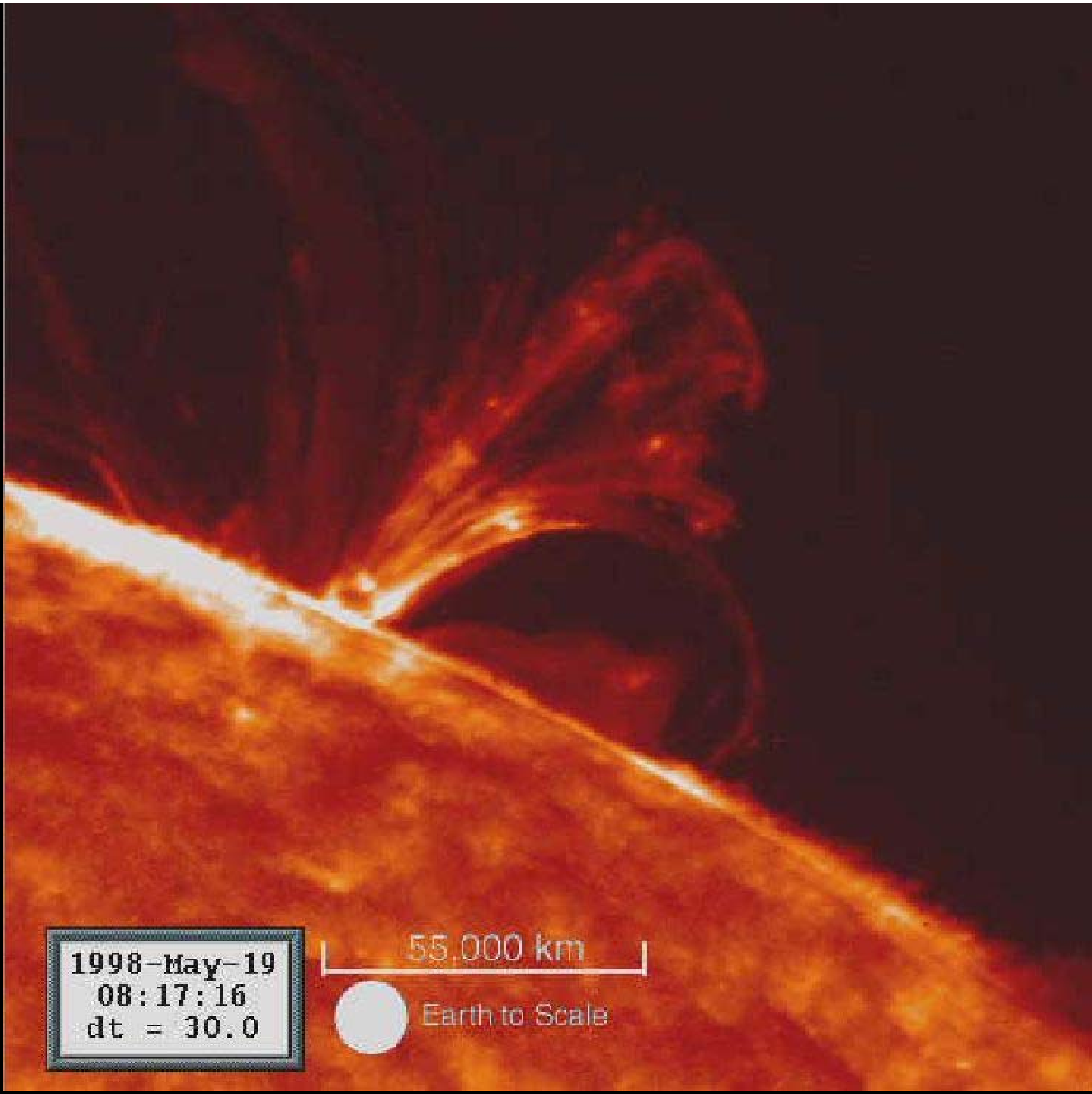


From NASA SOHO Satellite:

http://sohowww.nascom.nasa.gov/bestofsoho/Movies/EIT304_Apr98/EIT304_Apr98.mpg
http://sohowww.nascom.nasa.gov/bestofsoho/movies
See also Japanese Yohkoh satellite: http://www.lmsal.com/SXT/

Movie of a solar flare.

interaction of plasma
& magnetic field
loops above the surface
of the sun

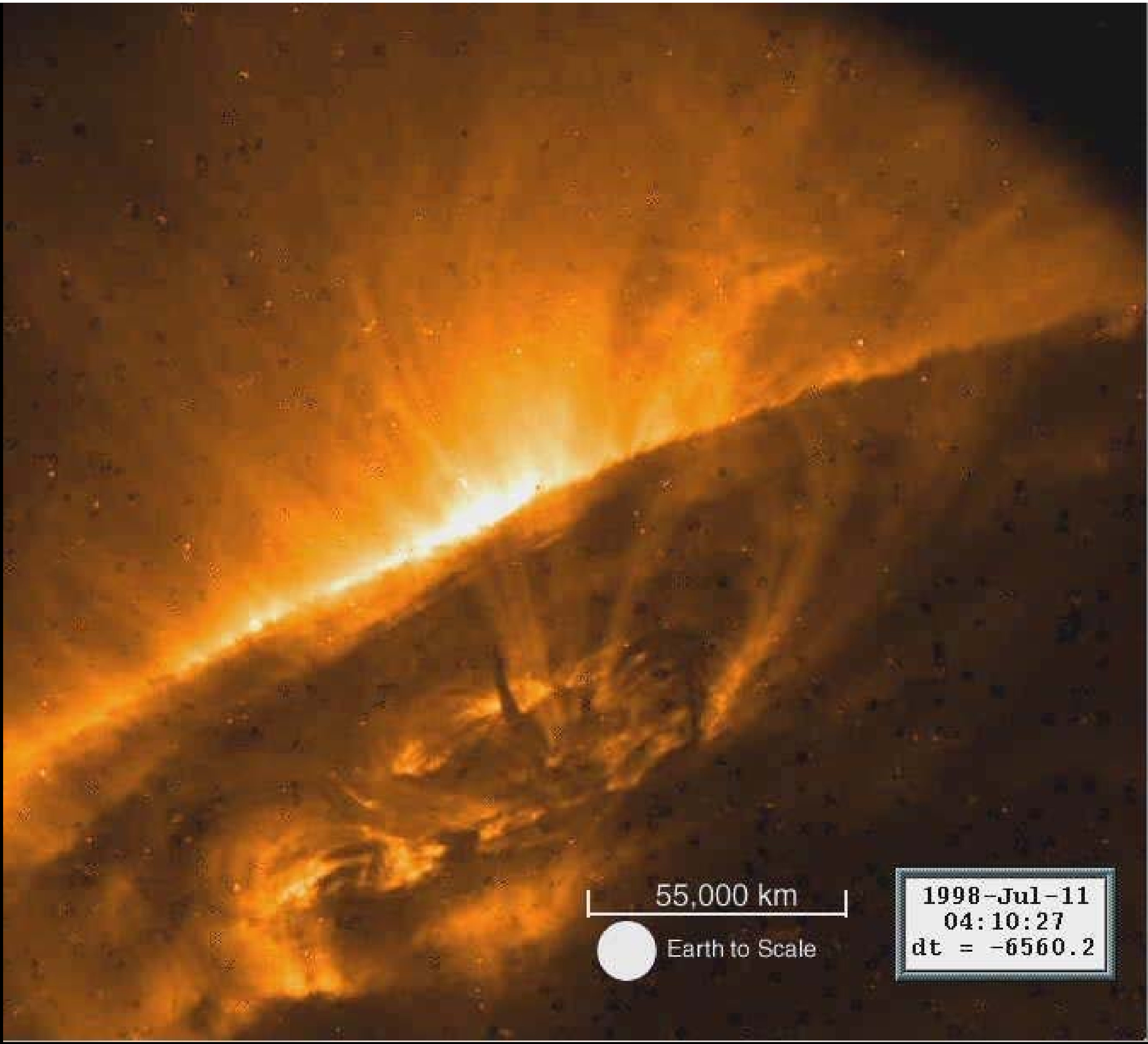


From NASA
TRACE Satellite

http://trace.lmsal.com/Science/ScientificResults/trace_cdrom/movie/flare_1216_color_lab.mov
http://trace.lmsal.com/Science/ScientificResults/trace_cdrom/html/mov_page.html
lots more stuff: http://trace.lmsal.com/POD/TRACEpod.html#movielist

Movie of
Coronal Mass Ejection.

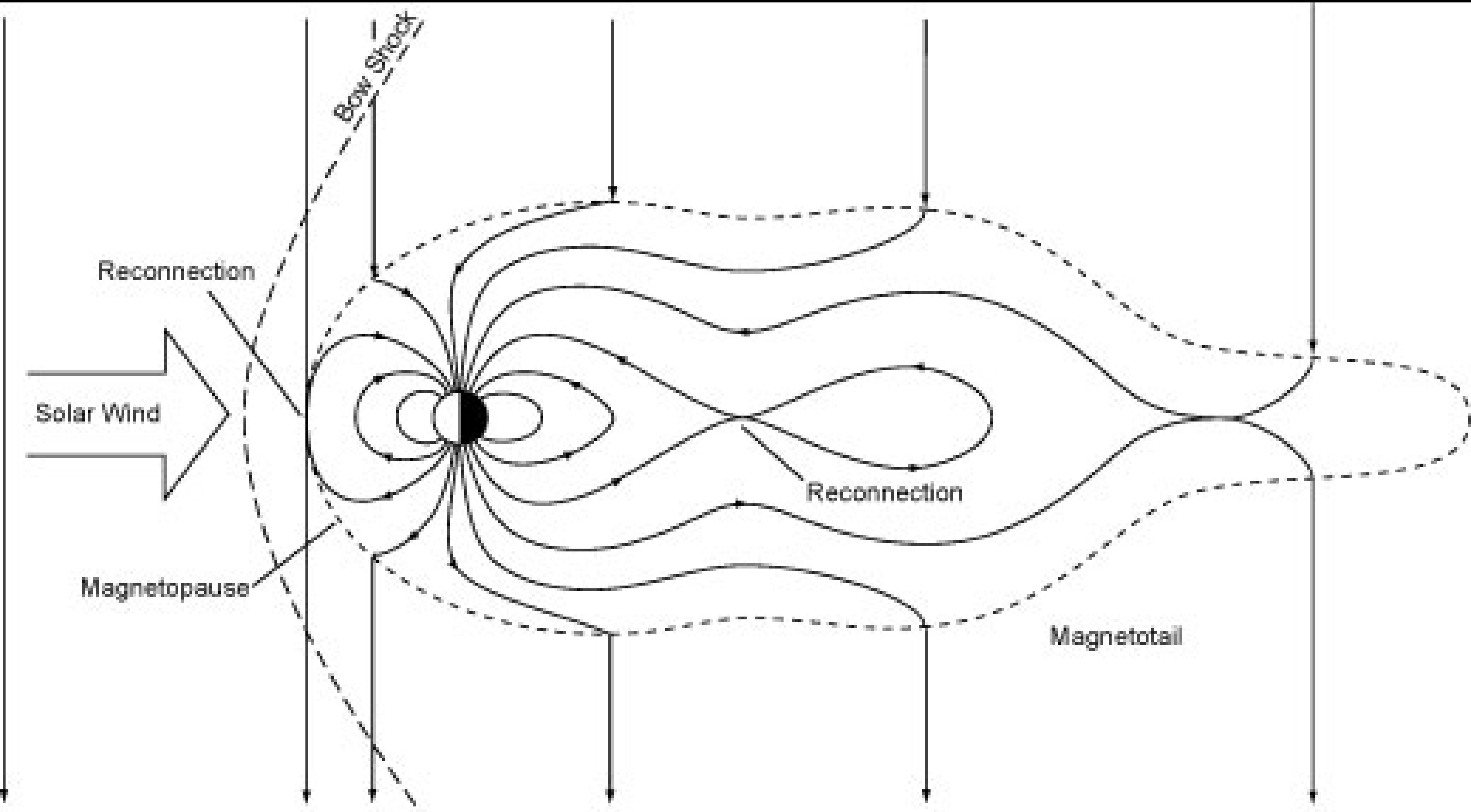
interaction of plasma
& magnetic field loops
above the surface of the sun



From NASA
TRACE Satellite

http://trace.lmsal.com/Science/ScientificResults/trace_cdrom/movie/cme_195_color_lab.mov
http://trace.lmsal.com/Science/ScientificResults/trace_cdrom/html/mov_page.html
lots more stuff: http://trace.lmsal.com/POD/TRACEpod.html#movielist

Earth’s magnetosphere protects from the solar wind. Magnetic reconnection in magnetotail observed to accelerate electrons to relativistic speeds



http://berkeley.edu/news/media/releases/2002/11/07_space.html http://en.wikipedia.org/wiki/Image:Magnetosphere_simple.jpg

Supernova blast wave

Plasma processes important in astrophysical shocks, particle acceleration, origin of cosmic rays, galactic magnetic fields, …

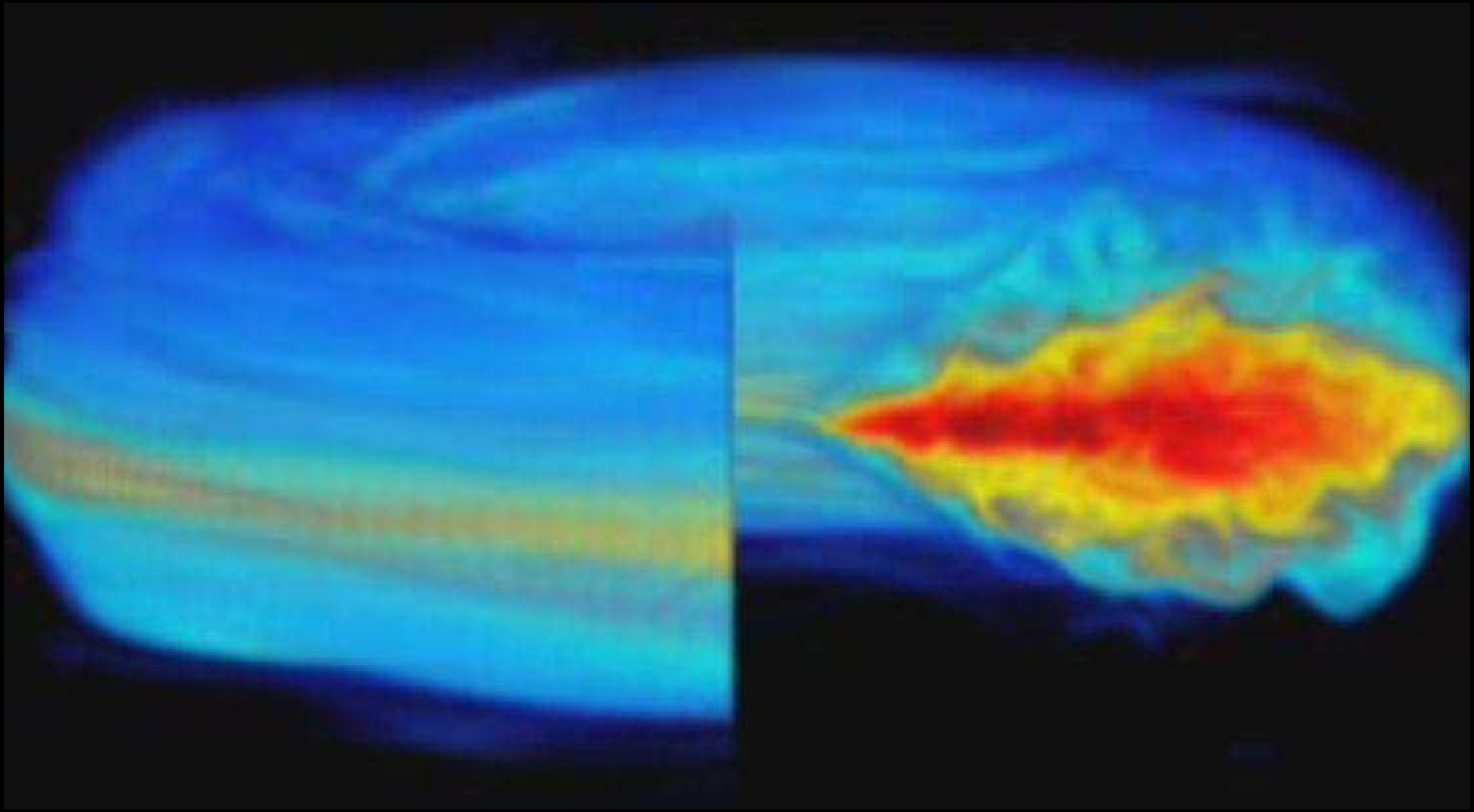
Cygnus Loop viewed by NASA Hubble Space Telescope
http://hubblesite.org/newscenter/archive/releases/1995/11

Star Birth

Hubble’s view of the Eagle Nebula

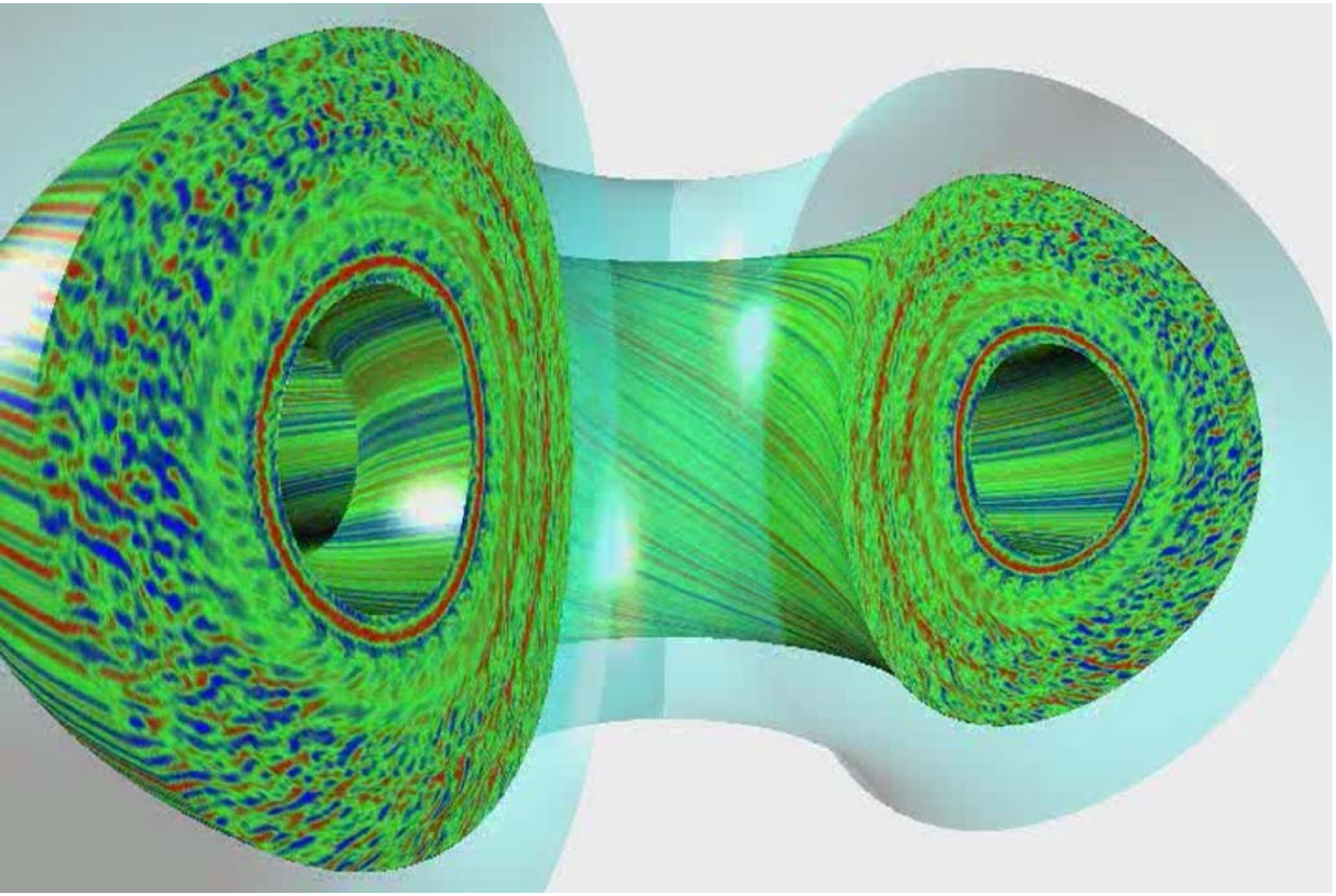
http://hubblesite.org/newscenter/archive/releases/1995/44

Plasma instability explains turbulence in accretion disks in astrophysics



Hawley & Balbus et al., Computer Simulation of Magneto-Rotational Instability Turbulence
http://www.astro.virginia.edu/~jh8h/
http://www.astro.virginia.edu/VITA/papers/torus3d/densityminchunk.mpg

5-D Gyrokinetic Simulation of Tokamak Plasma Turbulence with Candy & Waltz GYRO Code



Another example of complex plasma behavior.

Movie of density fluctuations from GYRO simulation
http://fusion.gat.com/THEORY/images/0/0f/N32o6d0.8.mpg
from http://fusion.gat.com/theory/Gyromovies

Waltz, Austin, Burrell, Candy, PoP 2006

500 radii x 32 complex toroidal modes (96 binormal grid points) x 10 parallel points along half-orbits x 8 energies x 16 v||/v, 12 hours on ORNL Cray X1E with 256 MSPs

Many Industrial/Commercial Applications of Plasmas

Processing: Surface Processing, Nonequilibrium (low pressure), Thermal (high pressure)
Volume Processing: Flue gas treatment, Metal recovery, Waste treatment
Chemical Synthesis: Plasma spraying, Diamond film deposition, Ceramic powders
Light Sources: High intensity discharge lamps, Low pressure lamps, Specialty sources
Surface Treatment: Ion implantation, Hardening, Welding, Cutting, Drilling
Space propulsion: plasma thrusters, fusion powered propulsion
Flat-Panel Displays: Field-emitter arrays, Plasma displays
Radiation Processing: Water purification, Plant growth
Switches: Electric Power, Pulsed power
Energy Convertors: MHD converters, Thermionic energy converters
Medicine: Surface treatment, Instrument sterilization
Beam Sources
Lasers: Free-electron lasers, X-ray lasers
Material Analysis
High-power RF sources

http://www.plasmacoalition.org/applications.htm

Many applications of plasmas

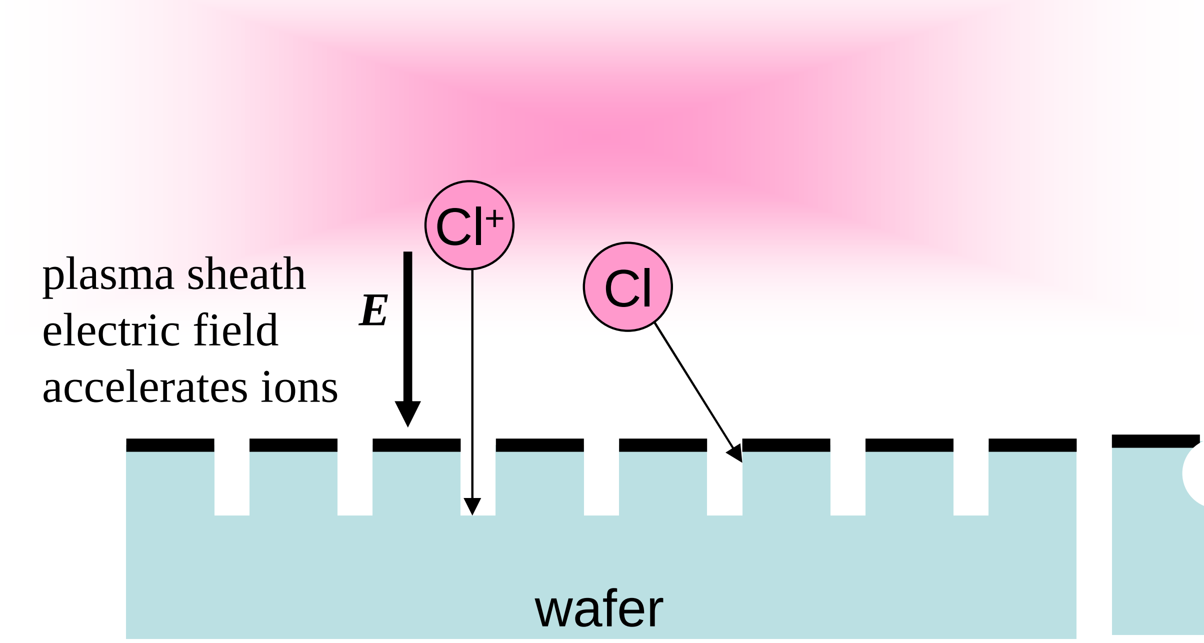
01—Plasma TV
02—Plasma-coated jet turbine blades
03—Plasma-manufactured LEDs in panel
04—Diamondlike plasma CVD eyeglass coating
05—Plasma ion-implanted artificial hip
06—Plasma laser-cut cloth
07—Plasma HID headlamps
08—Plasma-produced H_2 in fuel cell
09—Plasma-aided combustion
10—Plasma muffler
11—Plasma ozone water purification
12—Plasma-deposited LCD screen
13—Plasma-deposited silicon for solar cells
14—Plasma-processed microelectronics
15—Plasma-sterilization in pharmaceutical production
16—Plasma-treated polymers
17—Plasma-treated textiles
18—Plasma-treated heart stent
19—Plasma-deposited diffusion barriers for containers
20—Plasma-sputtered window glazing
21—Compact fluorescent plasma lamp

Plasmas in the Kitchen. Plasmas and the technologies they enable are pervasive in our everyday life. Each one of us touches or is touched by plasma-enabled technologies every day. Products from microelectronics, large-area displays, lighting, packaging, and solar cells to jet engine turbine blades and biocompatible human implants either directly use or are manufactured with, and in many cases would not exist without, the use of plasmas. The result is an improvement in our quality of life and economic competitiveness.

Plasma Science: Advancing Knowledge in the National Interest (2007), National Research Council,
http://books.nap.edu/openbook.php?record_id=11960&page=9

Plasma etching can make smaller features

Dry or Plasma Etching



$\mathrm{Si}(s) + 2\mathrm{Cl}(g) +$ ion energy $\rightarrow \mathrm{SiCl_2}(g)$

The directional ion energy drives the chemical reaction *only* at the bottom of the microscopic feature. Plasma also produces highly reactive ions.

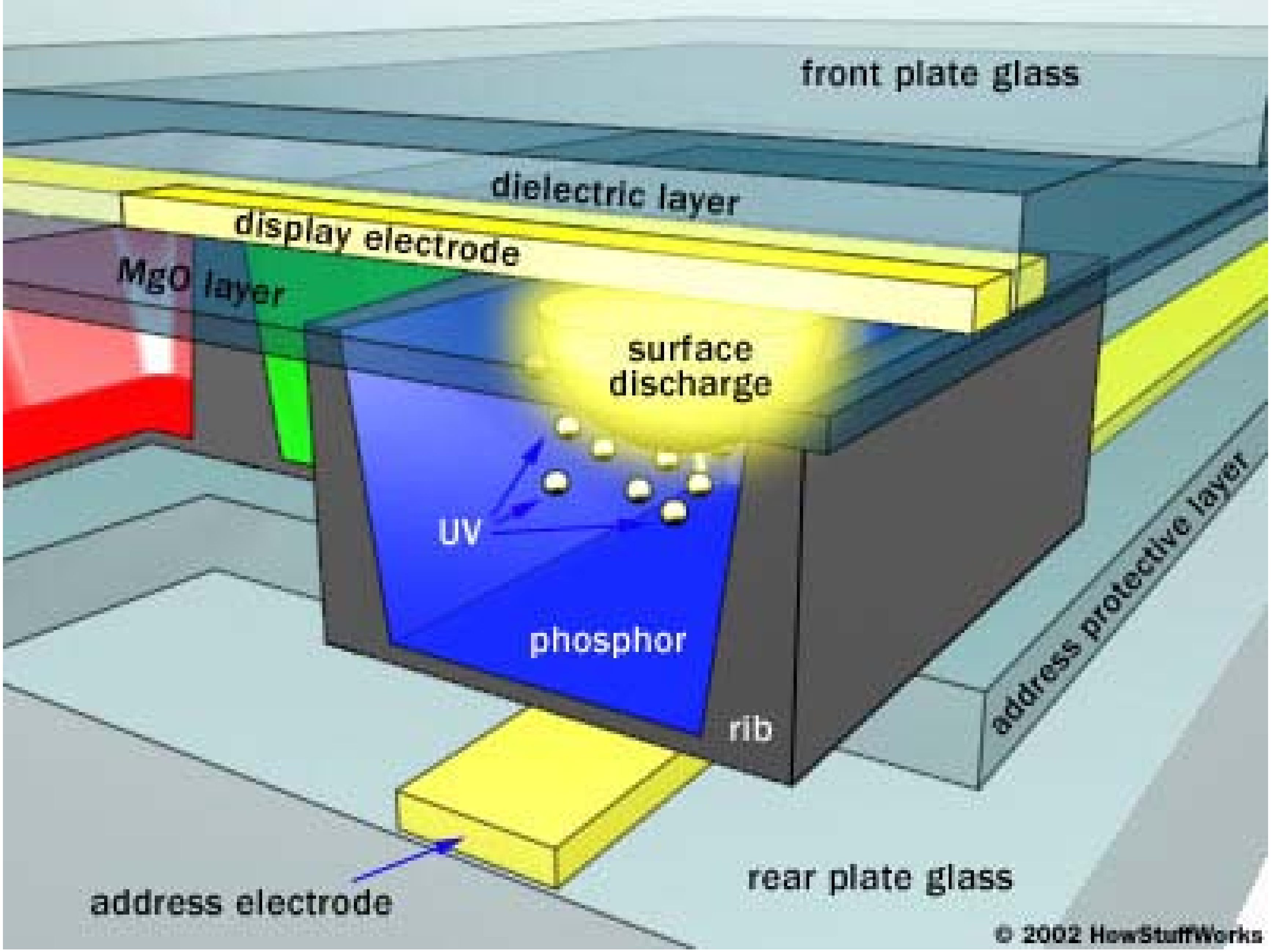
Wet Etching (in acid)

In wet chemistry, the chemical reaction occurs on all surfaces at the same rate. Very small features can not be microfabricated since they eventually overlap each other.

Based on Prof. Jeff Hopwood, Tufts Univ., http://www.ece.tufts.edu/~hopwood/lab/tools.htm

http://www.ece.tufts.edu/~hopwood/lab/images/Introduction to Plasmas (lectures 1 and 2).ppt

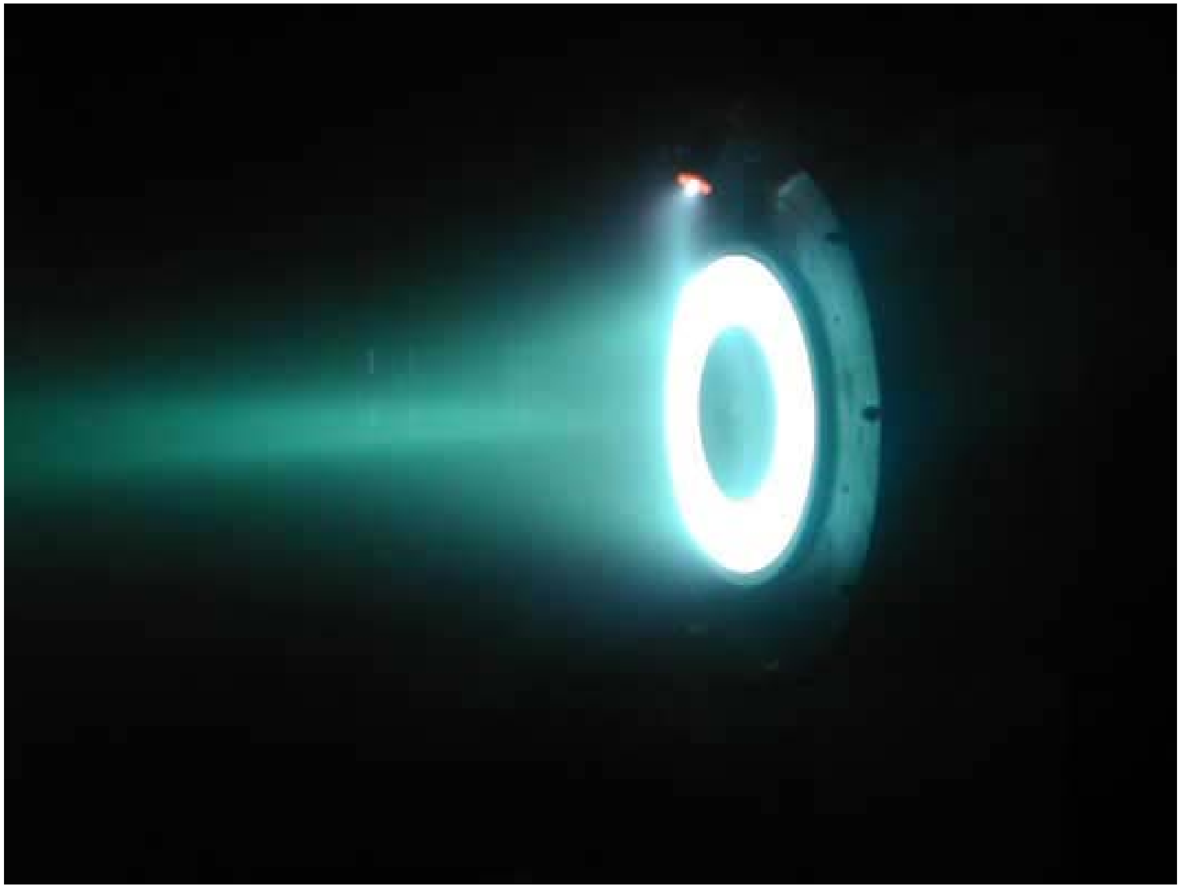
Plasma TV



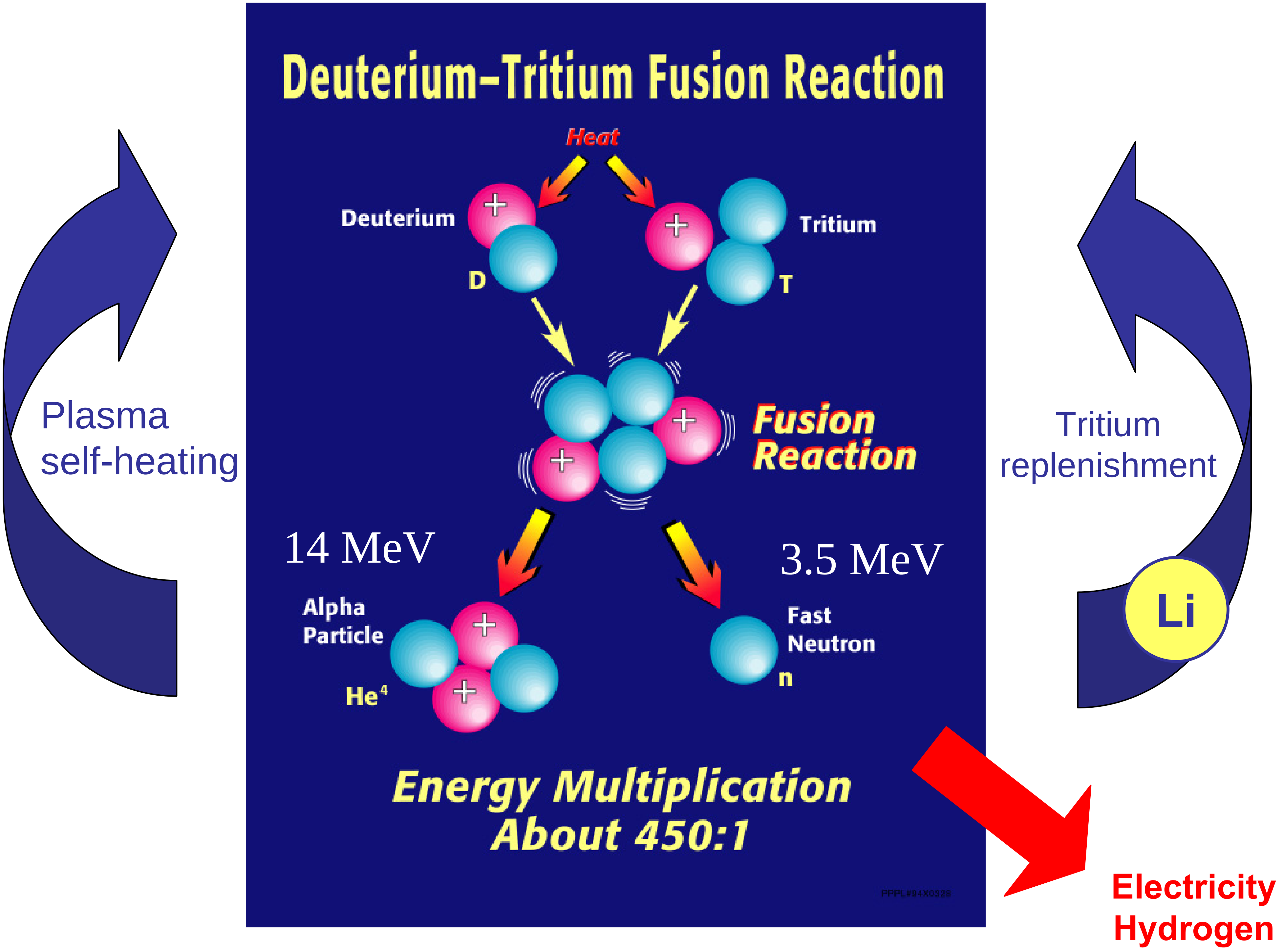
http://electronics.howstuffworks.com/plasma-display1.htm

Plasma Thrusters used on Satellites

Plasma thrusters have much higher exhaust velocity than chemical rockets: reduces amount of propellant that must be carried. Can be powered by solar panels, fission, or fusion.



Hall Thruster developed at PPPL



14 MeV

3.5 MeV

Plasma
self-heating

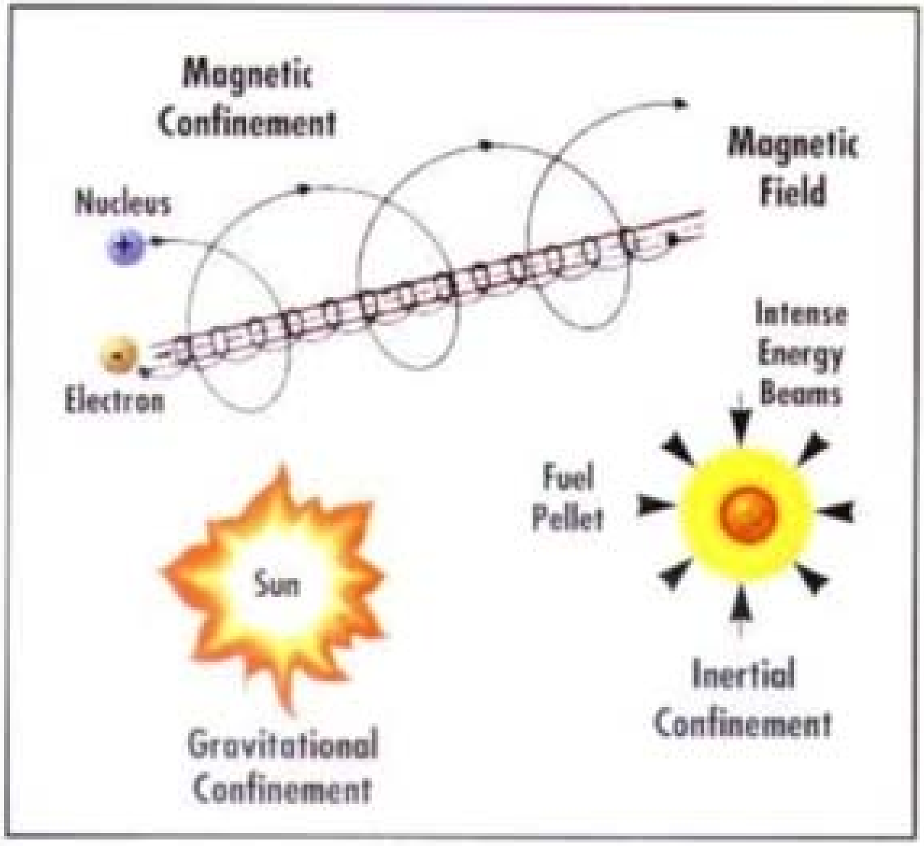
Tritium
replenishment

Electricity
Hydrogen

Plasma Confinement

magnetic

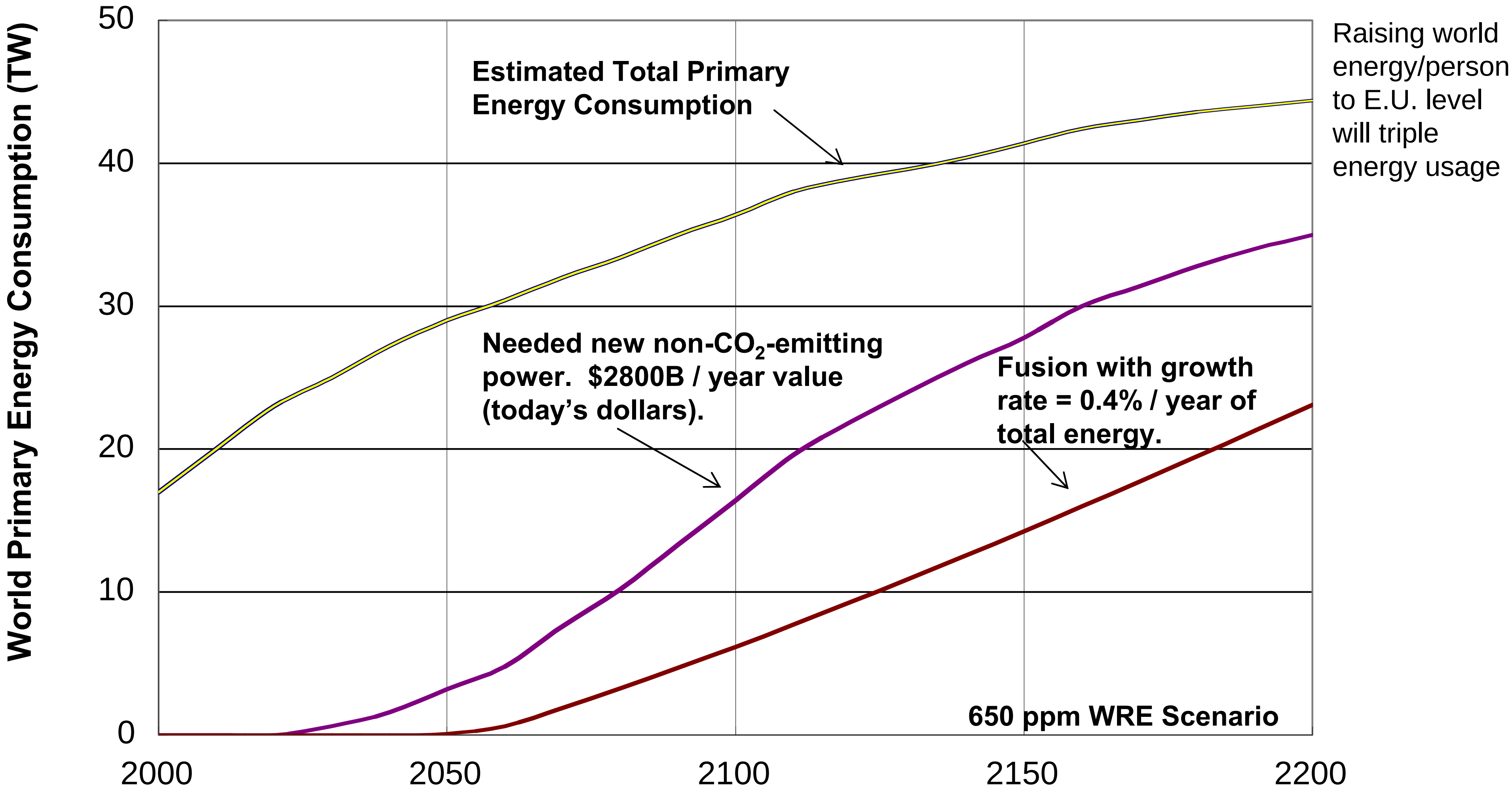
gravitational



“inertia”

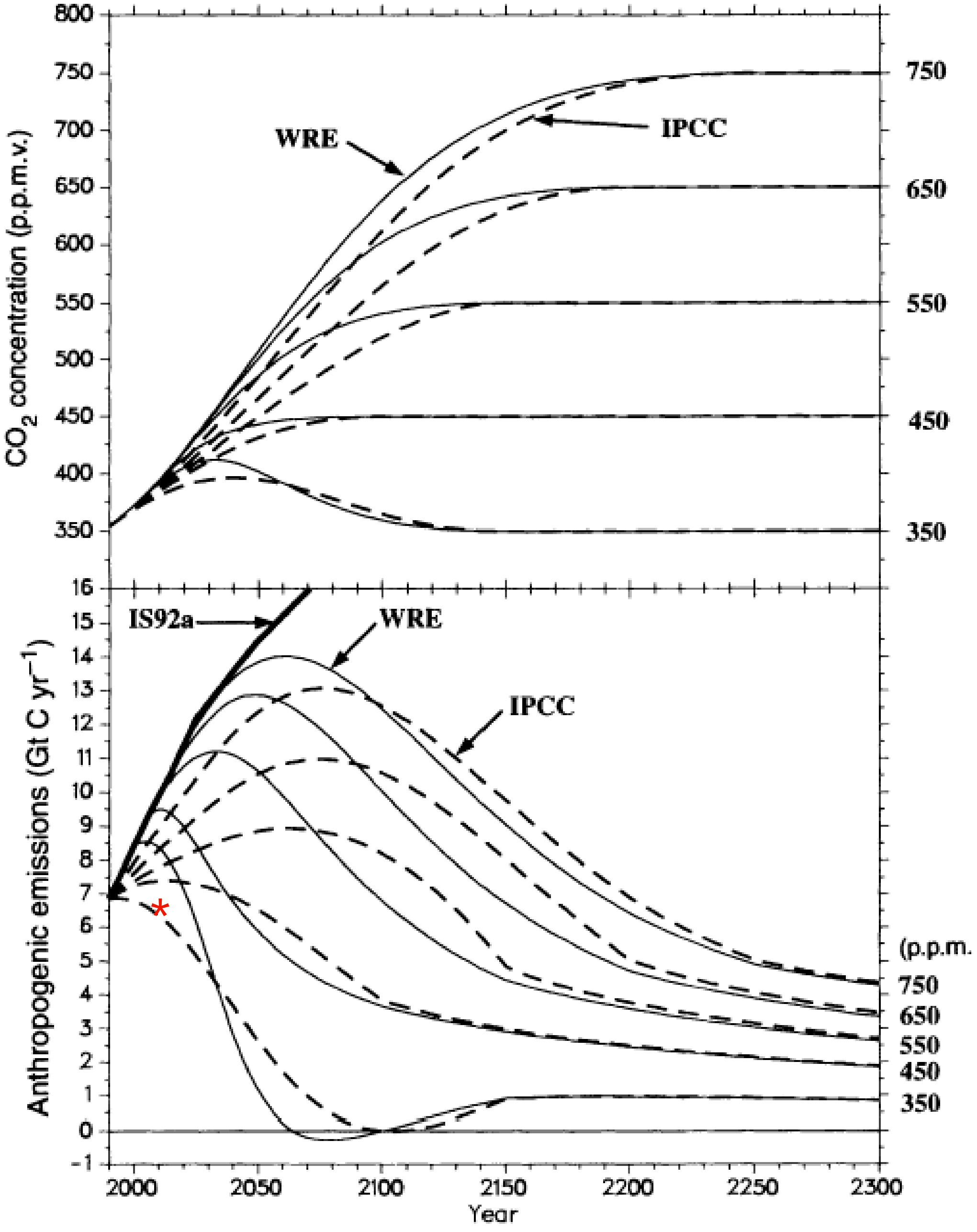
The Value of Fusion-Produced Energy is 12,000x Greater than the Development Cost

Return on investment still ~40:1 payoff after discounting for Net Present Value, 20% advantage over other energy sources, 50% chance of success, 1/3 payoff to U.S.



Total value ~ $296T at $0.02 per kWhr thermal ($FY2002)

Large CO2 Emissions cuts needed to stabilize CO2 & associated global warming



twice
preindustrial

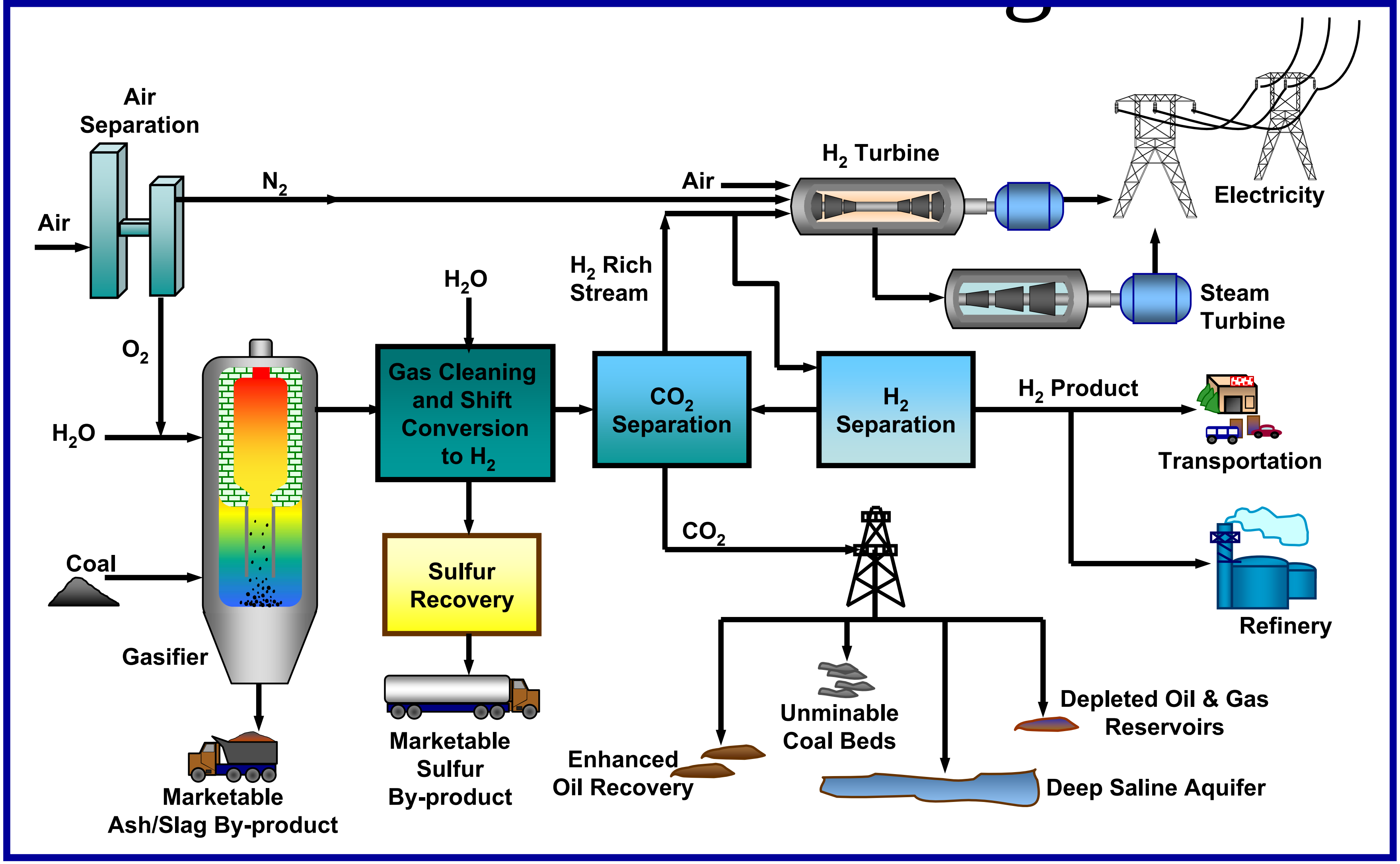
Raising world energy/person to E.U. level will triple energy usage

* Kyoto Accords: 2012 target 10% below 1990

Wigley, Richels, & Edmonds, Nature 379 (1996) 240.

GWH: Adequate reductions in CO2 over next 30-40 years probably possible with improved efficiency, windmills, fission, CO2 sequestration, etc.
But after 30-40 years, need fusion, or fission breeders, or ??

Future Gen Flow Diagram

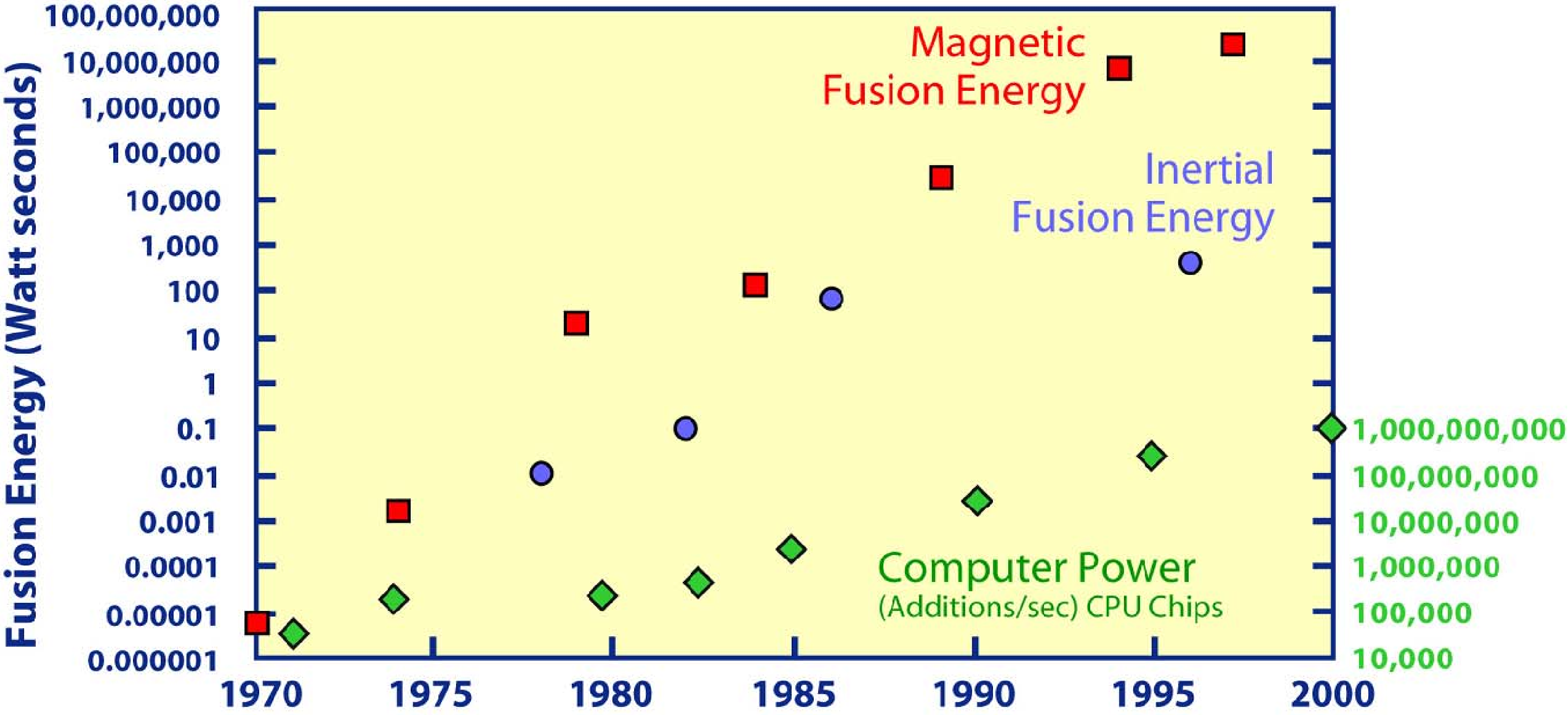


From Gary J. Stiegel, http://wvodyssey.nrcce.wvu.edu/2004/post_event/ppt/Stiegel_gasification.ppt

Fusion can be an Attractive Domestic Energy Source

- Abundant fuel, available to all nations
  - Deuterium and lithium easily available for thousands of years
- Environmental advantages
  - No carbon emissions, short-lived radioactivity
- Can't blow up, resistant to terrorist attack
  - Less than a minute's worth of fuel in the chamber
- Low risk of nuclear materials proliferation
  - No fissile or fertile materials required
- Compact relative to solar, wind and biomass
  - Modest land usage
- Not subject to daily, seasonal or regional weather variation, no requirement for local CO_2 sequestration.
  - Not limited in its contribution by need for large-scale energy storage or extreme-distance transmission
- Cost of power estimated similar to coal, fission
- Can produce electricity and hydrogen
  - Complements other nearer-term energy sources

Progress in Fusion Energy has Outpaced Computer Speed

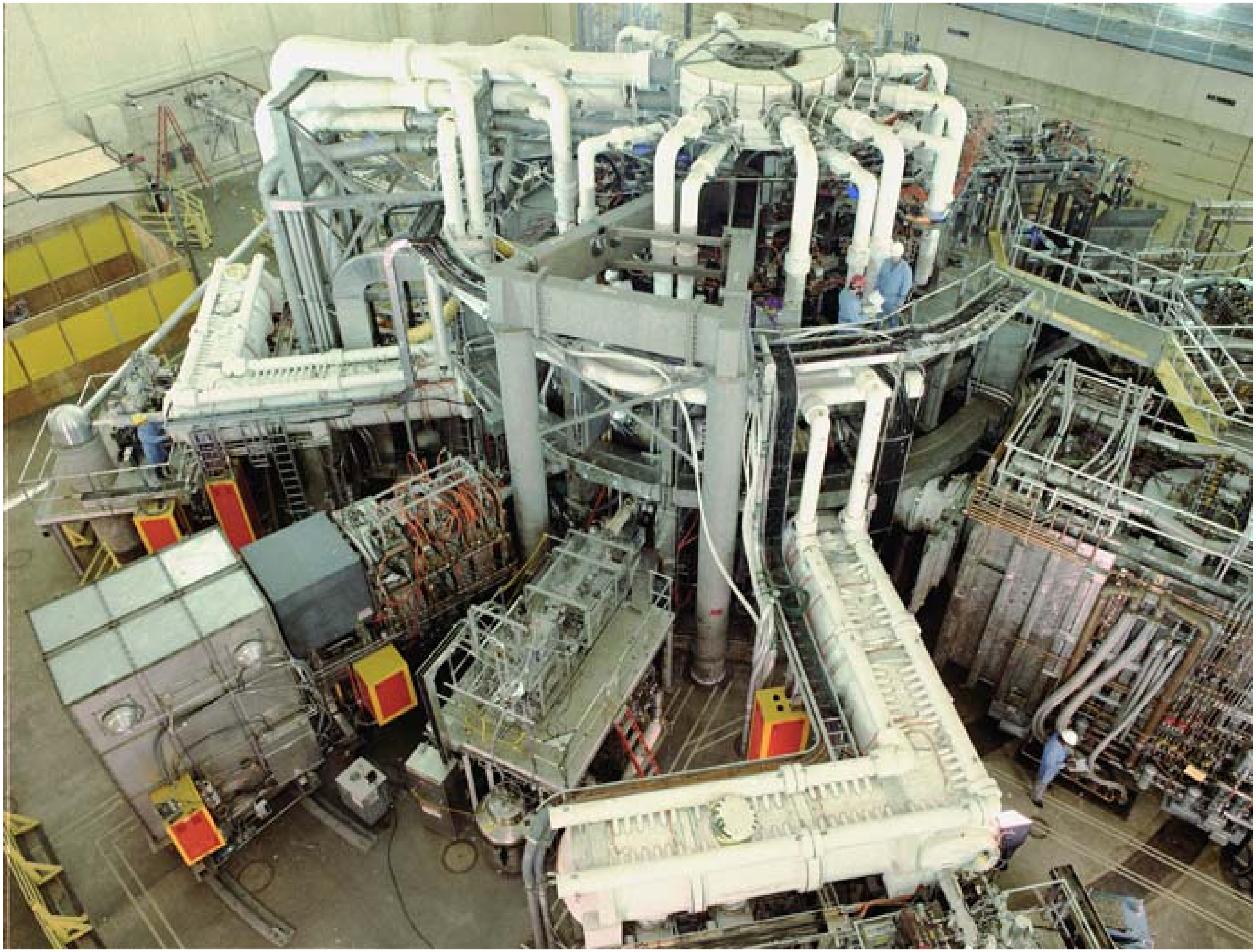


Some of the progress in computer speed can be attributed to plasma science.

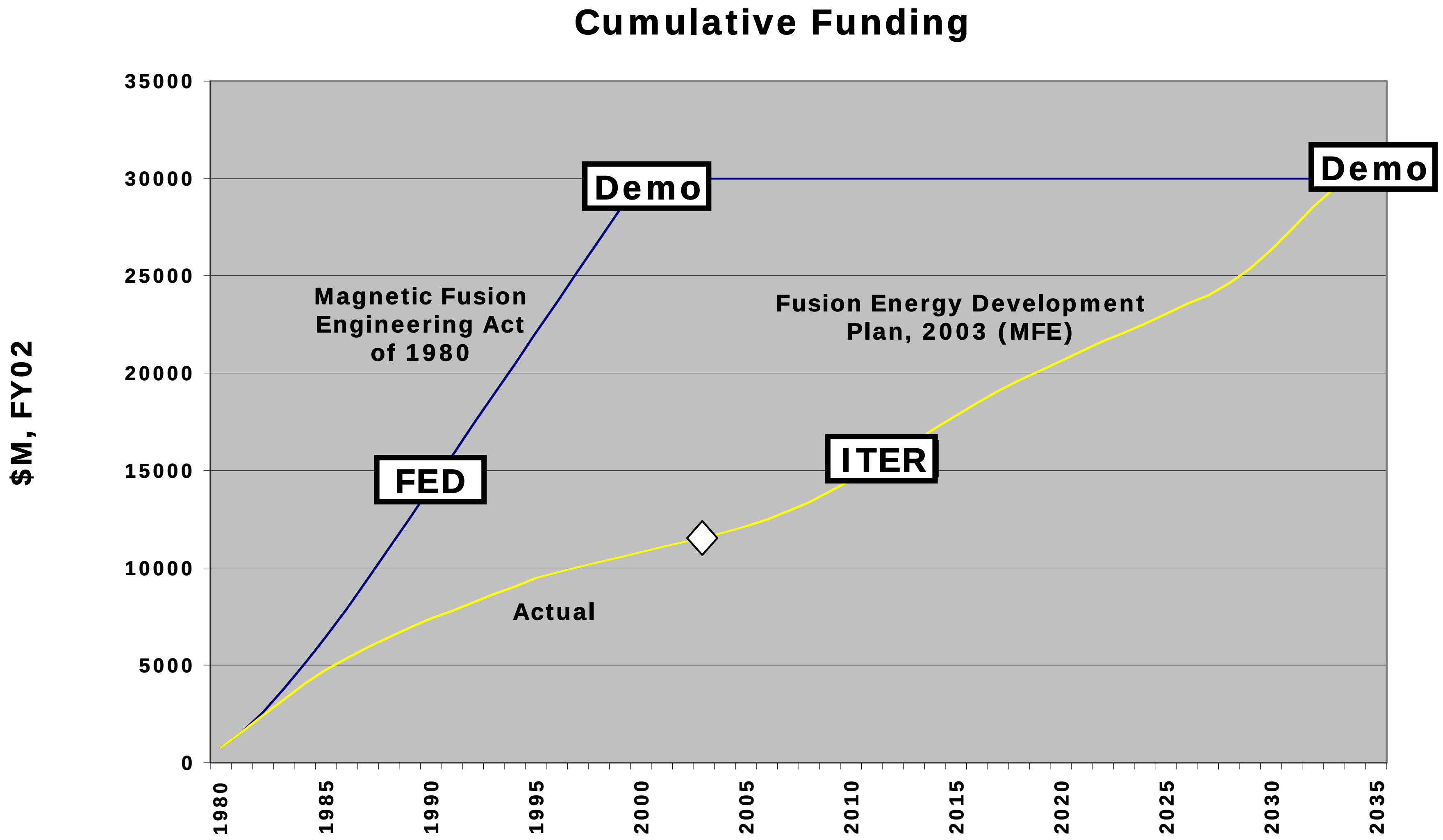
TFTR

Tokamak Fusion Test Reactor (1982-1997)

made 10 MW fusion power



The Estimated Development Cost for Fusion Energy is Essentially Unchanged since 1980



On budget,
if not on time.

$30B development cost tiny compared to >$100 Trillion energy needs of 21st century and potential costs of global warming. Still 40:1 payoff after discounting 50+ years.

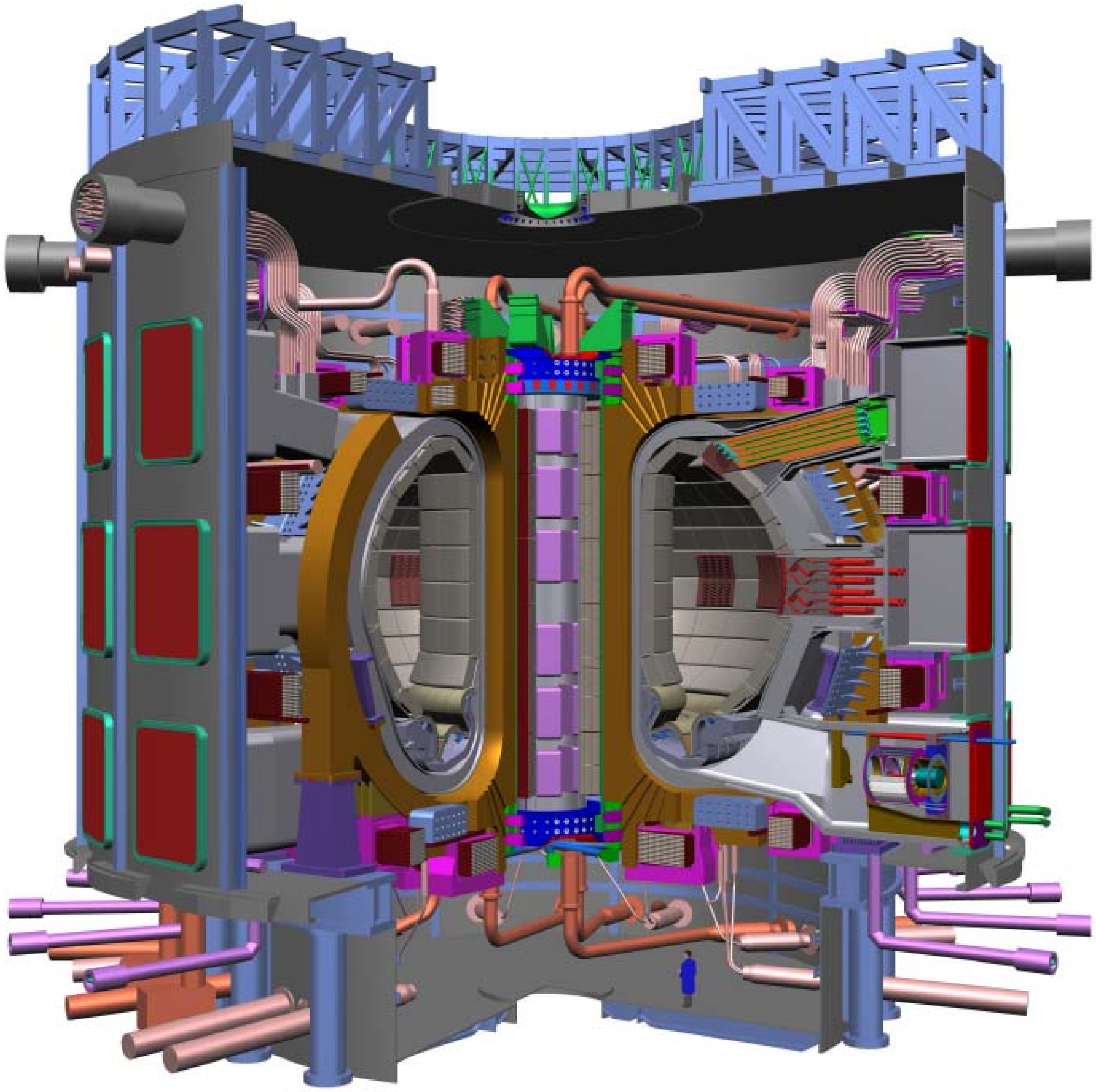


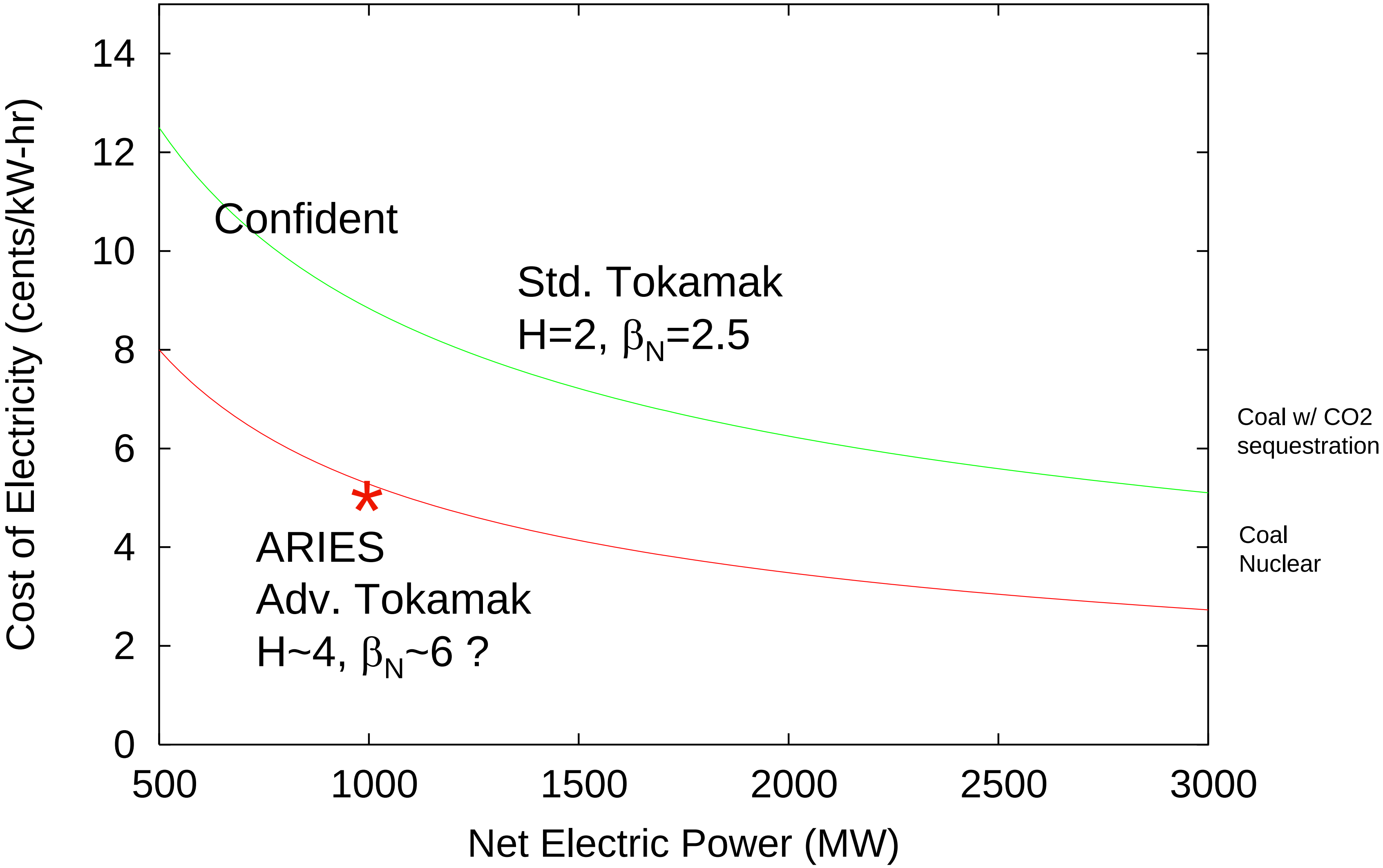
Figure 1.2.1-2 ITER Tokamak Cutaway

Parameter	400 MW	500 MW
R/a (m/m)	6.2/2.0	6.2/2.0
Volume (m^3)	831	831
Surface (m^2)	683	683
Sep.length (m)	18.2	18.2
$S_{cross\text{-}sect.}$ (m^2)	21.9	21.9
B_T (T)	5.3	5.3
I_P (MA)	15.0	15.0
κ_X/δ_X	1.85/0.48	1.85/0.48
κ_{95}/δ_{95}	1.70/0.33	1.70/0.33
l_i (3)	0.84	0.84
V_{loop} (mV)	75	75
q_{95}	3	3
β_N	1.8	2.0
$<n_e>$ (10^{19} m^{-3})	10.1	11.3
$<n_e>/n_G$	0.85	0.94
$<T_e>$ (keV)	8.8	8.9
$<T_i>$ (keV)	8.0	8.1
$<\beta_T>$ (%)	2.5	2.8
β_p	0.65	0.72

Parameter	400 MW	500 MW
$P_{RF} + P_{NB}$ (MW)	7 + 33	17 + 33
P_{OH} (MW)	1	1
P_{TOT} (MW)	121	151
P_{BRM} (MW)	21	26
P_{SYN} (MW)	8	8
P_{LINE} (MW)	18	27
P_{RAD} (MW)	47	61
P_{FUS} (MW)	400	500
$P_{LOSS}/P_{L\text{-}H}$	87/48	104/51
Q	10	10
τ_E (s)	3.7	3.4
W_{th} (MJ)	320	353
W_{fast} (MJ)	32	34
$H_{H98\,(y,2)}$	1.0	1.0
τ_{He}^{*}/τ_E	5	5
$Z_{eff,\,ave}$	1.66	1.72
$f_{He,axis\,/\,ave}$ (%)	4.3/3.2	4.4/3.2
$f_{Be,axis}$ (%)	2.0	2.0
$f_{Ar,axis}$ (%)	0.12	0.14

ITER Final Design Report 2001, http://www.iter.org/reports.htm

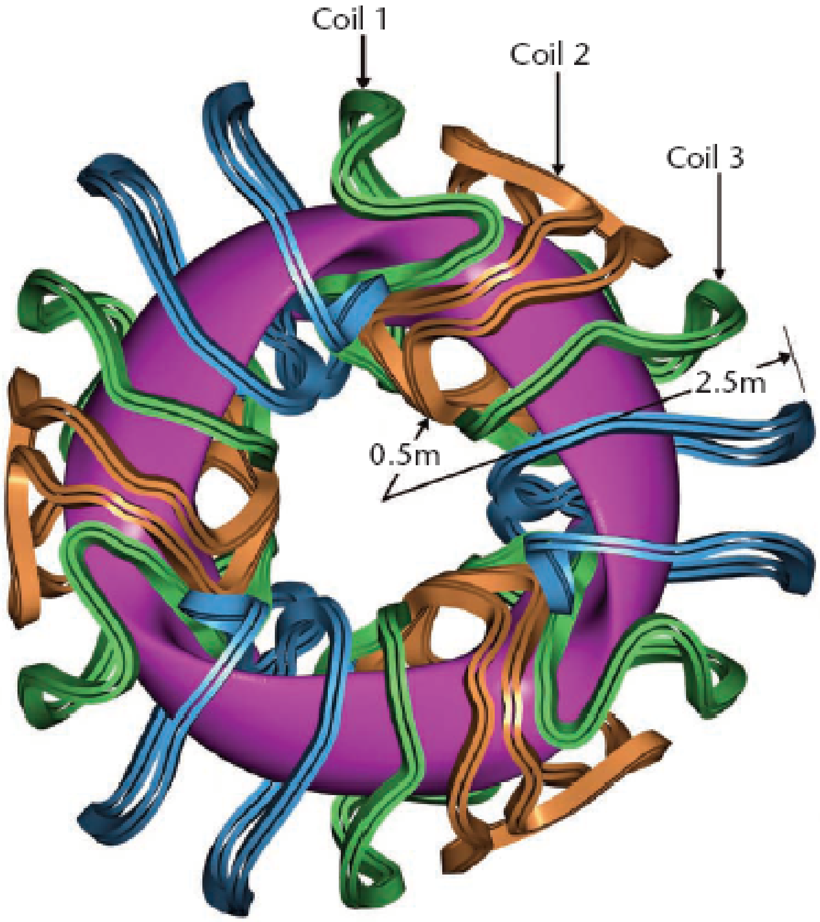
↓ turbulence & ↑ β could significantly improve fusion



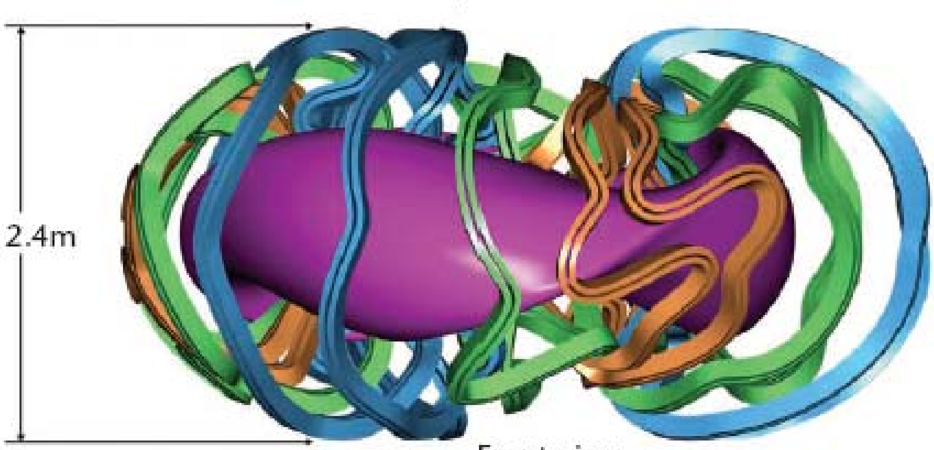
From Galambos, Perkins, Haney, & Mandrekas 1995 Nucl.Fus. (very good), scaled to match more detailed ARIES-AT reactor design study (2001), http://aries.ucsd.edu/ARIES/

Improved Stellarator Designs

- Magnetic field twist & shear provided by external coils, not plasma currents. Steady state & more stable. Appears to exceed Greenwald density limit, MHD beta limits, eliminate disruptions.
- Computer optimized designs much better than 1950-60 slide rules?
- Hidden symmetry discovered after 40 years: quasi-toroidal symmetry (of $\left|\vec{B}\right|$ in flux-coordinates)



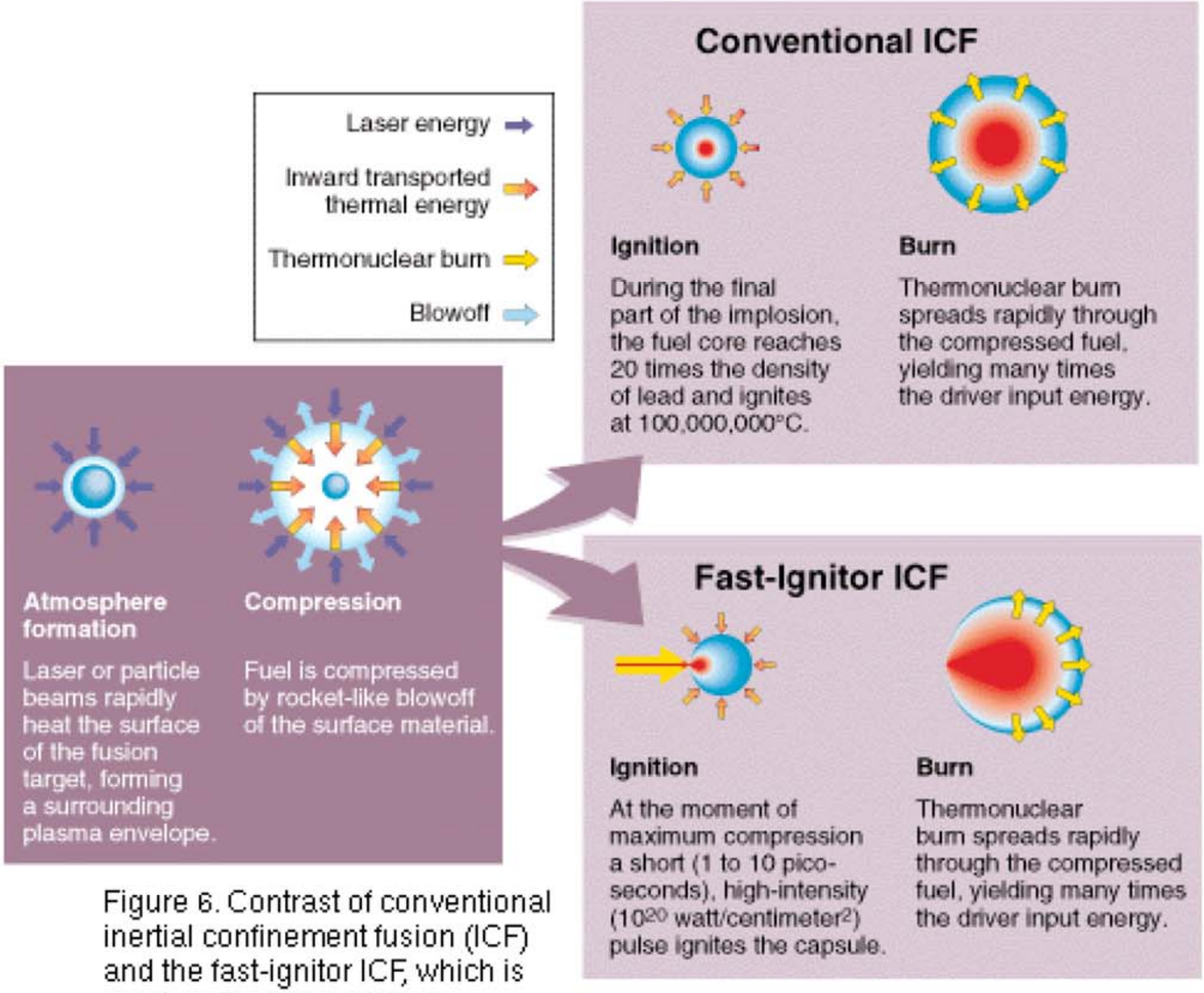
Top view



Front view


NSTX

Inertial Confinement Fusion



Atmosphere formation

Laser or particle beams rapidly heat the surface of the fusion target, forming a surrounding plasma envelope.

Compression

Fuel is compressed by rocket-like blowoff of the surface material.

Conventional ICF

Ignition

During the final part of the implosion, the fuel core reaches 20 times the density of lead and ignites at 100,000,000°C.

Burn

Thermonuclear burn spreads rapidly through the compressed fuel, yielding many times the driver input energy.

Fast-Ignitor ICF

Ignition

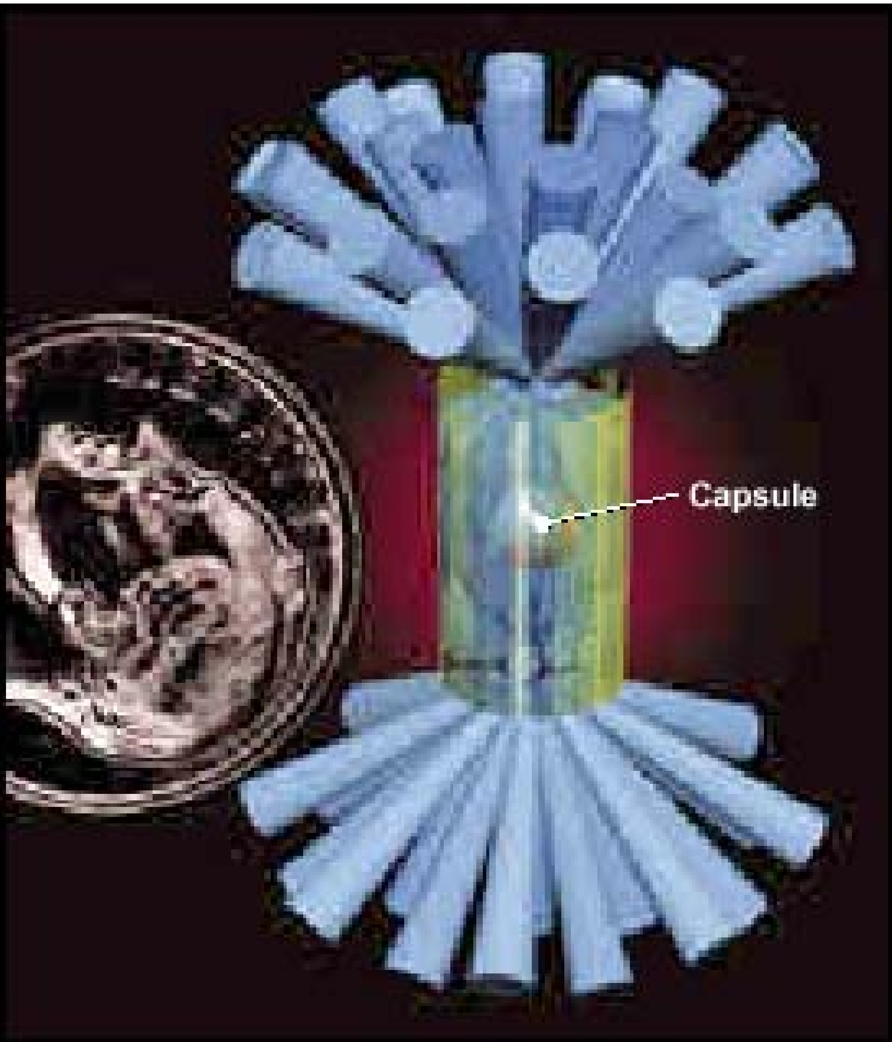
At the moment of maximum compression a short (1 to 10 picoseconds), high-intensity (10^{20} watt/centimeter^2) pulse ignites the capsule.

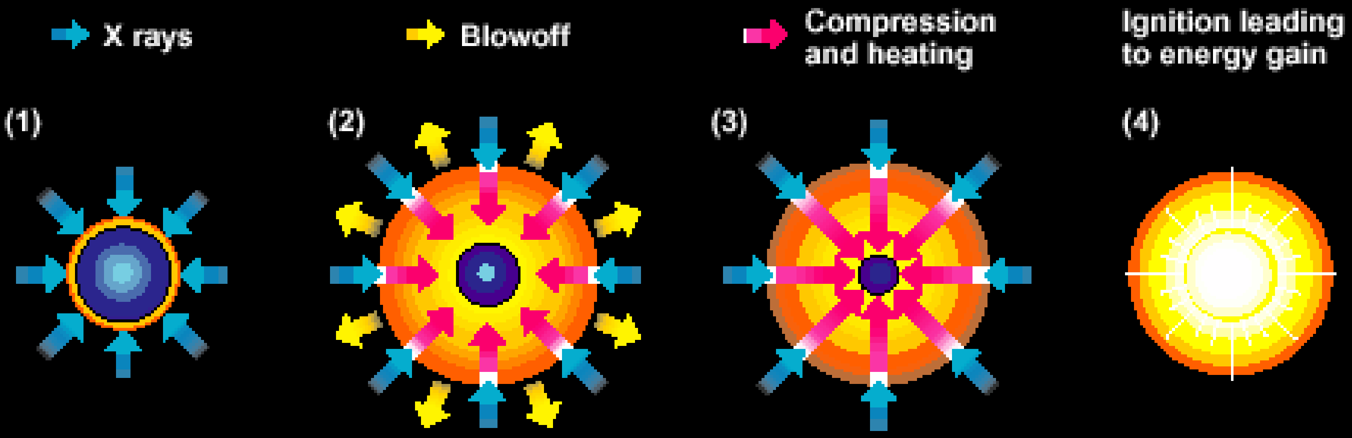
Burn

Thermonuclear burn spreads rapidly through the compressed fuel, yielding many times the driver input energy.

Figure 6. Contrast of conventional inertial confinement fusion (ICF) and the fast-ignitor ICF, which is used on the Petawatt laser.

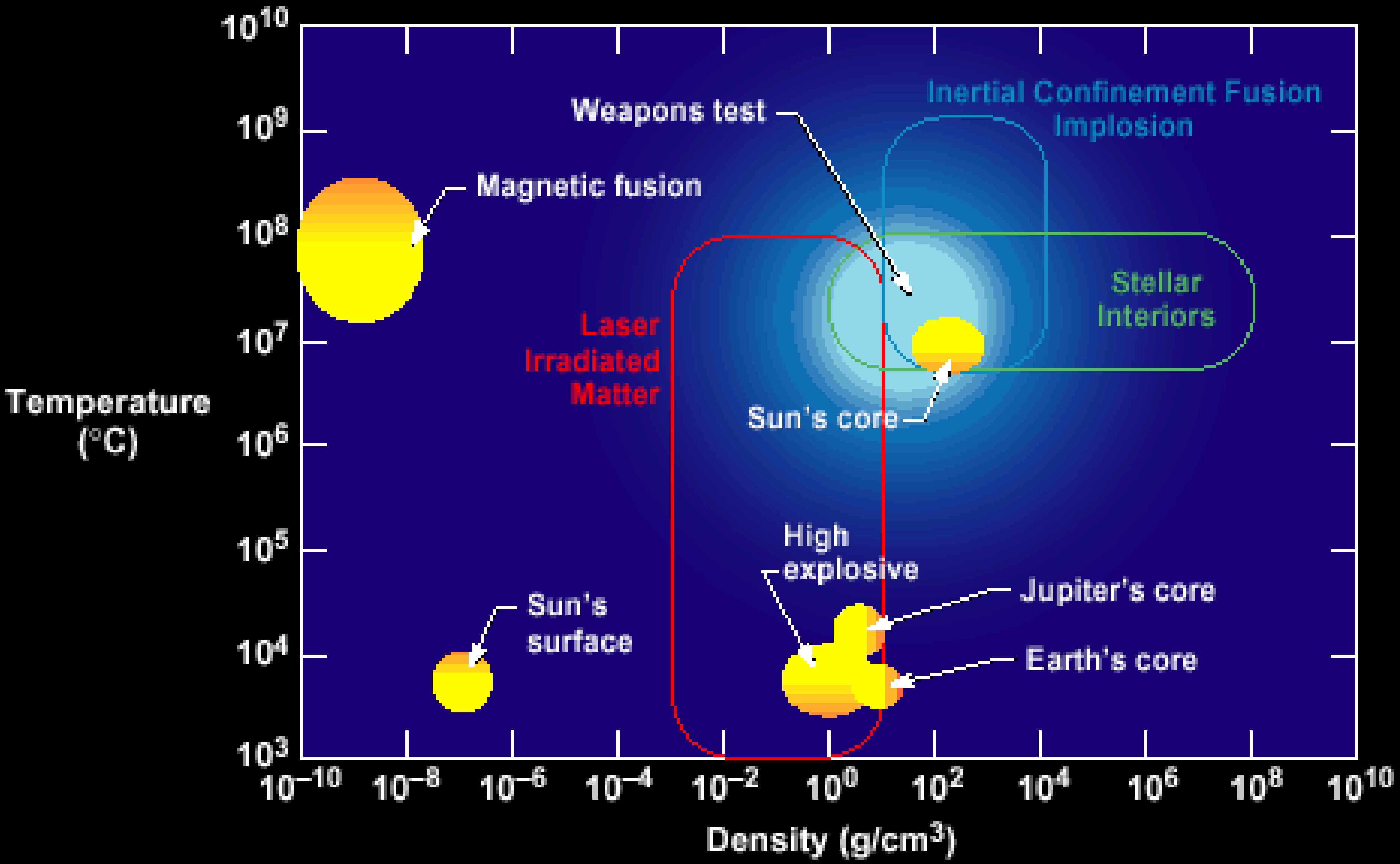
Using lasers to create high energy density conditions



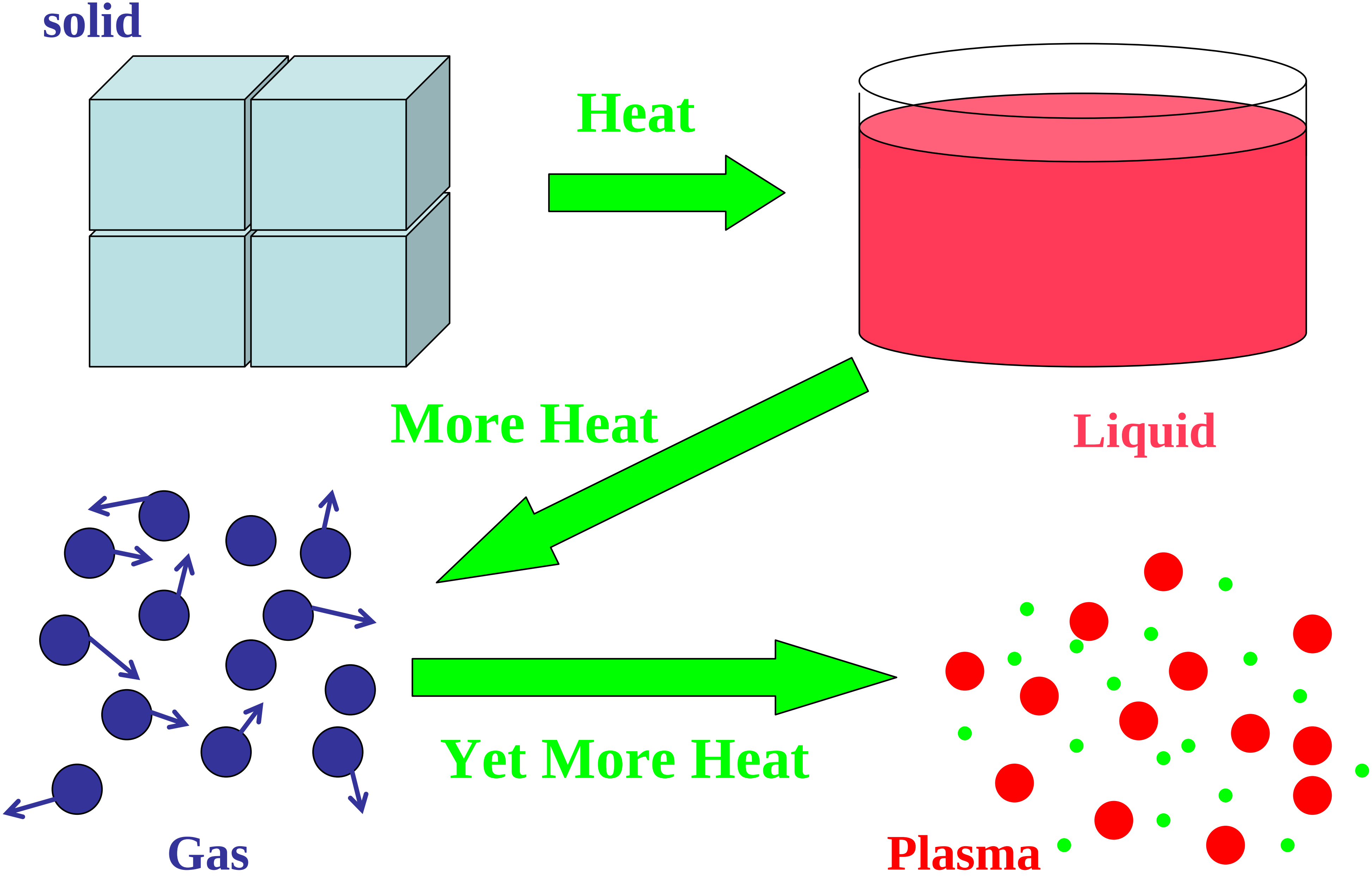


NIF Target Chamber

Plasma Regimes for Fusion



Plasma--4th State of Matter



solid

Heat

Liquid

More Heat

Gas

Yet More Heat

Plasma

States of Matter

1. Just an approximation, not a material property.

2. Depends on time scales, space scales, and physics of interest

Standard Definition of Plasma

- The standard definition of a plasma is as the 4th state of matter (solid, liquid, gas, plasmas), where the material has become so hot that electrons are no longer bound to individual nuclei. Thus a plasma is electrically conducting, and can exhibit collective dynamics.
- In other words, a plasma is an ionized gas.

- Implies that the potential energy of a particle with its nearest neighboring particles is weak compared to their kinetic energy (otherwise electrons would be bound to ions):

(see next page)

- This is the ideal “weakly-coupled plasma” limit.

- Even though the interaction between any pair of particles is typically weak, the collective interactions between many particles is strong.
2 examples: Debye Shielding & Plasma Oscillations.

Weak-coupling between nearest neighbor particles in a plasma

Typical distance between nearest neighbor particles $L_1 \sim n^{-1/3}$

I.e., a cube that contains on average 1 particle has a width L_1

such that $L_1^3 n \approx 1$

$$\frac{P.E.}{K.E.} = \frac{\text{Typical Potential Energy between nearest neighbors}}{\text{Typical Kinetic energy of a particle}}$$

$$\approx \frac{e\Phi}{\frac{1}{2}m\mathrm{v}^2} \approx \frac{e^2 / L_1}{\frac{1}{2}m\mathrm{v}^2} \sim \frac{e^2 n^{1/3}}{T}$$

for typical ITER parameters, $\mathrm{T} \approx 10\ \mathrm{keV}$, $\mathrm{n} \approx 10^{14} / cm^3$,

$\mathrm{P.E./K.E.} \sim 10^{-6}$

(T usually measured in energy units in plasma physics, so Boltzmann’s constant $k_B=1$)

Broader Definition of Plasma

- The electron temperature needs to be above ~0.3-1 eV in order to have most hydrogen ionized in thermal equilibrium. However, at lower temperatures can have weakly ionized plasmas (where plasma effects are still important), single species plasmas (pure electrons or pure ions, so there is no recombination), or non-equilibrium plasmas (at low density it takes a long time to recombine).

- Single-species non-neutral plasmas include intense charged particle beams where the self-interactions of the beam become important relative to external forces.

- A broader definition of a plasma could include matter which is electrically conducting even if the weak-coupling approximation doesn't hold. There are "strongly-coupled plasmas", "plasma crystal" states….

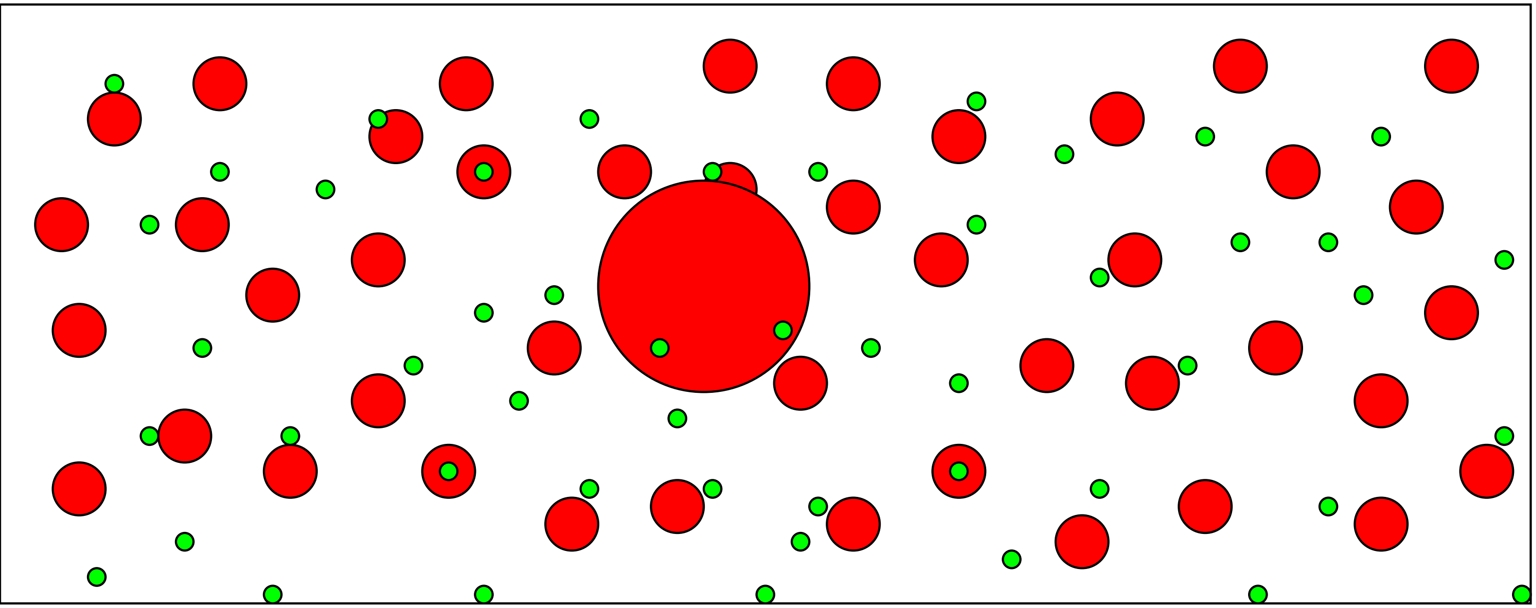
- An unconventional plasma at extreme conditions involving the strong nuclear force and not just electric forces: quark-gluon plasma ("Big Bang Goo", NYT headline for article on RHIC results by J. Glanz, plasma physicist turned journalist).

- However, here we will focus on the conventional or ideal limit of "weakly-coupled plasmas"

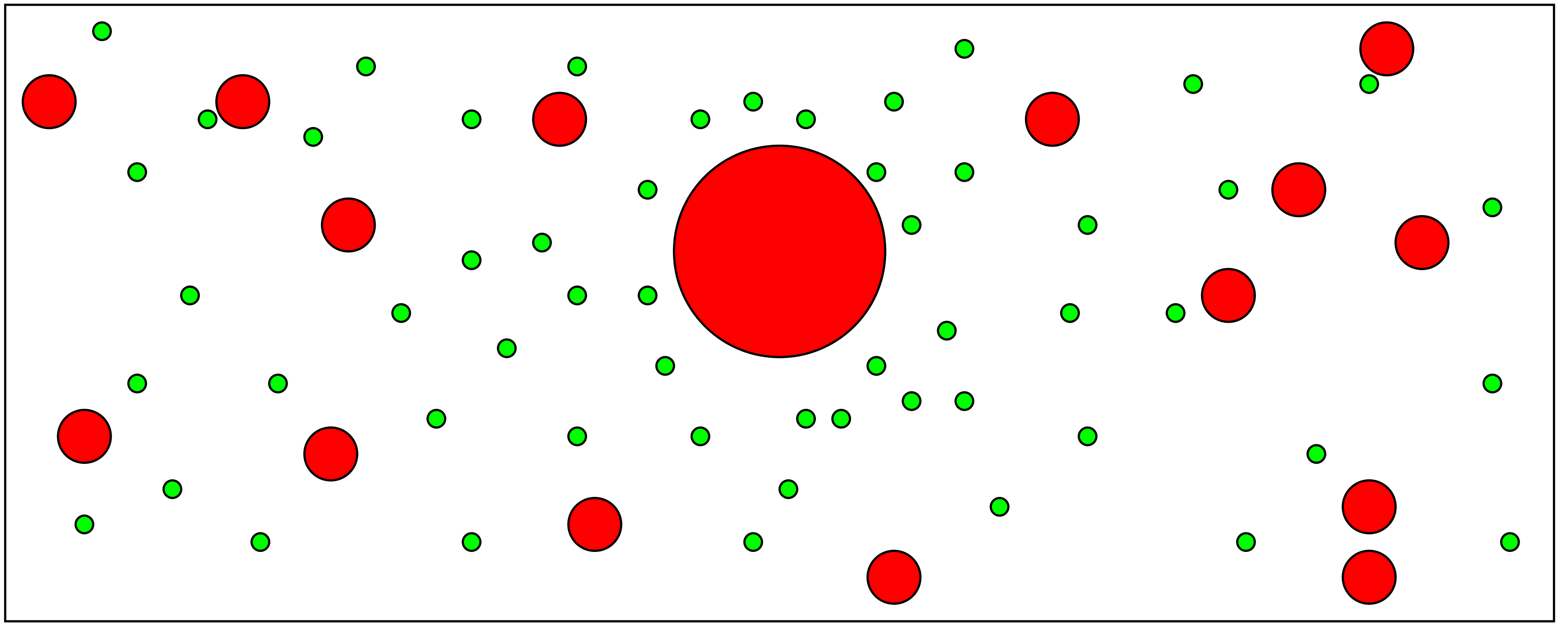
Properties of Plasma

1. Conducting medium, with many degrees of freedom
2. Shields electric fields
3. Supports many waves:
    1. vacuum waves, such as light waves
    2. Gas waves, such as sound waves
    3. A huge variety of new waves, based on electromagnetic coupling of constituent charged particles, and based on a variety of driving electric and magnetic fields

Plasma Shielding



Quasi-neutral plasma



Plasma Shielding

Plasma Debye Shielding

(cgs)

$$\boxed{\begin{aligned} &\nabla \bullet \vec{E} = 4\pi\left(\rho_T + \rho\right), \qquad \rho_T = Q\delta(\vec{r}) \\ &\vec{E} = -\nabla\Phi. \\ &\rho = n_i e - n_e e, \end{aligned}}$$

In equilibrium: $n_s = n_0 e^{-H/kT} = n_0 e^{-q_s\Phi/kT}$ Boltzmann/Gibbs

Linearize $n_e = n_0 + \tilde{n}, \qquad \Phi = \Phi_0 + \tilde{\Phi} = \tilde{\Phi}$

$$-\nabla^2\Phi = 4\pi Q\delta(\vec{r}) + 4\pi e n_0\left(e^{-e\Phi/kT} - e^{e\Phi/kT}\right)$$

$$e^{\mp e\Phi/kT} = 1 \mp e\Phi/kT + \dots$$

p.28 of NRL Plasma Formulary

$$-\nabla^2\Phi + \frac{1}{\lambda_D^2}\Phi = 4\pi Q\delta(\vec{r}), \qquad \lambda_D^{-2} \equiv \frac{4\pi\left(n_{0e} + n_{0i}\right)\mathrm{e}^2}{\mathrm{kT}}$$

$$\Phi = \frac{Q}{r}e^{-r/\lambda_D}$$

For typical ITER parameters, T=10 keV $n_e=10^{14}$ cm^{-3}:

$$\lambda_D \approx 0.5\times10^{-2}\,\mathrm{cm}$$

Physical implications of Debye shielding

- Although any individual particle is only slightly repelled or attracted by a particular nearby particle or imposed external charge, when this effect is added up over the many particles within a Debye radius, the net effect can be a strong shielding of charges on scales larger than a Debye radius.

- Another way to think about this: the weak-coupling property of nearest neighbor particles in a plasma means that the particles are uncorrelated (i.e., at random positions) to lowest order. But the weak correlation at next order can add up over many particles to give important collective effects on scales larger than a Debye radius.

The Plasma Parameter

A fundamental parameter used to characterize plasmas is the number of particles in a Debye sphere, Λ, a.k.a. “the plasma parameter:

$$\Lambda = n\frac{4\pi}{3}\lambda_D^3 \sim 10^8 \quad \text{for typical ITER parameters}$$

(A handy formula for Λ is on p. 29 of the NRL Plasma Formulary)

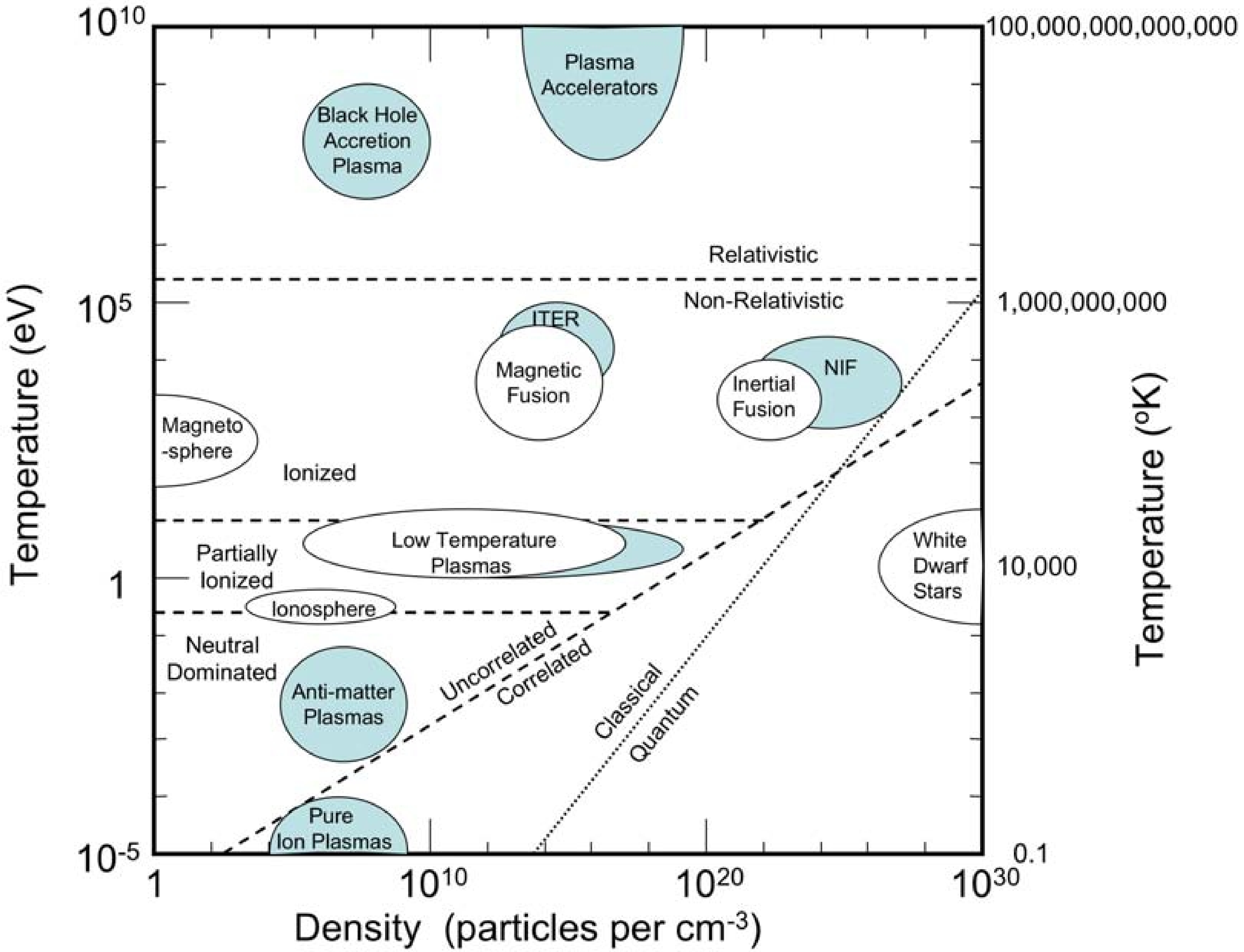
It turns out that the ratio of the potential energy between typical nearest neighbor particles to their typical kinetic energy (calculated a few slides back) can be expressed as

$$\frac{\text{Potential Energy}}{\text{Kinetic Energy}} \approx \frac{e^2 n^{1/3}}{T} = \frac{1}{(36\pi)^{1/3}\Lambda^{2/3}}$$

Thus Λ>>1 implies the plasma is in the weakly-coupled limit. We will find that it also implies that the mean free path is long compared to the Debye length.

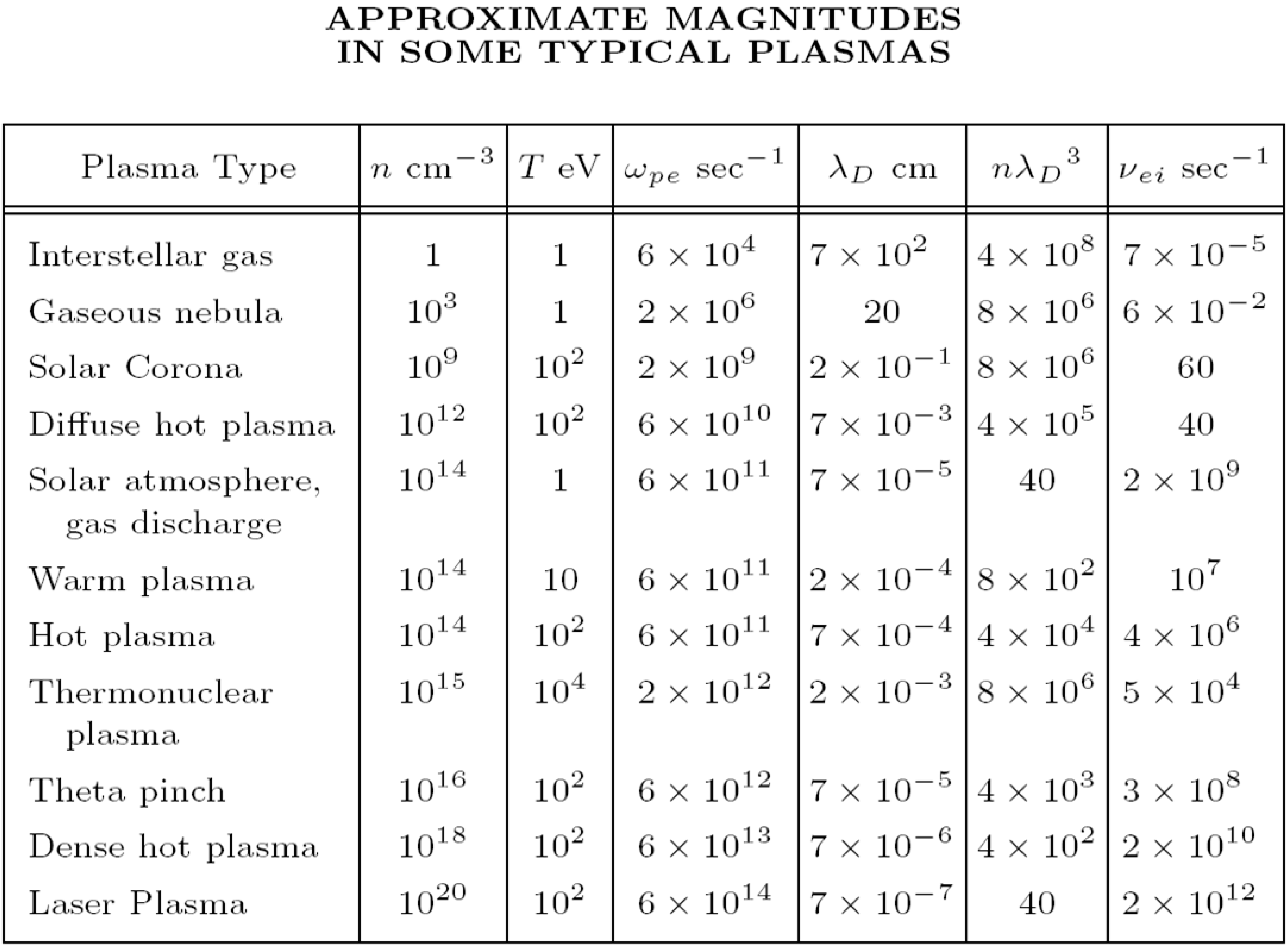
Wide range of possible plasma parameters.

Plasmas above the line marked “Uncorrelated-Correlated” correspond to $\Lambda >> 1$



Plasma Science: Advancing Knowledge in the National Interest (2007), National Research Council, http://books.nap.edu/openbook.php?record_id=11960&page=9

See also NRL Formulary version of this plot.

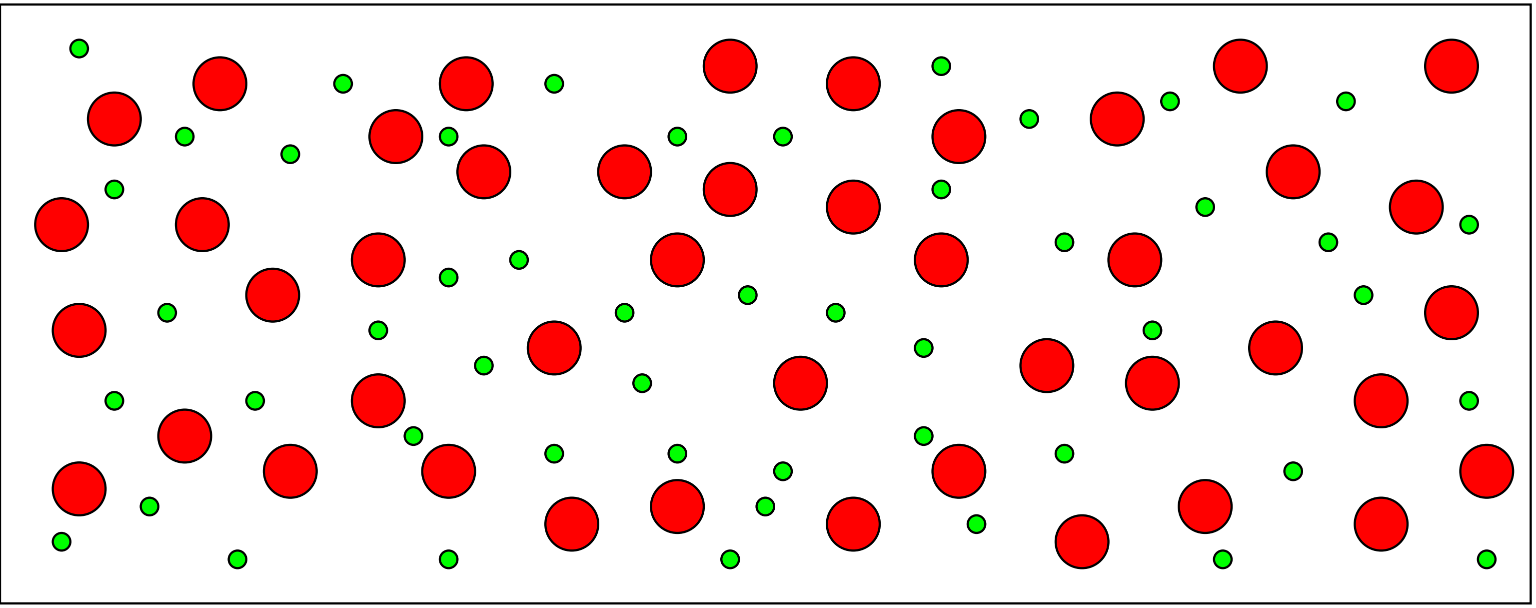


APPROXIMATE MAGNITUDES IN SOME TYPICAL PLASMAS

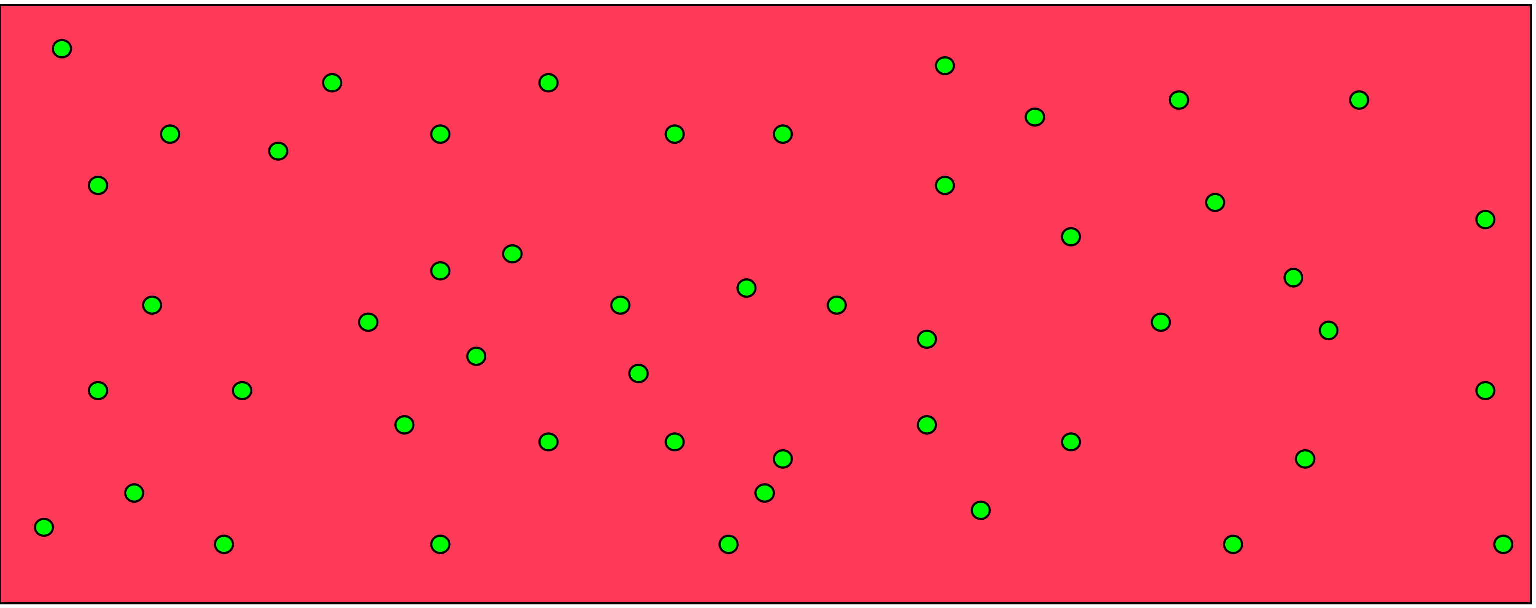
Plasma Type	n cm^{-3}	T eV	ω_{pe} sec^{-1}	λ_D cm	$n\lambda_D{}^3$	ν_{ei} sec^{-1}
Interstellar gas	1	1	6×10^4	7×10^2	4×10^8	7×10^{-5}
Gaseous nebula	10^3	1	2×10^6	20	8×10^6	6×10^{-2}
Solar Corona	10^9	10^2	2×10^9	2×10^{-1}	8×10^6	60
Diffuse hot plasma	10^{12}	10^2	6×10^{10}	7×10^{-3}	4×10^5	40
Solar atmosphere, gas discharge	10^{14}	1	6×10^{11}	7×10^{-5}	40	2×10^9
Warm plasma	10^{14}	10	6×10^{11}	2×10^{-4}	8×10^2	10^7
Hot plasma	10^{14}	10^2	6×10^{11}	7×10^{-4}	4×10^4	4×10^6
Thermonuclear plasma	10^{15}	10^4	2×10^{12}	2×10^{-3}	8×10^6	5×10^4
Theta pinch	10^{16}	10^2	6×10^{12}	7×10^{-5}	4×10^3	3×10^8
Dense hot plasma	10^{18}	10^2	6×10^{13}	7×10^{-6}	4×10^2	2×10^{10}
Laser Plasma	10^{20}	10^2	6×10^{14}	7×10^{-7}	40	2×10^{12}

From NRL Plasma Formulary (very useful)

Models of Plasma

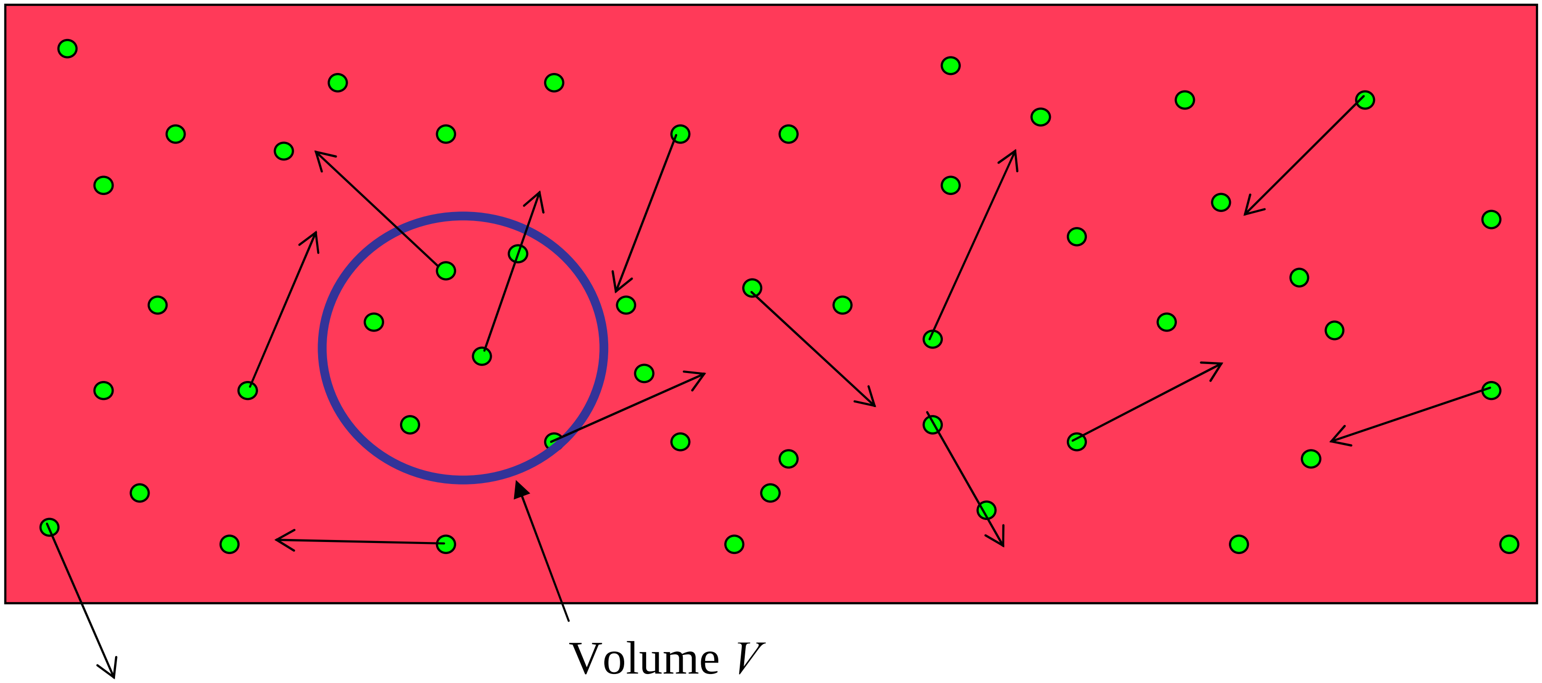


Quasi-neutral plasma



Ion neutralizing background

Fluid Model of Plasma



Ion neutralizing background

$$n(\vec{r},t) = \frac{\int_V d^3r \int d^3\mathrm{v}\, f(\vec{r},\vec{\mathrm{v}},t)}{V}$$

$$\vec{u}(\vec{r},t) = \frac{\int_V d^3r \int d^3\mathrm{v}\, f(\vec{r},\vec{\mathrm{v}},t)\vec{\mathrm{v}}}{\int_V d^3r \int d^3\mathrm{v}\, f(\vec{r},\vec{\mathrm{v}},t)}$$

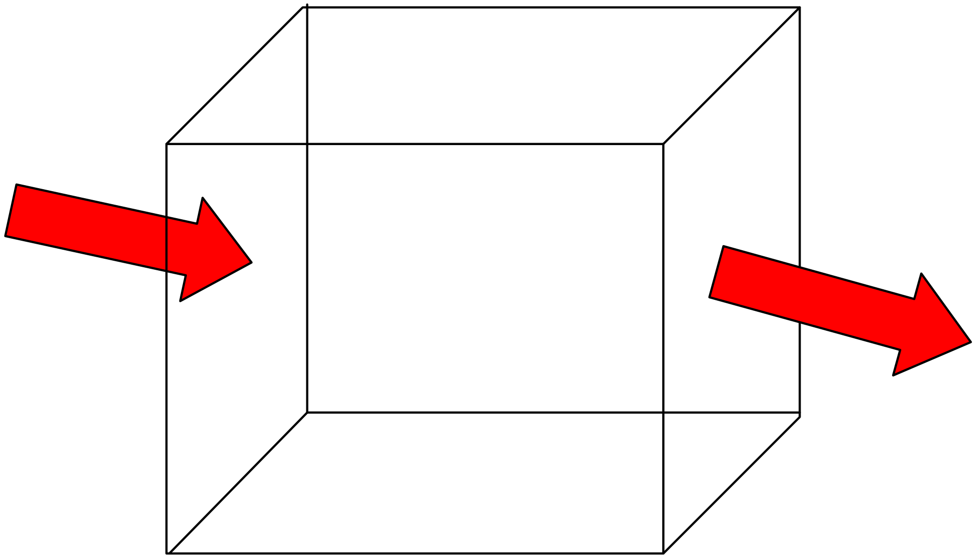
n = real-space density of particles

f = phase-space density of particles

v = particle velocity,

u = fluid velocity = average particle velocity

Fluid Equations



$$\vec{S}(\vec{r},t) = n\vec{u}$$

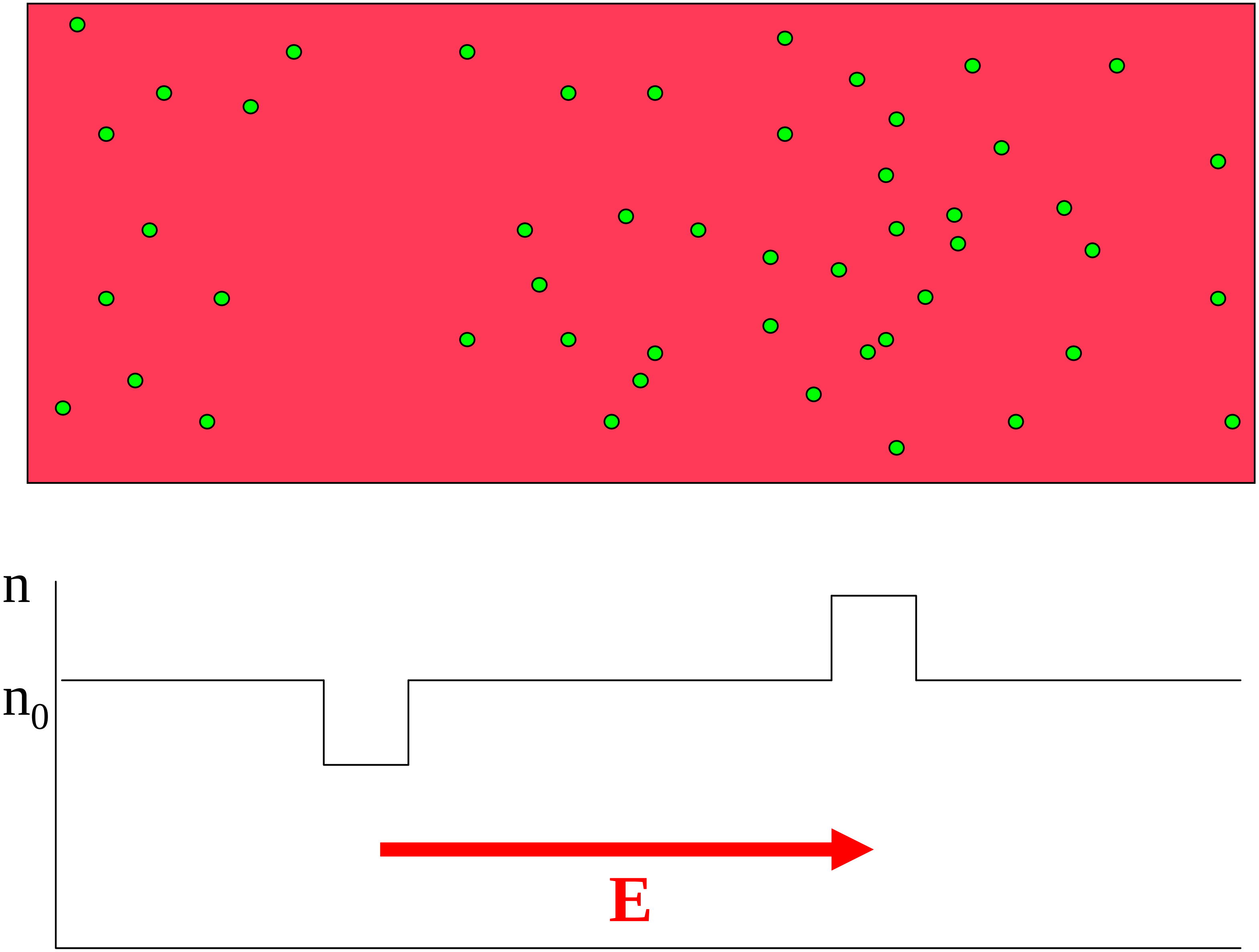
$$\vec{S}(\vec{r},t)$$

$$\frac{dN}{dt} = \int_V \vec{S} \cdot d\vec{A}$$

Continuity Equation

$$\frac{\partial}{\partial t} n + \nabla \cdot \left(n\vec{u} \right) = 0$$

Set up plasma oscillation



Cold Fluid Equations

$$\nabla \bullet \vec{E} = 4\pi e(n_0 - n_e)$$

Poisson's equation

$$\frac{\partial}{\partial t} n_e + \nabla \cdot \left(n_e \vec{\mathrm{v}}\right) = 0$$

Particle conservation

$$\frac{\partial}{\partial t} n_e m \vec{\mathrm{v}} + \nabla \cdot \left(n_e m \vec{\mathrm{v}} \vec{\mathrm{v}}\right) = -\underbrace{\nabla p_e}_{\to 0} + n_e q_e \left(\vec{E} + \frac{\vec{\mathrm{v}} \times \vec{B}}{c} \right)$$

Momentum conservation.
cold fluid limit p → 0,
& B=0 or $\vec{\mathrm{v}} \,||\, \vec{B}$.
ignore viscous tensor and drag terms.

(In the rest of these notes we will be dealing only with the fluid velocity, not individual particle velocities, and will denote the fluid velocity by v, not *u*.)

Plasma Oscillations (1)

$$\nabla \bullet \vec{E} = 4\pi e(n_0 - n_e)$$

Poisson's equation

$$\frac{\partial}{\partial t} n_e + \nabla \cdot n_e \vec{\mathrm{v}} = 0$$

Particle conservation

$$\frac{\partial}{\partial t} n_e m_e \vec{\mathrm{v}} + \nabla \cdot \left(n_e m_e \vec{\mathrm{v}}\vec{\mathrm{v}}\right) = -n_e e \vec{E}$$

Momentum conservation

Linearize
for small
perturbations

$$\boxed{\begin{aligned} n_e &= n_0 + \tilde{n}(\vec{r},t) \\ \vec{\mathrm{v}} &= \vec{\mathrm{v}}_0 + \tilde{\vec{\mathrm{v}}}(\vec{r},t) \\ \vec{E} &= \vec{E}_0 + \tilde{\vec{E}}(\vec{r},t) \end{aligned}}$$

Assume

$$\boxed{\begin{aligned} n_i &= n_0 \\ \vec{\mathrm{v}}_0 &= 0 \\ \vec{E}_0 &= 0 \end{aligned}}$$

$$\nabla \bullet \left(\vec{E}_0 + \tilde{\vec{E}}\right) = \nabla \bullet \tilde{\vec{E}} = -4\pi e \tilde{n}$$

Linearized
Poisson's equation

Plasma Oscillations (2)

Particle conservation

$$\frac{\partial}{\partial t}\left(\cancelto{0}{n_0} + \tilde{n}(\vec{r},t)\right) = -\nabla\cdot\left[\left(n_0 + \tilde{n}\right)\left(\vec{\mathrm{v}}_0 + \tilde{\vec{\mathrm{v}}}\right)\right]$$

$$= \cancelto{0}{-\nabla\cdot\left(n_0\vec{\mathrm{v}}_0\right)} \cancelto{0}{-\nabla\cdot\left(\tilde{n}\vec{\mathrm{v}}_0\right)} -\nabla\cdot\left(n_0\tilde{\vec{\mathrm{v}}}\right) \cancelto{0}{-\nabla\cdot\left(\tilde{n}\tilde{\vec{\mathrm{v}}}\right)}$$

$$= -n_0\nabla\cdot\tilde{\vec{\mathrm{v}}}$$

⇒

$$\frac{\partial\tilde{n}}{\partial t} = -n_0\nabla\cdot\tilde{\vec{\mathrm{v}}}$$

Linearized Particle Conservation Equation

Plasma Oscillations (3)

$$\frac{\partial}{\partial t} n_e m\vec{v} + \nabla\cdot\left(n_e m\vec{\mathrm{v}}\vec{\mathrm{v}}\right) = -n_e e\vec{E}$$

Momentum conservation

$$m\frac{\partial}{\partial t}\left(n_0+\tilde{n}\right)\left(\vec{\mathrm{v}}_0+\tilde{\vec{\mathrm{v}}}\right)+\nabla\cdot\cancelto{0}{\left(\left(n_0+\tilde{n}\right)m\tilde{\vec{\mathrm{v}}}\tilde{\vec{\mathrm{v}}}\right)}=-(n_0+\tilde{n})e\tilde{\vec{E}}$$

$$\Rightarrow\quad mn_0\frac{\partial\tilde{\vec{\mathrm{v}}}}{\partial t}=-n_0 e\tilde{\vec{E}}$$

Linearized momentum equation

Derivation of Cold Plasma Oscillations

$$\frac{\partial \tilde{n}}{\partial t} = -n_0 \nabla \cdot \tilde{\vec{\mathrm{v}}}$$ use $$m\frac{\partial \tilde{\vec{\mathrm{v}}}}{\partial t} = -e\tilde{\vec{E}}$$

or

$$\frac{\partial^2 \tilde{n}}{\partial t^2} = -n_0 \nabla \cdot \frac{\partial \tilde{\vec{\mathrm{v}}}}{\partial t} \quad = n_0 \nabla \cdot \frac{e}{m}\tilde{E}$$

use $$\nabla \bullet \tilde{E} = -4\pi e \tilde{n}$$

$$= -\omega_p^2 \tilde{n}$$

Plasma frequency $$\omega_p^2 = \frac{4\pi n_0 e^2}{m_e}$$

Plasma Oscillations

$$\nabla \bullet \vec{E} = 4\pi e(n_0 - n_e) = -4\pi e\tilde{n}$$ Poisson’s equation

$$\frac{\partial}{\partial t} n_e + \nabla \cdot n_e v = 0$$ Particle conservation

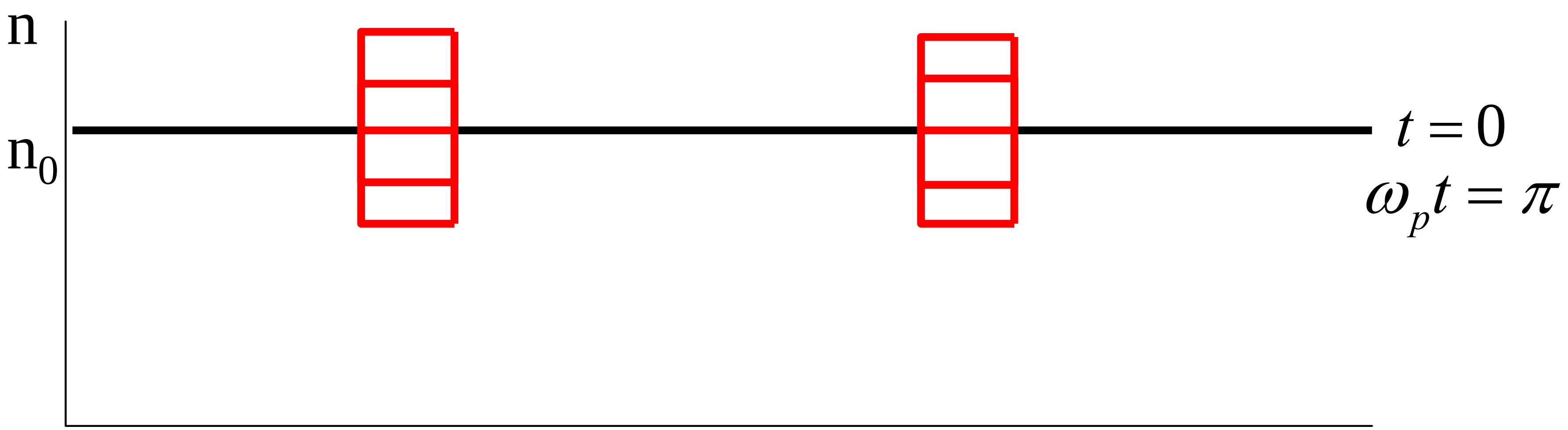
$$\frac{\partial}{\partial t} n_e m v + \nabla \cdot n_e m v v = eE$$ Momentum conservation

⇨

$$\boxed{\frac{\partial^2}{\partial t^2}\tilde{n} + \omega_p^2 \tilde{n} = 0}$$

$$\boxed{\tilde{n} = A(\vec{r})\cos\omega_p t + B(\vec{r})\sin\omega_p t}$$

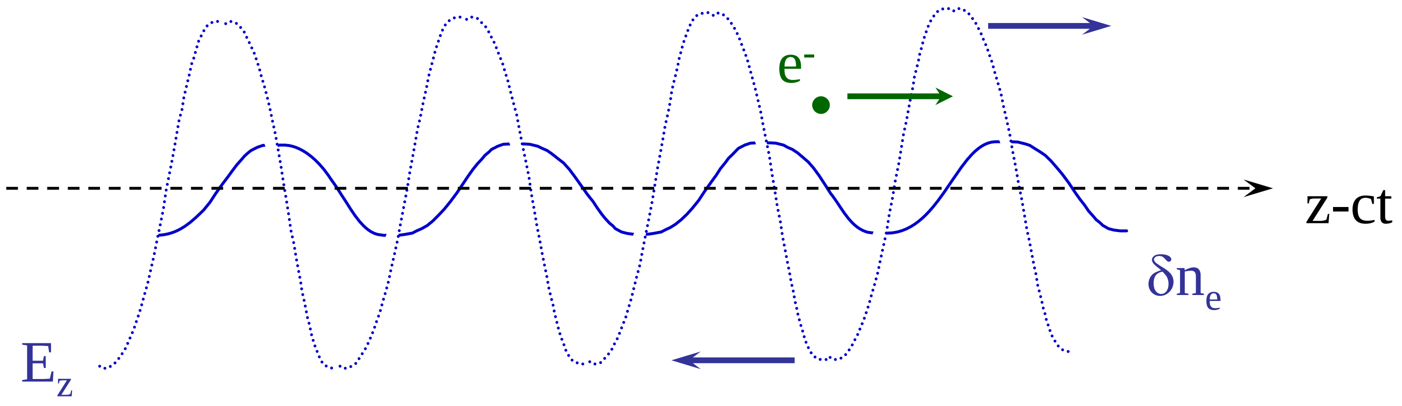
Set up plasma oscillation



Other possibilities:

$$\boxed{\Phi(x,t) = A(x)\cos[\omega_p\,(t - x/c)]}$$

Electron acceleration in a plasma wave

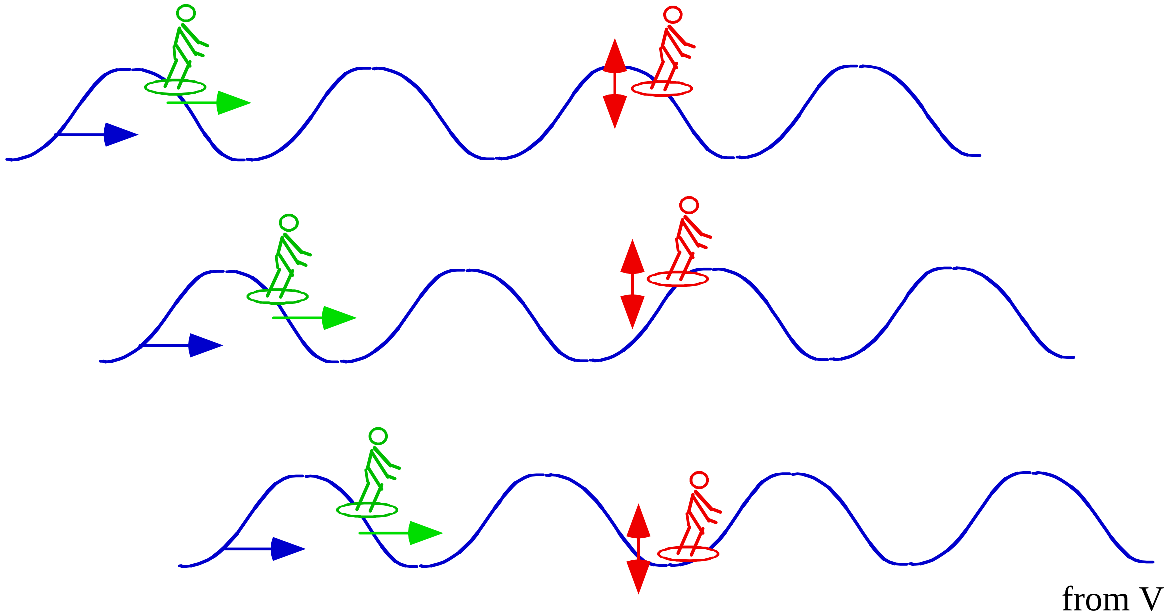


phase velocity -- arbitrary

Accelerate to TeV

Tajima and Dawson (1979)

Analogy:



from V.Malka

Accelerating Gradient in Plasma

Conventional Accelerator

Gradients ~ 20 MeV/m at 3GHz

1 TeV Collider requires 50 km

Peak gradients limited by breakdown

Plasma Accelerator

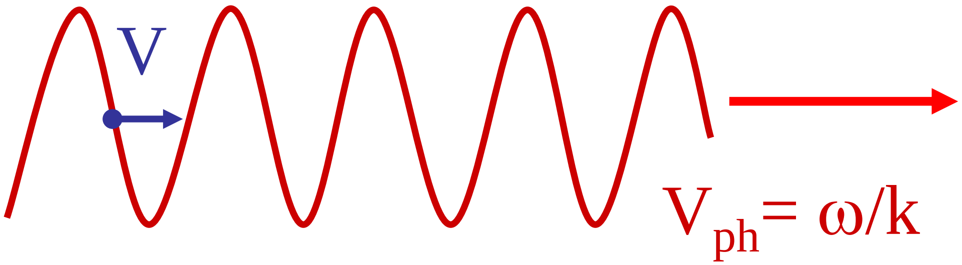
High fields, No breakdown

(Tajima and Dawson, 1979)

Example

$n_0 = 10^{18} \mathrm{cm}^{-3}$

eE=100 GeV/m



$$\nabla \bullet \vec{E} = -4\pi e\tilde{n}$$

$$\tilde{n}_{MAX} \approx n_0$$

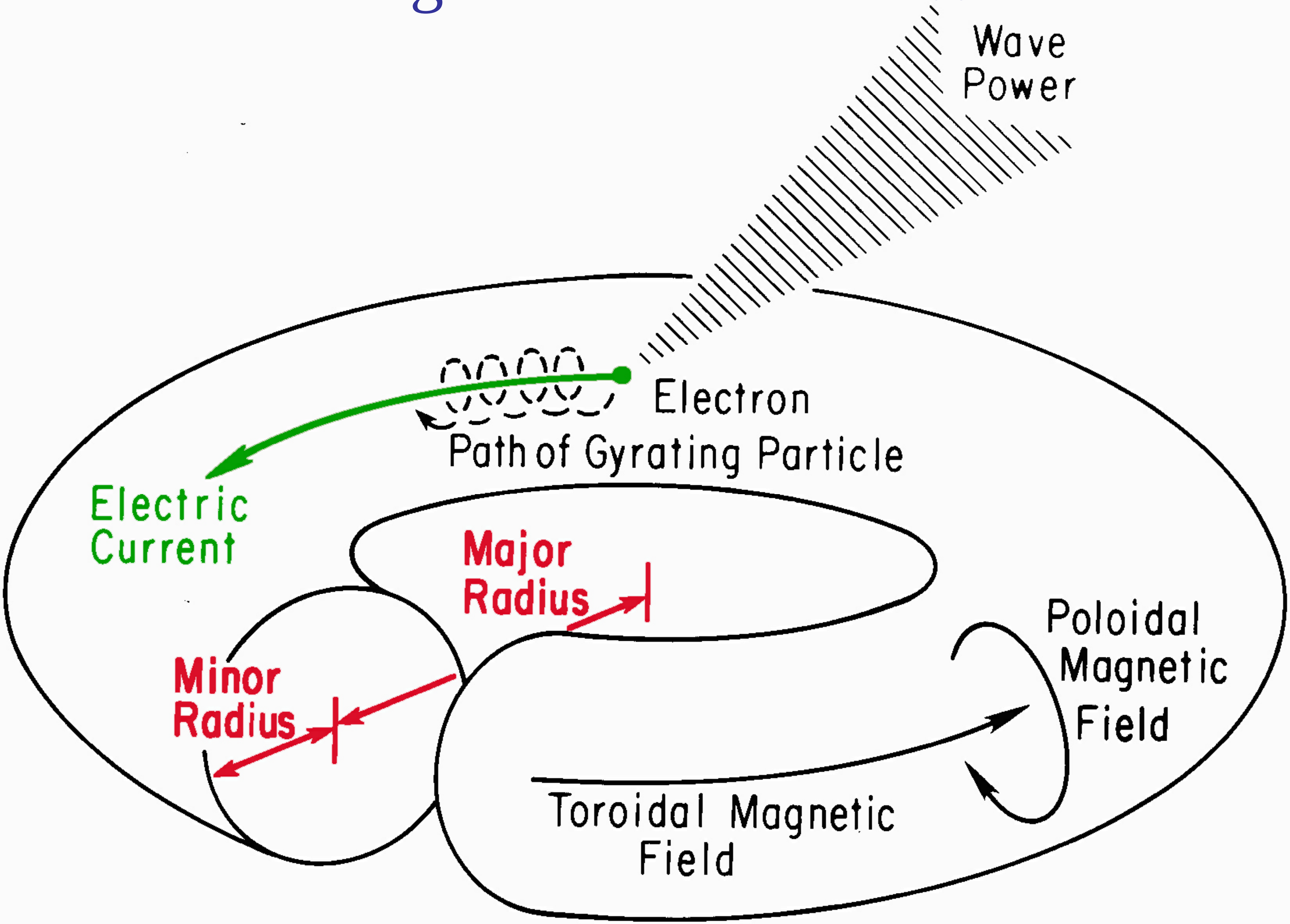
$$k = \frac{\omega_p}{c}$$

$$eE_{MAX} \approx \sqrt{n_0}\, GeV/cm$$

Note: For v << c , $\frac{v_{osc}}{c} \approx \frac{\tilde{n}}{n_0}$

Particles accelerated to relativistic energies, even as plasma motion is not

Heating a Tokamak with Waves

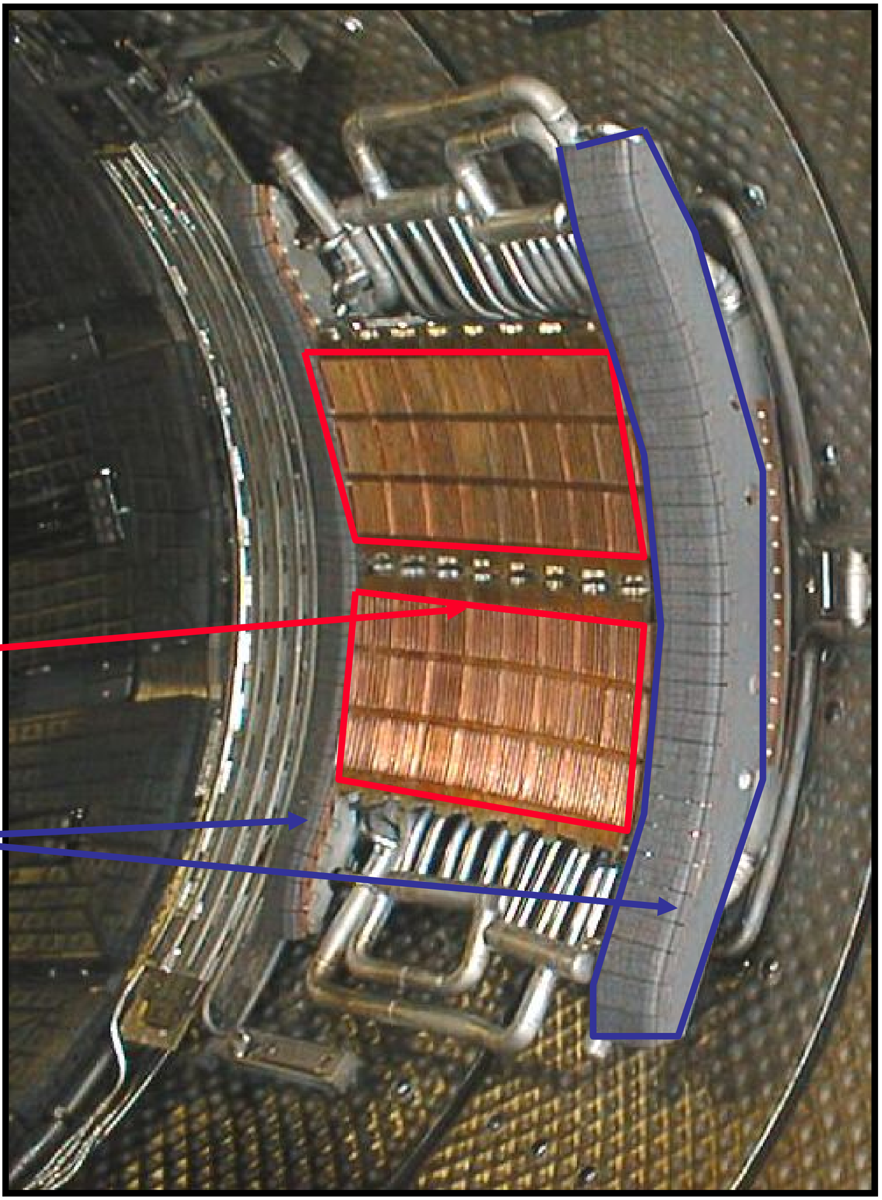


Tore Supra
new LH coupler (2001)

Antenna fully designed for long pulse operation:

4 MW, 1000 s, 3.7 GHz

(tested up to 3 MW, 9.5 s)



Limited power density

(25 MW/m^2 at full power and $n_{//0} = 2$)

Actively cooled side limiter

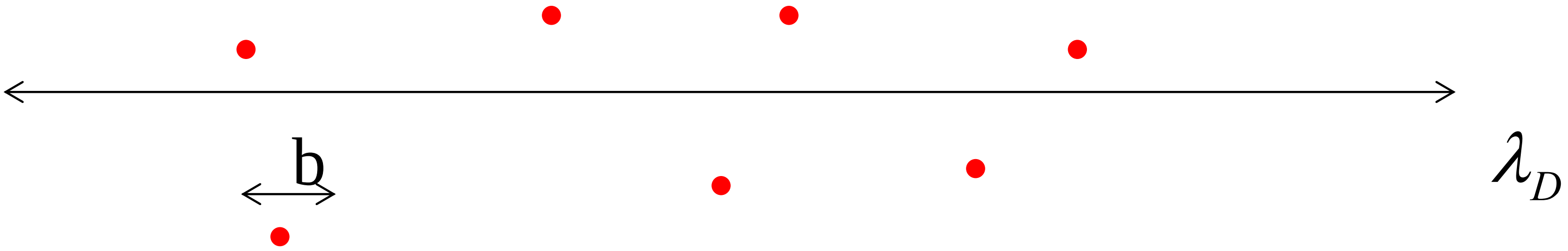
(exhaust capability:10 MW/m^2)

$1.7 \leq n_{//0} \leq 2.3$
Directivity: 70% ($n_{//0} = 2.0$)

48 active / 9 passive waveguides

Ideal Plasma

For ideal plasma: $\Lambda = n\lambda_D^3 >> 1 \Rightarrow \lambda_{mfp} >> \lambda_D >> n^{-1/3} >> b$

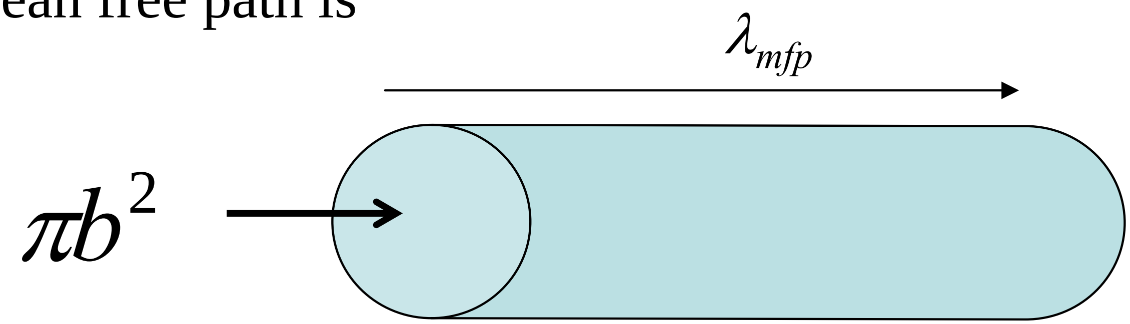


In order for two particles to undergo a 90-degree scattering off of each other, they must get within a distance b where the potential energy is comparable to the kinetic energy:

$$e^2 / b \approx T \qquad \text{or} \qquad b = e^2 / T$$

As a particle moves, it sweeps out a cylinder of cross-section πb^2 (the cross-section for scattering) . The probability that a particle will have undergone a 90-degree collision will be about unity if the volume of this cylinder contains about 1 particle: $\pi b^2 \lambda_{mfp} n = 1$. So the mean free path is

$$\lambda_{mfp} = \frac{1}{n\pi b^2}$$



Collisions are relatively weak in plasmas

It turns out that the final mean-free path (and collision rate) is enhanced by a factor of approximately $\ln(\Lambda)$, due to the dominance of small-angle scattering events. (Though they cause less scattering in a single event, they are more numerous than a single 90-degree scattering event). Typical $\ln(\Lambda) \sim 15$.

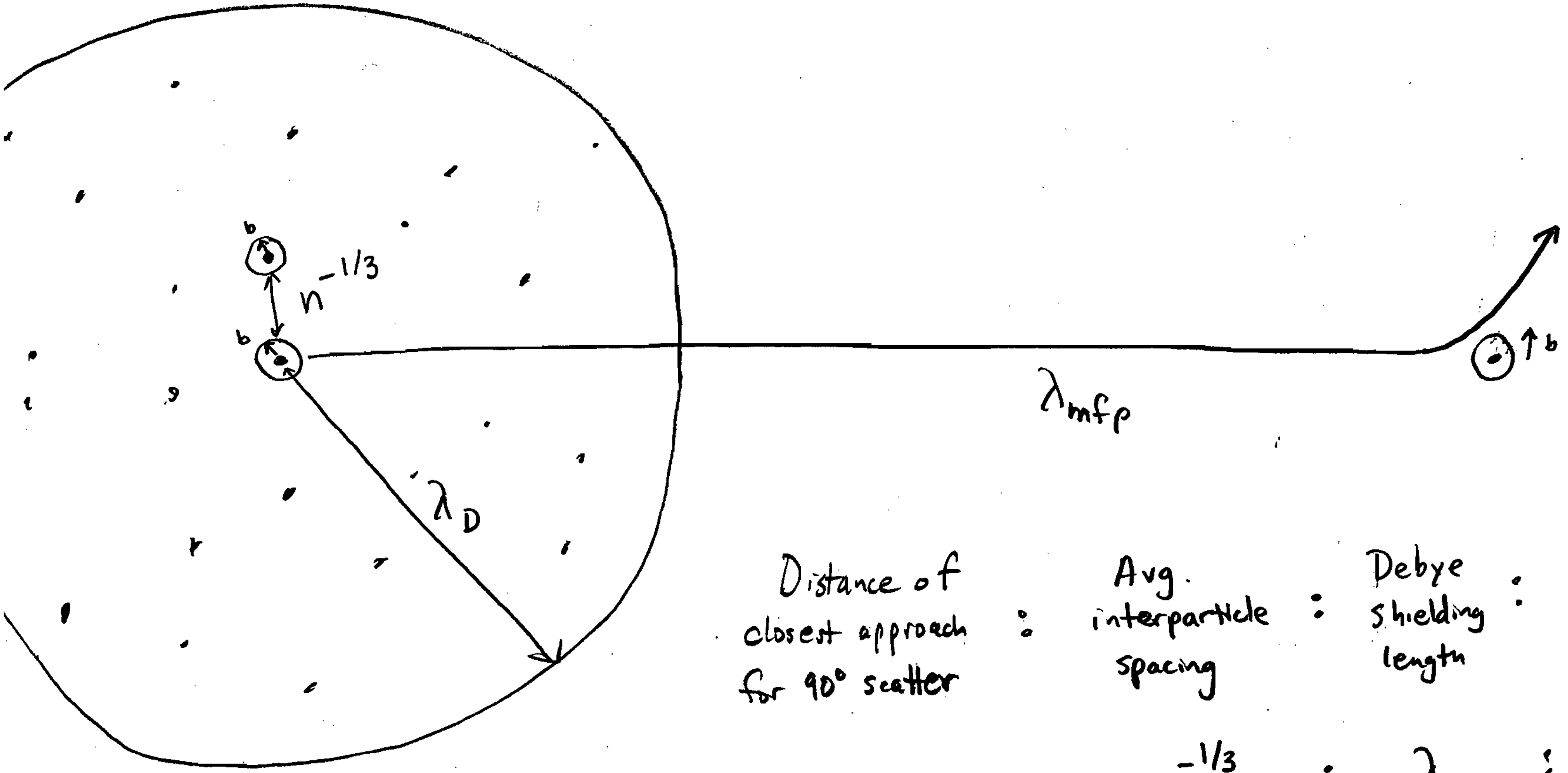
With a few lines of algebra, one can show that

$$\frac{\lambda_{mfp}}{\lambda_D} \sim \frac{\Lambda}{\ln(\Lambda)} \sim 10^7$$

Similarly, the electron plasma frequency is larger than the collision frequency by this ratio.

In hot magnetic fusion plasmas, λ_{mfp} can be of the order of kilometers, many times around the torus (thus standard fluid equations can break down). In colder denser plasmas, λ_{mfp} may be small compared to the device size, though still large compared to λ_D.

Review Fundamental Length Scales of a Plasma



≡ Average

Distance of closest approach for 90° scatter : Avg. interparticle spacing : Debye shielding length : Mean-free path between collisions.

$$b : n^{-1/3} : \lambda_D : \lambda_{mfp}$$

$$\Lambda^{-1} : \Lambda^{-1/3} : 1 : \frac{\Lambda}{\ln \Lambda}$$

$$10^{-6} : 10^{-2} : 1 : \frac{10^6}{15} \sim 10^5$$

$\Lambda \sim n \lambda_D^3$

= # of particles in Debye Sphere

$$\left.\begin{array}{l} \lambda_D \sim 10^{-2}\ \text{cm} \\ \lambda_{mfp} \sim 100\ \text{m} \end{array}\right\} \text{TFTR}$$

You should be able to derive Debye shielding, Boltzmann response.

Implications:

- Binary interactions weak (consider typical nearest neighbors)!

$$\frac{P.E.}{K.E.} \sim \frac{\frac{e^2}{n^{-1/3}}}{mv^2} \sim \frac{e^2 n^{1/3}}{T} \sim \frac{1}{\Lambda^{2/3}}$$

- Collective interactions strong (Quasineutrality, Alfvén waves, low resistivity, ...)

$$\frac{\text{P. E. of Debye Cloud}}{T} \sim \frac{\frac{\Lambda e^2}{\lambda_D}}{T} \sim 1$$

Fundamental Time Scales

You should be able to derive electron plasma oscillations

$$\omega^2 = \omega_{pe}^2 \quad (+ \; 3k^2 v_{te}^2 + \dots + \text{Landau damping})$$

$$\omega_{pe}^2 = \frac{4\pi n_e e^2}{m_e}$$

$$\lambda_D = \frac{v_{te}}{\omega_{pe}} \qquad v_{te} = \sqrt{\frac{T_e}{m_e}}$$

Collision frequency $\frac{\nu}{\omega_{pe}} \sim \frac{\ln \Lambda}{\Lambda} \ll 1$

Further Plasma References

- www.plasmacoalition.org
- NRL Plasma Formulary: http://wwwppd.nrl.navy.mil/nrlformulary/
- Plasma Science: Advancing Knowledge in the National Interest (2007), National Research Council, http://books.nap.edu/openbook.php?record_id=11960&page=9
- www.pppl.gov
- many more…

- Textbooks:
- F. F. Chen simplest introduction with many physical insights
- Goldston & Rutherford, somewhat more advanced, but still for beginning graduate student or upper level undergraduate
- Many others, some much more mathematical or advanced:
Hazeltine & Waelbroeck, Friedberg, Boyd & Sanderson, Dendy, Bittencourt, Wesson, Krall & Trivelpiece, Miyamoto, Ichimaru, Kulsrud, Spitzer, Stix, others
- Blandford & Thorne's draft book has chapters on plasma physics:
http://www.pma.caltech.edu/Courses/ph136/yr2006/text.html