Some Properties of Landau-Fluid Models of Kinetic MHD

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Kulsrud / Kruskal-Oberman/ Chew-Goldberger-Low Kinetic MHD

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0,$$

$$\rho \frac{\partial \mathbf{V}}{\partial t} + \rho \left(\mathbf{V} \cdot \nabla \right) \mathbf{V} = \frac{(\nabla \times \mathbf{B}) \times \mathbf{B}}{4\pi} - \nabla \cdot \mathbf{P} + \mathbf{F_g},$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left(\mathbf{V} \times \mathbf{B} \right),$$

$$\mathbf{P} = p_\perp \mathbf{I} + \left(p_\parallel - p_\perp \right) \hat{\mathbf{b}}\hat{\mathbf{b}},$$

$$\frac{\partial f}{\partial t} + \left(v_\parallel \hat{\mathbf{b}} + \mathbf{v}_E \right) \cdot \nabla f + \left(-\hat{\mathbf{b}} \cdot \frac{D\mathbf{v}_E}{Dt} - \mu \hat{\mathbf{b}} \cdot \nabla B + \frac{e}{m} (E_\parallel + F_{g\parallel}/e) \right) \frac{\partial f}{\partial v_\parallel} = C\left(f \right),$$

Drift-kinetic equation: similar to gyro-kinetic equation but without FLR, and includes compressional Alfven wave / fast-wave

Mirror force hidden in CGL pressure tensor

$$\mathbf{P} = p_\perp \mathbf{1} + (p_\parallel - p_\perp)\hat{b}\hat{b}$$

- Parallel component of force balance (stationary u=0 equil.):

$$0 = -[\nabla \cdot \mathbf{P}] \cdot \hat{b} = -\nabla_\parallel p_\parallel - (p_\perp - p_\parallel)\frac{\nabla_\parallel B}{B}$$

$p_\perp - p_\parallel > 0$ corresponds to particles trapped in magnetic well, $\nabla_\parallel p_\parallel \neq 0$

i.d.:

$$\begin{aligned}\nabla \cdot \left(\hat{b}\hat{b}\right) \cdot \hat{b} &= \nabla \cdot \hat{b} + \left(\hat{b} \cdot \nabla \hat{b}\right) \cdot \hat{b} \\ &= \nabla \cdot \hat{b} + \hat{b} \cdot \nabla\left(\frac{1}{2}\left|\hat{b}\right|^2\right) \\ &= -\frac{\hat{b} \cdot \nabla B}{B}\end{aligned}$$

- Note: mirror force independent of magnitude of B, important for arbitrarily weak B!

Evolution of the Pressure Tensor

$$\rho B \frac{d}{dt}\left(\frac{p_\perp}{\rho B}\right) = -\nabla\cdot(\hat{\mathbf{b}} q_\perp) - q_\perp \nabla\cdot\hat{\mathbf{b}}$$

adiabatic invariance
of $\mu \propto mv^2_\perp/B \sim T_\perp/B$

$$\frac{\rho^3}{B^2}\frac{d}{dt}\left(\frac{p_\| B^2}{\rho^3}\right) = -\nabla\cdot(\hat{\mathbf{b}} q_\|) + 2q_\perp \nabla\cdot\hat{\mathbf{b}},$$

$$\frac{p_\| B^2}{\rho^3} \propto \frac{T_\| B^2}{\rho^2}$$

Only parallel
compression affects $T_\|$

$q_\perp = q_\| = 0$ CGL or Double Adiabatic Theory

$$q_\| = -n \mathrm{v}_t \frac{\mathrm{v}_t^2}{\mathrm{v}_t |\mathrm{k}_\|| + \nu} \nabla_\| T_\|$$

Closure Models for heat flux (temp. gradients wiped out on ~ a crossing time) => multipole approx. to Landau damping.

recovers Braginskii/Chapman-Enskog in large collision frequency ν limit

Real-space form of heat conduction integral

Closure in k space: $$q_{\|} = -n \mathrm{v}_t \frac{\mathrm{v}_t^2}{\mathrm{v}_t |\mathrm{k}_{\|}| + \nu} i k_{\|} T_{\|}$$

Fourier-transform, get a non-local heat conduction integral along magnetic field lines:

$$q_{\|}(z) = -n_0 \mathrm{v}_t \int_0^{\infty} dz' \frac{T(z+z') - T(z-z')}{z'} \frac{1}{1 + z'^2 / \lambda_{mfp}^2}$$

(incl. collisions, in Snyder, Hammett, Dorland, Phys. Plasmas 1997)

Non-localLity means $-q_{\|}(z) dT/dz > 0$ not guaranteed everywhere, but can show that total entropy S satisfies $dS/dt > 0$

Landau-fluid closure approximations originally derived for small-amplitude turbulence in core of fusion devices with stiff magnetic field: fast evaluation using FFTs. For edge turbulence, and for astrophysical applications, could benefit from nonlinear extensions (work in progress, at least some nonlinear improvements look feasible...), & need to integrate along fluctuating magnetic fields.

Fully kinetic/gyrokinetic simulations more rigorous than fluid approach & becoming very powerful, but continued interest in using Landau-fluid closure approximations

In fusion research, edge turbulence is high priority and very challenging. Critical problem needs multiple codes to attack it and cross-check each other.

Must span both collisional and moderately collisionless plasmas, and wide range of time and space scales.

Extended fluid approach would allow higher resolution and/or faster simulations. [However, apparent speed advantage over kinetic simulations reduced by need to evaluate non-local heat integral, and the fact that kinetic simulations of core turbulence have found they can converge with relatively few velocity grid points (~10 energies, ~20 pitch angles) using high-order velocity integration and other advanced algorithms.]

Caveats: Landau-fluid closures are approximations & are inaccurate in some regimes unless many fluid moments are kept. best for strong-turbulence regimes where instabilities are basically in a fluid-like regime & nonlinearly couple to Landau-damped modes. Weak-turbulence regimes harder. Several papers on limitations of Landau-fluid approx. and extensions, e.g. to neoclassical effects…

Form of pressure equations used (avoid divide by B):

$$\frac{\partial p_{\parallel_s}}{\partial t} + \nabla \cdot (\mathbf{U} p_{\parallel_s}) + \nabla \cdot (\hat{\mathbf{b}} q_{\parallel_s}) + 2 p_{\parallel_s} \hat{\mathbf{b}} \cdot \nabla \mathbf{U} \cdot \hat{\mathbf{b}} - 2 q_{\perp_s} \nabla \cdot \hat{\mathbf{b}}$$

$$= -\tfrac{2}{3} \nu_s (p_{\parallel_s} - p_{\perp_s}),$$

$$\frac{\partial p_{\perp_s}}{\partial t} + \nabla \cdot (\mathbf{U} p_{\perp_s}) + \nabla \cdot (\hat{\mathbf{b}} q_{\perp_s}) + p_{\perp_s} \nabla \cdot \mathbf{U} - p_{\perp_s} \hat{\mathbf{b}} \cdot \nabla \mathbf{U} \cdot \hat{\mathbf{b}}$$

$$+ q_{\perp_s} \nabla \cdot \hat{\mathbf{b}} = -\tfrac{1}{3} \nu_s (p_{\perp_s} - p_{\parallel_s}),$$

Alt. form: average $p=(2p_{\perp} + p_{\|})/3$ and
Pressure difference $\delta p = p_{\|} - p_{\perp}$

$$\mathbf{P} = p\mathbf{1} + \mathbf{\Pi} = p\mathbf{1} + \delta p(3\hat{b}\hat{b} - \mathbf{1})/3$$

$$\frac{dp_s}{dt} + \frac{5}{3}p_s \nabla \cdot \mathbf{U} = -\frac{2}{3}\nabla \cdot (\hat{\mathbf{b}} q_s) - \frac{2}{3}\mathbf{\Pi_s} : \nabla \mathbf{U},$$

$$\frac{d\,\delta p_s}{dt} + \frac{5}{3}\delta p_s \nabla \cdot \mathbf{U} + \mathbf{\Pi_s} : \nabla \mathbf{U} + 3p_s \hat{\mathbf{b}} \cdot \nabla \mathbf{U} \cdot \hat{\mathbf{b}} - p_s \nabla \cdot \mathbf{U}$$
$$- 3q_{\perp_s} \nabla \cdot \mathbf{U} + \nabla \cdot [\hat{\mathbf{b}}(q_{\|_s} - q_{\perp_s})] = -\nu_s \delta p_s ,$$

High collision frequency limit:

$$\delta p_{1s} = -\frac{p_{0s}}{\nu_s}(3\hat{\mathbf{b}} \cdot \nabla \mathbf{U} \cdot \hat{\mathbf{b}} - \nabla \cdot \mathbf{U}).$$

Agrees well with Braginskii’s anisotropic viscosity (5-25% diffs).

Landau-MHD approx. contain at least some key mirror physics

Previous PIC simulation of mirror thought observation that $T_\perp$ is anti-correlated with B was counter-intuitive. ($T_{||}$=const.)

Explanation clear: although μ conservation implies $v_\perp^2 \sim B$ (at moving particle position), particles with high μ are more easily trapped in regions of small B…

Snyder, Hammett, Dorland 1997 Landau-MHD closures:

$$q_{||} \propto -n \mathrm{v}_{\mathrm{t}} \frac{1}{|k_{||}|} \nabla_{||} T_{||}$$

$$q_{\perp} \propto -n \mathrm{v}_{\mathrm{t}} \frac{1}{|k_{||}|} \left[\nabla_{||} T_{\perp} - \frac{T_\perp}{T_{||}} \left(T_\perp - T_{||} \right) \frac{\nabla_{||} B}{B} \right]$$

In reality, $1/|k_{||}|$ is an integral operator when Fourier-transformed to real space, but assume $|k_{||}| =$ const. and solve for $q_{||} = q_\perp = 0$ equil. solutions:

$$\nabla_{||} T_{||} = 0 \qquad\qquad \frac{\nabla_{||} T_\perp}{T_\perp} = \frac{\nabla_{||} n}{n} = -\frac{\left(T_\perp - T_{||} \right)}{T_\perp} \frac{\nabla_{||} B}{B}$$

Landau-fluid approx. agrees well with kinetic mirror growth rate and threshold, much better than CGL

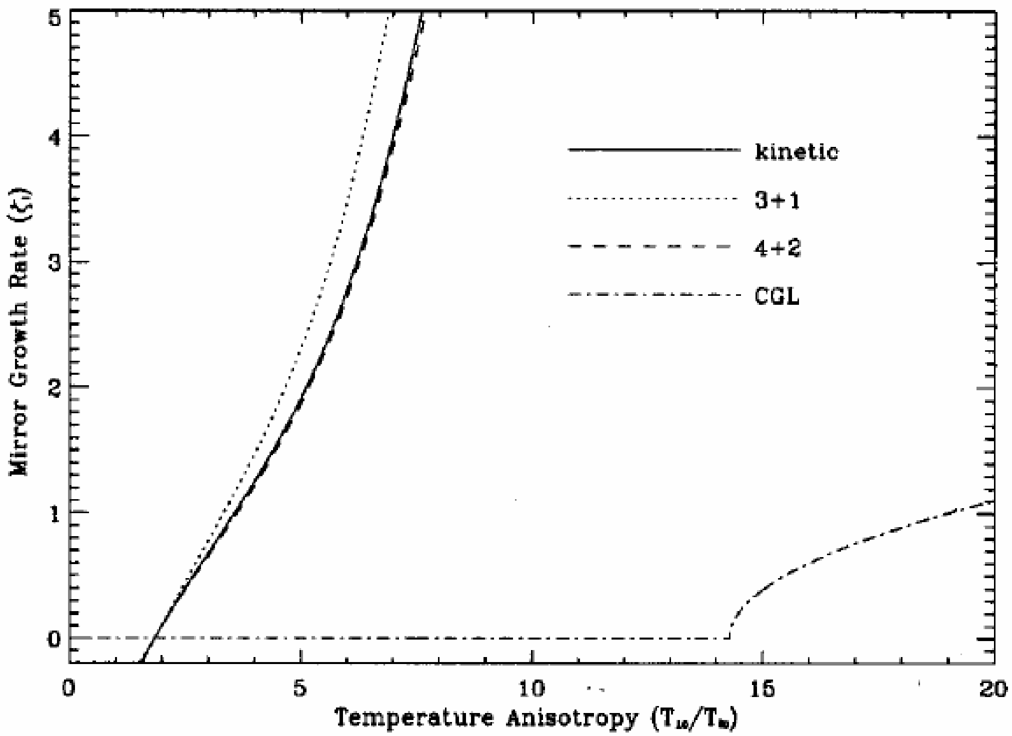

FIG. 5. The linear growth rate of the mirror instability ($k_\perp^2 \gg k_\parallel^2$) as predicted by kinetic theory, $3+1$ and $4+2$ Landau MHD models, and CGL theory (ideal MHD cannot predict the mirror growth rate as it posits an isotropic pressure). The normalized growth rate $[\zeta_i = \mathrm{Im}(\omega)/\sqrt{2}|k_\parallel| v_{T_{\parallel_i}}]$ is plotted versus the temperature anisotropy $(T_{\perp_0}/T_{\parallel_0})$ at constant $\beta = \{(2/3)p_{\perp_0} + (1/3)p_{\parallel_0}\}/(B_0^2/8\pi)$. The parameters chosen are $Z=1$, $T_{\perp_{0i}} = T_{\perp_{0e}}$, $T_{\parallel_{0i}} = T_{\parallel_{0e}}$, $\beta = 1$, and $\sqrt{m_i/m_e} = 40$.

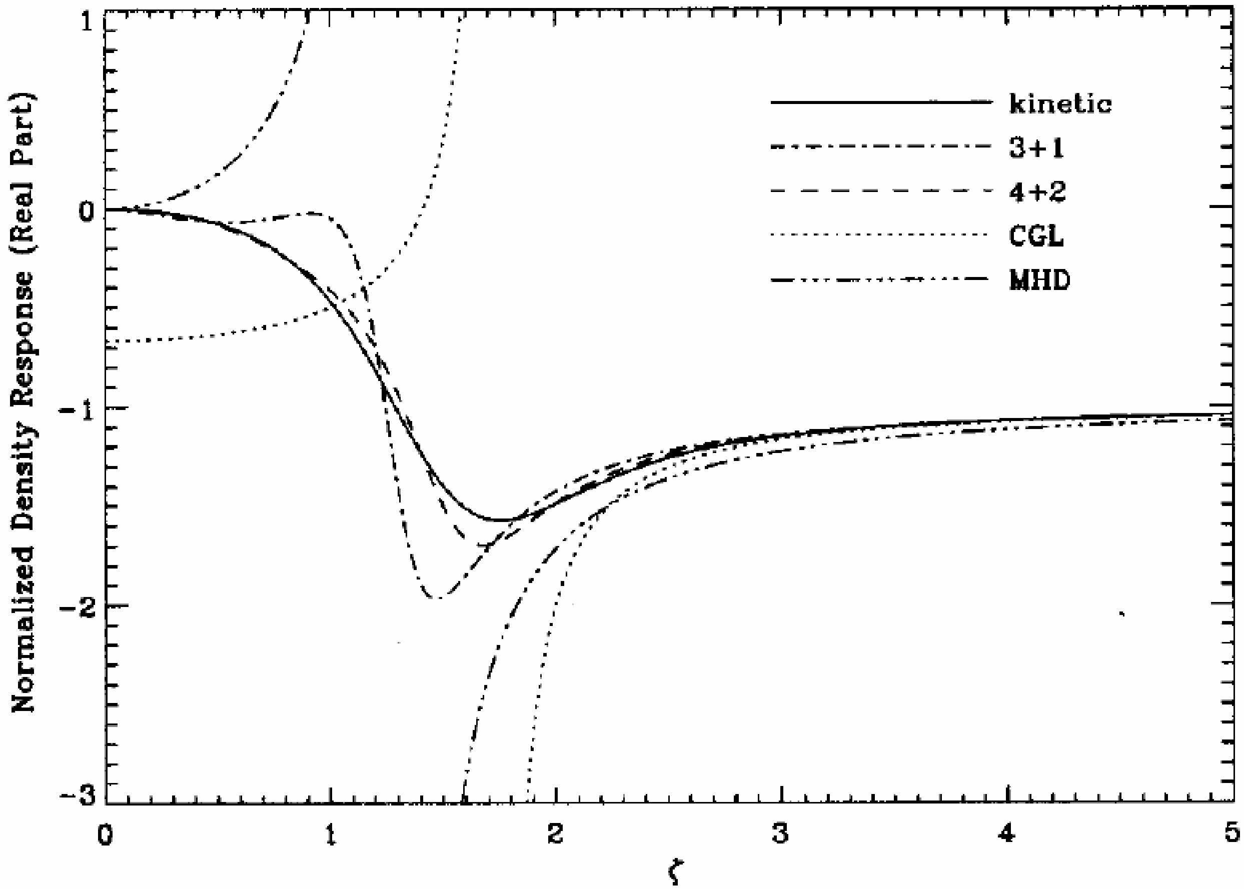

FIG. 1. The real part of the normalized linear density response $(n_1/ik_x\xi_x n_0)$, versus real normalized frequency $(\zeta_i=\omega/\sqrt{2}|k_\parallel|v_{T_{\parallel_i}})$. The 3+1 and 4+2 moment Landau MHD models are compared with linear kinetic theory. Predictions of CGL theory and ideal MHD theory are also shown. Parameters chosen are $Z=1$, $T_{\perp_0}/T_{\parallel_0}=1$, $T_{\perp_{0i}}=T_{\perp_{0e}}$, $T_{\parallel_{0i}}=T_{\parallel_{0e}}$, and $\sqrt{m_i/m_e}=40$.

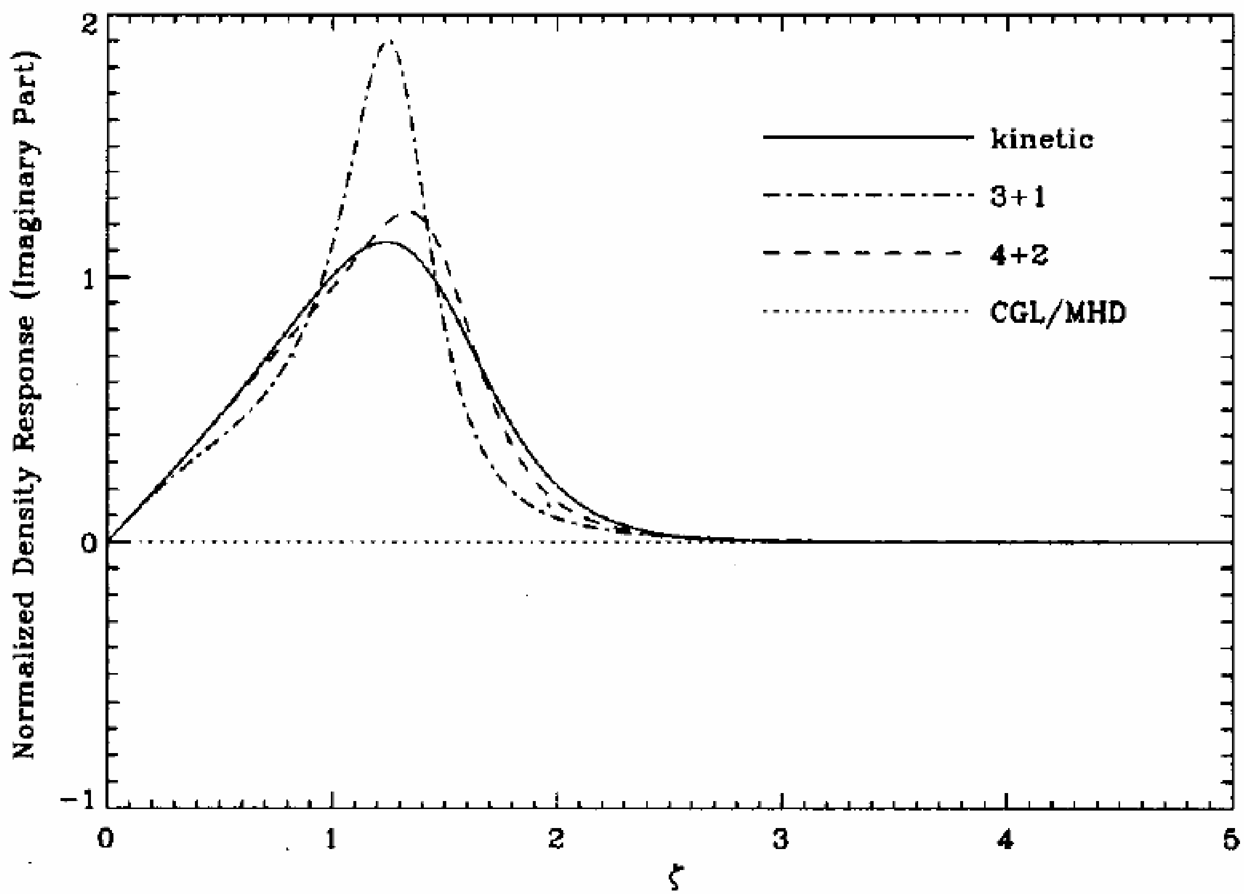

FIG. 2. The imaginary part of the normalized linear density response $(n_1/ik_x\xi_x n_0)$, versus real normalized frequency $(\zeta_i=\omega/\sqrt{2}|k_\parallel|v_{T_{\parallel_i}})$. The 3 + 1 and 4 + 2 moment Landau MHD models are compared with linear kinetic theory. Both CGL theory and Ideal MHD theory predict no imaginary density response. Parameters are identical to those in Fig. 1.

Subgrid models for Mirror modes?

- In process of comparing Landau-MHD approx. with full PIC simulations of mirror modes. Caveat: most previous PIC simulations done with large initial pressure anisotropy and so break μ invariance…
- Mirror mode unstable if $(p_\perp - p_{||})/ p_{||} > 1/\beta_{||}$ but is low frequency and can't break μ invariance unless $(p_\perp - p_{||})/ p_{||} > 7/\beta_{||}$
- Growth rate of mirror modes ~ k in MHD limit. Need hyperviscosity or FLR to cutoff small scales in simulation?
- Try:
  - Where $(p_\perp - p_{||})/ p_{||} < 7/\beta_{||}$, rely on interaction with mirror modes naturally contained in simulation of Landau-fluid/MHD equations.
  - Where $(p_\perp - p_{||})/ p_{||} > 7/\beta_{||}$, introduce rapid scattering to model ultra-high frequency sub-grid modes that break μ invariance