# Thoughts on Continuum Algorithms.

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at PMP "1st Annual Summer Frontier Center"

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① Gyrofluids : 6 velocity moments
GK : ~400 velocities
} $\chi_i$ within a factor of ~2.

Can one reduce # of v's in GK code?

Can one use 20–100 v's ?

Adjustable to check convergence...

② "Recurrence problem"

(Present codes have some numerical dissipation to cope: slight upwinding, slight off-center implicit, collisions... )

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⇒ Hyper collision operator ⇒ ② eliminate recurrence
can reduce numerical dissipation elsewhere
① reduce # of v's.

# Coping with well-known "Recurrence problem"

in this "Continuum code" ("Vlasov code", finite-difference "Eulerian")

(related, in a way, to noise in a Lagrangian PIC (Particle-in-cell) code or to Landau damping)

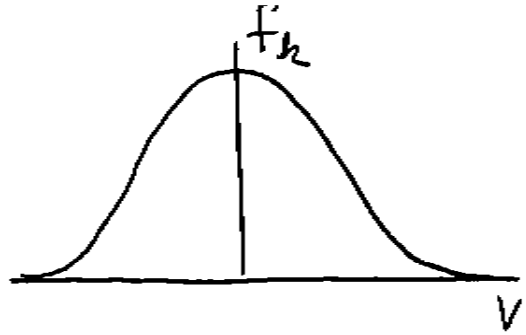
Consider simple 1-D free streaming:

$$\frac{\partial f}{\partial t} + v \frac{\partial f}{\partial z} = 0$$

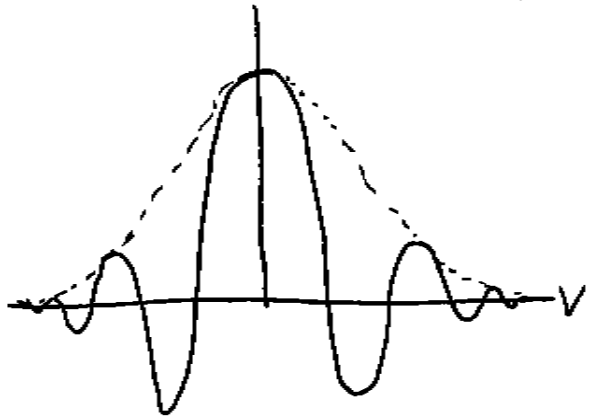
$$\frac{\partial f_k}{\partial t} = -ikv\, f_k$$

$$f_k(t) = e^{-ikvt} \underbrace{e^{-\frac{v^2}{2v_t^2}}}_{f_k(v,\, t=0)}$$

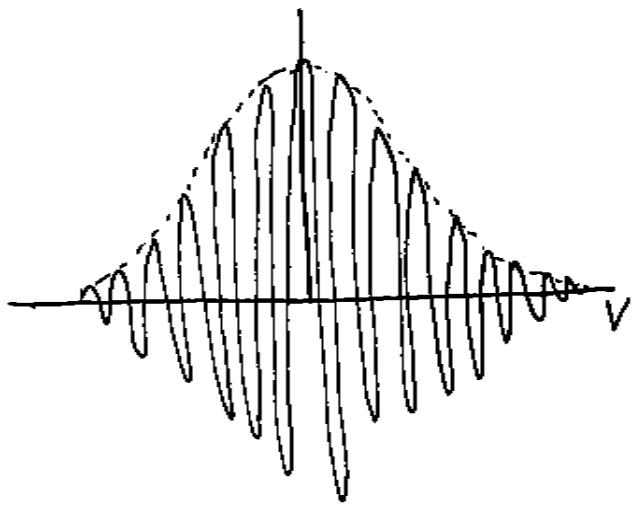
$t=0$




$t=t_1$




$t=4t_1$




Need high velocity resolution!

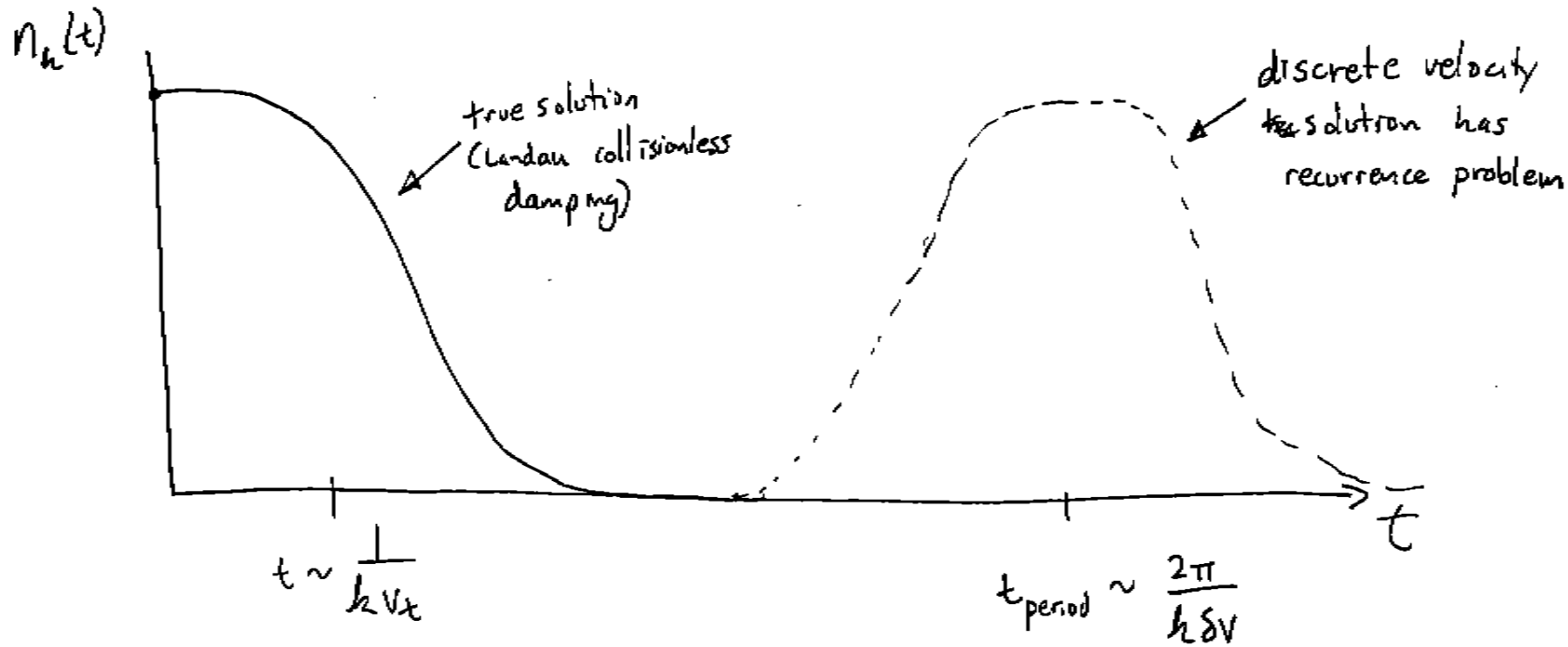
Physically: small $v$ scales damped by collisions.

$$n(t) = \int dv\, f_k \propto e^{-k^2 v_t^2 t^2/2}$$

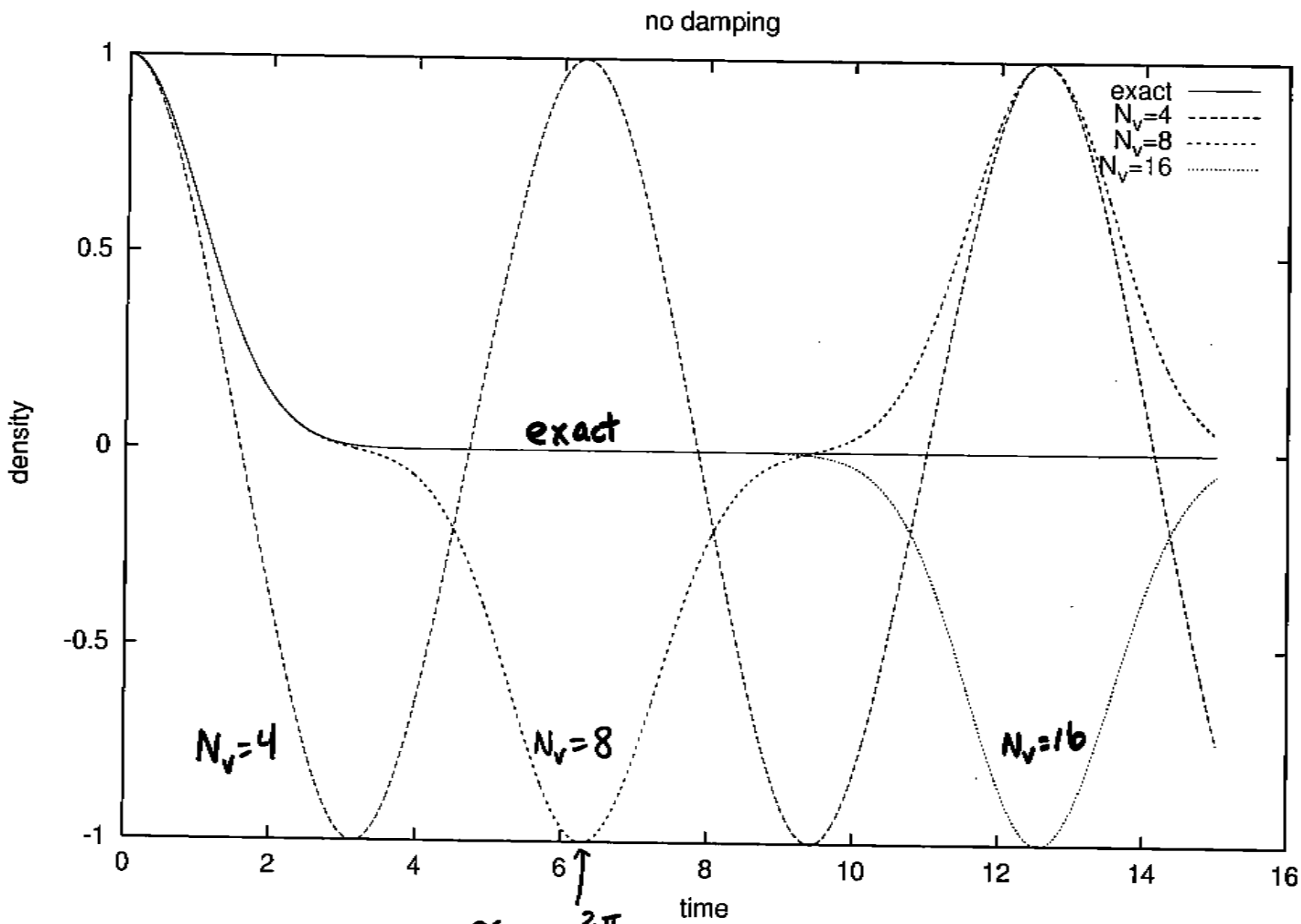
But if a discrete velocity grid is used:

$$n_k(t) = \sum_j \delta v\, \underbrace{e^{-ikv_j t}}_{\text{Phases decohere temporarily}}\, e^{-\frac{v^2}{2v_t^2}}$$

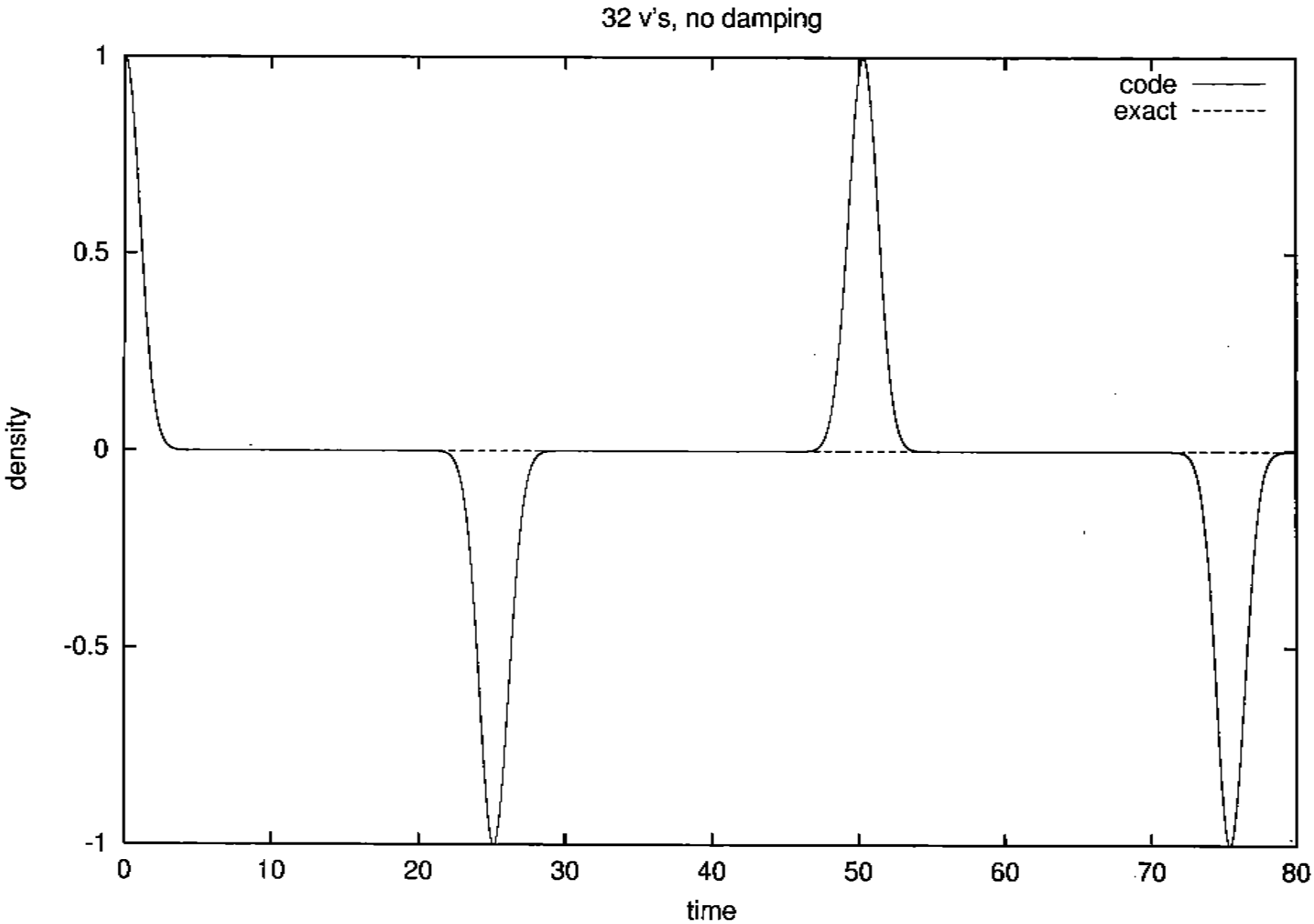
Periodic $t = \frac{2\pi}{k\delta v}$



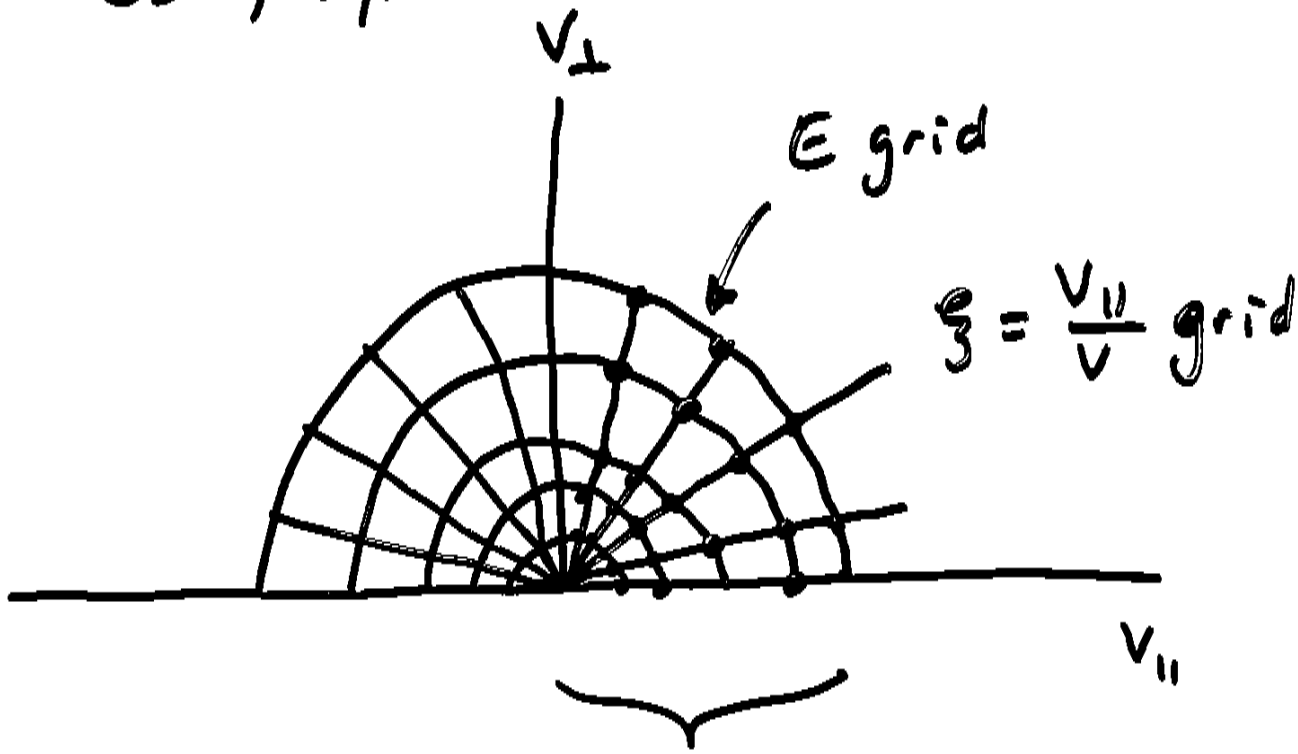
$$f \propto e^{-v^2/2v_t^2}\, e^{-ikvt} \qquad n_{exact} = \int dv\, f = e^{-k^2 v_t^2 t^2/2}$$




$$\tau_{echo} = \frac{2\pi}{|k|\Delta v} \qquad v \in [-4v_t, 4v_t] \qquad \Delta v = \frac{8v_t}{N_v}$$


32 v's, no damping
code
exact
density
time
1
0.5
0
-0.5
-1
0
10
20
30
40
50
60
70
80

GS2, Gyro




Set of $v_\parallel$'s not on a uniform grid.

$$n \approx \sum_j \delta v_{\parallel j} \underbrace{e^{-ik_\parallel v_j t}}_{} f_j$$

hard to get phases to coherently add

$\Rightarrow$ get $\frac{1}{N_{v_\parallel}}$ "noise"

Smooth out small velocity scales, w/ std. collision operator:

$$\frac{\partial f}{\partial t} + \text{standard gyrokinetic terms} = \nu_2 V_t^2 \frac{\partial^2 f}{\partial v_\parallel^2}$$

(+ momentum + energy conserving terms)

Or Hyper-collision operator:

$$= -\nu_4 V_t^4 \frac{\partial^4 f}{\partial v_\parallel^4}$$

Advantages: less sensitive to choice of $\nu_4$

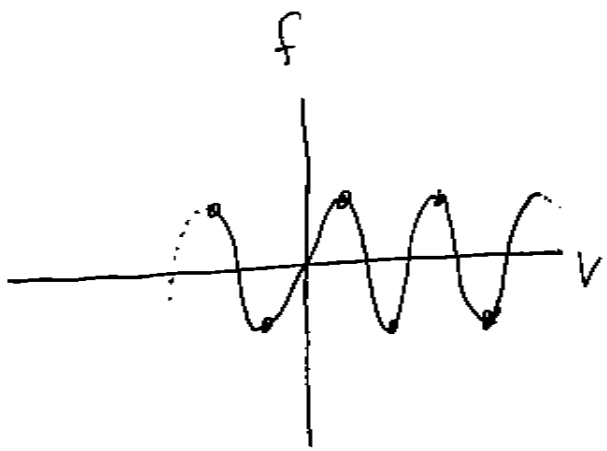
Conserves lowest 4 moments $\int dv_\parallel \, v_\parallel^{0-3} f$

# Minimum Hyper-Damping rate requirement:
(confirmed numerically)

Need

Damping rate at barely resolved (Nyquist limit) scale $>$ Phase-mixing rate at which information should transfer to unresolved scales.

$$\nu_4 \, V_t^4 \, \frac{16}{(\Delta v)^4} > \underbrace{k \, \Delta V}_{\text{Frequency difference between adjacent grid points.}}$$




$$f_j \propto e^{i k v_j t}$$

# Free Streaming + Hyper Collisions

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial z} = (-1)^n \nu_{2n} V_t^{2n} \frac{\partial^{2n} f}{\partial v^{2n}}$$

Fourier trans. in space:

$$\frac{\partial f}{\partial t} = -ikvf + (-1)^n \nu_{2n} V_t^{2n} \frac{\partial^{2n} f}{\partial v^{2n}}$$

w/ init. condition $f(v, t=0) = f_0(v)$

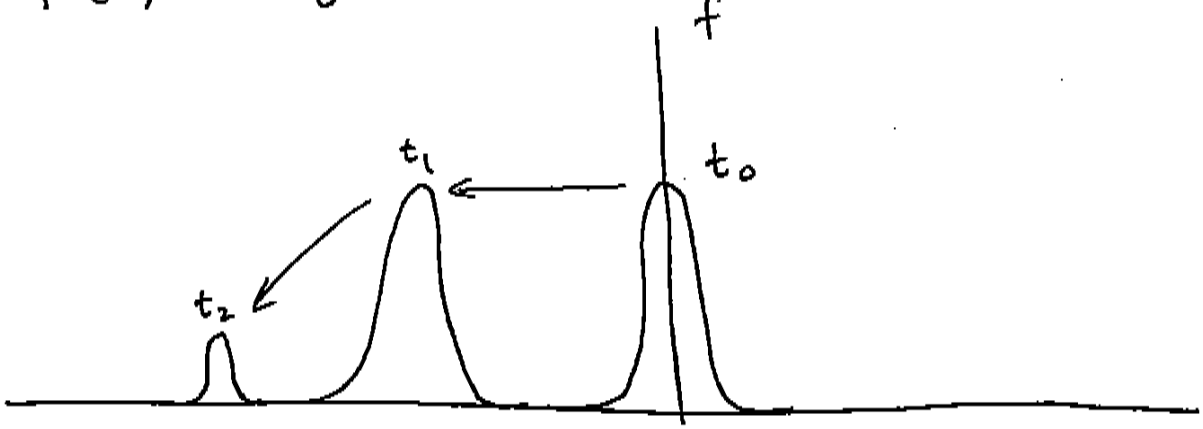
Fourier trans. in velocity:

$$f(v,t) = \frac{1}{\sqrt{2\pi}} \int ds\, e^{isv}\, \hat{f}(s,t)$$

$$\frac{\partial \hat{f}}{\partial t} = k\frac{\partial \hat{f}}{\partial s} - \nu_{2n} V_t^{2n} s^{2n} \hat{f}$$

Exact Soln. (Parker + ~~Mattor?~~ Carditti)

$$\hat{f}(s,t) = g(s+kt)\, e^{\frac{\nu}{k} V_t^{2n} \frac{s^{2n+1}}{2n+1}}$$



Want to damp $\hat{f}$ before it reaches Nyquist limit $|\xi_{max}| = \frac{\pi}{\delta v}$

& "reflects" & gives an "aliased echo" error.

Minimum damping (to keep echo error $< 10^{-3}$):

$$\boxed{\nu_{2n} > 1.2\,(2n+1)\left(\frac{\delta v}{2 V_t}\right)^{2n+1} |k|\, V_t}$$

Landau-Fluid models:

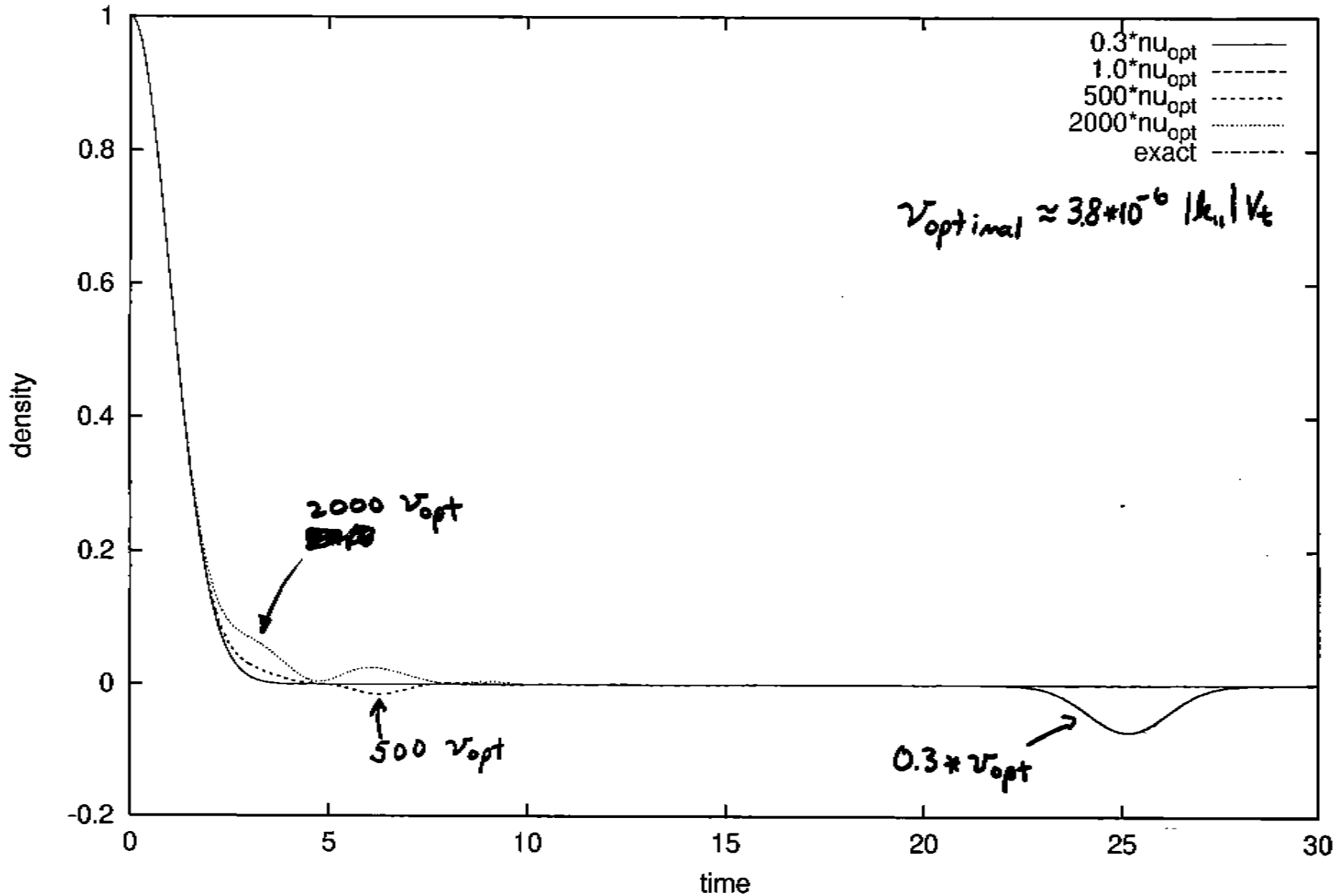
Apply damping $\propto |k|\, V_t$ in the highest Fluid moments ($\approx H_3$ Hermite polynomial $\approx$ hyper collisions with $2n = 4$).

$$\boxed{\nu_6 \gtrsim 4 * 10^{-6}\, |k|\, V_t \quad \text{at} \quad \delta v = \frac{V_t}{4}}$$

Has very little effect on $\delta v \sim V_t$ scales.

Use $k = k_{max}$ & apply to all modes $\Rightarrow$ avoid FFT's.

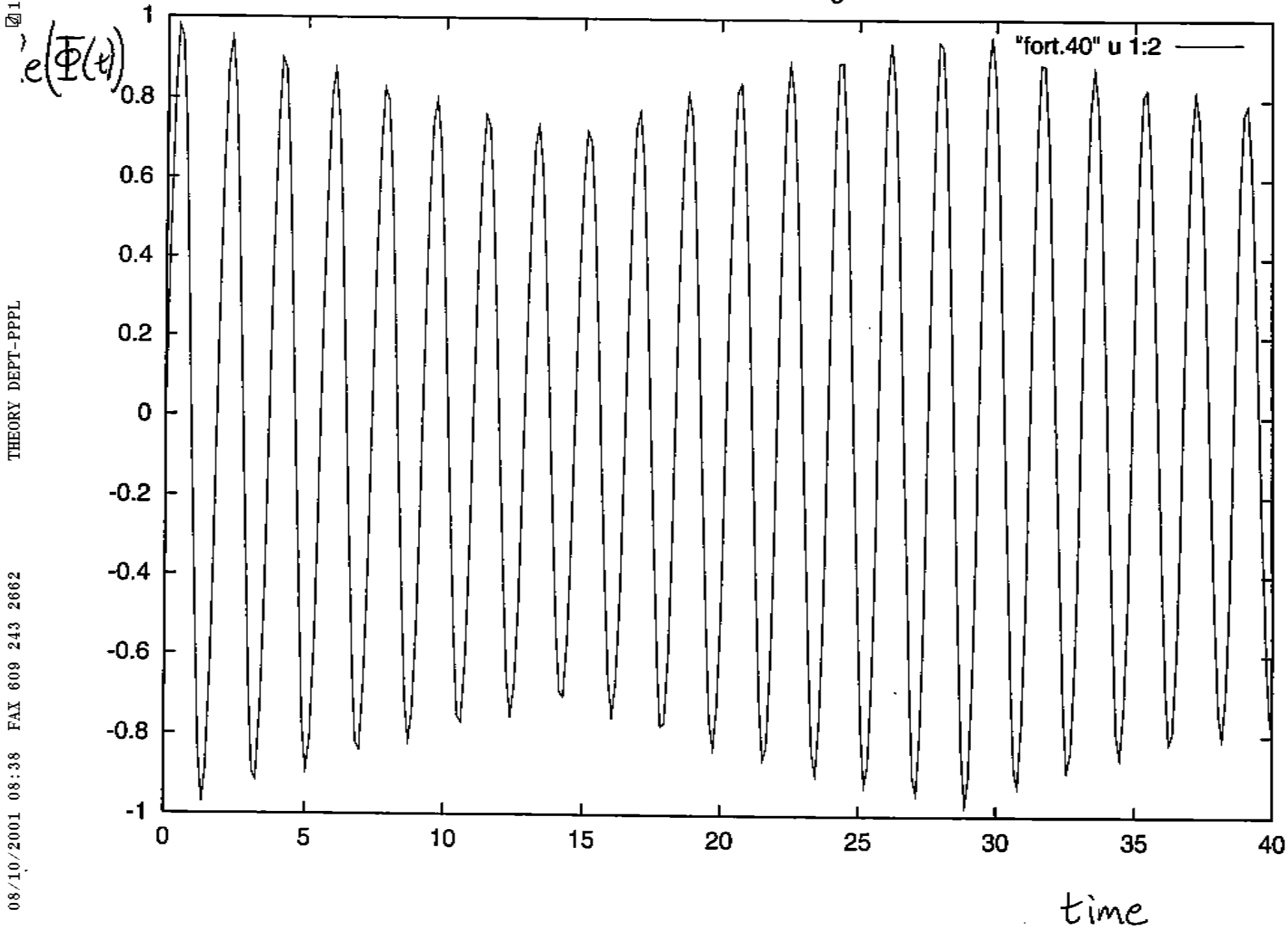
Results insensitive to factor $\sim 10^3$ variation in $\frac{\partial^6 f}{\partial v^6}$ coefficient



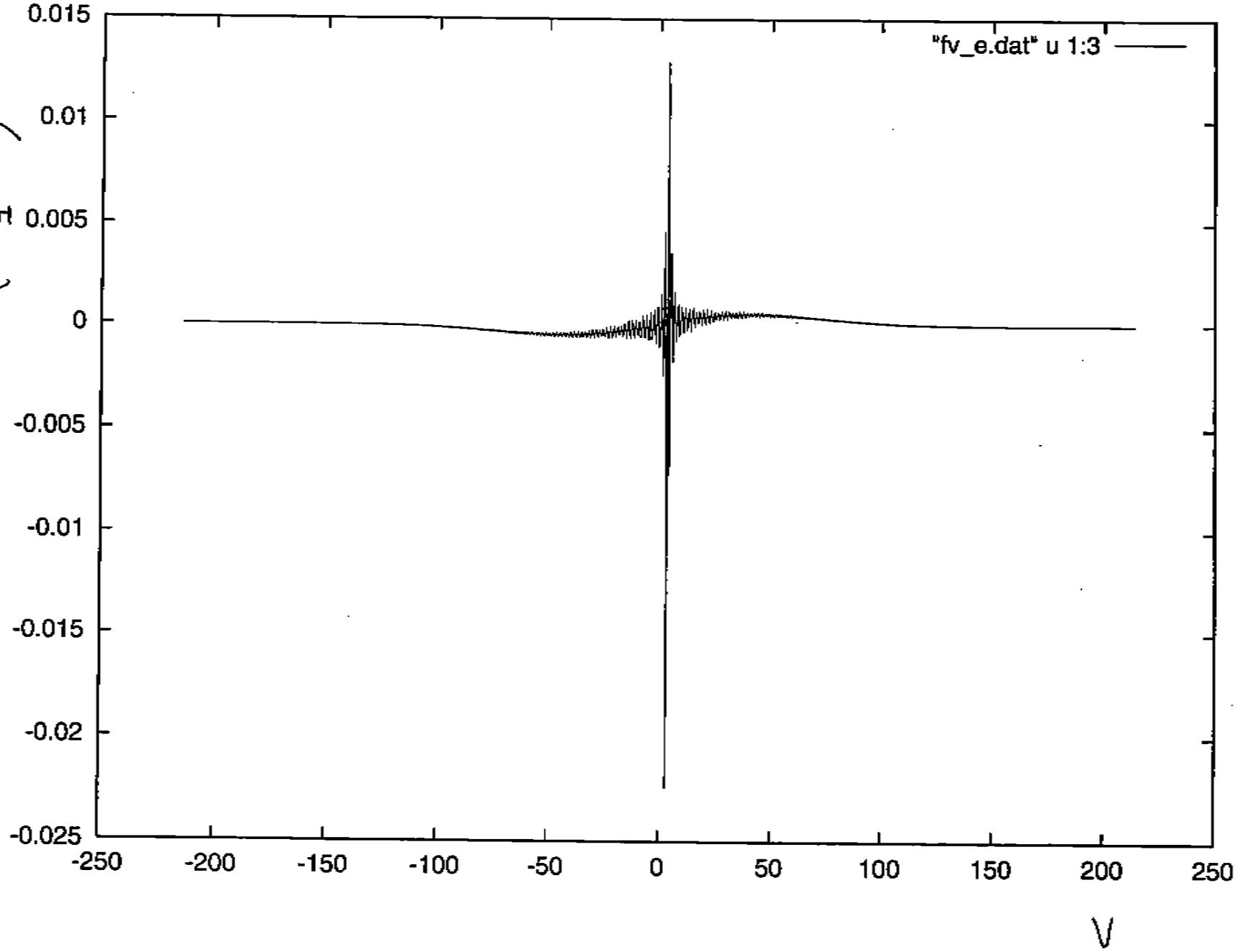

$\nu_{optimal} \approx 3.8 * 10^{-6} \, |k_{\parallel}| \, V_t$

$N_v = 32 \qquad \frac{\Delta v}{v_t} \approx \frac{1}{4}$

Recurrence Problem (no net damping) at $\nu_4 = \nu_2 = 0$



$\nu_4 = \nu_2 = 0$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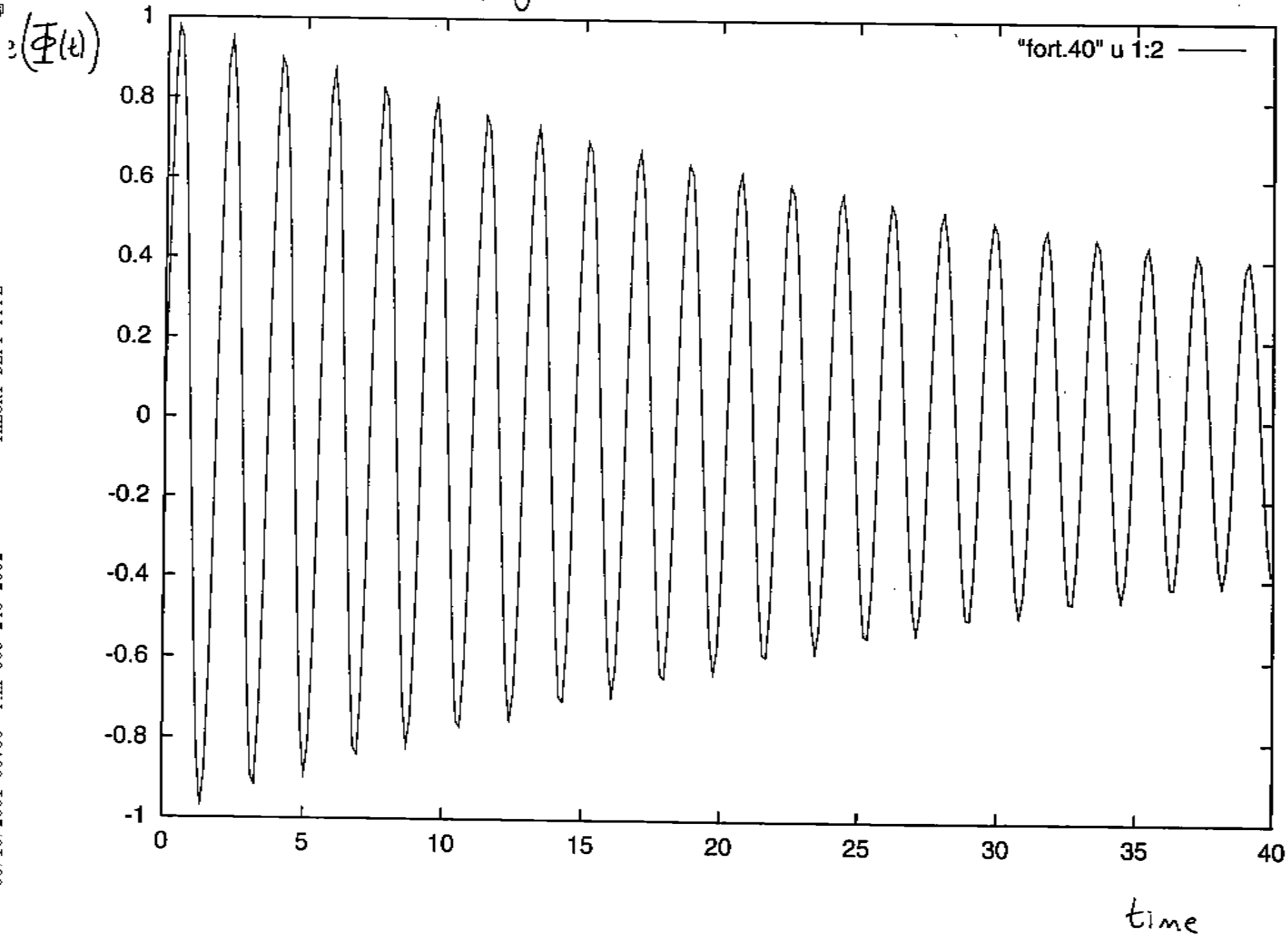
0.01
0.005
0
-0.005
-0.01
-0.015
-0.02
-0.025
-250
-200
-150
-100
-50
0
50
100
150
200
250
$\mathrm{Im}\left(\frac{f_e(v)}{\Phi}\right)$
V

Landau damping okay at $\nu_4 = 10^{-8}$

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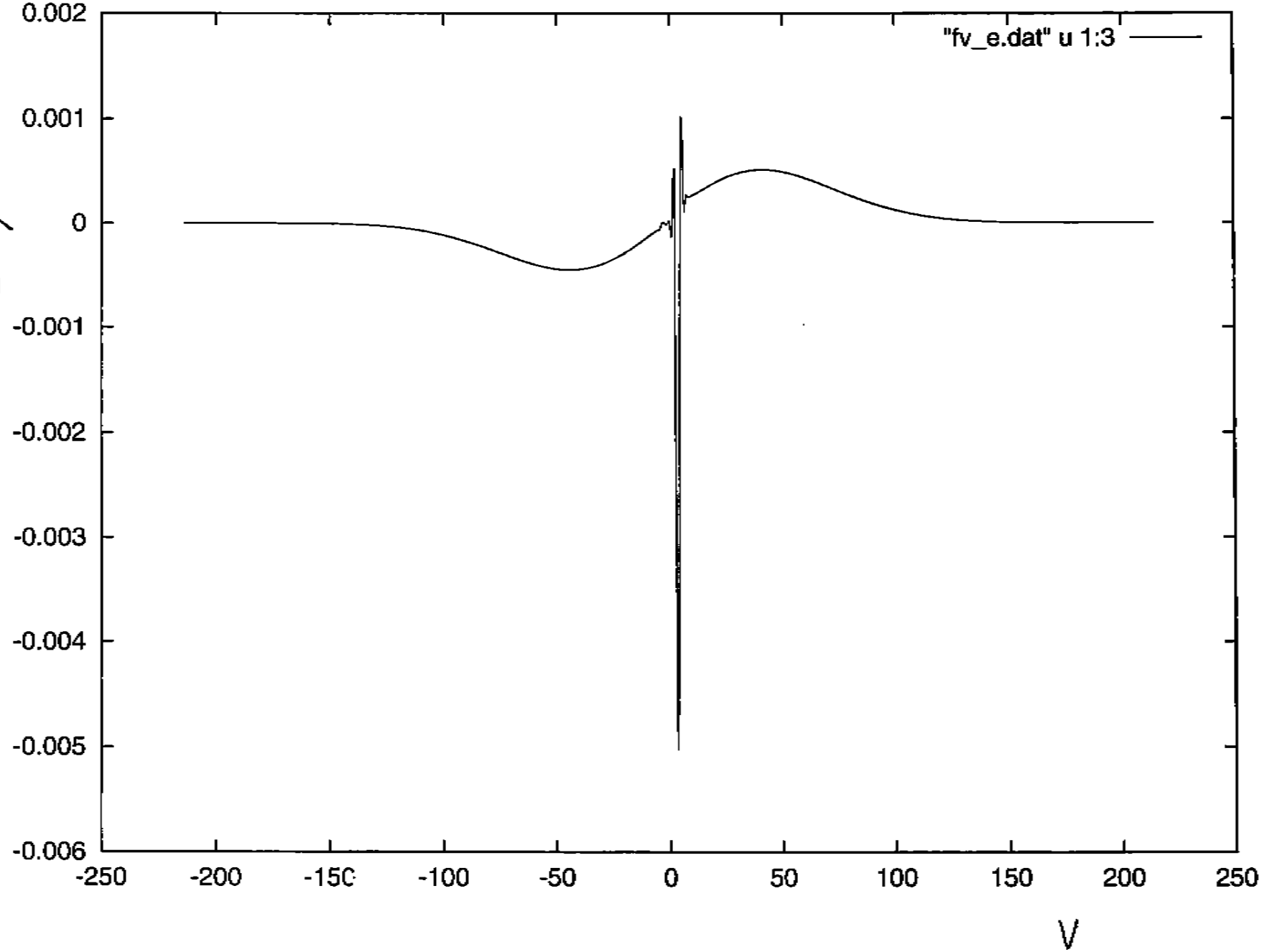
$\nu_4 = 10^{-8}$


"fv_e.dat" u 1:3
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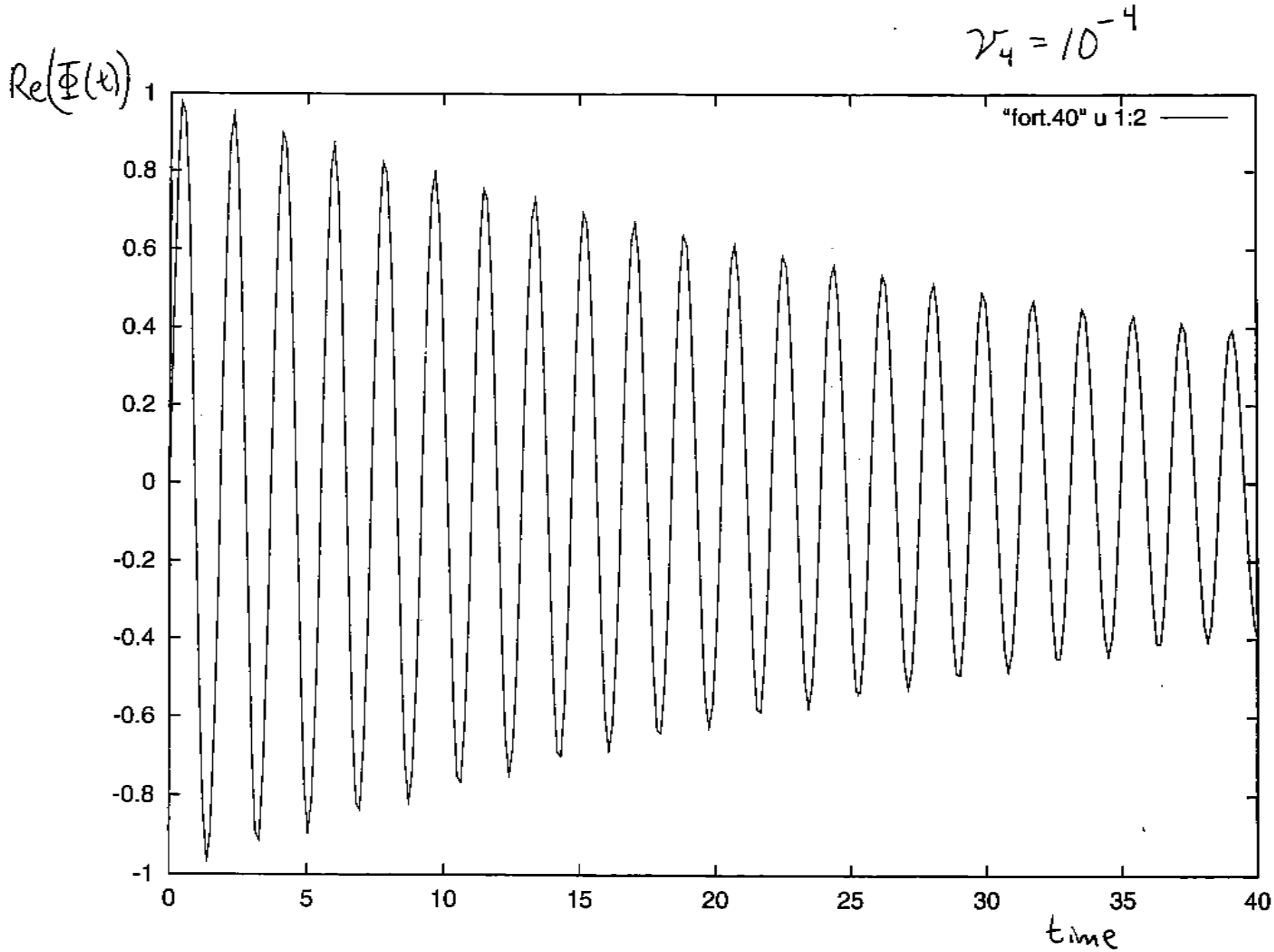
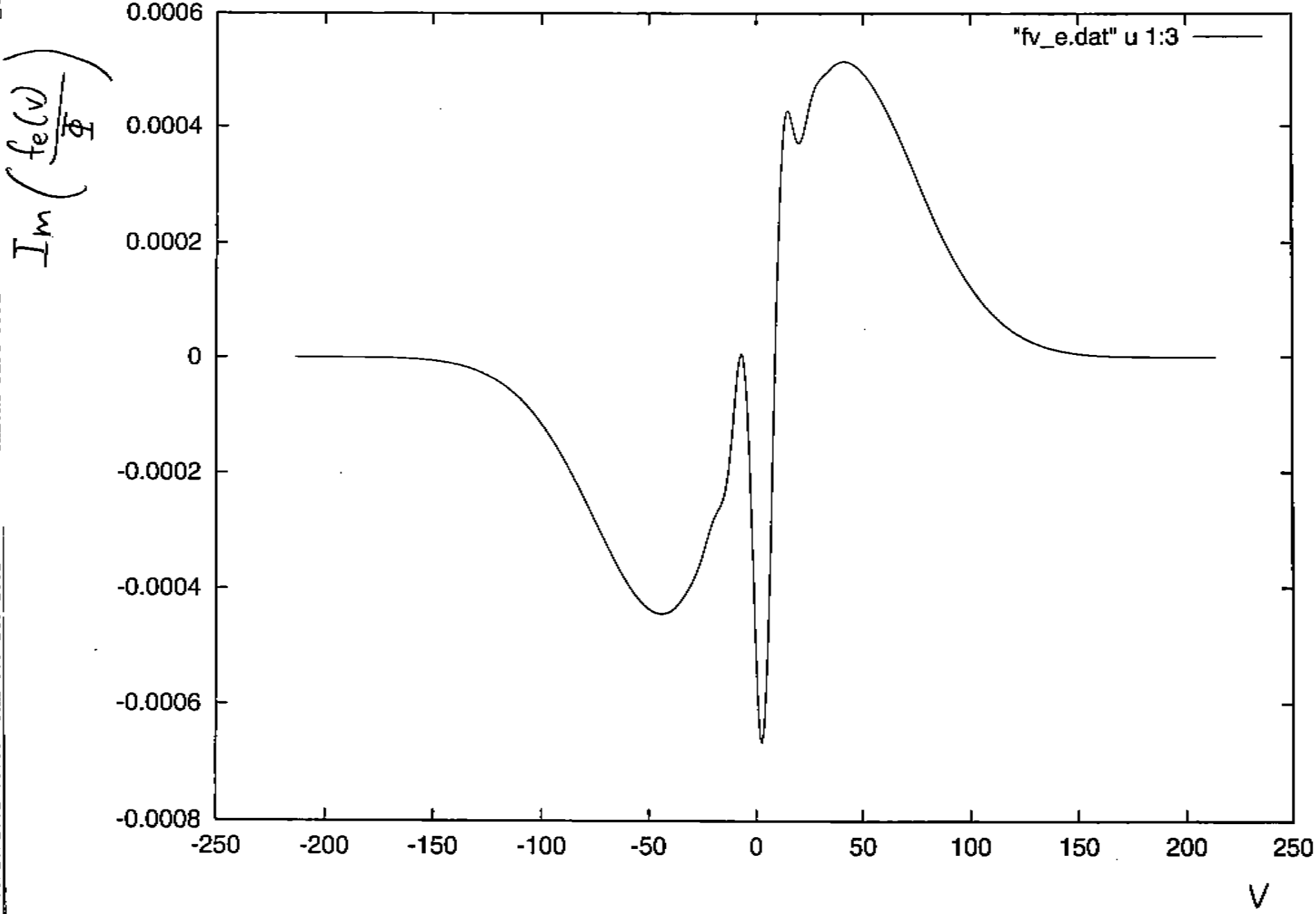
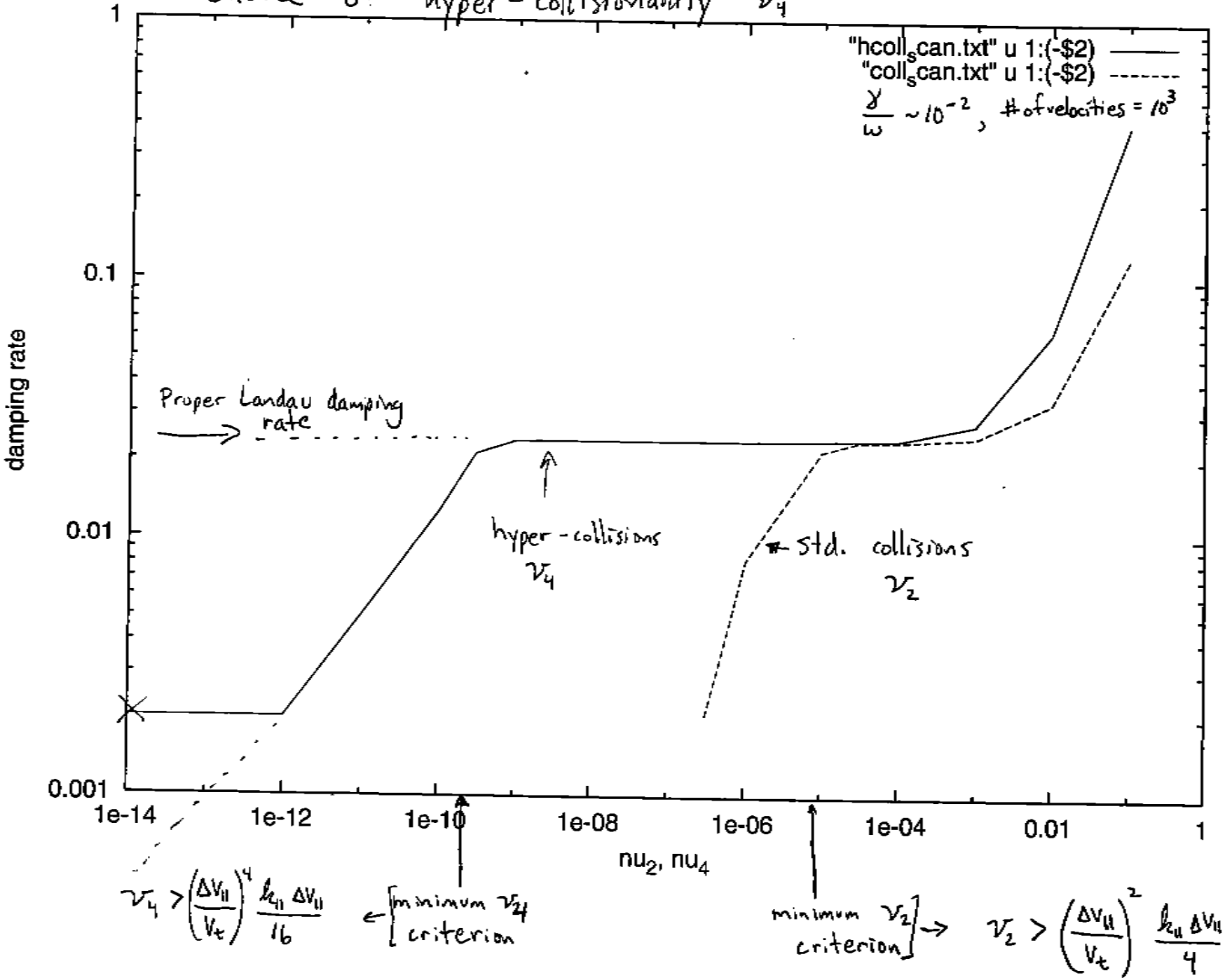
0.001
0
-0.001
-0.002
-0.003
-0.004
-0.005
-0.006
-250
-200
-150
-100
-50
0
50
100
150
200
250
V
$\mathrm{Im}\left(\frac{\tau_e(V)}{\Phi}\right)$


$\nu_4 = 10^{-4}$
Re(Φ(t))
"fort.40" u 1:2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
5
10
15
20
25
30
35
40
time

$\nu_4 = 10^{-4}$


"fv_e.dat" u 1:3
0.0006
0.0004
0.0002
0
-0.0002
-0.0004
-0.0006
-0.0008
-250
-200
-150
-100
-50
0
50
100
150
200
250
V
$\mathrm{Im}\left(\frac{f_e(v)}{\Phi}\right)$

Electron Landau Damping of Alfven wave, insensitive to choice of hyper-collisionality $\nu_4$



# $\frac{c}{\omega_{pe}}$ in Jenko/Scott Formulation

1-D electromagnetic

$$\frac{\partial f}{\partial t} + v_{\parallel}\frac{\partial f}{\partial z} = \left(-\frac{\partial \Phi}{\partial z} - \frac{1}{c}\frac{\partial A_{\parallel}}{\partial t}\right)\frac{q v_{\parallel}}{T} F_M$$

Jenko & Scott

$$f = g - \frac{1}{c} A_{\parallel} \frac{q v_{\parallel}}{T} F_M$$

$$\frac{\partial g}{\partial t} + v_{\parallel}\frac{\partial g}{\partial z} = -\frac{\partial \Phi}{\partial z}\frac{q v_{\parallel}}{T} F_M + v_{\parallel}^2 \frac{\partial A_{\parallel}}{\partial z}\frac{q}{cT} F_M$$

Modified Ampere's Law:

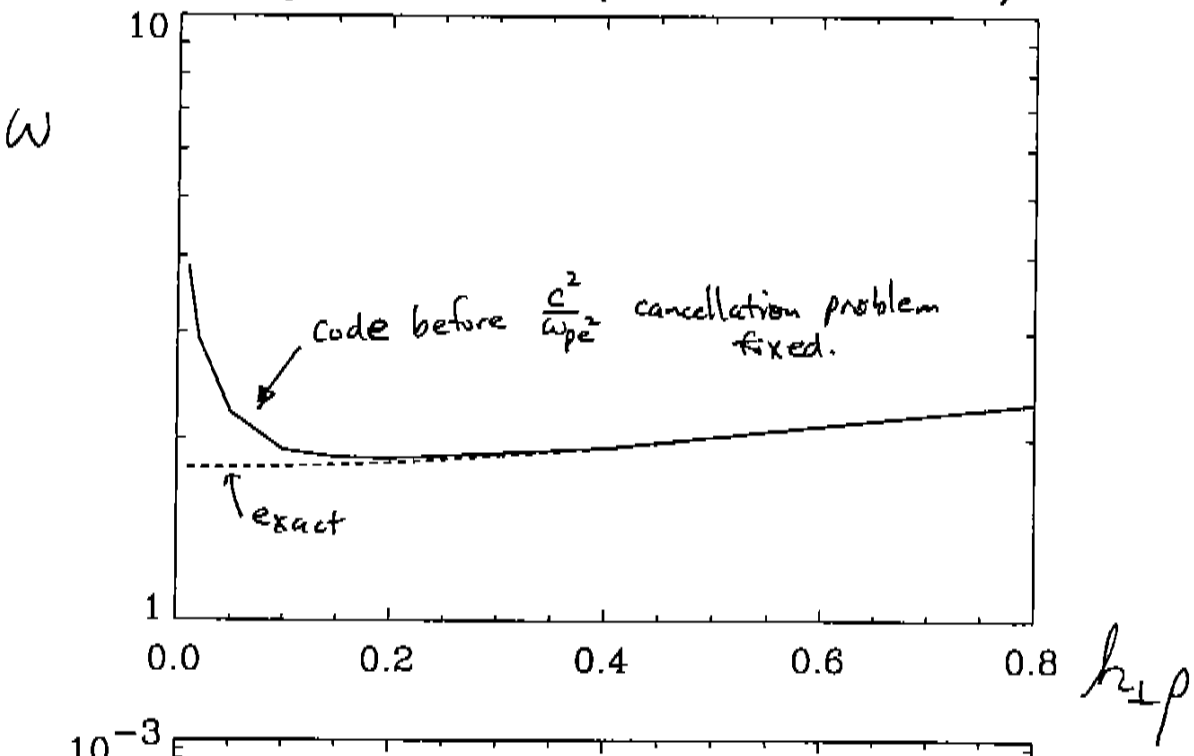
$$\left[k_{\perp}^2 + \frac{1}{c^2}\underbrace{\sum_s \frac{4\pi q^2 \int d^3v\, F_M v_{\parallel}^2}{T}}_{\text{Analytically} = \omega_{pe}^2}\right] A_{\parallel} = \frac{4\pi}{c}\sum_s q \int d^3v\, g\, v_{\parallel}$$

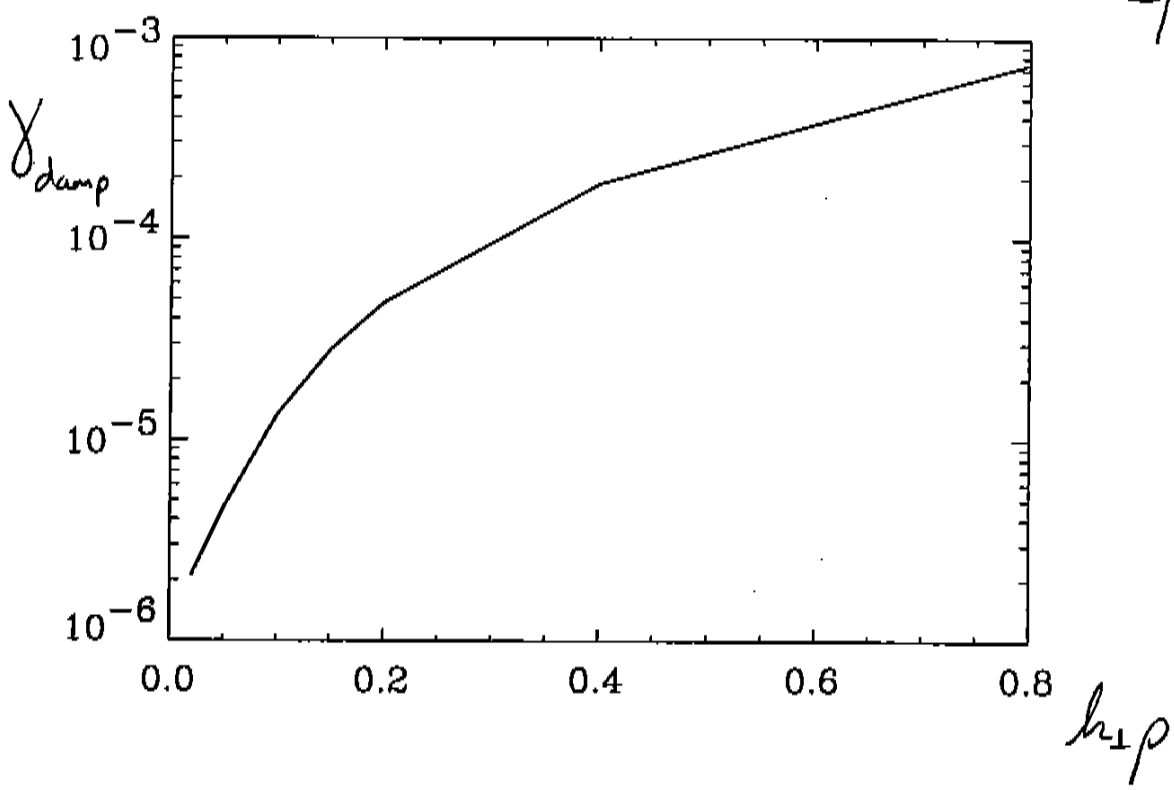
Need to do these integrals on equal footing to get cancellations right. Fixed low $k_{\perp}$ problem Jenko pointed out.

# Kinetic shear Alfvén waves: MHD limit

$\beta_e = 0.1$

*Fully kinetic model (Jenko & Scott 1999)*



# Conclusions:

* Kotschenreuther's implicit (& fast) algorithm for linear terms recovers plasma dispersion Z function

  if $\frac{\omega \Delta t}{2} << 1$ & $\frac{k \Delta z}{2} << 1$

  i.e. Courant condition for fastest particles $V_{max} \Delta t < \Delta z$ can be violated & still keep accuracy for lower phase velocity waves.

* Hyper-collisions has little effect on long v scales, eliminates recurrence problems at small v scales, & reduces # of v grid points needed.

* Future work: Alternative noise filters (or projection operators) to further reduce # of v points needed?