

Continuum Gyrokinetic Simulations of Turbulence in Magnetized Plasmas

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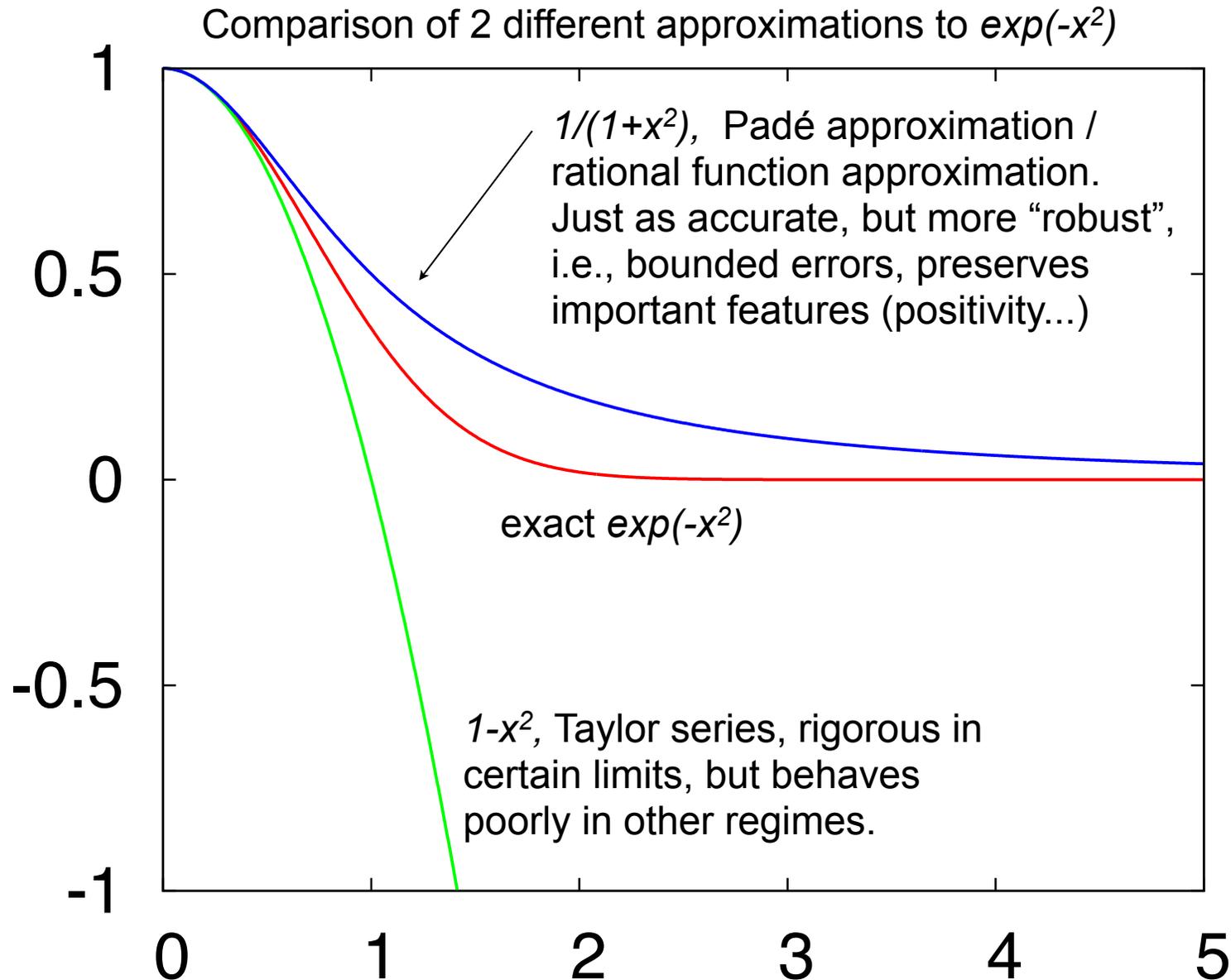
Outline:

- * Brief status of magnetic fusion research & interesting ideas being pursued
- * Summary of the gyrokinetic equations
- * Brief description of 3 of the most widely used gyrokinetic codes, the continuum codes GS2, GYRO, and GENE. Example of results from such codes.
- * Description of various algorithms used in these codes.

Richness of Study of Algorithms

- Vast zoo of algorithms, in part because developed for different applications in different fields (difficulties translating jargon between fields)
- Different algorithms are optimal for different applications, tradeoffs in accuracy, speed, complexity, conservation properties, bounds on solutions (positivity, non-oscillatory, etc.), preservation of other properties, efficiency on different computer architectures, overall robustness.
- Independent codes and/or different algorithms can be useful cross-checks.
- Get deeper insight into equations when trying to actually solve them on a computer

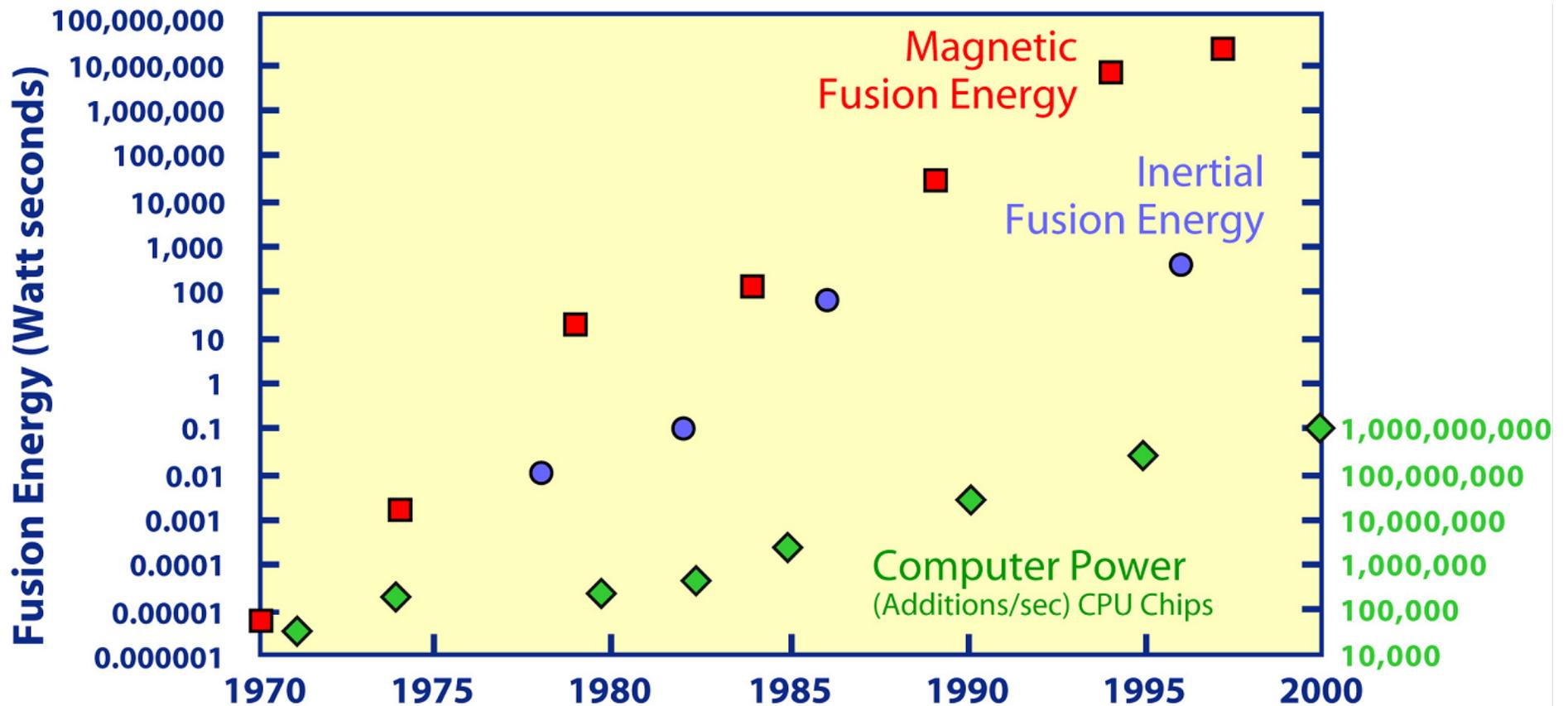
“Robustness”: a Useful Quality for Algorithms



(This may seem like a trivial example, but is related to approximations for gyroaveraging or for plasma Z-function. Illustrates issues that can arise from discretizing an operator. Related to implicit/explicit methods.)

Progress in Fusion Energy has Outpaced Computer Speed

10 MW (TFTR, Princeton, US)
16 MW (JET, Culham, US)
enough for ~ 5000 people



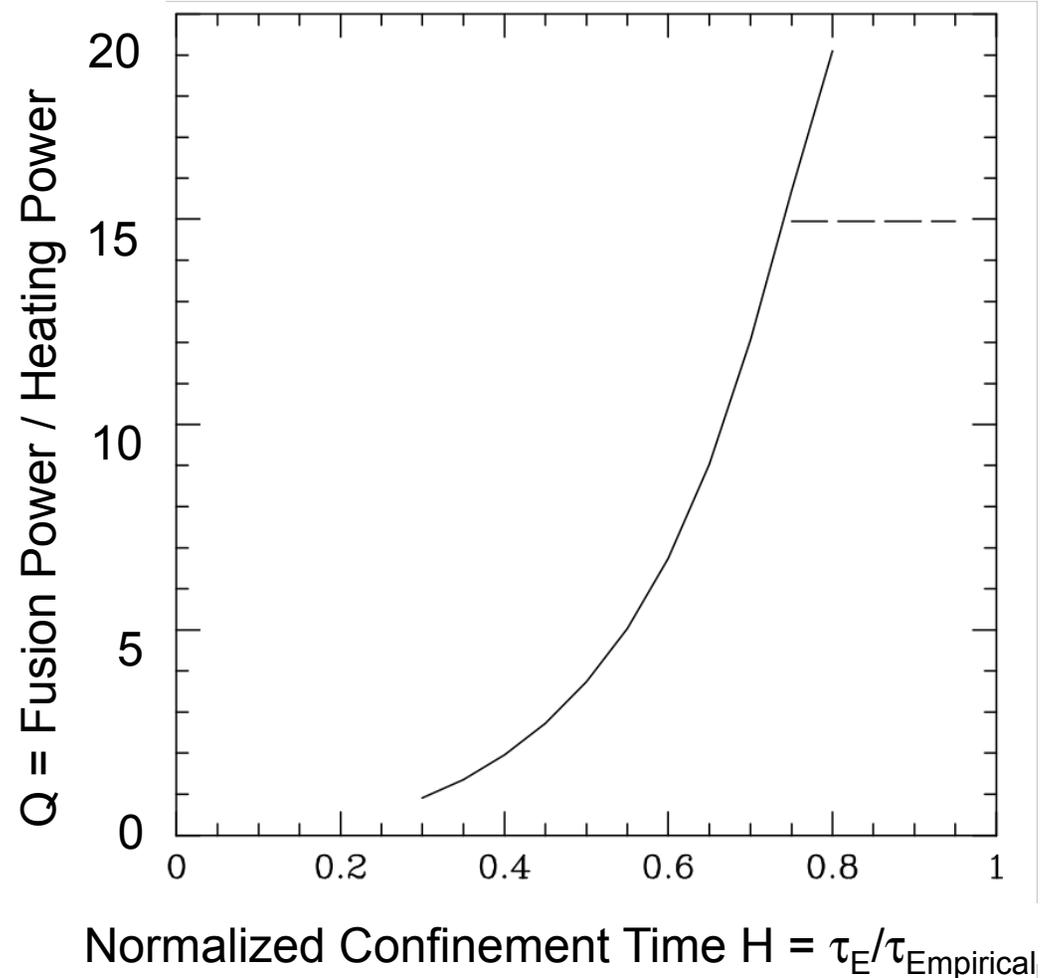
Some of the progress in computer speed can be attributed to plasma science.

Fusion performance depends sensitively on confinement

If we can find ways to reduce turbulence and improve the confinement time, then we can significantly reduce the necessary size at fixed gain Q , beta, shape, $n/n_{\text{Greenwald}}$ and thus reduce capital cost:

$$\text{Cost} \sim R^2 \sim 1 / H^{4.7}$$

a 30% improvement in confinement is a factor of 3 on the cost.



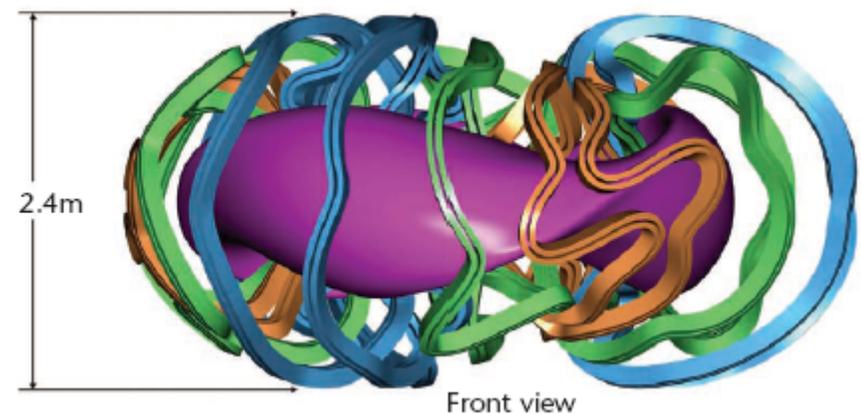
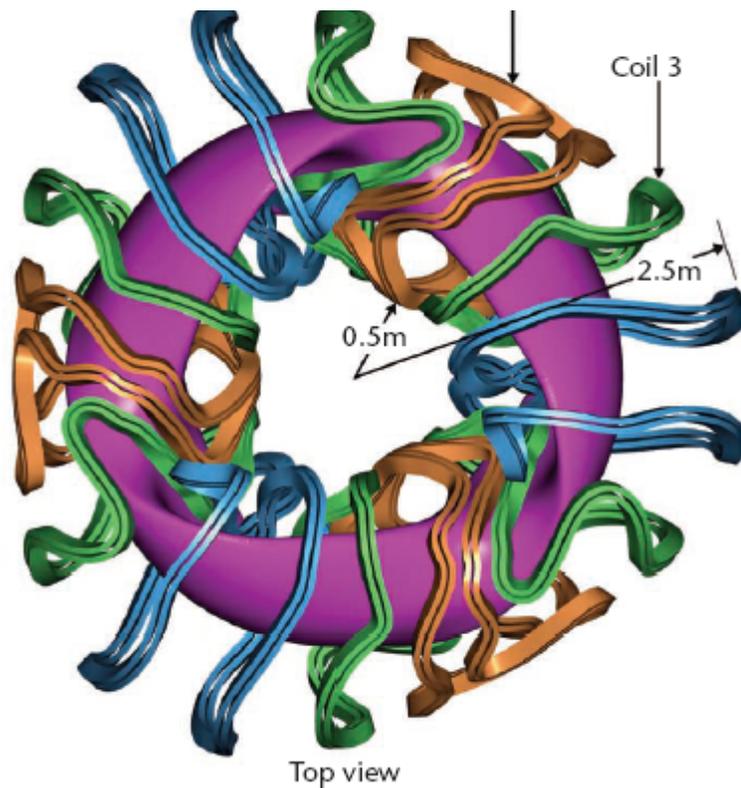
Caveats: lower bound in size set by blanket thickness. Lowering net COE best if MHD pressure limits also improve with improved confinement. Other limits also: power load on divertor & wall, ...

Interesting Ideas To Try To Improve Fusion

- * **Liquid metal (lithium) coatings on walls:** (1) protects solid wall (2) absorbs incident plasma, reduces recycling of cold neutrals back to plasma, raises edge temperature & improves global performance. TFTR found: ~2 keV edge temperature. NSTX, LTX: more lithium is better, where is the limit?
- * **Spherical Tokamaks (STs)** appear to be able to suppress much of the ion turbulence: PPPL & Culham upgrading 1 --> 2 MA to test scaling
- * **Advanced tokamaks**, methods to control Edge Localized Modes, higher plasma shaping, alternative operating regimes (reverse q profile or “hybrid”)
- * **Tokamaks spontaneously spin:** this sheared flow can reduce temperature-gradient driven turbulence. Can we enhance this with updown-asymmetric tokamaks (maybe?) or non-stellarator-symmetric **stellarators with quasi-toroidal symmetry** (maybe not?)?
- * **Many possible stellarator designs, room for further optimization:** Mynick, Xanthopoulos et al. (PRL, 2010). Proll, Helander, et al. (PRL 2012) recently demonstrated a “quasi-isodynamic” configuration in which all trapped particles have averaged good curvature. Eliminates trapped-electron instabilities, combine with lithium to eliminate turbulence?

Improved Stellarators Being Studied

- Originally invented by Spitzer ('51), the unique idea when fusion declassified ('58)
- Mostly abandoned for tokamaks in '69. But computer optimized designs now much better than slide rules. Now studying cost reductions.
- Breakthrough: Quasi-symmetry (& omnigenicity) discovered in 1990's: don't need vector \mathbf{B} symmetric exactly toroidally, $|\mathbf{B}|$ symmetric in field-aligned coordinates sufficient to be as good as tokamak.
- Magnetic field twist & shear provided by external coils, not plasma currents, inherently steady-state. Stellarator expts. find they don't have Greenwald density limit or hard beta limit & don't disrupt. Quasi-symmetry allows plasma spin to reduce turbulence? Other ways to reduce turbulence?



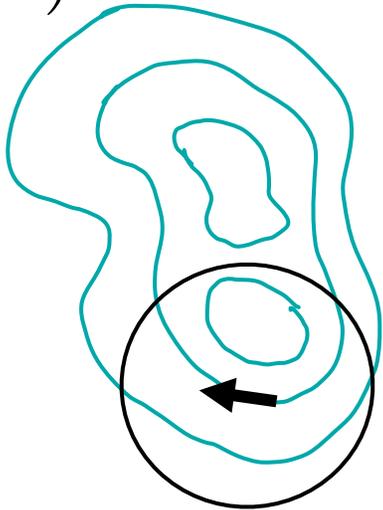
Future Advances in Robotic Manufacturing Could Significantly Reduce Cost of Fusion Energy

- * It seems that over the next 20 years there will be continued radical leaps forward in robotic manufacturing capabilities.
- * Of course this might benefit other energy sources too, but perhaps it will benefit fusion more:
- * Many key fusion components (superconducting coils, vacuum vessel) are large and complicated and can't be mass-produced at present in a factory and shipped to a power plant.
- * Instead of relying on robots in factories and shipping parts out, bring the robots to the construction site.
- * Future robots could be quickly reconfigured from one task to another: complex, high-precision tasks that at present aren't done in high enough volume to justify robotic automation could be done robotically in the future.

Effects and meaning of gyroaveraging

if low frequencies $\omega \ll$ cyclotron frequency (Ω_c),
 \rightarrow average over particle gyration, treat particles
as rings of charge in spatially varying fields

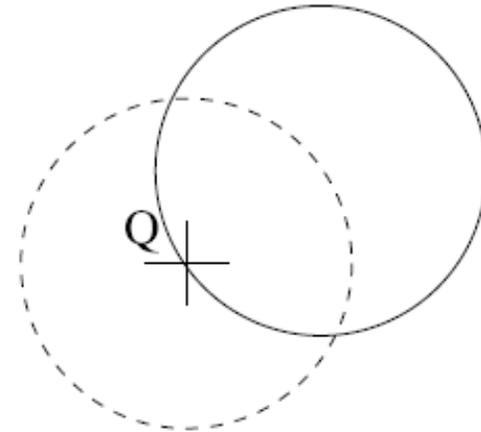
$$\phi(\mathbf{x})$$



$$\mathbf{E} \times \mathbf{B} \rightarrow -\nabla \overline{\phi} \times \mathbf{B}$$



potential averaged
around particle orbit,
no Taylor expansion.



Subtlety in solving Maxwell's equation /
quasineutrality:

When calculating charge at point Q,
have to sum over all particles whose
guiding centers are on the dashed line,
& have to include small variation of particle
density around gyro-orbit in response to
potential (\rightarrow polarization shielding)

Two main approaches to deriving gyrokinetics

Roots in early work by J. B. Taylor, E. Frieman, P. Rutherford, P. Catto.

Advantage: Analytically eliminates high frequency (ω_{pe} , Ω_{ce} , Ω_{ci}), ultra-short scale (λ_{Debye}) phenomena, focus on low frequency kinetic-MHD & drift-wave microinstabilities.

Fundamental idea: average over fast gyration of particles in magnetic field, while allowing fluctuations on the gyroradius scale. Realization that this was feasible in the presence of gyro-scale turbulence was surprising at the time. μ is still conserved even with fluctuations with $k_{\perp} \rho \sim k_{\perp} v_{\perp} / \Omega \sim 1$. **Triumph of sophisticated asymptotic analysis and physical insight.**

(1) Local “ δf ”, 2-scale, asymptotic expansion. Original method in break-through paper by Frieman and Chen 1982. Systematic expansion through 2nd order turbulence and 3th order transport equations: Sugama and Horton 1998, Abel, Schekochihin, Cowley et al. 2012. (δf here is different than the δf numerical method, which is a separate technique for noise reduction in PIC codes.)

(2) Global “*full F*”, mixed-scales, Lagrangian/Hamiltonian coordinate transformations, removing high-frequencies order by order. Roots in work by Littlejohn. Dubin & Krommes, Hahm, Brizard, Scott. Brizard and Hahm, Rev. Mod. Phys. 2007. Sugama 2000.

$$\epsilon \sim \frac{\omega}{\Omega} \sim \frac{f}{L} \sim \frac{f^2}{f_0} \sim \frac{e\Phi}{T_0} \quad \text{but} \quad h_{\perp} \rho \sim \mathcal{O}(1)$$

$$\sim \frac{h_{\parallel}}{h_{\perp}}$$

Introduce two scales, L & h_{\perp} :

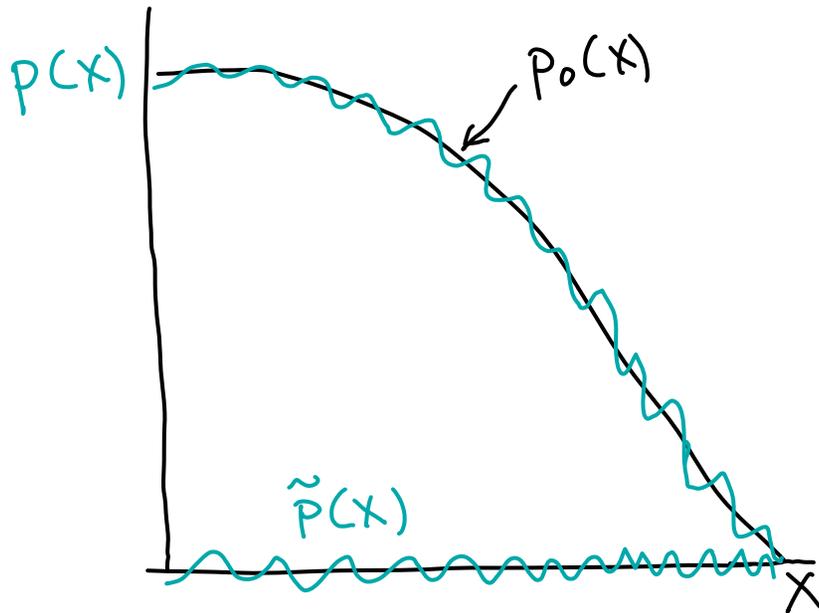
$$\nabla \rho = \underbrace{\nabla p_0}_{\frac{p_0}{L}} + \underbrace{\nabla \tilde{p}}_{\sim h_{\perp} \tilde{p}}$$

even though

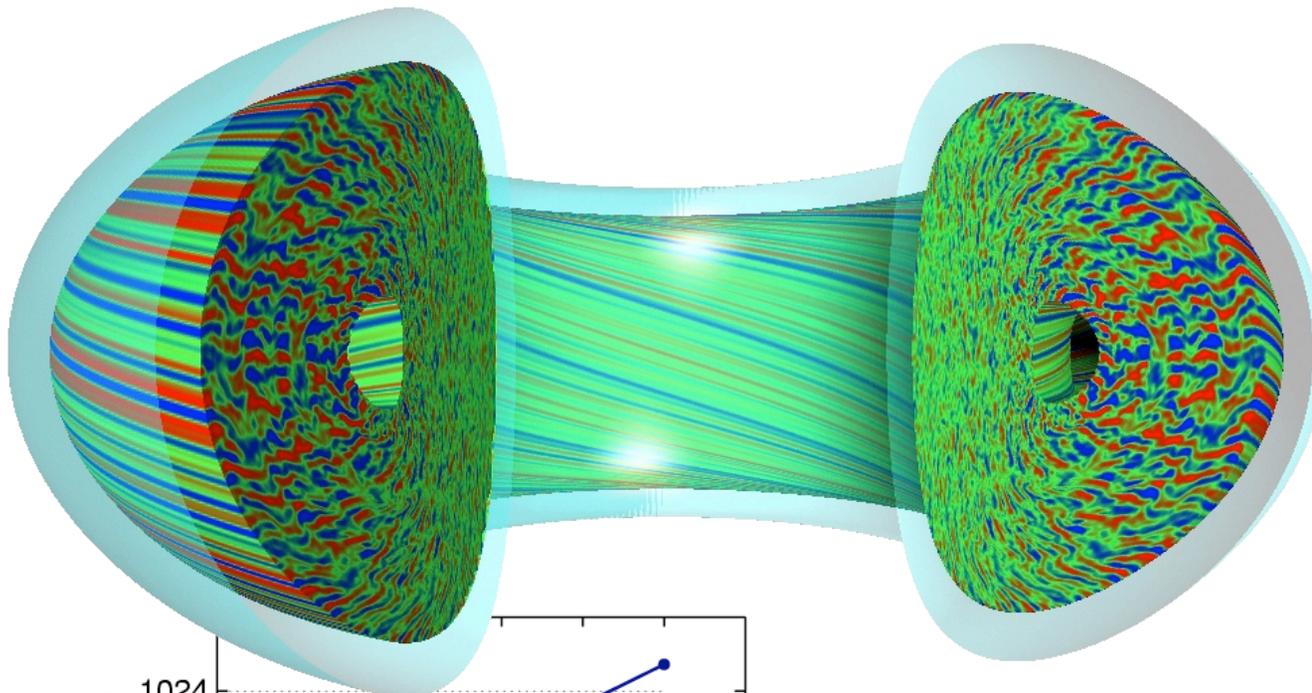
$$\tilde{f} \ll f_0$$

$$\nabla \tilde{f} \sim \nabla f_0$$

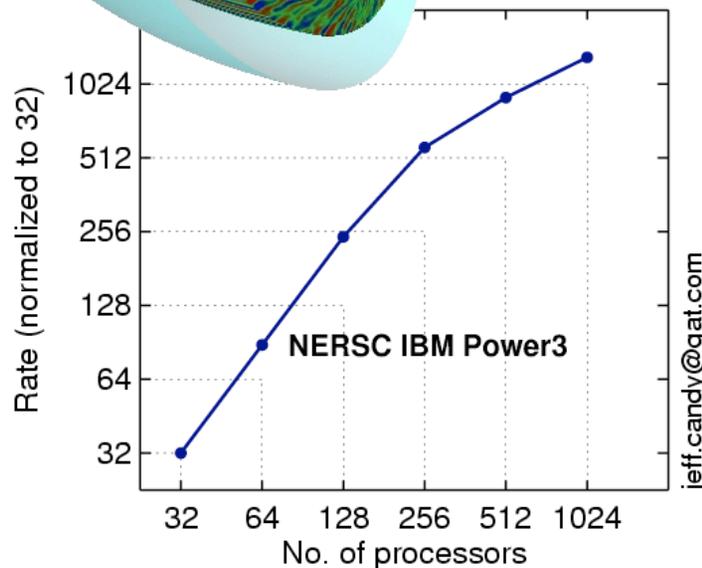
perturbations can locally
flatten gradients, nonlinearities
important



Keep arbitrary $h_{\perp} \rho$,
FLR to all orders



(old figure, these codes now scale to 100k+ processors for some problems.)



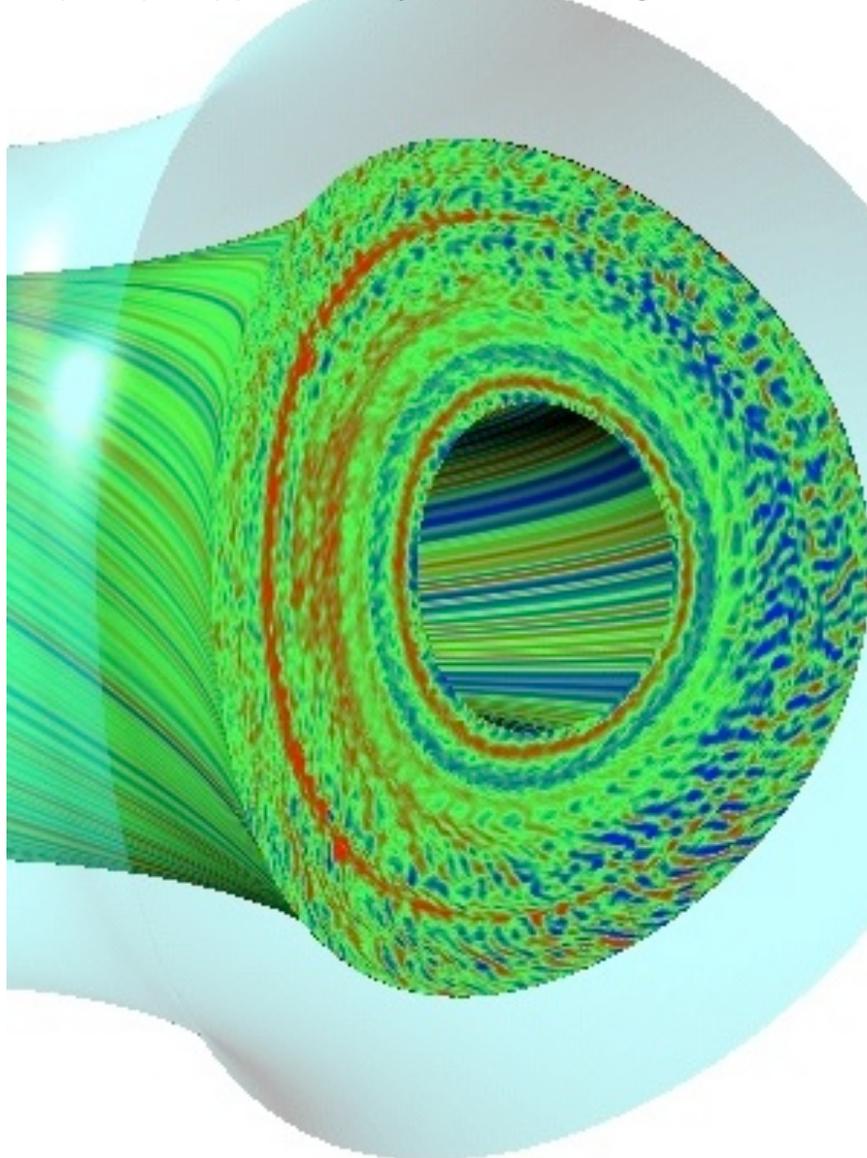
GYRO gives superlinear scaling up to 1024 processors on FIXED problem size.



Comprehensive 5-D computer simulations of core plasma turbulence being developed by Plasma Microturbulence Project. Candy & Waltz (GA) movies shown: d3d.n16.2x_0.6_fly.mpg & supercyclone.mpg, from http://fusion.gat.com/comp/parallel/gyro_gallery.html (also at <http://w3.pppl.gov/~hammett/refs/2004>).

Fairly Comprehensive 5-D Gyrokinetic Turbulence Codes Have Been Developed

small scale, small amplitude density fluctuations (<1%) suppressed by reversed magnetic shear



- Solve for the particle distribution function $f(r, \theta, \alpha, E, \mu, t)$ (avg. over gyration: 6D \rightarrow 5D)
- 500 radii x 32 complex toroidal modes (96 grid points in real space) x 10 parallel points along half-orbits x 8 energies x 16 v_{\parallel}/v
12 hours on ORNL Cray X1E with 256 MSPs
- Realistic toroidal geometry, kinetic ions & electrons, finite- β electro-magnetic fluctuations, collisions. Sophisticated algorithms.

Continuum/Eulerian Approach to Electromagnetic Gyrokinetic Turbulence

GS2 (Dorland & Kotschenreuther) <http://gs2.sourceforge.net>

GENE (Jenko, Gorler, Xanthopoulos, and others) <http://gene.rzg.mpg.de>

GYRO (Candy & Waltz) <https://fusion.gat.com/theory/Gyro>

These codes are widely used by many to study plasma turbulence in fusion devices, and are currently the most comprehensive gyrokinetic codes available:

- Gyrokinetic ions (multiple species) & adiabatic/drift-kinetic/gyrokinetic electrons
 - Trapped and passing electrons (and ions) for Trapped Electron Mode
 - Complete linearized Landau collisions or gyro-averaged with model field terms
 - Finite beta magnetic fluctuations as well as electrostatic fluctuations (important for kinetic-ballooning modes, microtearing, magnetic flutter contribution to transport)
 - General shaped tokamak geometry
 - Equilibrium ExB and parallel velocity shear
 - Finite- ρ_* non-local/global effects (profile shear stabilization, nonlocal transport)
- (except for GS2, which focuses on the local limit)

Nevertheless, a lot of interesting work remains to be done: more tests against experiments, particle & momentum transport, transport barrier formation, shaping effects, understand scalings, couple to transport codes for complete predictive ability. Edge turbulence difficult, need new codes.

Overview of Algorithms used in GK continuum codes

- All 3 of the major, comprehensive gyrokinetic codes that are being widely used for comparisons to experiments are “continuum” (Eulerian) codes, essentially solving on a grid in 5D phase space, to find time evolution of the 5-D particle distribution function $f(\mathbf{x}, \mu, v_{\parallel}, t)$
- “typical” grid (local, moderate-rez) 96x48x32 spatial, 10x20 velocity, x 3 species for 10^4 time steps. Convergence studies find velocity grid is adequately resolved.
- (PIC-based (Lagrangian) gyrokinetic codes are being used some. Also some semi-Lagrangian codes. Not time to discuss here.)
- Highly optimized for particular problems: drift-wave microturbulence in core region of tokamaks (and certain astrophysics turbulence and reconnection problems). $E \times B$ nonlinearity strong (mixing in space), E_{\parallel} nonlinearity weak (particle acceleration).
- Continuum codes are using a mixture of advanced algorithms: (high order) finite difference in some directions, (pseudo-)spectral in other directions. High order Gaussian algorithms for velocity integrals. Various types of implicit/explicit, or fully explicit time advance.
- All codes use highly-efficient field-aligned coordinates, essential for turbulence with $k_{\parallel} / k_{\perp} \sim \rho_* \sim \rho / a \sim 10^{-2} \text{ -- } 10^{-3}$.

Algorithms used in GS2

- GS2 (Dorland, Kotschenreuther, et al.): First nonlinear GK code that could handle magnetic fluctuations. Based on earlier linear GK code by Kotschenreuther: All linear dynamics fully implicit in time. (Crank-Nicolson) Interesting trick to make implicit solve practical.
- Numerical implicit algorithm reproduces analytic bounce-averaged result even when electron dynamics is not resolved. (Uses equivalent (E, μ) coordinates).
- 3rd order Adams-Bashforth for time advance of nonlinear terms.
- Nonlinear terms evaluated with pseudo-spectral methods.
- Thin flux tube domain to focus on small scale turbulence.
- Finite difference along magnetic field lines, (pseudo-)spectral for dynamics perpendicular to field lines. Parallel linear terms: a 2nd order version of Compact Finite Differencing (Beam & Warming). Unusual: time and space differenced symmetrically, bi-diagonal advection matrices easy to invert, but phase velocity increases with $k \Delta x$ so maximum $\omega \sim \pi/\Delta t$: always have to be implicit.
- Gaussian methods used for numerical velocity integrals: Given p points in general, can fit a polynomial to them to get order p accuracy. Gaussian methods optimizes the location of the points to give $2p-1$ order accuracy.
- Trapped and passing parts of velocity space are integrated separately, to allow (near) discontinuities across the trapped-passing boundaries.

Kotschenreuther's Implicit Trick

- The discretized linear gyrokinetic equation (after Fourier-transforming the two perpendicular directions, as done in GS2) still involves the distribution function f at $N_{v\parallel} \times N_{v\perp} \times N_z$ grid points (where N_z is the number of grid points along the field line). A direct inversion would lead to the need to solve an $(N_{v\parallel} \times N_{v\perp} \times N_z) \times (N_{v\parallel} \times N_{v\perp} \times N_z)$ matrix problem, an expensive task. The LU factorization of this matrix is not sparse (the electric field depends on velocity integrals of f).
- Kotschenreuther (CPC, 1995) found a trick to exactly factor this into much easier problems: first solving an $N_z \times N_z$ matrix problem (involving the plasma response at position i to a perturbation at position j) to implicitly find the electric field at the future time step, and then solving $N_{v\parallel} \times N_{v\perp}$ matrix problems, but where each is a simple bidiagonal matrix (or more generally, a simple banded matrix) of size $N_z \times N_z$, to solve for the distribution function at the future time.
- (Details described in E. Belli's 2006 Ph.D. thesis, which studied possible iterative solvers for the intermediate step. It is challenging to find efficient iterative solvers for hyperbolic problems, need good preconditioners and algorithms (multigrid, FFTs).)

Time-advancement Algorithms used in GENE and GYRO

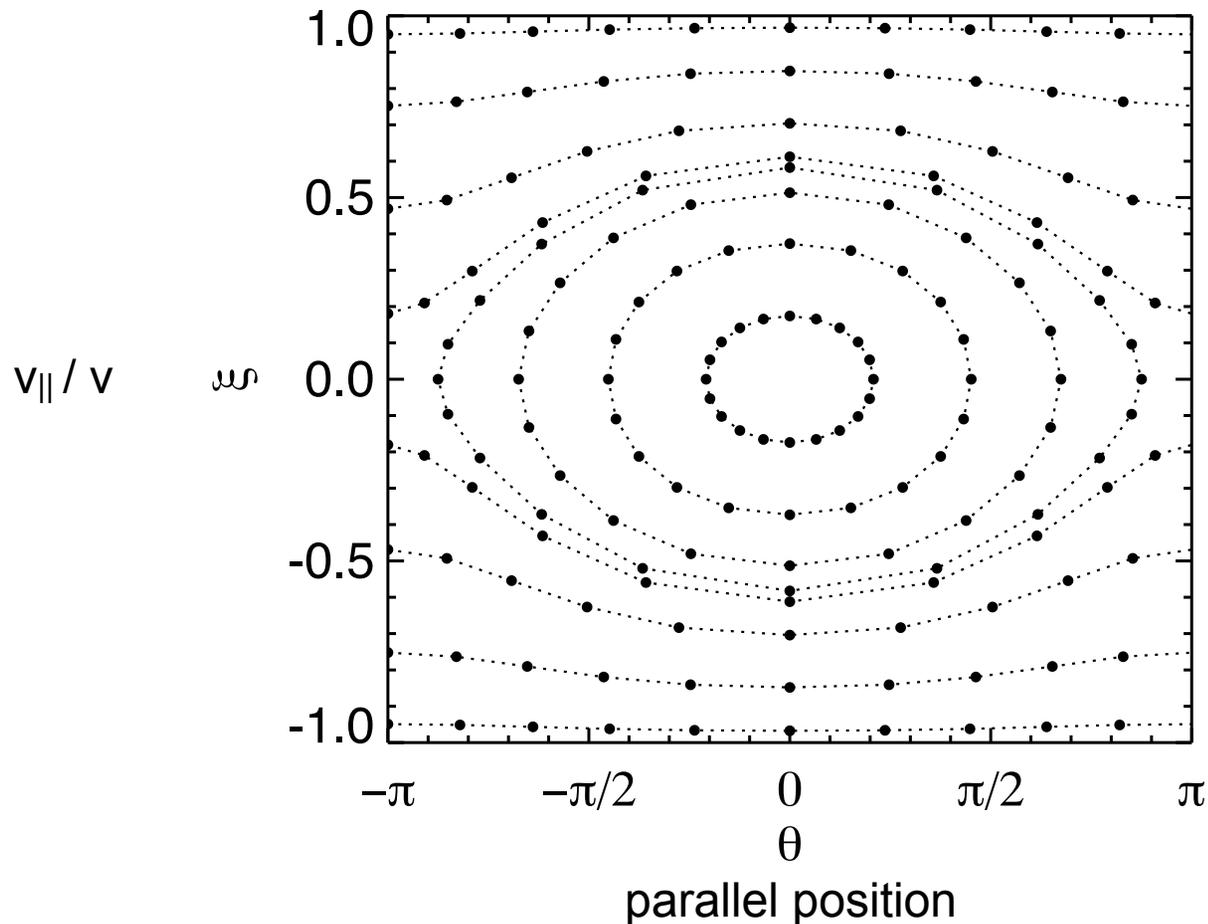
- GYRO and GENE web sites have extensive documentation. GYRO technical documentation in particular provides extensive documentation of its algorithms.
- GENE (Jenko, Goerler, et al.): Fully explicit time advance. Requires smaller Δt to resolve fast parallel electron dynamics, but makes the algorithms more local and easier to parallelize. Highly optimized to work on very large numbers of processors.
- GENE options include 6-stage 4th-order Runge-Kutta with larger stable time step.
- GYRO: IMEX-RK-SSP Implicit/Explicit Runge-Kutta Strong-Stability-Preserving time advance options, with implicit parallel electron dynamics. Uses UFMPACK/MUMPS direct sparse matrix solvers for some of the solvers.
- Designed flexibly so different time-step algorithms can be implemented fairly easily.

Other Algorithms used in GENE and GYRO

- GYRO and GENE also use high-order Gaussian integration methods, at least for the energy coordinate.
- GYRO and GENE do similar high-order finite difference of radial and parallel motion, including upwind biasing, or equivalently, some hyper diffusion $\propto \nabla^4$ or ∇^6 that introduce dissipation near grid scale. Equivalent to 3rd or 5th order upwind differencing.
- GENE also has a “hypercollision” term to represent damping at small scales in velocity space.
- P. Morel, A. Bañón, et al. 2012 GENE paper explore a Germano-based dynamic subgrid model. (more of a physics model than a numerical algorithm per se)
-

GYRO “equal bounce-time grid”

- GENE uses (v_{\parallel}, μ) coordinates, GYRO essentially uses (E, μ) coordinates (or $(E, \mu/E)$ coordinates) but uses an unusual parallel discretization, which gives equal spacing in “bounce-time”. This provides uniform accuracy, avoiding a loss of accuracy problem near the turning points with other (E, μ) coordinates. It somewhat complicates the collision operator and field solver, which he deals with by other (finite-element-based) methods.



Nonlinear Algorithms used in GENE and GYRO

- GYRO: First continuum code with non-local/global capabilities. GENE can now do this as well.

Many problems in physics can be written in terms of Poisson brackets, because of the underlying Hamiltonian structure of the problem. The Vlasov equation can be written as

$$\frac{\partial f}{\partial t} = \{H, f\} = \frac{\partial H}{\partial x} \frac{\partial f}{\partial p} - \frac{\partial H}{\partial p} \frac{\partial f}{\partial x}$$

as can the gyrokinetic equation. For example, the $\mathbf{E} \times \mathbf{B}$ nonlinearity can be written as $\propto \hat{b} \times \nabla \phi \cdot \nabla f \propto \{\phi, f\}$. One can multiply the Poisson bracket by either argument (H or f) and integrate over all phase space and it vanishes, leading to energy conservation and a quantity related to entropy conservation (or to enstrophy in 2-D hydrodynamics).

Nonlinear Algorithms used in GENE and GYRO (2)

- The pseudo-spectral method used for the nonlinear terms in GS2 exactly preserve these invariants. (Dealiasing removes just enough energy to avoid the aliasing errors that would have generated excess energy, and restores energy conservation.)
- The famous Arakawa finite difference algorithm (described in a special issue of JCP on their top algorithm papers) also preserves these invariants.
- GYRO is spectral in the toroidal direction and finite difference in the radial direction (as is the global version of GENE). They use a special mixed finite-difference/spectral version of Arakawa that also preserves these invariants.
- Should note that conservation properties are not the only important criterion and are not all equally important. Energy conservation is more fundamental. The entropy-like invariant will be lost when even a small amount of dissipation is included and the solution cascades to the dissipation scale (or to scales smaller than can be resolved numerically).

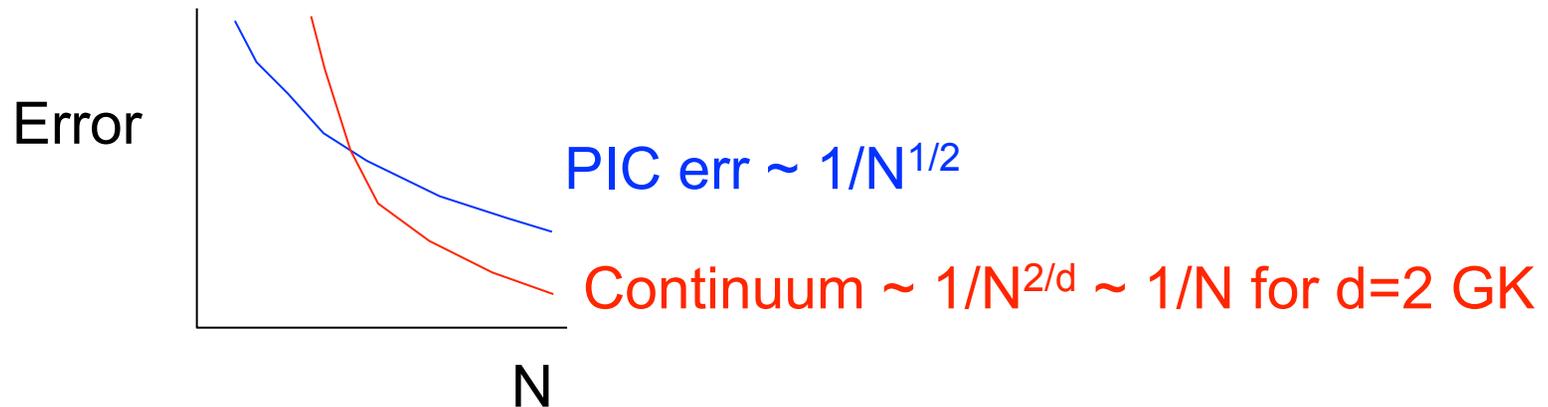
Major Theoretical & Algorithmic Speedups

relative to simplest brute force PIC algorithm, fully resolved ($\Delta t \sim 1/\omega_{pe}$, $\Delta x \sim \lambda_{De}$), for ITER $\rho_* = \rho/a \sim 1/700$

- Nonlinear gyrokinetic equation
 - ion polarization shielding eliminates plasma freq. $\omega_{pe}/\Omega_{ci} \sim m_i/m_e$ x10³
 - ion polarization eliminates ρ_e & Debye scales $(\rho_i/\lambda_{De})^3$ x10⁵
 - average over fast ion gyration (& field-aligned), $\Omega_{ci}/\omega_* \sim 1/\rho_*$ x10³
- Continuum or δf PIC, reduces noise, $(f_0/\delta f)^2 \sim 1/\rho_*^2$ x10⁶
- Field-aligned coordinates (nonlinear extension of ballooning coord.)
 - $\Delta_{||} / (\Delta_{\perp} q R / a) \sim a / (q R \rho_*)$ x70
- Flux-tube / Toroidal annulus wedge, \downarrow simulation volume
 - $k_{\theta} \rho_i = 0, 0.05, 0.1, \dots, 1.0$
 $n = 0, 15, 30, \dots, 300$ (i.e., 1/15 of toroidal direction) x15
 - $L_r \sim a/5 \sim 140 \rho \sim 10$ correlation lengths x5
- High-order / spectral algorithms in 5-D, $2^5 \times 2$ x64
- Implicit electrons x5-50
- **Total combined speedup of all algorithms** **x10²³**
- Massively parallel computers (Moore's law 1982-2007) x10⁵

PIC & Continuum algorithm comparisons

- Very different algorithms (Lagrangian / Monte-Carlo random sampling vs. Eulerian / optimized integration) with different numerical properties. Very useful to have independent algorithms to cross check each other, particularly for the types of difficult problems we study.
- Modern “Continuum” codes use a range of advanced CFD algorithms (pseudo-spectral, high-order upwind, finite elements, discontinuous Galerkin, Arakawa,...) not just simple 2cd order methods on a grid.
- Both PIC & continuum codes need similar spatial resolution for electromagnetic/gravitational fields. But use different methods for velocity integrals to calculate charge/current densities needed for fields.
- Error vs. N (# particles/cell or velocity grid points) (assuming simple 2cd order integration for continuum)



- Continuum appears asymptotically more efficient for gyrokinetics ($d=2$) and full Vlasov ($d=3$ velocity space). PIC may be better for problems with large “signal” where larger noise can be tolerated. Continuum may be better for problems where low noise is needed (e.g. near marginal stability).
- Some PIC simulations of reconnection or tokamak edge plasmas now with 1000 particles/cell \rightarrow 6000 quantities/cell (x & v for each particle). Usually use “finite-size-particles” with smoothing of fields over 3 adjacent cells in each direction (similarly, “force-softening” in N-body tree codes) \rightarrow equivalent to $\sim 56^3$ velocity space per resolved region.

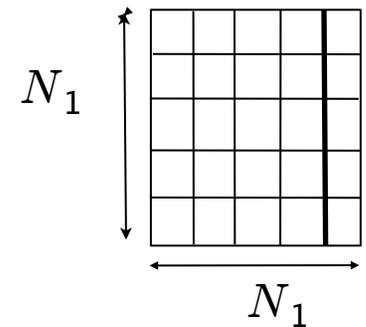
PIC & Continuum algorithm comparisons

Both PIC & continuum codes need comparable spatial resolution to represent electromagnetic/gravitational fields. But use different methods to do velocity integrals to calculate charge/current densities needed to find fields.

Convergence rates for d-dimensional integral, where $N = N_1^d$:

2cd order (midpoint) Eulerian: $\epsilon \sim (\Delta x)^2 \sim \frac{C_2}{N_1^2} \sim \frac{C_2}{N^{2/d}}$

Monte Carlo sampling: $\epsilon \sim \frac{C_{MC}}{N_{\text{particles}}^{1/2}}$



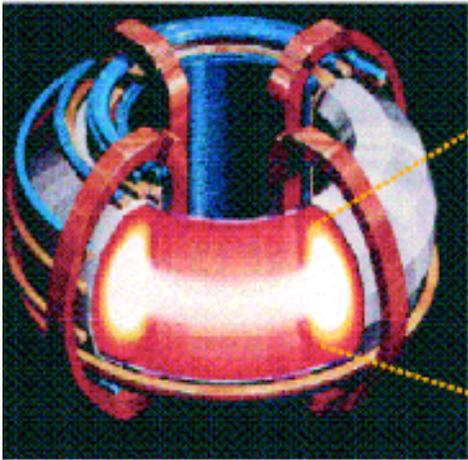
Continuum methods appear competitive/better for $d \leq 4$.

Caveats:

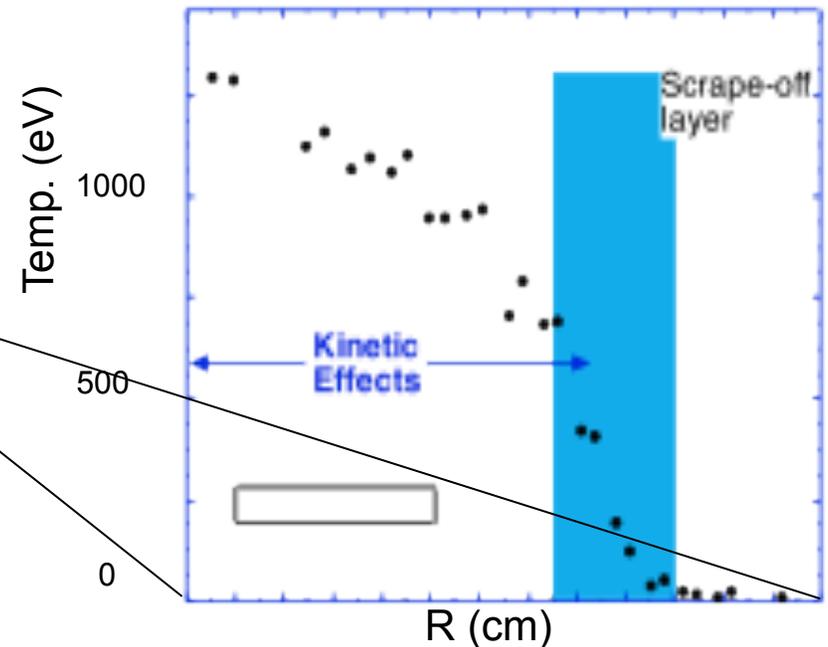
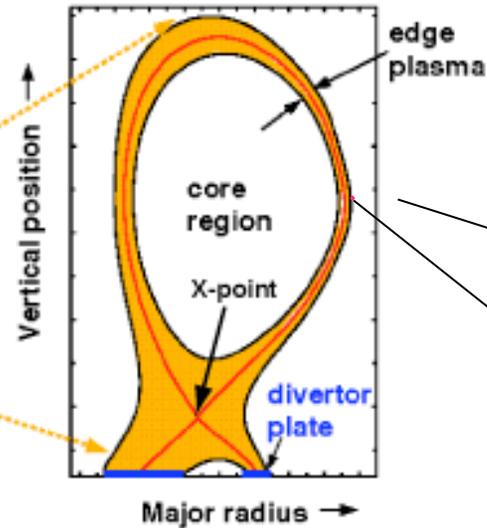
- (1) coefficients highly problem-dependent
- (2) Don't need same resolution in all directions,
- (3) Modern continuum codes use higher-order/spectral methods: DG uses Gaussian integration with optimized non-uniform spacing of p points per Δx per cell: $\epsilon \sim (\Delta x)^{2p}$
- (4) Focused here on velocity integration methods, but algorithms also differ in how they solve particle motion or solve for distribution function.

Edge region very difficult

Tokamak magnetic fusion device



Simulated edge-plasma region



Edge pedestal temperature profile near the edge of an H-mode discharge in the DIII-D tokamak. [Porter2000]. Pedestal is shaded region.

Major extensions to gyrokinetic codes needed to handle additional complications of edge region of tokamaks (& stellarators):

open & closed field lines, steep gradients near beta limit, electric & magnetic fluctuations, strong shear-flow layers, steep-gradients and large amplitude fluctuations, positivity constraints, wide range of collisionality, non-axisymmetric RMP coils, plasma-wall interactions, strong sources and sinks in atomic physics.

Future ideas ?

- Interesting future possibilities: Recent versions of DG (discontinuous Galerkin) algorithms that also preserve the quadratic invariants of the Poisson bracket (with centered fluxes), and even with upwind fluxes still preserve energy conservation. Might be able to merge DG with limiters to more robustly handle the large amplitude fluctuations of the edge region while still having good conservation properties.
- More ideas?

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- * Brief description of 3 of the most widely used gyrokinetic codes, the continuum codes GS2, GYRO, and GENE. Example of results from such codes.
- * Description of various algorithms used in these codes.

Key references:

- This talk: <http://w3.pppl.gov/~hammett/talks>
- The Center for the Study of Plasma Microturbulence (CSPM), a DOE Scientific Discovery Through Advanced Computing (SciDAC) project
<https://fusion.gat.com/theory/Cspm>
- The 3 major comprehensive, continuum gyrokinetic codes:
 - GYRO: <https://fusion.gat.com/theory/Gyro>
 - GENE: <http://gene.rzg.mpg.de>
 - GS2: <http://gyrokinetics.sourceforge.net>
- Krommes (2012), “The Gyrokinetic Description of Microturbulence in Magnetized Plasmas”, Annual Reviews of Fluid Mechanics.

Selected Further References

- Extensive fusion library, fusion history, reactor design studies, etc: <http://fire.pppl.gov>
- This talk: <http://w3.pppl.gov/~hammett/talks>
- Center for the Study of Plasma Microturbulence <https://web.gat.com/theory/Cspm>
- DOE Scientific Discovery Through Advanced Computing <http://www.scidac.gov>
- GYRO code and movies <http://fusion.gat.com/comp/parallel/gyro.html>
& <http://w3.pppl.gov/~hammett/talks/2004>
- GENE gyrokinetic turbulence code <http://gene.rzg.mpg.de>
- GS2 gyrokinetic code <http://gs2.sourceforge.net>
- Center for Multiscale Plasma Dynamics <http://cmpd.umd.edu/>
- My gyrofluid & gyrokinetic plasma turbulence references: <http://w3.pppl.gov/~hammett/papers/>
- "ENDING THE ENERGY STALEMATE: A Bipartisan Strategy to Meet America's Energy Challenges", The National Commission on Energy Policy, December 2004. <http://www.energycommission.org/>
- "Anomalous Transport Scaling in the DIII-D Tokamak Matched by Supercomputer Simulation", Candy & Waltz, Phys. Rev. Lett. 2003
- "Burning plasma projections using drift-wave transport models and scalings for the H-mode pedestal", Kinsey et al., Nucl. Fusion 2003
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