Intro to Simulations & Bad-Curvature-Driven Instabilities in Magnetic Confinement Fusion

---

Greg Hammett
Princeton Plasma Physics Laboratory
Astro-Fusion-Plasma Workshop, PCTS April 8-12 2013

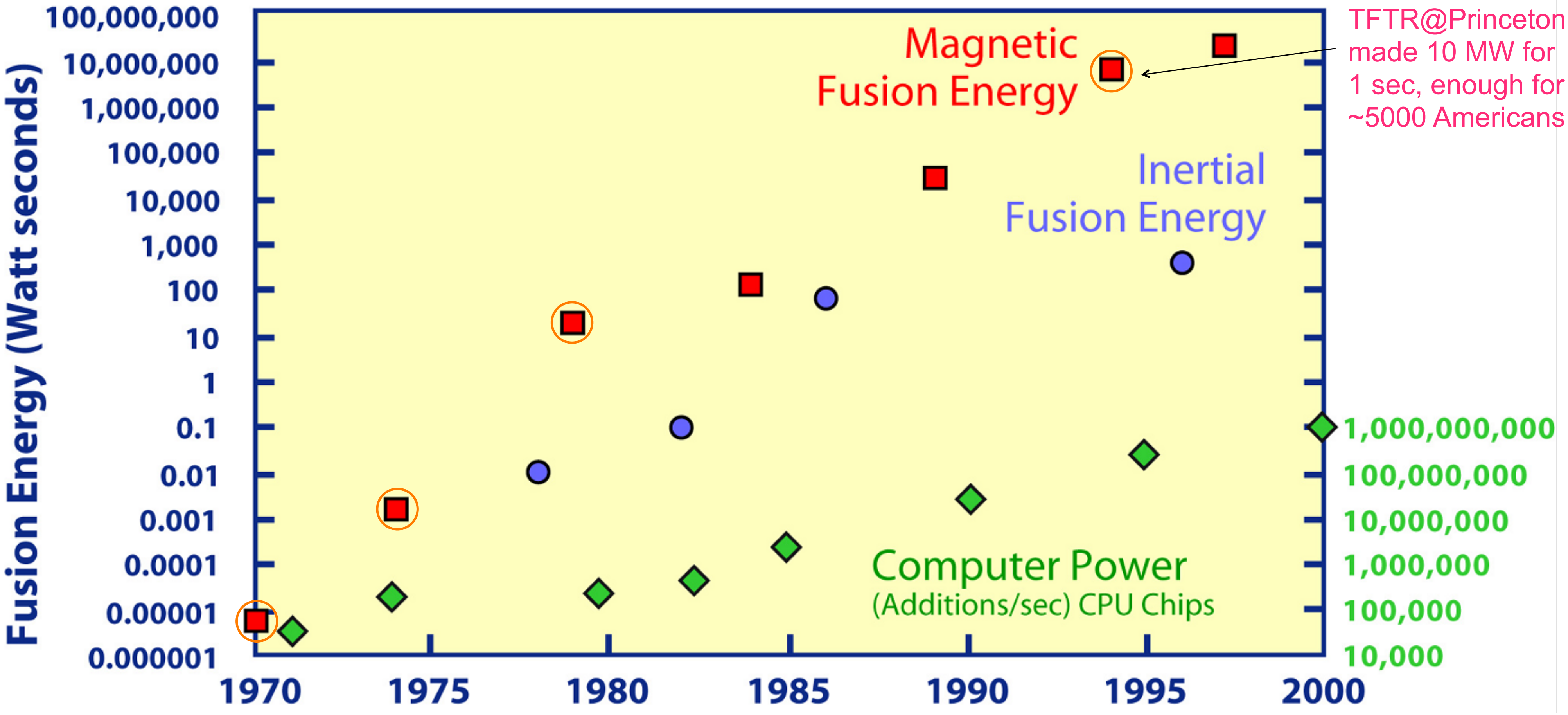
Fusion Intro Outline

- My perspective on status of fusion energy research
- Physical picture of instabilities driven by “bad curvature” / effective gravity
- Various methods to improve fusion being studied
- Simulation techniques developed in fusion that may be useful in astro/solar
  - comprehensive 5D continuum simulations of gyrokinetic microturbulence fairly successful in main part of fusion plasmas
  - new project exploring advanced continuum algorithms (discontinuous Galerkin, ...) for edge region, Vlasov-Boltzmann / Hamiltonian problems (A. Hakim poster)

My Perspective on Fusion Energy

- Need to pursue many alternative energy sources. All have tradeoffs & uncertainties. Challenging to supply all energy needed in the long term. Energy demand expected to triple throughout the century as poor countries continue to develop.
- Fusion energy is hard, but it’s an important problem, we’ve been making progress, and there are interesting ideas to pursue that could make it more competitive:
  - “advanced tokamak” regimes, spherical torus
  - spontaneous spinning reduces turbulence?
  - Liquid metal walls: handle power loads better, “black hole” absorbing wall reduces cold neutral recycling & improves performance. LTX, NSTX, …
  - Stellarators: After 40+ years of research, a hidden symmetry discovered that improves performance
  - other ideas, long shots.

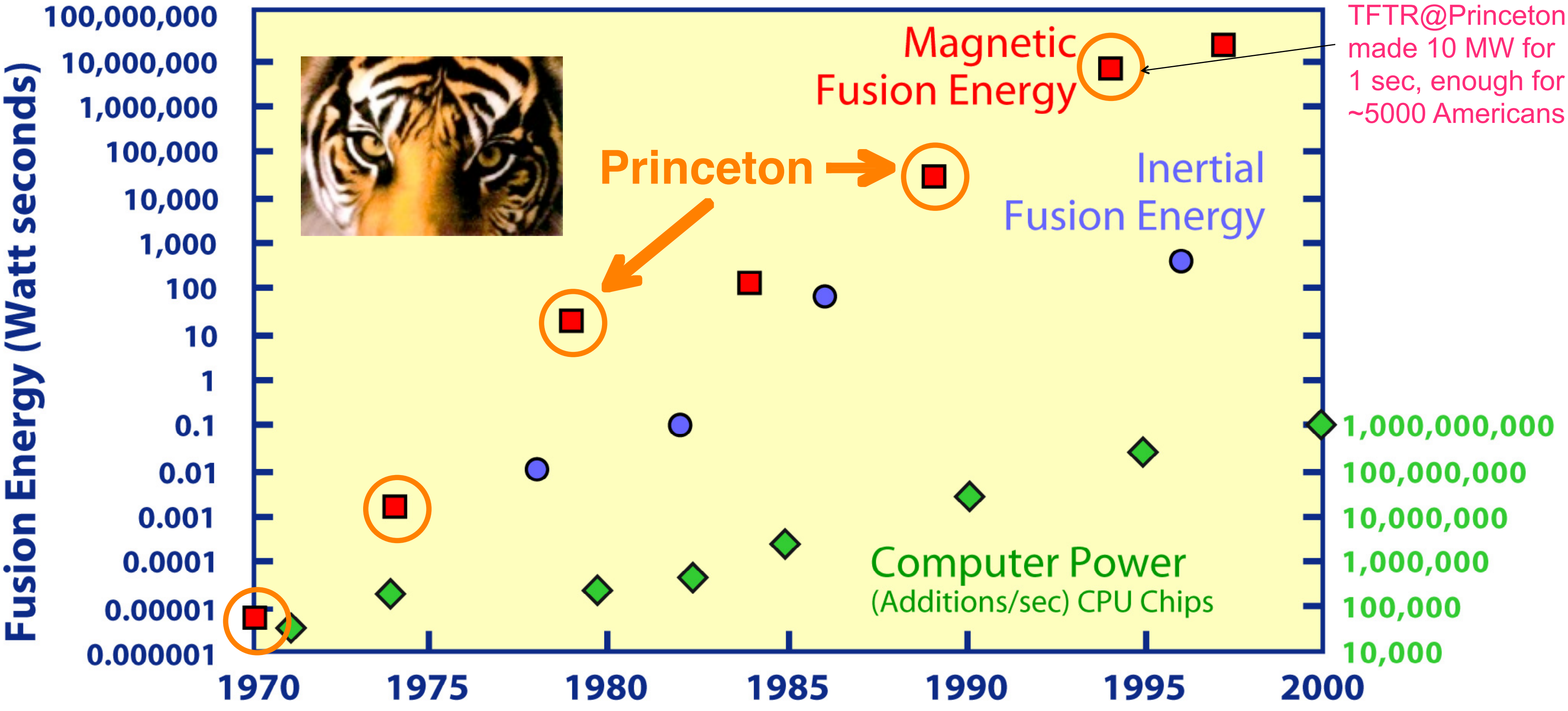
Progress in Fusion Energy has Outpaced Computer Speed



TFTR@Princeton made 10 MW for 1 sec, enough for ~5000 Americans

Some of the progress in computer speed can be attributed to plasma science.

Progress in Fusion Energy has Outpaced Computer Speed

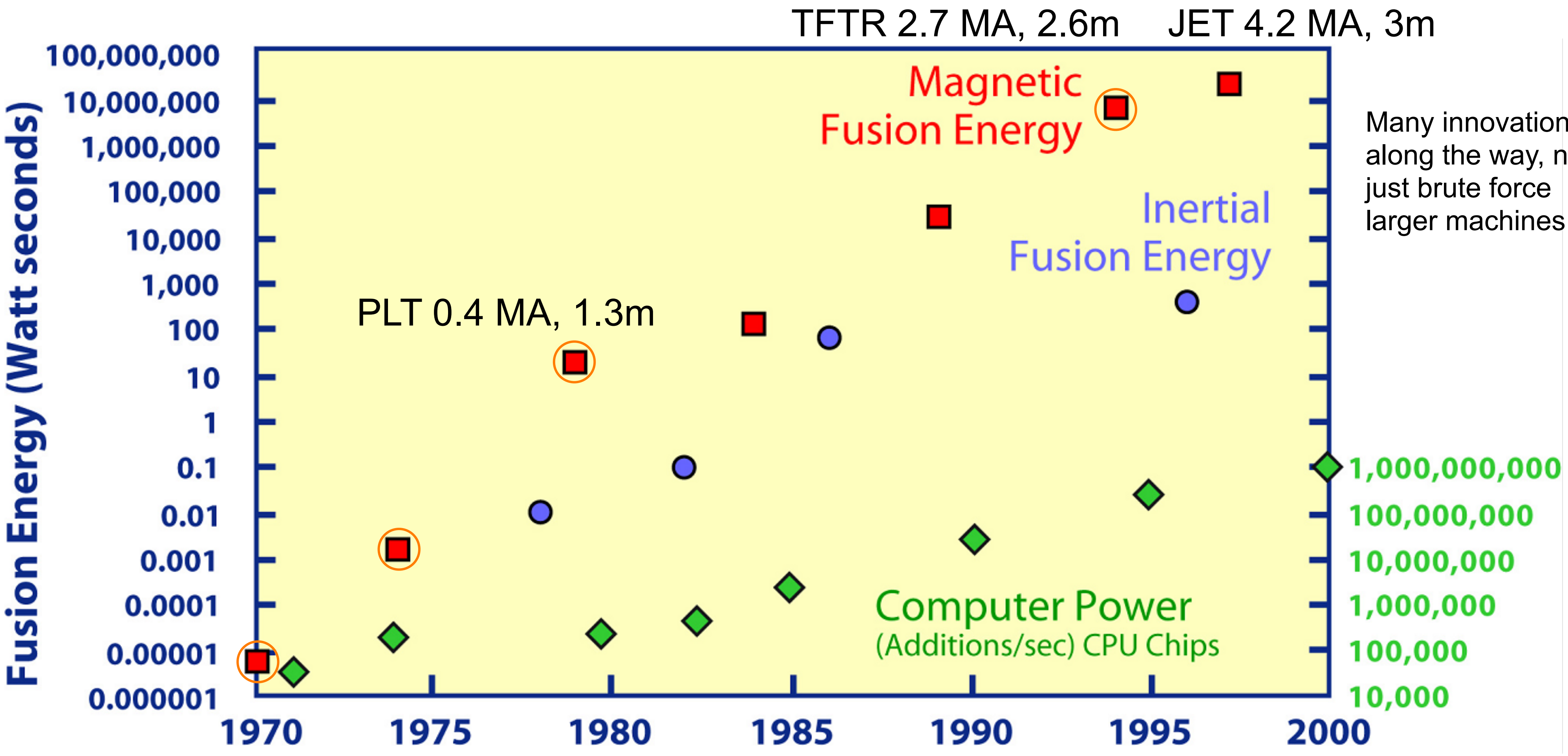


TFTR@Princeton made 10 MW for 1 sec, enough for ~5000 Americans

Some of the progress in computer speed can be attributed to plasma science.

Progress in Fusion Energy has Outpaced Computer Speed

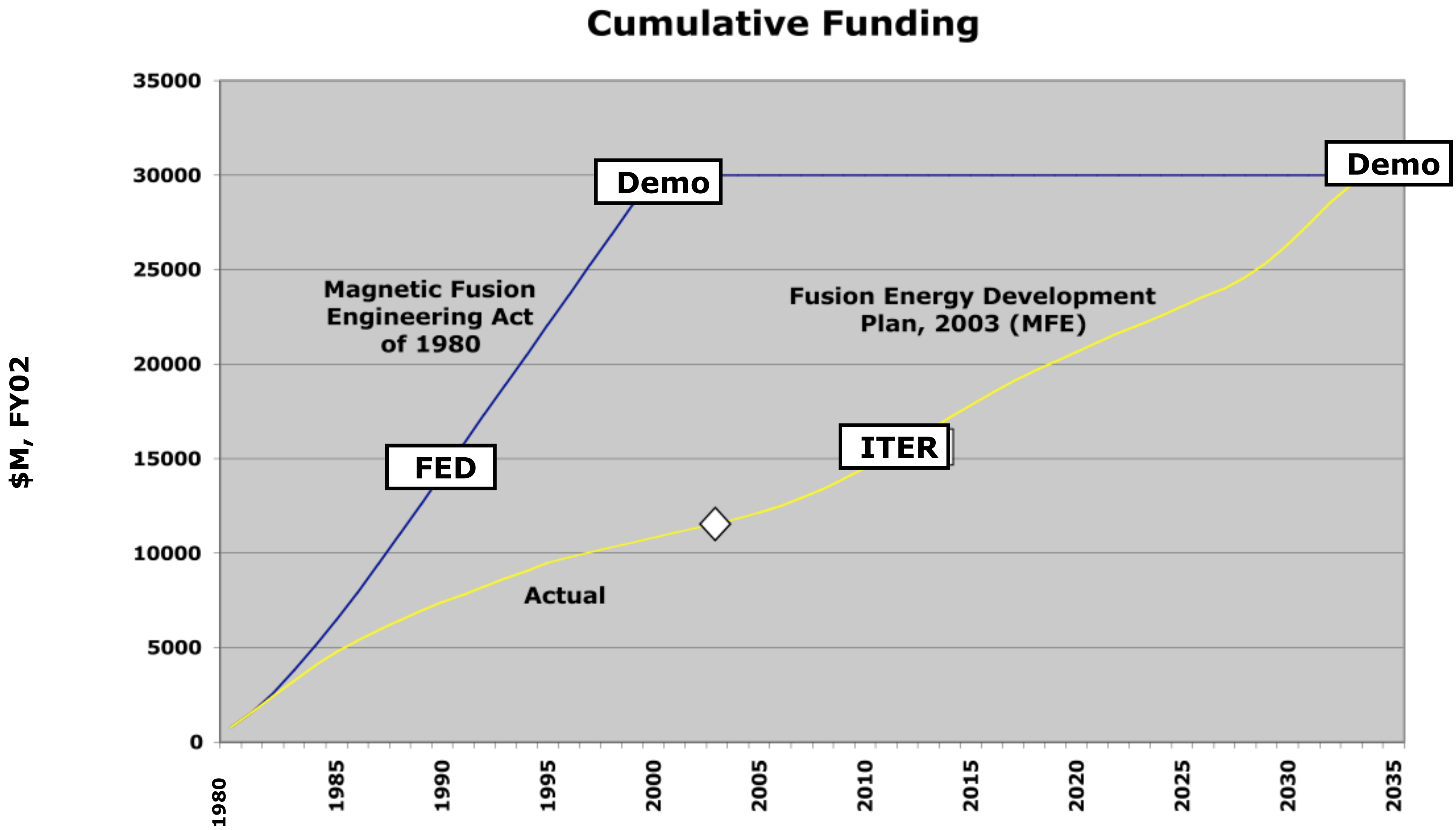
→ ○
ITER 15 MA, 6.2m



Many innovations along the way, not just brute force larger machines.

ITER goal: 200 GJ/pulse (500 MW = 30 x JET's power 16 MW, for 400x longer), 10^7 MJ/day of fusion heat).
NIF goal: 20 MJ/pulse (and /day) of fusion heat.

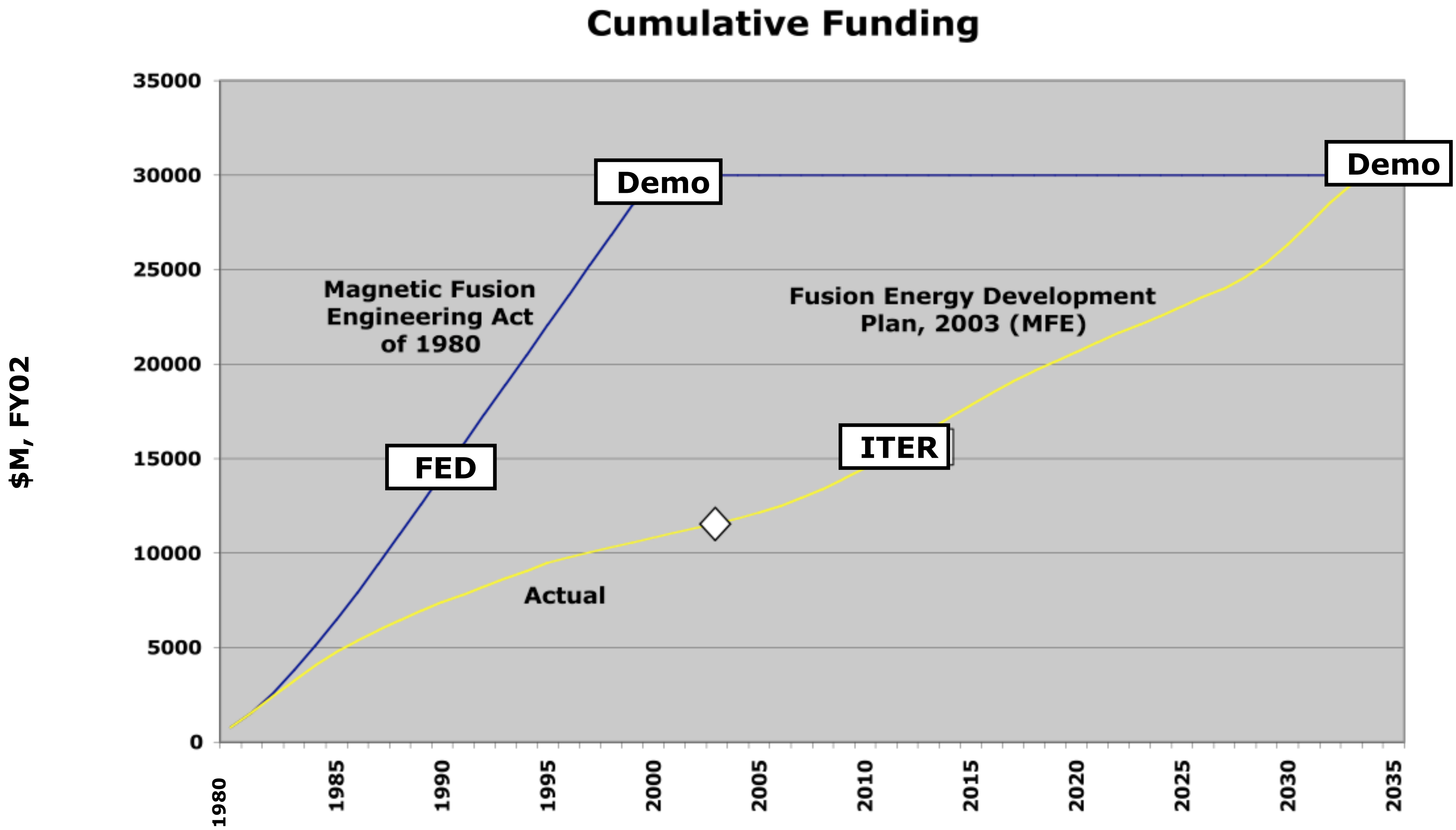
Fusion Could Be Done In A Shorter Time Scale If Sufficient Budget Eventually Provided



$30-$90B development cost is tiny compared to >$100 Trillion energy needs of 21st century & potential costs of global warming. (Apollo program ~ $100B.) Still 40:1 payoff after discounting 50+ years.

based on slide from R.J. Goldston

Fusion Could Be Done In A Shorter Time Scale If Sufficient Budget Eventually Provided

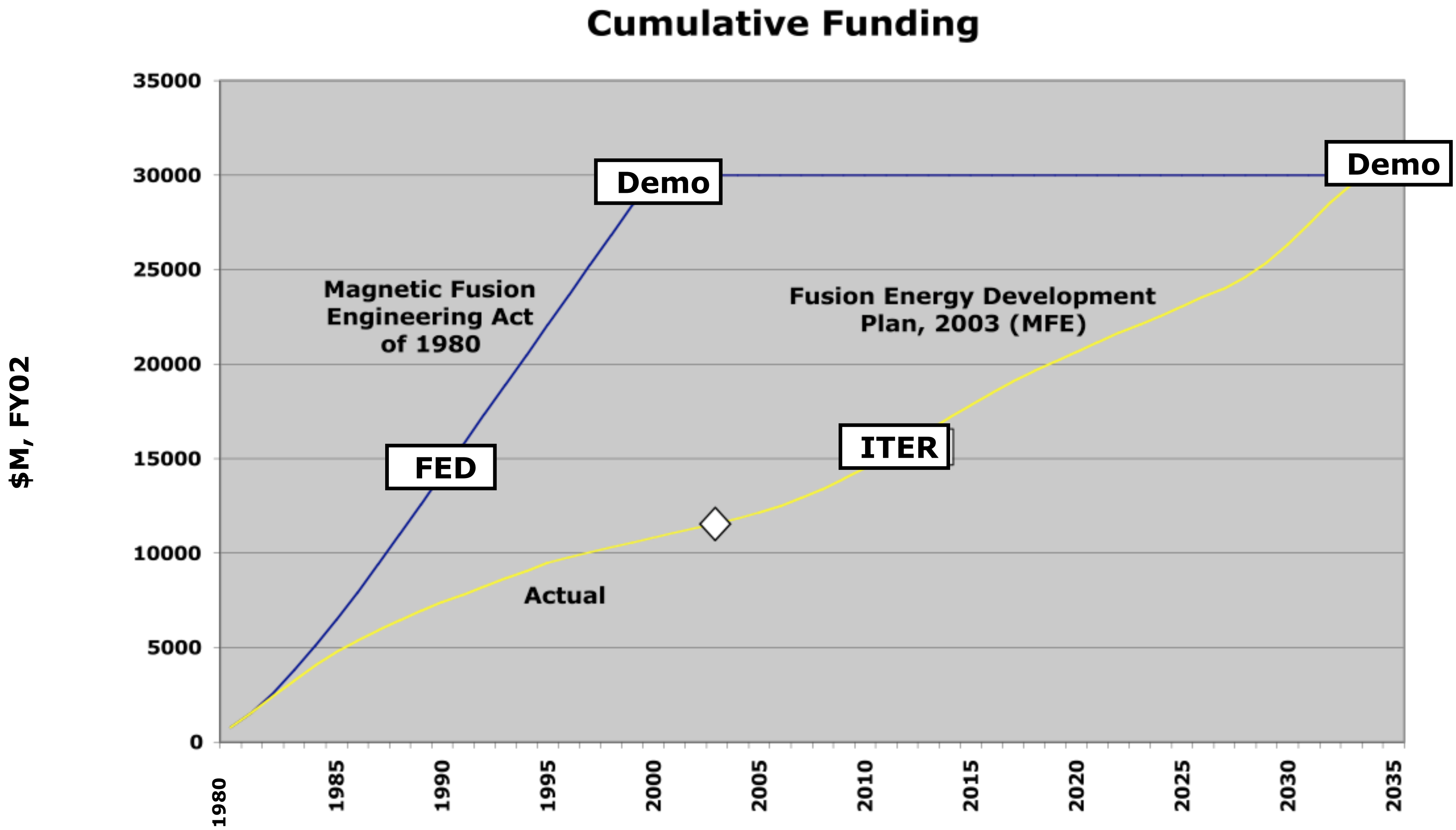


Einstein: Time is relative,

$30-$90B development cost is tiny compared to >$100 Trillion energy needs of 21st century & potential costs of global warming. (Apollo program ~ $100B.) Still 40:1 payoff after discounting 50+ years.

based on slide from R.J. Goldston

Fusion Could Be Done In A Shorter Time Scale If Sufficient Budget Eventually Provided



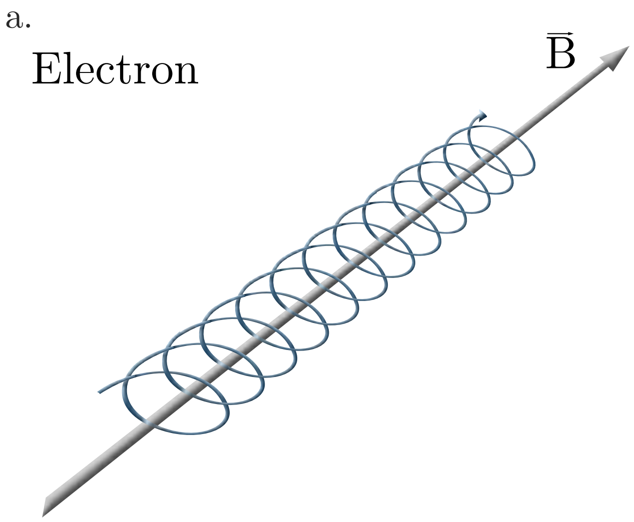
Einstein: Time is relative, Measure time in $$

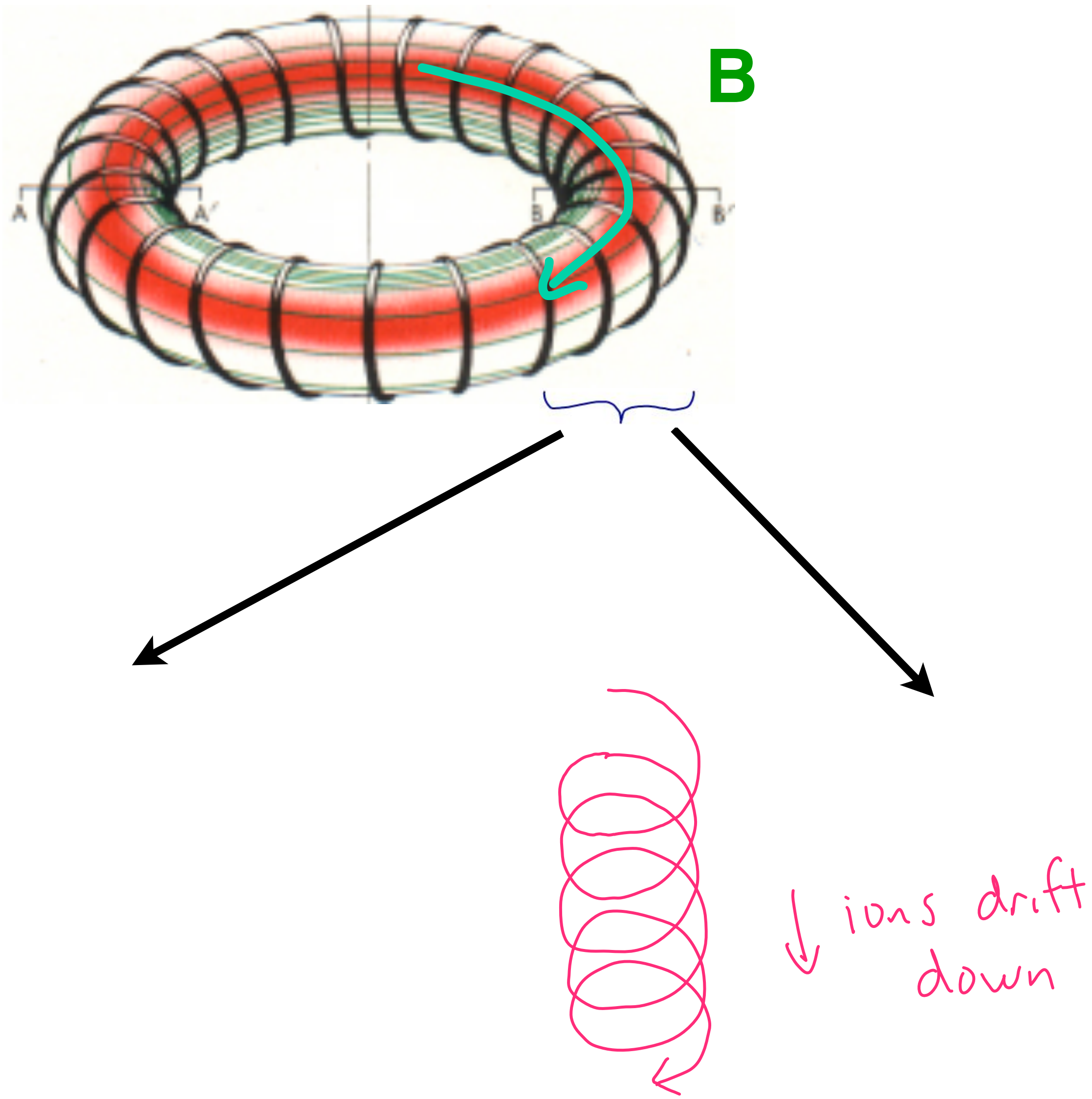
$30-$90B development cost is tiny compared to >$100 Trillion energy needs of 21st century & potential costs of global warming. (Apollo program ~ $100B.) Still 40:1 payoff after discounting 50+ years.

based on slide from R.J. Goldston

A Crash Course in Magnetic Confinement (in 3 slides)

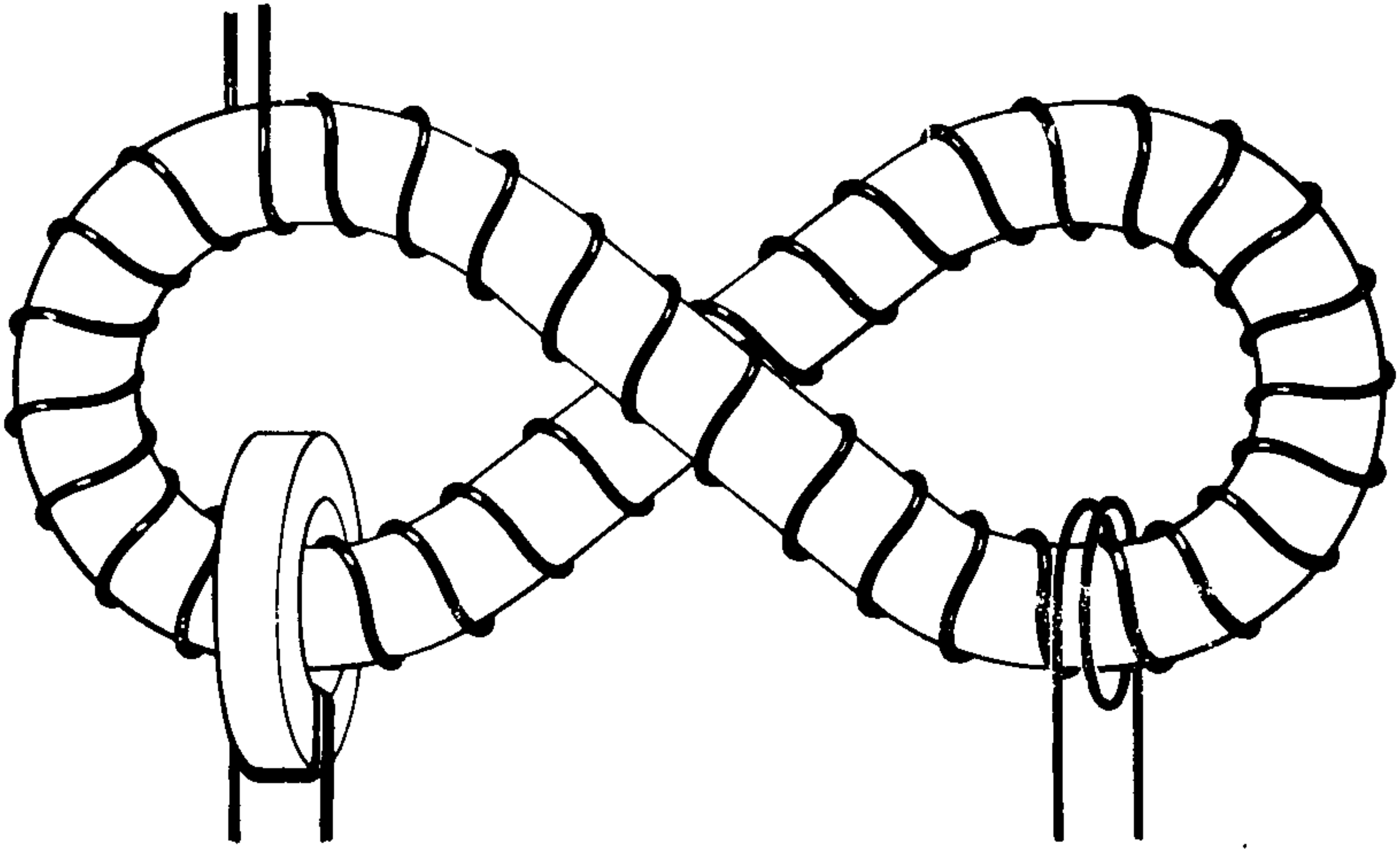
Particles have helical orbits in B field, not confined along B. Try to fix by wrapping B into a torus.





Fermi (~1946): but now B ~ 1/R, so particles will drift out:

Spitzer’s stellarator solution: twist torus into figure-8 to cancel drifts and confine particles.

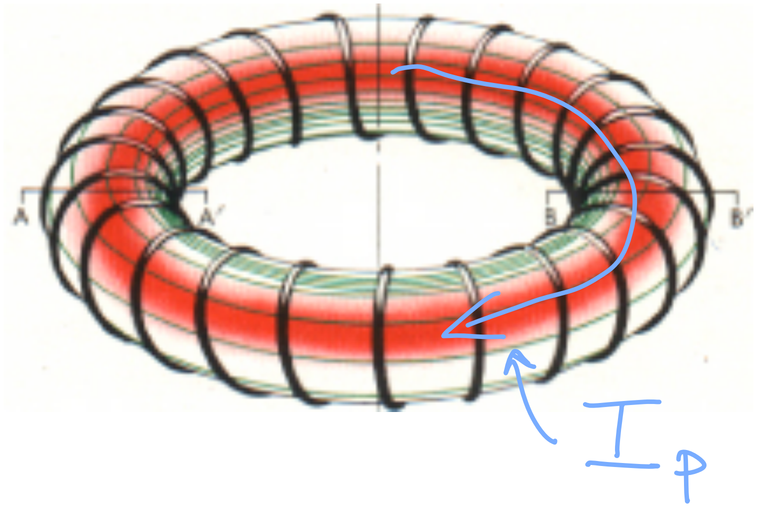


When fusion declassified in ’57 in the West and the USSR, the stellarator was the unique invention.

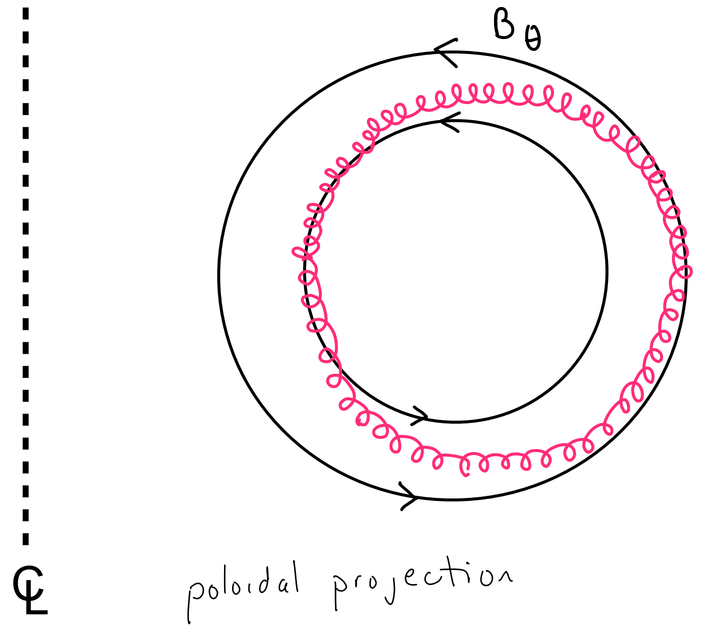
R. A. Ellis, Jr,, Princeton Alumni Weekly, Sept. 19, 1958

Cure problems by twisting the $\underset{\sim}{B}$ field

Induce a current in plasma:

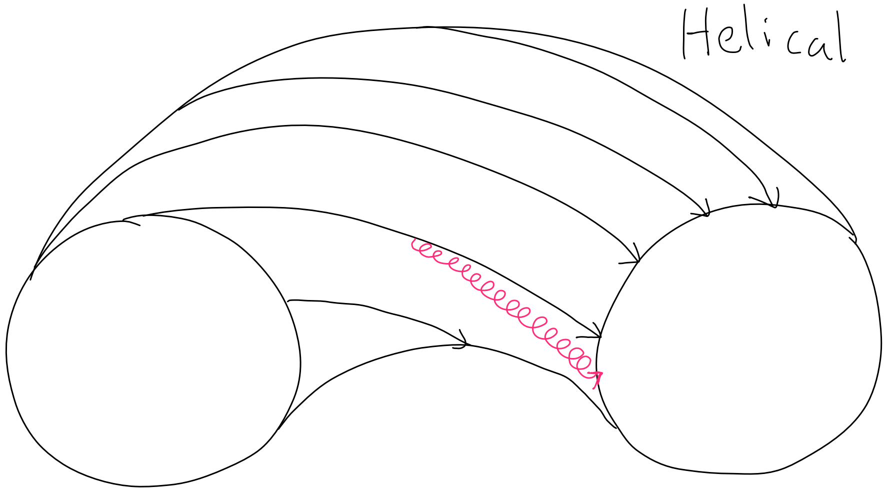


Ion motion along twisting $\underset{\sim}{B}$ field + downward drift:



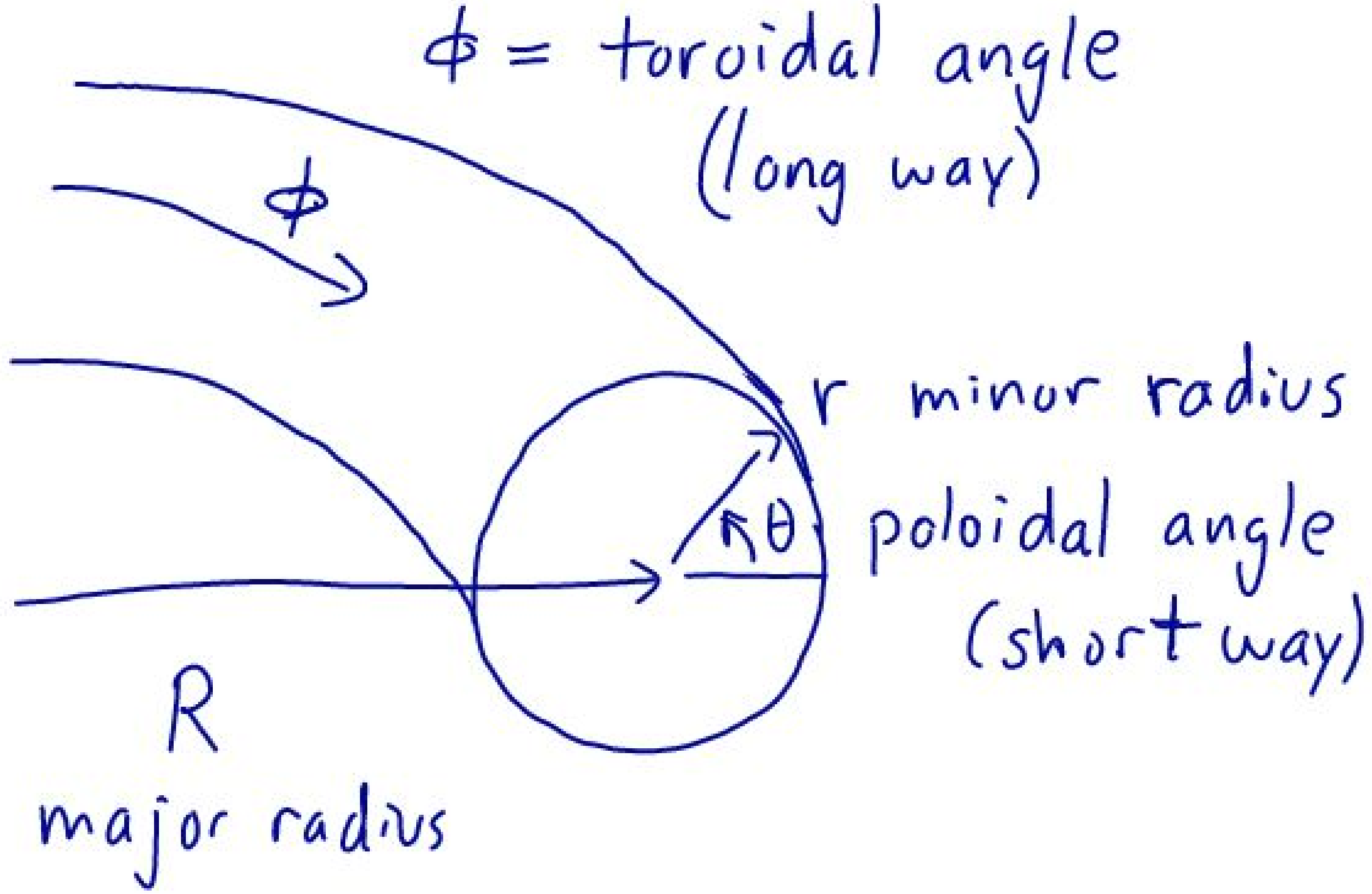
poloidal projection

Helical twist in $\vec{B}$



Perspective view

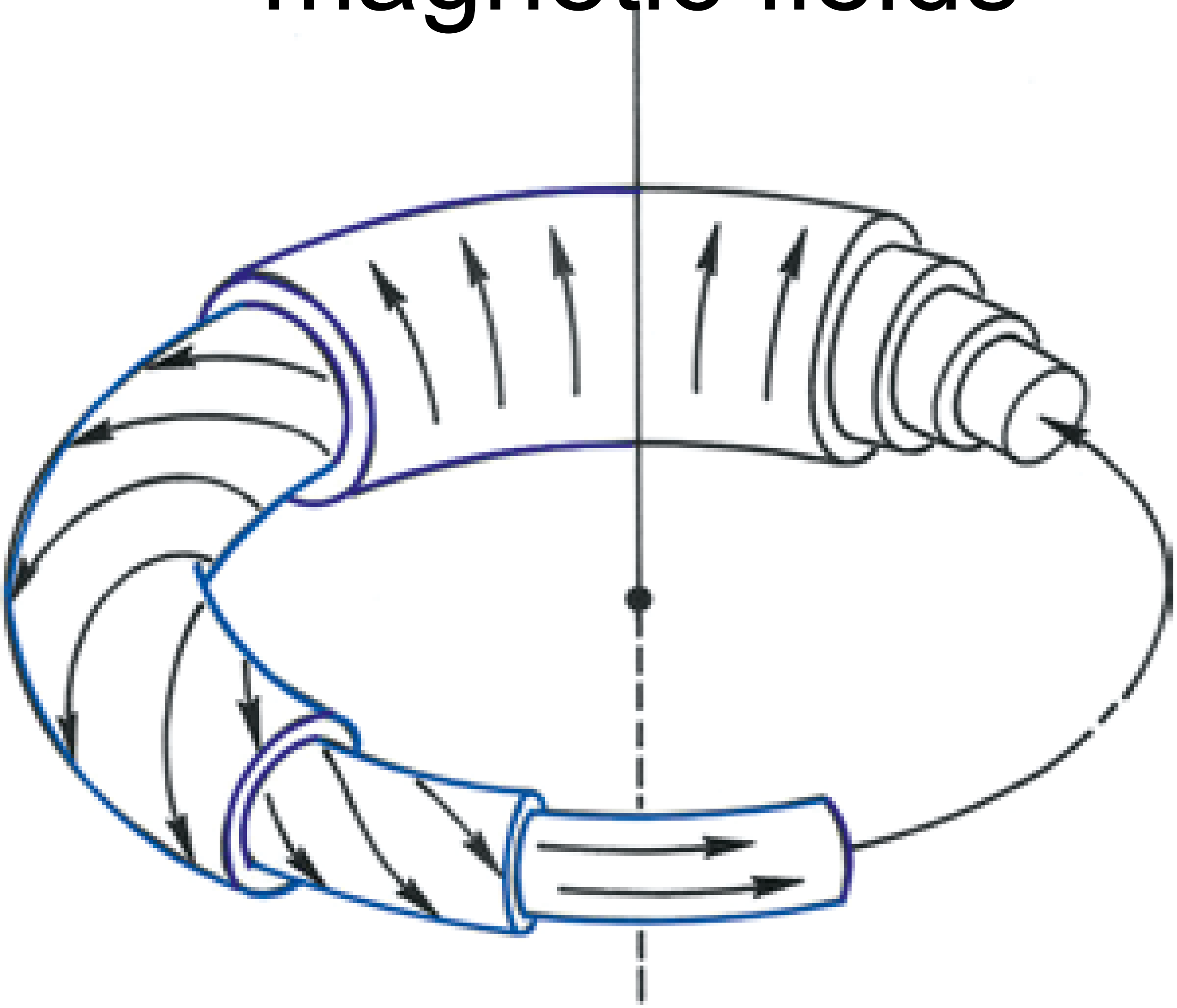
Tokamak Geometry



$$q = \frac{d\phi}{d\theta} = \frac{r}{R}\frac{B_\phi}{B_\theta}$$

q = “safety factor” = magnetic winding number. Follow field line q times toroidally, will get 1 poloidal twist.

Torus with sheared helical magnetic fields



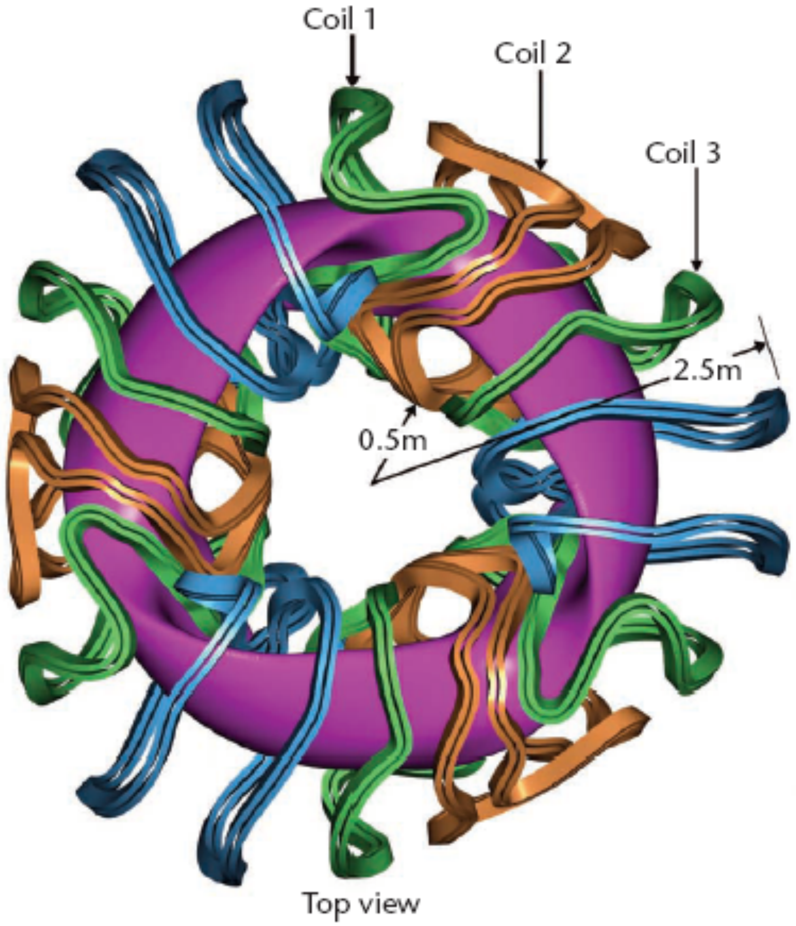
Extreme example, magnetic field is mostly in toroidal direction in standard tokamak.

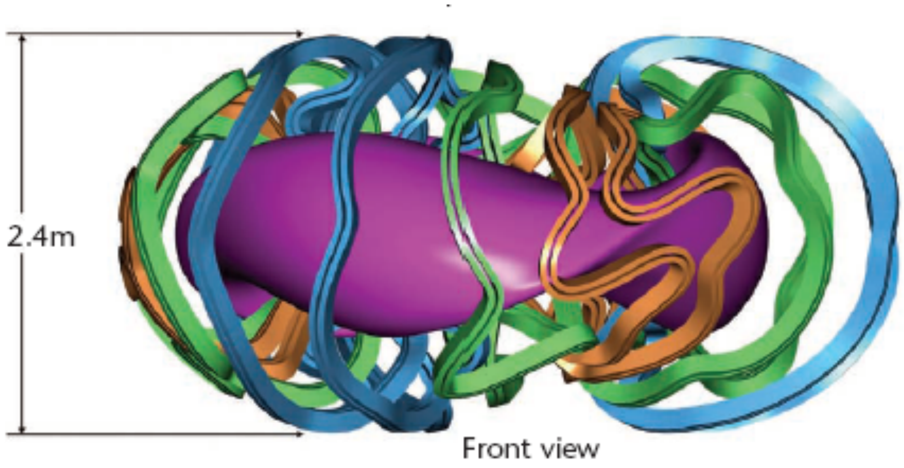
magnetic shear can help stabilize instabilities
(negative & zero average shear can be better, average ≠ local shear)

From F.F. Chen, “An Indispensable Truth”, 2011

Modern stellarators

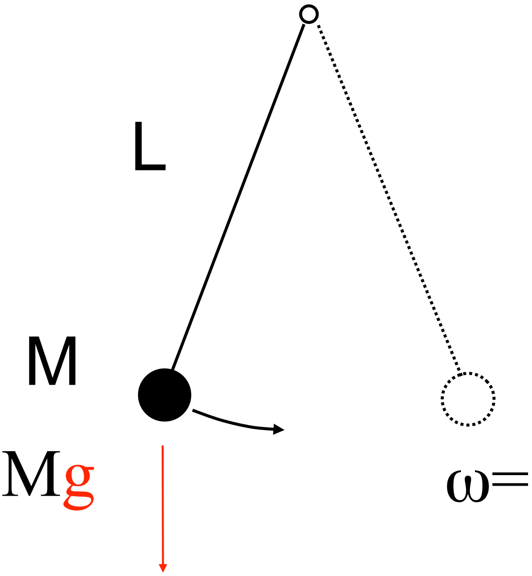
Spitzer later realized that particles can be confined by a net poloidal twist in the magnetic field produced by higher-order toroidal asymmetries. Eventually evolved into modern stellarator with modular, unlinked coils (here is the 3-period NCSX design).





Simple Physical Pictures Underlying Gyrokinetic & MHD Instabilities

Stable Pendulum

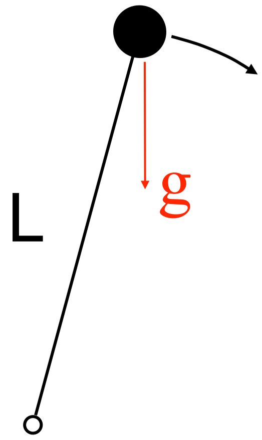


$F=M\mathbf{g}$

$\omega=(g/L)^{1/2}$

Unstable Inverted Pendulum

(rigid rod)

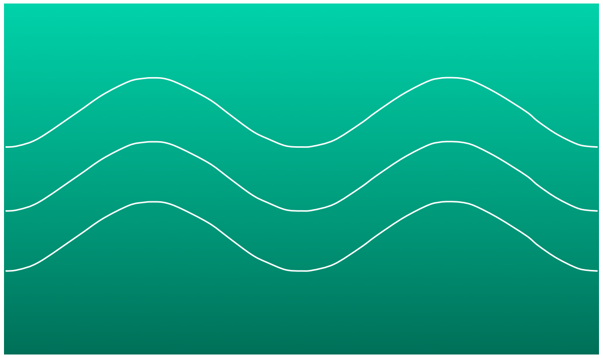


$\omega= (-g/|L|)^{1/2} = i(g/|L|)^{1/2} = i\gamma$

↑ Instability

Density-stratified Fluid

$\rho=\exp(-y/L)$

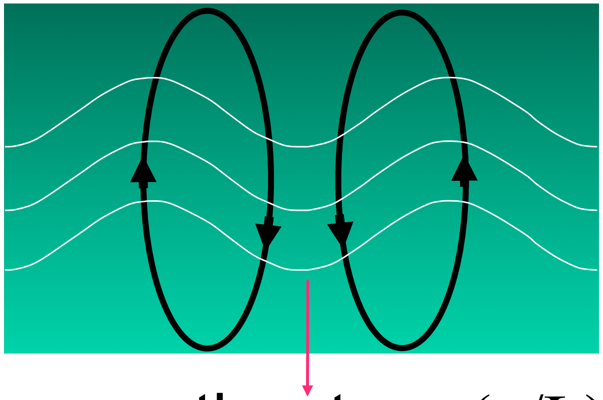


stable $\omega=(g/L)^{1/2}$

Inverted-density fluid

⇒Rayleigh-Taylor Instability

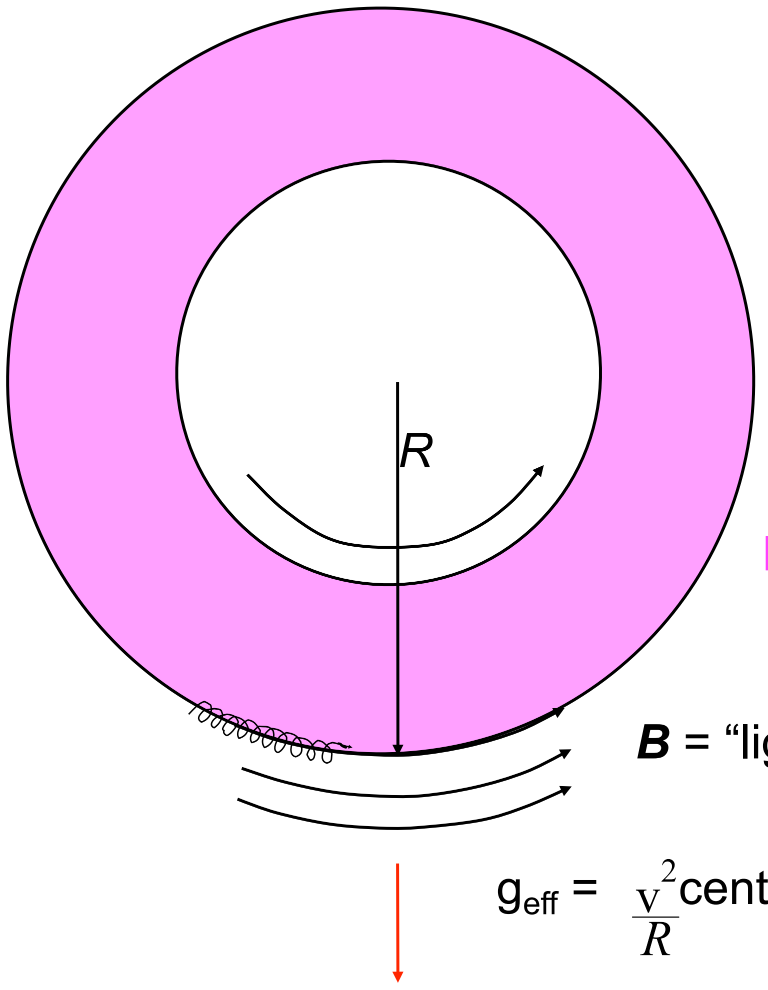
$\rho=\exp(y/L)$



Max growth rate $\gamma=(g/L)^{1/2}$

“Bad Curvature” instability in plasmas ≈ Inverted Pendulum / Rayleigh-Taylor Instability

Top view of toroidal plasma:



$g_{eff} = \frac{v^2}{R}$ centrifugal force

Growth rate:

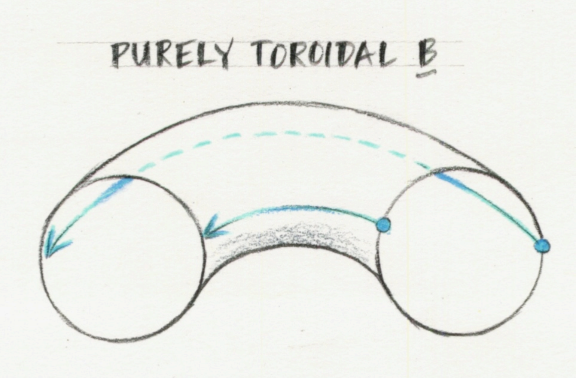
$$\gamma = \sqrt{\frac{g_{eff}}{L}} = \sqrt{\frac{v_t^2}{RL}} = \frac{v_t}{\sqrt{RL}}$$

Similar instability mechanism
in MHD & drift/microinstabilities

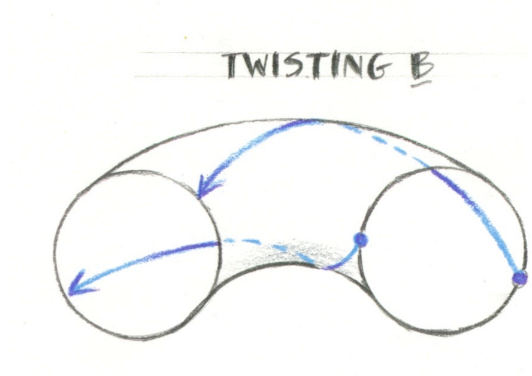
1/L = ∇p/p in MHD,
∝ combination of ∇n & ∇T
in microinstabilities.

The Secret for Stabilizing Bad-Curvature Instabilities

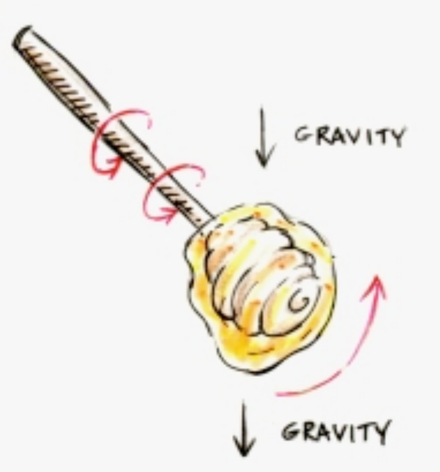
Twist in ***B*** carries plasma from bad curvature region to good curvature region:



Unstable

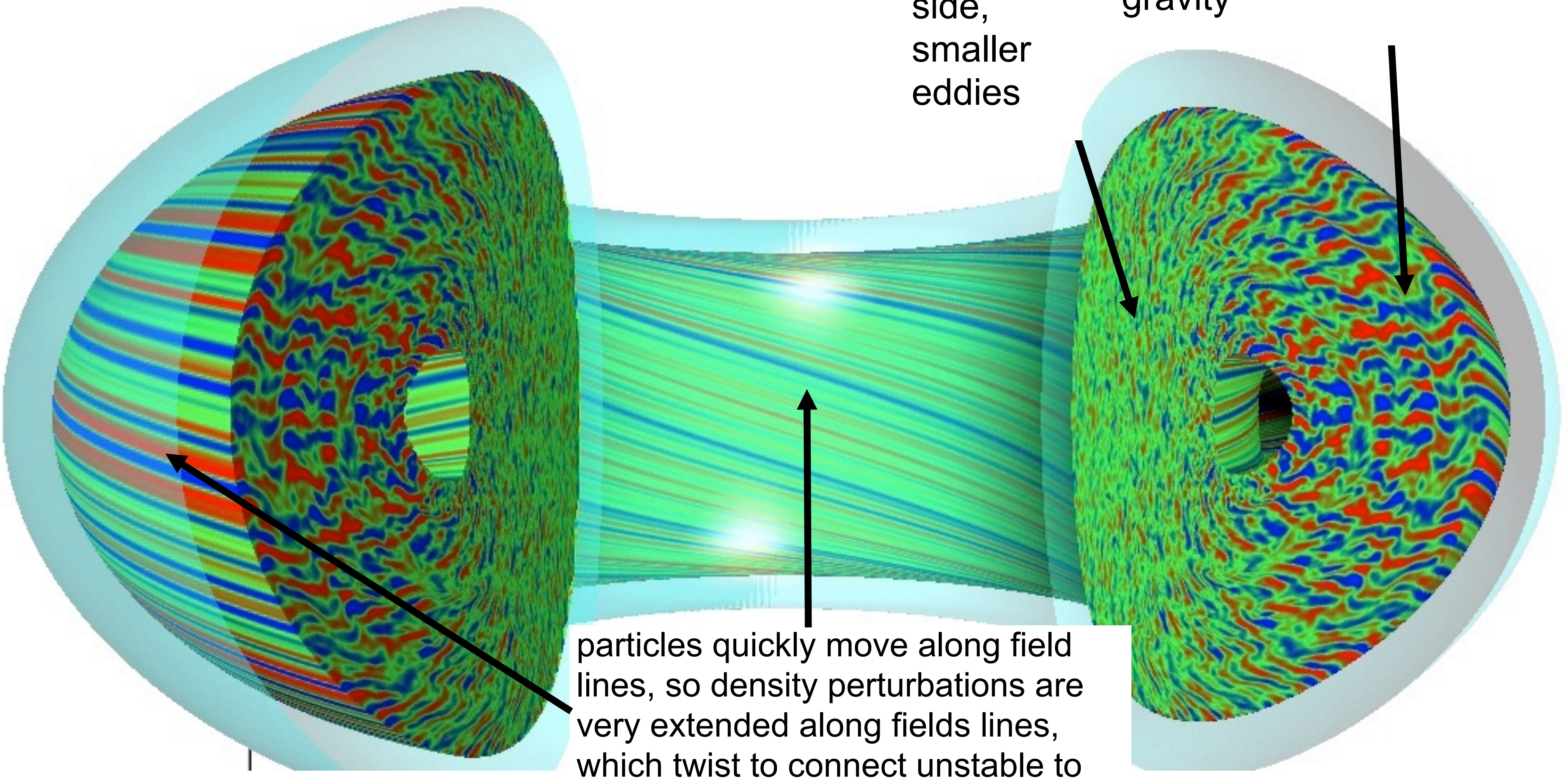


Stable



Similar to how twirling a honey dipper can prevent honey from dripping.

These physical mechanisms can be seen in gyrokinetic simulations and movies

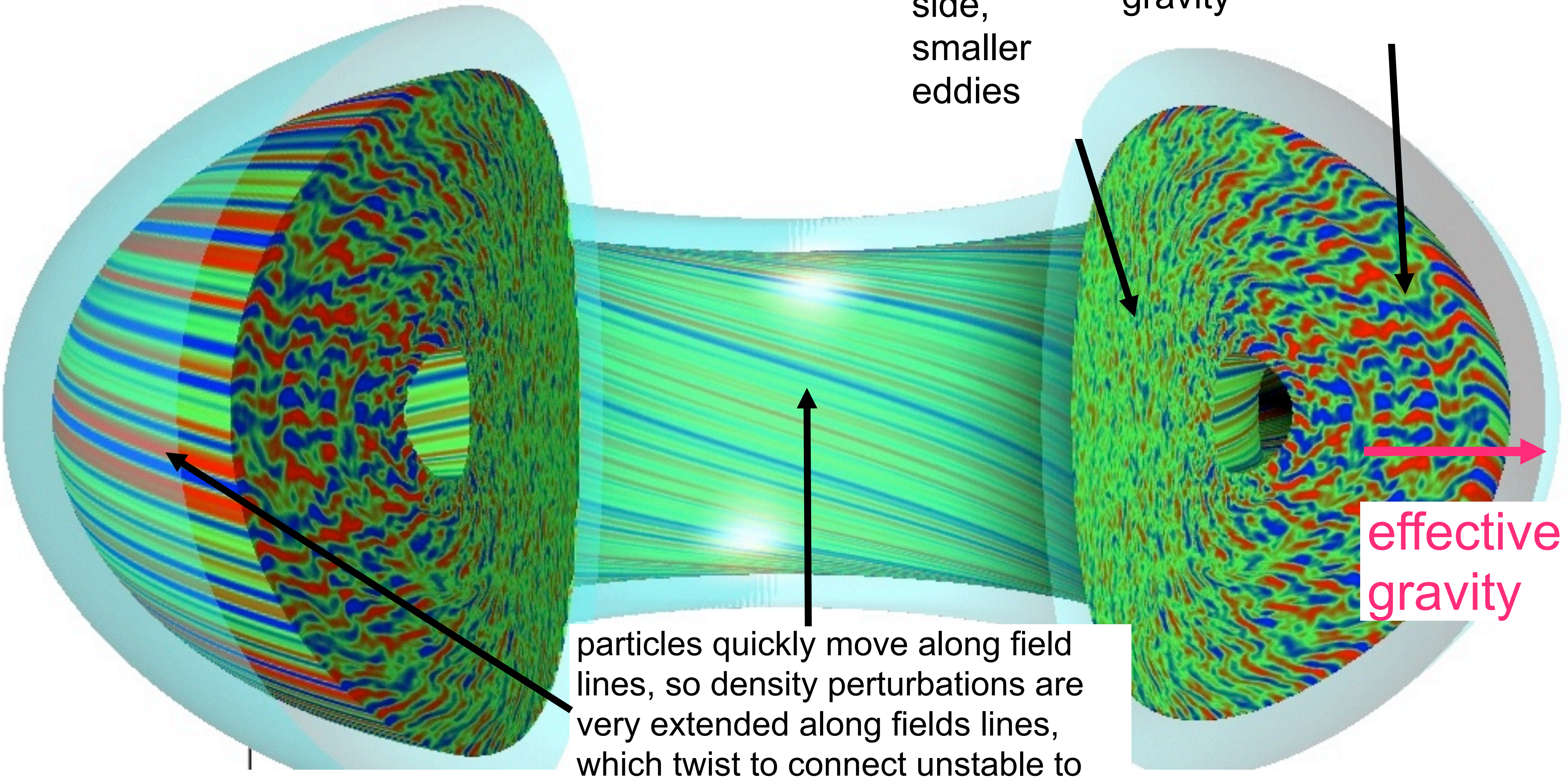


Stable side, smaller eddies

Unstable bad-curvature side, eddies point out, direction of effective gravity

particles quickly move along field lines, so density perturbations are very extended along fields lines, which twist to connect unstable to stable side

These physical mechanisms can be seen in gyrokinetic simulations and movies



Stable side, smaller eddies

Unstable bad-curvature side, eddies point out, direction of effective gravity

effective gravity

particles quickly move along field lines, so density perturbations are very extended along fields lines, which twist to connect unstable to stable side

Bad-curvature mechanism for both MHD & Drift-type instabilites

- MHD: magnetic field lines & plasma move together, local bad curvature instability must be faster than Alfven wave propagation to good curvature side:

$$\gamma > \frac{v_A}{qR} \qquad \Rightarrow \qquad q^2 R \frac{\beta}{L_p} > \text{const.}$$

familiar MHD instability parameter

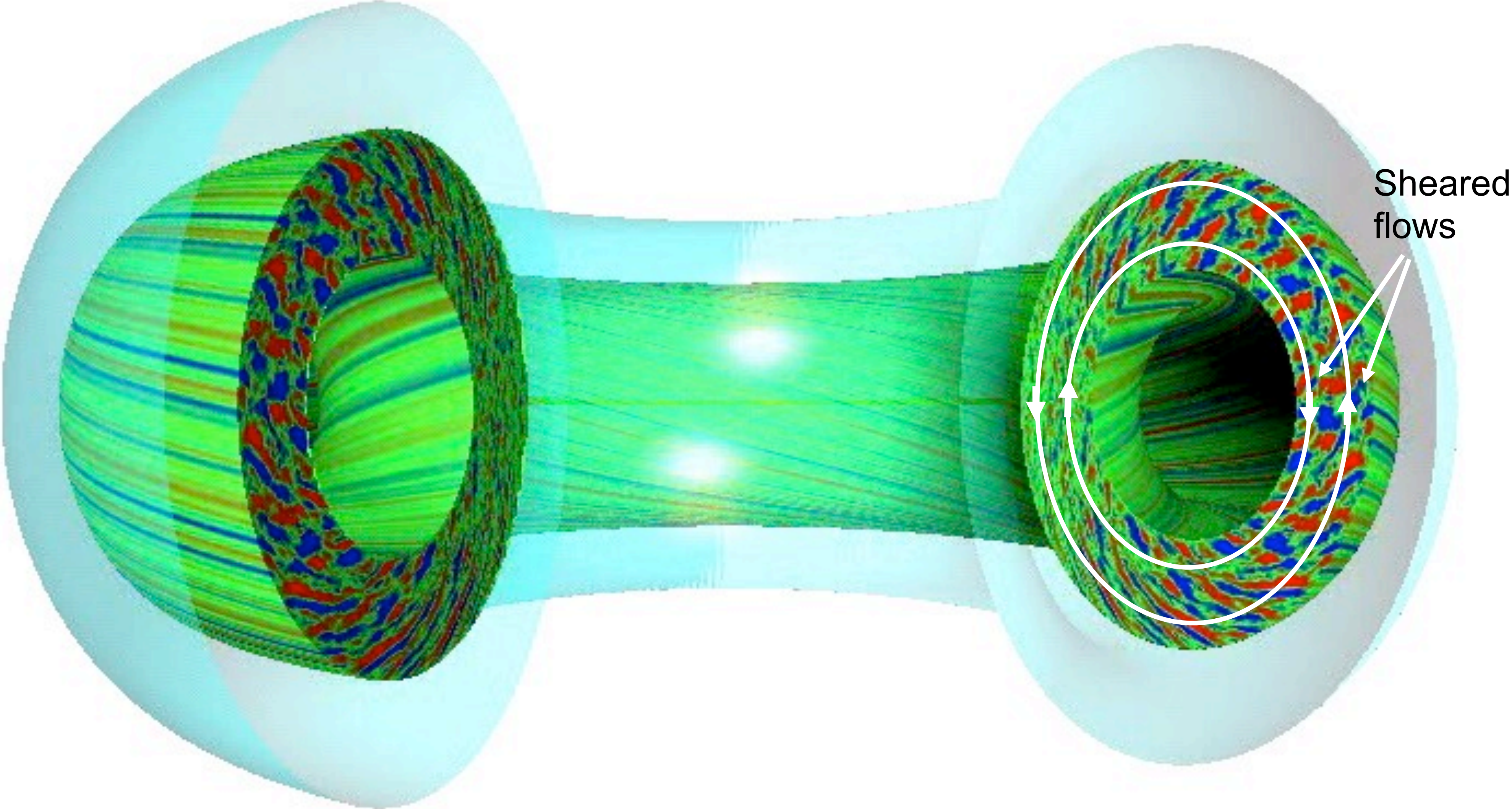
- Drift waves / gyrokinetics: $k_\perp \rho$ FLR corrections decouple magnetic field & plasma, --> electrostatic ExB flows, instability must be faster than sound wave propagation to good curvature side:

$$\gamma > \frac{v_t}{qR} \qquad \Rightarrow \qquad \frac{R}{L_p} > \frac{\text{const.}}{q^2}$$

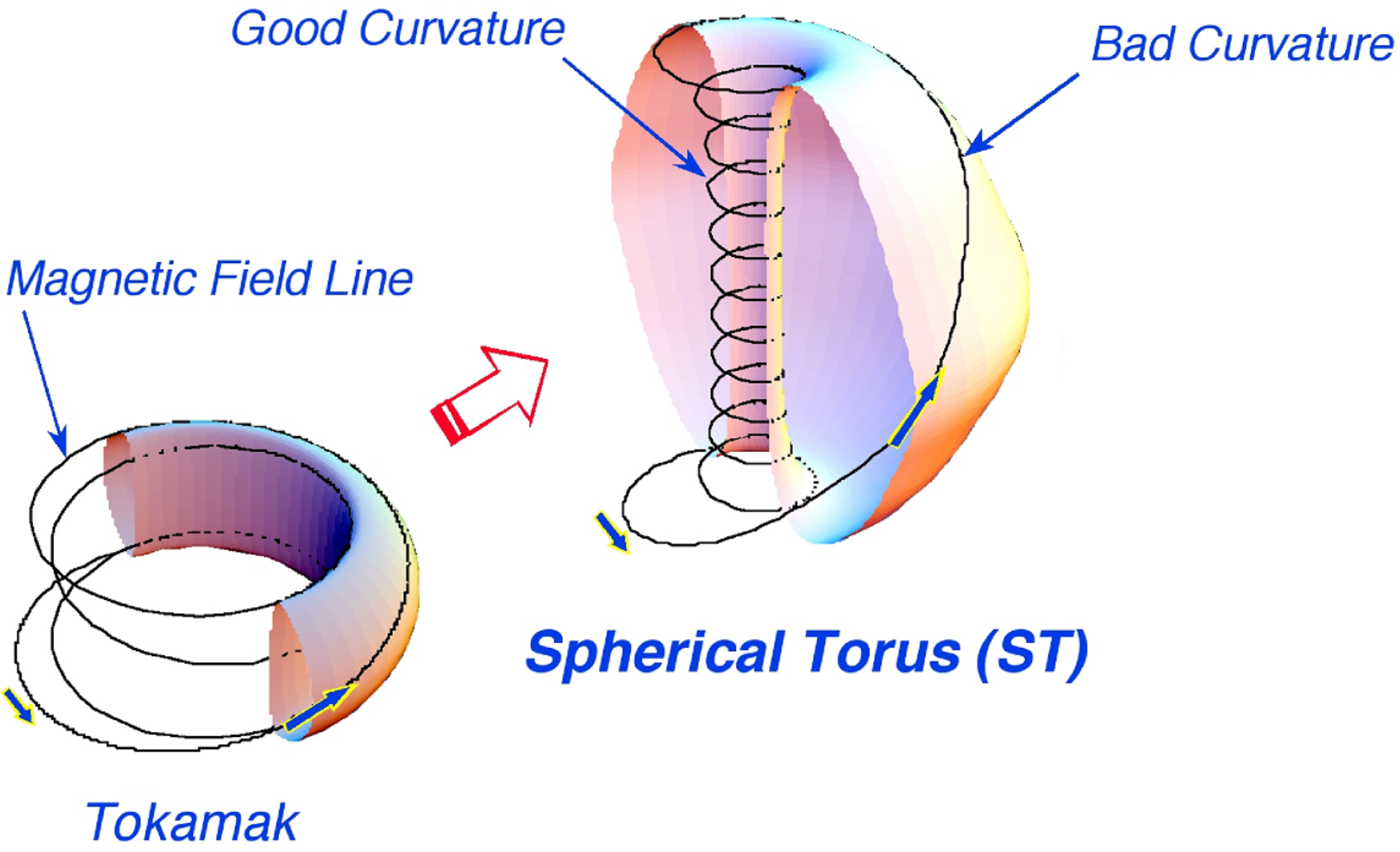
other mechanisms stabilize at high q

These physical pictures help explain how sheared flows & negative magnetic shear can be stabilizing.

Movie http://fusion.gat.com/THEORY/images/3/35/D3d.n16.2x_0.6_fly.mpg from http://fusion.gat.com/theory/Gyromovies shows contour plots of density fluctuations in a cut-away view of a GYRO simulation (Candy & Waltz, GA). This movie illustrates the physical mechanisms described in the last few slides. It also illustrates the important effect of sheared flows in breaking up and limiting the turbulent eddies. Long-wavelength equilibrium sheared flows in this case are driven primarily by external toroidal beam injection. (The movie is made in the frame of reference rotating with the plasma in the middle of the simulation. Barber pole effect makes the dominantly-toroidal rotation appear poloidal..) Short-wavelength, turbulent-driven flows also play important role in nonlinear saturation.

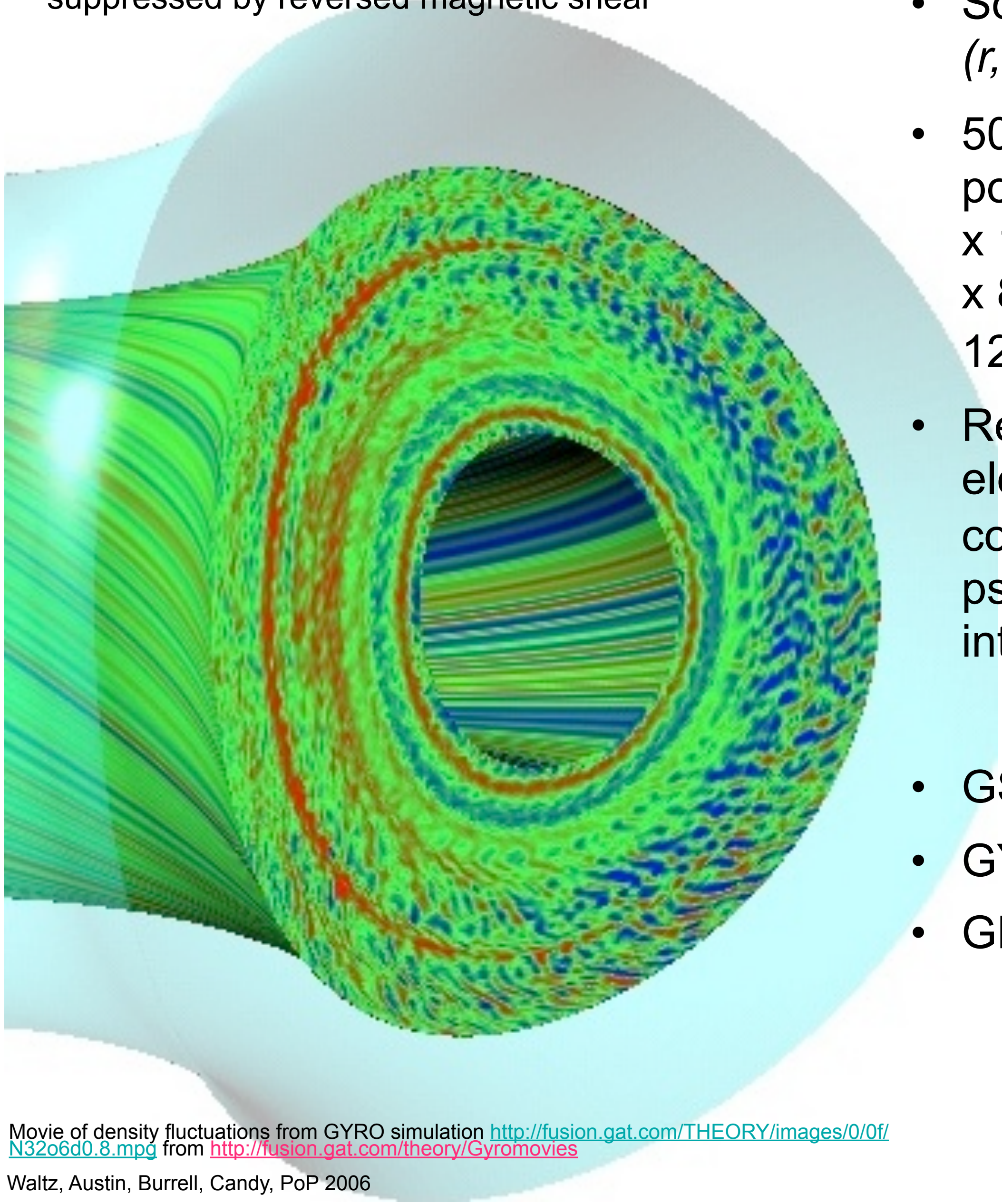


Spherical Torus has improved confinement and pressure limits (but less room in center for coils)



Fairly Comprehensive 5-D Gyrokinetic Turbulence Codes Have Been Developed

small scale, small amplitude density fluctuations (<1%) suppressed by reversed magnetic shear



Movie of density fluctuations from GYRO simulation http://fusion.gat.com/THEORY/images/0/0f/N32o6d0.8.mpg from http://fusion.gat.com/theory/Gyromovies

Waltz, Austin, Burrell, Candy, PoP 2006

- Solve for the particle distribution function f $(r,\theta,\alpha,E,\mu,t)$ (avg. over gyration: 6D → 5D)
- 500 radii x 32 complex toroidal modes (96 grid points in real space)
  x 10 parallel points along half-orbits
  x 8 energies x 16 $v_{||}/v$
  12 hours on ORNL Cray X1E with 256 MSPs
- Realistic toroidal geometry, kinetic ions & electrons, finite-β electro-magnetic fluctuations, collisions. Sophisticated algorithms: mixed pseudo-spectral, high-order Gauss-Legendre integration in velocity space, ...

- GS2 (Dorland & Kotschenreuther)
- GYRO (Candy & Waltz)
- GENE (Jenko et al.)

The electrostatic gyrokinetic equation, in a "full-f" drift-kinetic-like form, for the gyro-averaged, guiding-center distribution function $\bar{f}(\vec{R}, v_{||}, \mu, t) = \bar{f}_0 + \delta\bar{f}$:

$$\boxed{\frac{\partial \bar{f}}{\partial t} + (v_{\parallel}\hat{\mathbf{b}} + \mathbf{v}_E + \mathbf{v}_d)\cdot\nabla\bar{f} + \left(\frac{q}{m}E_{\parallel} - \mu\nabla_{\parallel}B + v_{\parallel}(\hat{\mathbf{b}}\cdot\nabla\hat{\mathbf{b}})\cdot\mathbf{v}_E\right)\frac{\partial\bar{f}}{\partial v_{\parallel}} = 0}$$

$\underset{\sim}{V}_E = -\frac{c}{B}\nabla\langle\Phi\rangle\times\hat{b}$

$E_{\parallel} = -\hat{b}\cdot\nabla\langle\Phi\rangle$

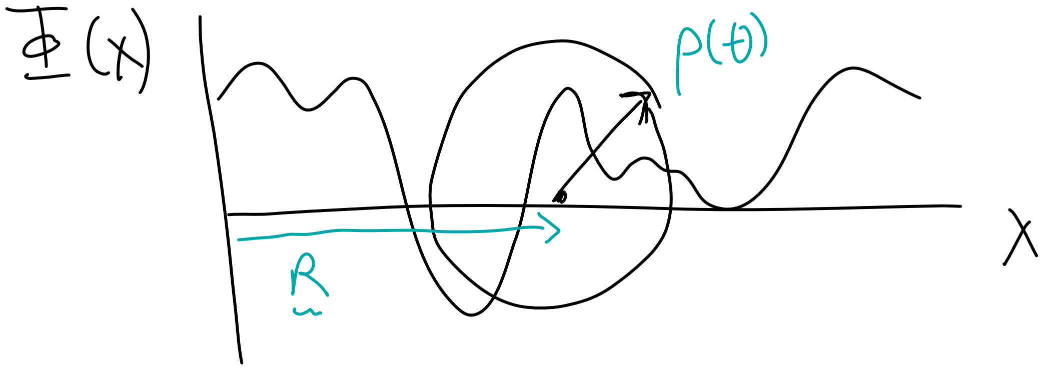
$\mu = \frac{1}{2}\frac{v_\perp^2}{B}$

using gyroaveraged potential:

$$\langle\phi\rangle(\vec{R}) = \frac{1}{2\pi}\int d\theta\,\phi(\vec{R}+\vec{\rho}(\theta))$$
$$= \frac{1}{2\pi}\int d\theta\sum_{\vec{k}}\phi_{\vec{k}}\,e^{i\vec{k}\cdot(\vec{R}+\vec{\rho}(\theta))}$$
$$= \sum_{\vec{k}} J_0(k_\perp\rho)\phi_{\vec{k}}\,e^{i\vec{k}\cdot\vec{R}} = J_0\phi$$

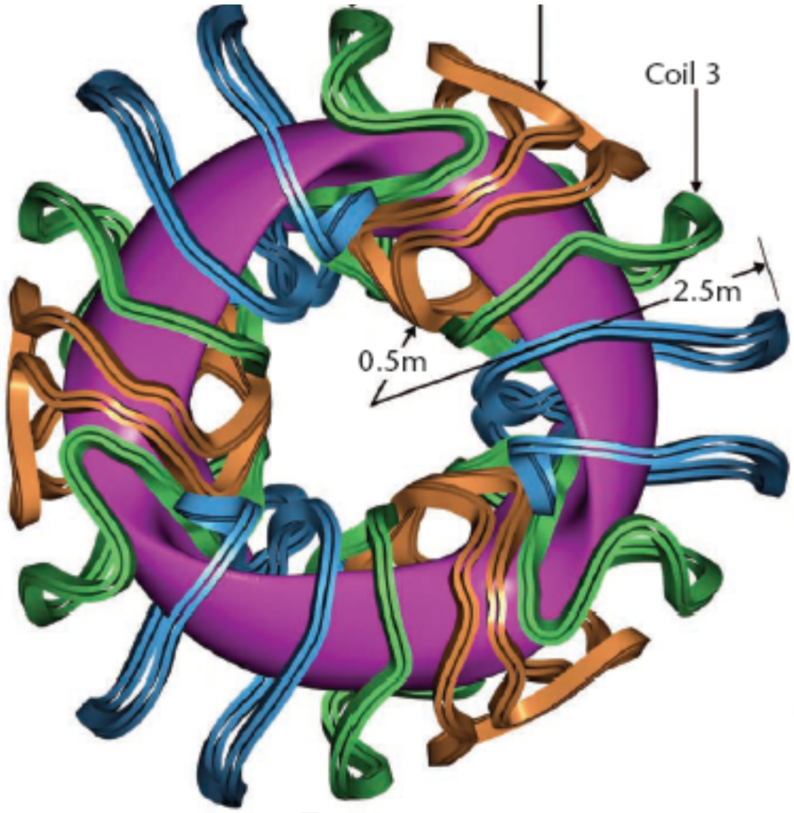
$\underset{\sim}{V}_d$ = ∇B + curvature drift

$$v_d = \frac{v_{||}^2}{\Omega}\hat{b}\times(\hat{b}\cdot\nabla\hat{b}) + \frac{\mu}{\Omega}\hat{b}\times\nabla B$$

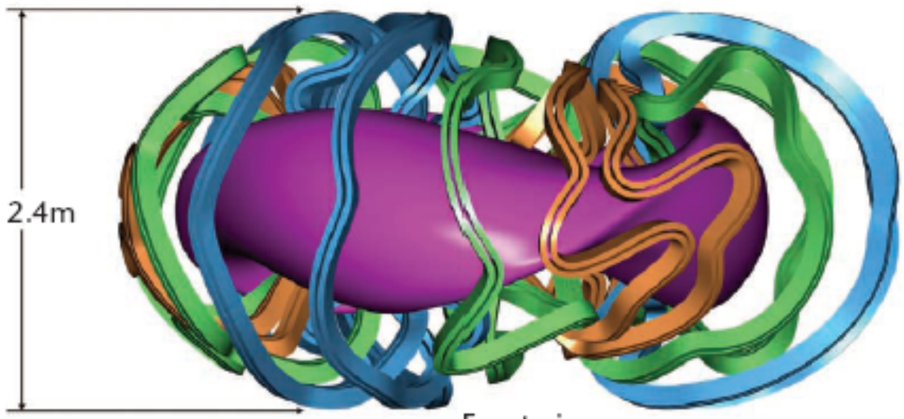


Improved Stellarators Being Studied

- Originally invented by Spitzer ('51), the unique idea when fusion declassified ('57)
- Mostly abandoned for tokamaks in '69. But computer optimized designs now much better than slide rules. (Robotic advances could bring down manufacturing cost.)
- Quasi-symmetry discovered in late 90's: don't need vector ***B*** exactly symmetric toroidally, |***B***| symmetric in field-aligned coordinates sufficient to be as good as tokamak.
- Magnetic field twist & shear provided by external coils, not plasma currents, inherently steady-state. Stellarator expts. don't have Greenwald density limit or hard beta limit & don't disrupt.



Top view



Front view

Computational methods used in fusion may be useful in astro, & vice versa, with caveats

- astro and solar physics codes often more general for plasmas in a wide range of parameter space ($\beta >> 1$, $\beta << 1$, λ_{mfp} large or small, shocks, relativistic, dynamic equilibria, fast instabilities, ...)
- fusion codes often optimized for challenges in particular parameter regimes ($\beta < 1$ or $<<1$, non-relativistic, near-equilibria with very slow instabilities compared to Alfven or cyclotron time, no shocks, high resolution around rational surfaces to resolve reconnection or small-scale drift-wave turbulence, ...)

M3D-C^1 is implicit MHD code with high-order finite elements in 3D

Several techniques are used to obtain a well conditioned 3D matrix with a small condition number for iterative solution each time step.

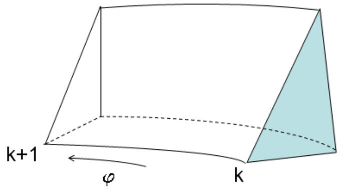
1. Helmholtz decomposition of velocity field to approximately capture each MHD wave in a single velocity variable.

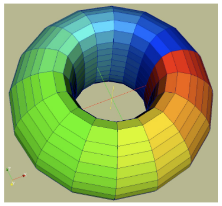
$$\mathbf{V} = R^2\nabla U \times \nabla\phi + R^2\omega\nabla\phi + \frac{1}{R^2}\nabla_\perp\chi$$

2. Schur-complement technique to eliminate perturbed pressures and magnetic fields in favor of velocity variables

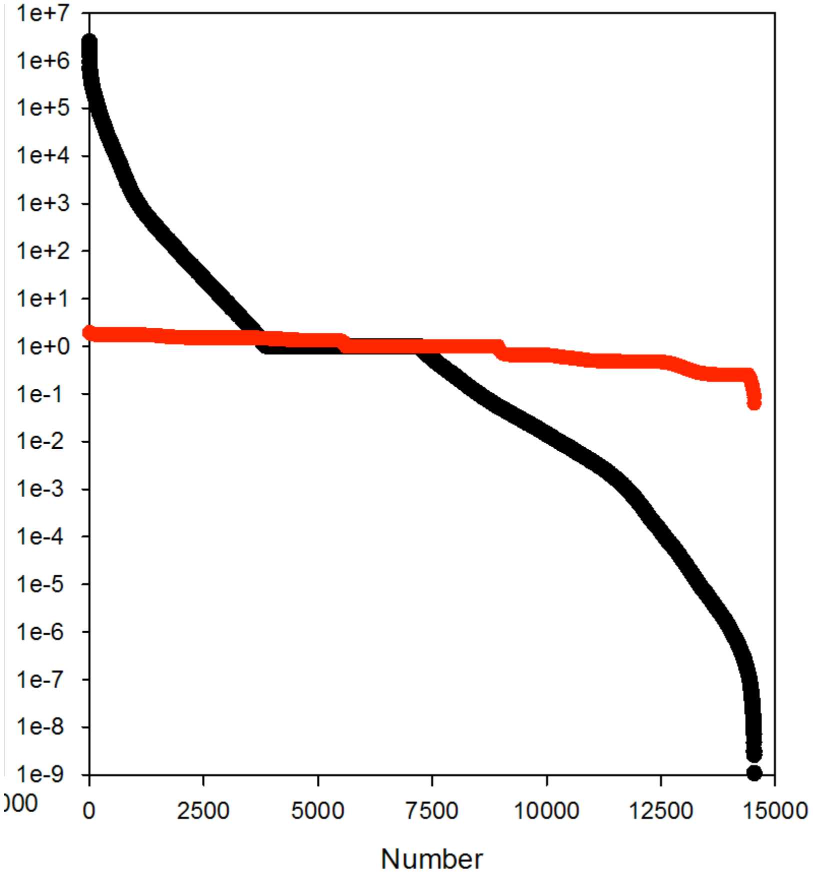
3. Annihilation operators applied to momentum equation to approximately diagonalize

4. Block-Jacobi preconditioner solves perpendicular dimensions directly.





Eigenvalues of 3D Matrix **BEFORE** and **AFTER** preconditioning steps 2-4



Condition number reduced from 10^{15} to 30 !!

S. Jardin (PPPL)

δf algorithm for reducing noise in PIC simulations

- Developed in fusion for reducing noise in gyrokinetic PIC simulations of small-amplitude drift wave fluctuations (Kotschenreuther, Dimits, Parker, Aydemir, Krommes, Alfrey, Y. Chen, Hatzky, ...)
- Break up $F = F_0 + \delta f$ into known smooth F_0 and small corrections δf, treating only δf with weighted particles

$$\delta f(\vec{x}, \vec{v}, t) = \sum_i w_i(t)\, \delta(\vec{x} - \vec{x}_i(t))\, \delta(\vec{v} - \vec{v}_i(t))$$

- Reduces rms noise level by factor of $|F_0/\delta f| \sim 10^2$, reduces number of particles required by factor of $|F_0/\delta f|^2 \sim 10^4$.
- Implemented by M. Kunz in new hybrid code (discussed w/ Belova & Hammett) to study mirror/firehose instabilities in astro plasmas

Landau-Fluid Closures

Originally developed for fusion drift-wave applications, applied by Sharma, Hammett, Quataert, Stone to study kinetic effects on MRI turbulence and resulting strong electron heating (via mirror/firehose instabilities).

Can model Landau damping and other kinetic effects in fluid equations by introducing thermal diffusion along field lines that depends on wavenumber k:

$$D = \frac{v_t^2}{v_t |k_{||}| + \nu}$$

large ν --> recover
Braginskii fluid limit.

Converges: use a dissipative closure after keeping N fluid moments --> N-pole Pade approximation to the Z function. (also keeps more nonlinear effects.)

Fourier transform to real space, get a non-local heat flux:

$$q(z) = -n_0 v_{t0} \int_0^\infty dz' \frac{T(z+z') - T(z-z')}{z'} \frac{1}{1 + z'^2/\lambda_{mfp}^2}$$

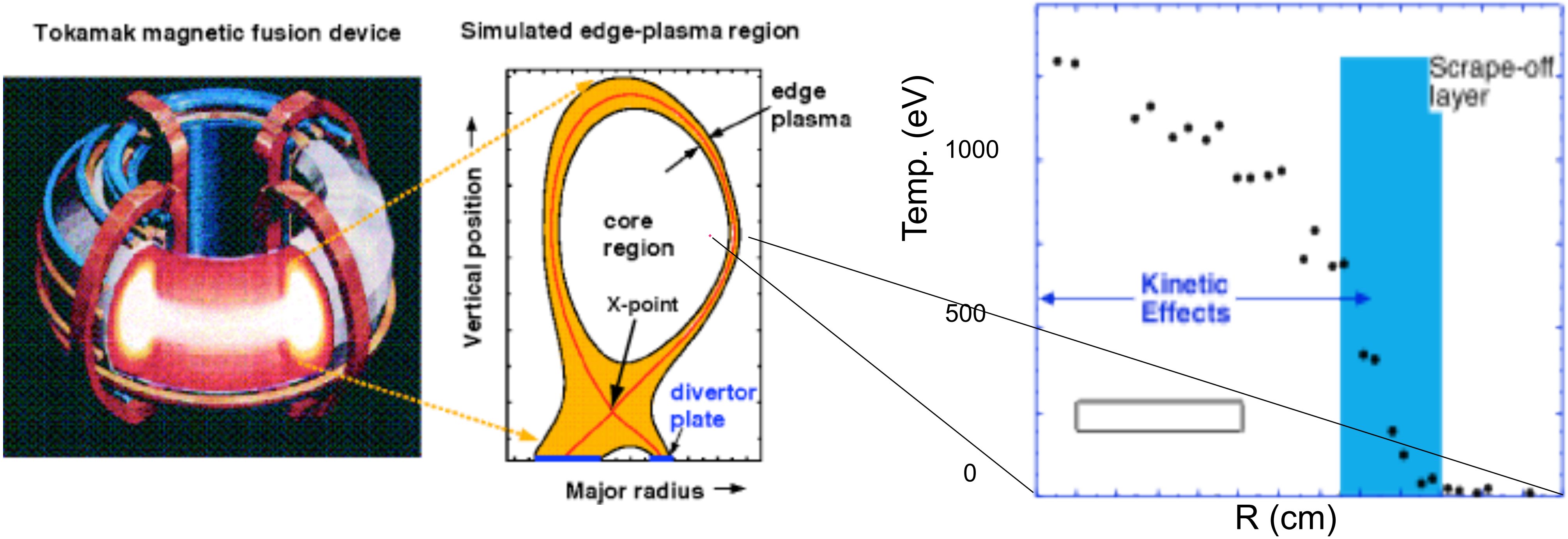
New: can use Fast Multipole Methods? (Chacon & del-Castillo-Negrete). Ideas on possibles ways to generalize to stronger parallel nonlinearities. How to generalize to weak/no magnetic fields (3D)? Reconnection?

Snyder, Hammett, Dorland (1998), Sharma, Hammett, Stone, Quataert

Discontinuous Galerkin Algorithms for Hamiltonian/ Kinetic Problems

(See Ammar Hakim's poster for details)

Edge region very difficult



Edge pedestal temperature profile near the edge of an H-mode discharge in the DIII-D tokamak. [Porter*2000*]. Pedestal is shaded region.

Major extensions to gyrokinetic codes needed to handle additional complications of edge region of tokamaks (& stellarators):

> open & closed field lines, steep gradients near beta limit, electric & magnetic fluctuations, strong shear-flow layers, steep-gradients and large amplitude fluctuations, positivity constraints, wide range of collisionality, non-axisymmetric RMP coils, plasma-wall interactions, strong sources and sinks in atomic physics.

A new code with these capabilities might also be useful for a wider range of astrophysics and other applications.

General goal: new robust (gyro)kinetic code benefiting from several advanced continuum algorithms

New continuum code using combination of advanced algorithms that could help it be significantly more efficient and robust, particularly on coarse velocity space grids. Advanced algorithms include:

- certain versions of discontinuous Galerkin (DG) methods that are quite efficient and have good conservation properties (subtle for kinetic Hamiltonian problems), while allowing certain types of limiters (help preserve positivity).

- Maxwellian-weighted (or more general) basis functions,

- subgrid / hypercollision models to model phase-mixing and turbulent mixing to unresolved scales (handles recurrence issues).

DG combines some advantages of Finite Volume (FV) with Finite Element accuracy:
FV interpolates p uniformly-spaced points to get p order accuracy
DG interpolates p optimally-located points to get 2p-1 order accuracy

(DG has lower phase-errors like Finite Elements / Compact Finite Differencing, but calculations are local like FV, explicit code easier to parallelize.)

General goal: new robust (gyro)kinetic code benefiting from several advanced continuum algorithms

New continuum code using combination of advanced algorithms that could help it be significantly more efficient and robust, particularly on coarse velocity space grids. Advanced algorithms include:

- certain versions of discontinuous Galerkin (DG) methods that are quite efficient and have good conservation properties (subtle for kinetic Hamiltonian problems), while allowing certain types of limiters (help preserve positivity).

- Maxwellian-weighted (or more general) basis functions,

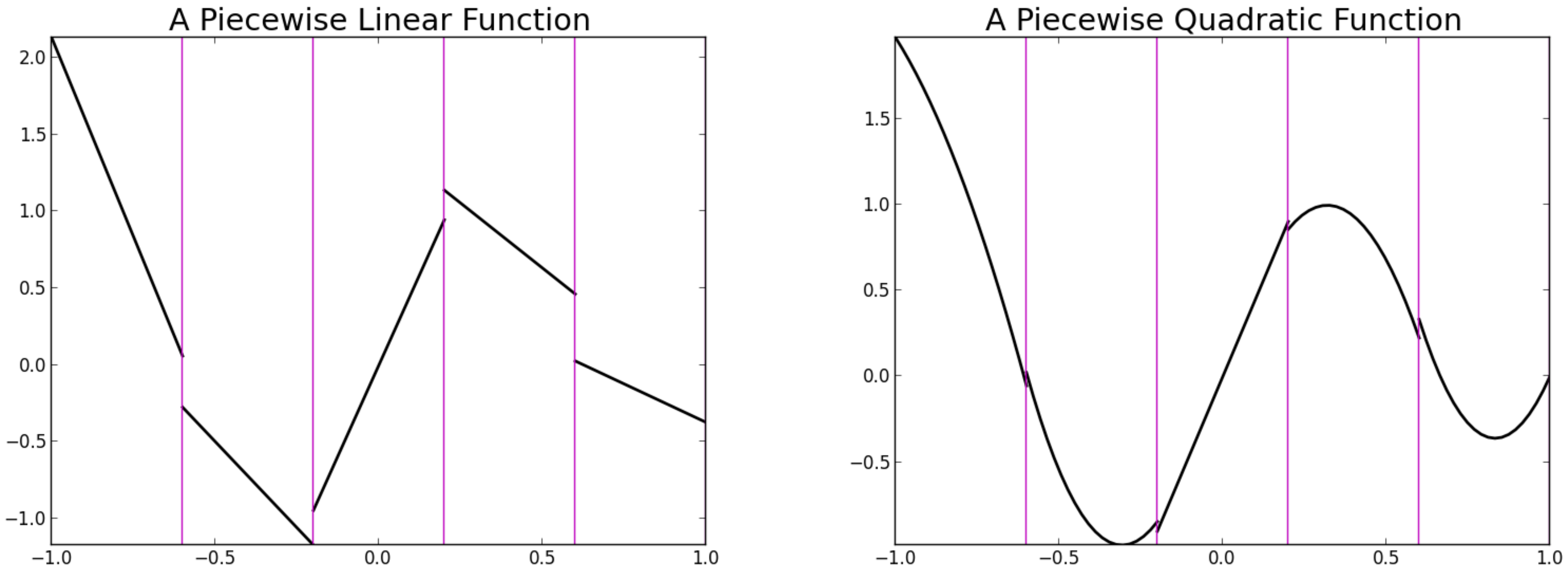
- subgrid / hypercollision models to model phase-mixing and turbulent mixing to unresolved scales (handles recurrence issues).

DG combines some advantages of Finite Volume (FV) with Finite Element accuracy:
FV interpolates p uniformly-spaced points to get p order accuracy
DG interpolates p optimally-located points to get 2p-1 order accuracy

Research software projects hard to predict, but hope/goal: be able to run very quickly at coarse velocity resolution like a fluid code & get qualitatively useful robust results, but be able to ramp up velocity resolution to get a rigorous kinetic solution and check convergence when needed.

Discontinuous Galerkin (DG) Concepts

- Don't get hung up on the "discontinuous" name. Key idea: DG generalizes finite volume approach to advance higher moments of the solution instead of just the cell average. (i.e., evolve higher order basis functions in each cell).
- Lowest order DG is a Finite volume method, corresponding to knowing a piecewise constant representation in each cell, but can always interpolate between adjacent cells to reconstruct a higher-order and/or smooth solution when needed.
- Edge/pedestal gyrokinetic turbulence: very challenging, 5D problem. Benefits from all tricks we can find: Factor of 2 reduction in resolution in each direction --> 64x speedup.



A Piecewise Linear Function

A Piecewise Quadratic Function

Figure: The best L_2 fit of $x^4 + \sin(5x)$ with piecewise linear (left) and quadratic (right) basis functions.

Continuum kinetic code might be useful for some problems where low noise levels are needed

- PIC algorithms work well for many types of problems, particularly where there is a large amplitude effect (like shocks or strong instabilities) so that moderate particle noise is not too bothersome.
- A continuum kinetic code may be useful for problems where a very low noise level is needed, perhaps near marginal stability where instabilities saturate at a small amplitude.
- May also be useful at intermediate collisionality where kinetic corrections are needed, but collisional smoothing at small velocity scales reduces resolution requirements in velocity space.

- In any case, these are difficult and complex problems, and it is helpful to have independent codes with different algorithms to cross-check each other.

Summary

- My perspective on status of fusion energy research
  - hard, but important and we are making progress. There are good ideas about how to improve further.
- Physical picture of instabilities driven by “bad curvature” / effective gravity
- Various methods to improve fusion being studied
  - advanced tokamak regimes, liquid metal walls, new stellarator designs
- Simulation techniques developed in fusion that may be useful in astro/solar
  - comprehensive 5D continuum simulations of gyrokinetic microturbulence fairly successful in main part of fusion plasmas, being applied to some astro problems: heating in the tail of an Alfven turbulent cascade, cosmic ray transport, ...
  - new project exploring advanced continuum algorithms (discontinuous Galerkin, ...) for edge region, Vlasov-Boltzmann / Hamiltonian problems (A. Hakim poster)